Rare *B* decays at e^+e^- colliders

Rahul Tiwary^{1,*}

¹Tata Institute of Fundamental Research, Mumbai 40005, India

Abstract. The rare decays of B mesons offer a prime opportunity to test predictions of the Standard Model and to search for effects beyond the Standard Model. The coherent production of *B* mesons at e^+e^- colliders provides a clean experimental setup for these studies. In this paper, we discuss the recent results of rare *B* decays from the Belle and Belle II experiments.

1 Introduction

Flavour-Changing Neutral Currents (FCNC) transitions of *B* mesons are identified as rare decays, characterized by a branching fraction of 10^{-5} or below. These FCNC transitions are forbidden at the tree level [1] and proceed through electroweak-loop diagrams, with additional suppression possible due to Cabibo-Kobayashi-Maskwa (CKM) matrix factors [2, 3]. The SM contribution to these processes is small, making them highly sensitive to beyond-SM (BSM) physics. BSM particles can contribute to the loop, such as charged Higgs [4], or mediate the process at the tree level, such as leptoquarks [5]. For instance, the $B \rightarrow Kv\bar{v}$ transition has relatively precise rate predictions since it is not affected by the theoretical uncertainties that arise from photon exchange in $b \rightarrow s\ell\ell$ processes [6]. The $B^+ \rightarrow K^+v\bar{v}$ decay rate can be significantly modified by non-SM particles like leptoquarks [7] and could involve decays into undetectable particles such as an axion [8] or a dark-sector mediator [9].

BSM searches using radiative decays of *B* mesons to exclusive final states, such as $B \to K^*\gamma$ and $B \to \rho\gamma$, are promising. The $b \to s/d\gamma$ operator is the dominant contributor to these decays, facilitating the distinction between SM and BSM physics contributions [10]. The $B \to K^*\gamma$ transition, first observed by the CLEO collaboration [11], has the SM branching fraction of the order of 10^{-5} [12, 13]. With advancements in accelerators and new generation colliders, precision measurements of such transitions are now feasible. The $B \to \rho\gamma$ transition, however, is further suppressed due to CKM factors, with a typical branching fraction of around 10^{-7} [14], offering a complementary ground for BSM searches compared to $B \to K^*\gamma$. Additionally, the double radiative decay of *B* mesons to a $\gamma\gamma$ final state is the most suppressed decay discussed here, with an SM prediction for the branching fraction around 10^{-8} [4] and experimental upper limits still at 10^{-7} [15]. The results discussed in this paper are based on measurement of rare *B* decays performed using 711 fb^{-1} of data collected by the Belle II experiment.

The remainder of this document is arranged as follows: Section 2 provides a brief description of the Belle and Belle II detectors. Section 3 summarizes the search for the rare decay $B \rightarrow Kv\bar{v}$ using Belle II data. Section 4 discusses the measurement of observables

^{*}e-mail: rahul.tiwary@tifr.res.in

for $B \to K^* \gamma$ using Belle II data. Section 5 presents the study of $B \to \rho \gamma$ decay using the combined Belle and Belle II datasets. Finally, Section 6 describes the search for $B \to \gamma \gamma$ decay using the combined Belle and Belle II datasets.

2 The Belle and Belle II detectors

The Belle detector [16, 17] was a large-solid-angle spectrometer that operated at the KEKB asymmetric-energy e^+e^- collider [18, 19]. The energies of the electron and positron beams were 8.0 GeV and 3.5 GeV, respectively. The detector consisted of a silicon-strip vertex detector, a central drift chamber, an array of aerogel Cherenkov counters and time-of-flight scintillation counters for identification of charged particles, and a CsI(Tl)-based electromagnetic calorimeter (ECL), all of which were surrounded by a superconducting solenoid coil providing a magnetic field of 1.5 T. An iron flux return yoke located outside the coil instrumented with resistive-plate chambers to facilitate the detection of K_L^0 mesons and to identify muons.

Belle II [20] is an upgraded version of Belle and located at the SuperKEKB [21] e^+e^- collider. The energies of electron and positron beams are 7.0 GeV and 4.0 GeV, respectively. The Belle II detector includes two layers of silicon pixel sensors, four layers of double-sided silicon-strip vertex detectors [22] and an upgraded 56-layer central drift chamber. The second layer of the pixel detector covers only one-sixth of the azimuthal angle in the data used for the results presented in these proceedings. Two types of Cherenkov-light detector systems surround the drift chamber: an azimuthal array of time-of-propagation detectors for the barrel region and an aerogel ring-imaging Cherenkov detector for the forward endcap region. Belle II reuses the ECL of Belle along with its solenoid and the iron flux return yoke; the latter is equipped with both resistive-plate chamber and plastic scintillator modules to detect K_L^0 mesons and muons. The *z* axis of the laboratory frame is defined as the solenoid axis, where the positive direction is along the electron beam. This convention applies both to Belle and Belle II.

3 Evidence for $B^+ \to K^+ \nu \overline{\nu}$ decays

This section discusses the measurement of $B^+ \to K^+ \nu \bar{\nu}$ decays performed using 362 fb⁻¹ data collected by the Belle II experiment [23]. The decay $B^+ \to K^+ \nu \bar{\nu}$ is particularly challenging to study due to the presence of only a single charged track in the final state. The SM predicts the branching fraction for this decay to be $\mathcal{B}(SM) = (5.58 \pm 0.37) \times 10^{-6}$ [24]. This decay channel is one of the cleanest within the SM and is highly suitable for indirect searches for new physics (NP) [25–27]. New physics could significantly alter the rate of this decay. To date, no direct observation of this decay has been made, with previous results providing only upper limits. Belle II offers distinct advantages for studying this decay, including constraints from well-known initial state kinematics and a lower average multiplicity at the $\Upsilon(4S)$ compared to hadronic collisions, enhancing the ability to isolate and analyze the decay events.

At Belle II, the *B* meson production is through the $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\overline{B}$ process. Here, the kinematics of the partner *B* meson (B_{tag}) can be used to constrain the kinematics of the event and infer the properties of the signal *B* meson. The current analysis uses two complementary methodologies to study $B^+ \rightarrow K^+ \nu \overline{\nu}$ decays. The first method employs inclusive tagging for analysis (ITA), where the B_{tag} is reconstructed alongside the signal *B* meson in an inclusive manner [28]. The second technique uses the well-known hadronic tagging for analysis (HTA) [29, 30], in which the B_{tag} is reconstructed in a hadronic final state. The HTA study helps to verify the newer ITA approach. The HTA delivers tighter background control but poorer signal reconstruction efficiency, whereas the ITA produces greater signal reconstruction efficiency with looser background control. Overall, the ITA analysis yields a higher sensitivity. Despite employing the same data set, the two studies employ statistically independent datasets, resulting in strong and credible findings.

Tracks produced near the e^+e^- interaction point (IP) are selected based on the requirements $|d_r| < 0.5$ cm and $|d_z| < 3.0$ (4) cm for ITA (HTA) analysis. Here, d_r (d_z) denotes the track's transverse (longitudinal) impact parameters. The Kaon tracks registering at least 20 hits in the CDC are identified using PID likelihoods based on information coming from various subdetectors. Lastly, candidates are required to register at least one hit in the pixel detector to improve the impact parameter resolution and reject background events.

With the assumption that the signal *B* meson is produced at rest in the e^+e^- c.m. frame, the invariant mass squared of the neutrino pair is computed as:

$$q_{rec}^2 = s/(4c^4) + M_K^2 + \sqrt{s}E_K^*/c^4.$$
(1)

Here, M_K denotes the known mass of K^+ meson and E_K^* is the reconstructed energy of the kaon in the c.m. frame, and s is the beam energy in the c.m. frame. The candidate with the lowest q_{rec}^2 is retained for further analysis. In the ITA method, the remaining tracks and clusters not associated with the signal kaon are denoted as rest-of-event (ROE).

The HTA also requires the full reconstruction of the B_{tag} to be reconstructed into one of 36 hadronic final states. This is accomplished through the full event interpretation (FEI) [31] algorithm. The FEI algorithm is a hierarchical multivariate approach to reconstructing B_{tag} candidates. The final-state particles are reconstructed using the tracks and energy deposits in the ECL, which are combined into intermediate particles until the whole decay chain has been reconstructed.

The background suppression strategy is based on Boosted Decision Tree (BDT) [32] algorithms. The BDTs are trained on a simulated sample using the following input variables: event shape, kinematics of charged kaon, the kinematic properties of the ROE (for the ITA) and extra tracks and extra photons (for the HTA), and the variables of B_{tag} for the HTA. The ITA method uses two BDTs, the first one to act as an event filter and the second one for the final event selection.

The signal efficiency for the ITA method is validated using the $B^+ \to K^+ J/\psi (\to \mu^+ \mu^-)$ control channel. In the reconstruction of the control channel, the muon candidates are removed, and the K^+ momentum is scaled to match the expected kinematics of a three-body decay to mimic the signal mode. Validation studies show good agreement between data and simulation for the control channel. The modeling of $e^+e^- \to q\bar{q}$ ($q \in (u, d, s, c)$) continuum background events is corrected using the off-resonance data sample collected 60 MeV below the $\Upsilon(4S)$ resonance, using the method described in [33].

Dedicated studies are performed to validate the possible background contribution from *B* decays. The *B* decays involving K_L mesons are of particular interest because they are poorly understood and the detector response can be mis-modeled, with K_L potentially faking missing energy. The dominant backgrounds come from semi-leptonic *B* decays and events in which one or more $K_L^{0,s}$ escape detection. The K_L^0 reconstruction efficiency is validated using $e^+e^- \rightarrow \phi(\rightarrow K_S^0 K_L^0)\gamma_{ISR}$ events. The $B \rightarrow D \rightarrow K_L^0 X$ decays are quantified using a pion-enriched sideband obtained by reversing the Kaon likelihood requirement. The threebody $B^+ \rightarrow K^+ K_L^0 K_L^0$ decays are modeled using the Dalitz spectra of $B^+ \rightarrow K^+ K_S^0 K_S^0$ decays measured by BaBar [34], assuming equal probabilities for the two decays. Similar strategies are employed to estimate and validate the background contributions from $B^+ \rightarrow K^+ K_L^0 K_S^0$ and $B^+ \rightarrow K^+ v \overline{v}$ decays. As a closure test, the branching fraction of $B^+ \rightarrow K^0 \pi^+$ decay was

measured: $\mathcal{B}(B^+ \to K^0 \pi^+) = (2.5 \pm 0.5) \times 10^{-5}$, and the result was found to be compatible with the world average [35].

The signal yields were extracted by fitting the output of the BDT, which separates signals from backgrounds. The ITA analysis also used the q_{rec}^2 as the second fit variable. The corresponding signal strength, μ defined as the ratio of measured branching ratio over the SM expectation, is 2.2 for HTA and 5.4 for ITA. The branching fraction obtained from ITA method is $\mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) = (2.7 \pm 0.5 \pm 0.5) \times 10^{-5}$ with a significance of 3.5 standard deviations with respect to the background only hypothesis, and 2.9 standard deviations with respect to the SM expectation. Similarly, the HTA yields $\mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) = (1.1 \pm {}^{+0.9}_{-0.8} \pm {}^{+0.8}_{-0.5}) \times 10^{-5}$, which is compatible with the background only hypothesis at 1.1 standard deviations and in agreement with the SM at 0.6 standard deviations. The overlap of data sample between the two analyses is small (2%), hence the results are combined by removing the common events from ITA to obtain

$$\mu = 4.6 \pm 1.0 \pm 0.9,\tag{2}$$

$$\mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) = (2.3 \pm 0.5 \pm {}^{+0.5}_{-0.4}) \times 10^{-5}.$$
(3)

The combined result has a significance of 3.5 standard deviations with respect to the background-only hypothesis, and it is 2.7 standard deviations above the SM expectation.

4 First results of $B \rightarrow K^* \gamma$ decays from Belle II

This section presents the results from the first measurement of $B \to K^* \gamma$ decays performed using 362 fb⁻¹ of data collected by the Belle II experiment. The SM prediction for the $B \to K^* \gamma$ branching fraction includes large uncertainties due to form factors [12, 13]. However, observables like \mathcal{ACP} and Δ_{0+} are clean because the form factor contributions and certain experimental systematics cancel out in these ratios [36, 37]. In the SM, isospin asymmetry (Δ_{0+}) is predicted to have a small positive value, ranging from approximately 2.7% [37] to 8.0% [38] with a typical uncertainty of about 2%. Beyond SM (BSM) effects can also shift it to a negative value [39]. The SM prediction for the *CP* asymmetry ($\mathcal{A_{CP}}$) is small, just a few percent [39, 40], whereas BSM contributions can enhance it to over 10%[41]. A recent measurement by the Belle experiment [42] using a dataset of $771 \times 10^6 B\overline{B}$ events reported a non-zero Δ_{0+} with a significance of 3.1 standard deviations.

The $B \to K^* \gamma$ decays are reconstructed in four final states depending on the K^* , namely $K^{*0} \to K^+ \pi^-$, $K^{*0} \to K^0_S \pi^0$, $K^{*+} \to K^+ \pi^0$, and $K^{*+} \to K^0_S \pi^+$. The reconstruction takes a hierarchical approach, starting from the final state particles. Similar to the $B^+ \to K^+ \nu \bar{\nu}$ analysis, we select tracks originating from near the IP using selections in impact parameters $|d_r| < 0.5$ cm and $|d_z| < 2.0$ cm. These tracks are required to register at least 20 hits in the CDC and are classified as either K^+ or π^+ using PID likelihoods that are calculated using information from various detectors. The high-energy photons coming directly from the *B* decay are reconstructed from clusters present in both the barrel and forward endcap regions of the ECL, with energies satisfying $1.4 \text{ GeV} < E^*_{\gamma} < 3.4 \text{ GEV}$. Selections on the ECL shower shape are applied to retain isolated photons, and a BDT trained on Zernike moments [43] is employed to separate high-energy photons from K^0_L clusters. The difference between the photon time and the event time must be less than $2 \mu s$ to suppress out-of-time photons from the beam background.

The K_S^0 candidates are reconstructed from pairs of oppositely charged tracks assumed to be pions and constrained to originate from a common vertex. Candidates failing the vertex fit are excluded. Selections on d_r , d_z , and PID criteria are not applied to these tracks. The invariant mass of the K_S^0 candidate must be within $10 \text{ MeV}/c^2$ of the known K_S^0 mass. Additional selections are applied to the kinematic variables of the K_S^0 candidates to suppress background. The π^0 candidates are reconstructed from pairs of photons, each having an energy greater than 80, 30, or 60 MeV, depending on whether the photon is detected in the forward, barrel, or backward region, respectively, of the ECL. The π^0 candidates are required to have a diphoton invariant mass $(m_{\gamma\gamma})$ in the range $120 < m_{\gamma\gamma} < 145 \text{ MeV}/c^2$. Further selections are applied to the kinematic variables of π^0 and the photons of π^0 candidates to suppress contributions from backgrounds.

The K^* candidate is reconstructed from $K^+\pi^-$, $K_S^0\pi^0$, $K^+\pi^0$, or $K_S^0\pi^+$ combinations. We retain K^* candidates with an invariant mass within 75 MeV/ c^2 of the known K^* mass. Finally, a K^* candidate is combined with a high-energy photon candidate to form a *B* meson. A vertex fit [44] is subsequently applied to the entire *B* decay chain, with the *B* meson production vertex constrained to the IP. The χ^2 probability for the vertex fit is required to be greater than 0.1 % to reduce random combinatorial events. Furthermore, we apply selections to two kinematic variables: $M_{\rm bc} (\equiv \sqrt{s/4 - (\vec{p}_B^*)^2}) > 5.23 \,{\rm GeV/c^2}$ and $|\Delta E (\equiv E_B^* - \sqrt{s}/2)| < 0.3 \,{\rm GeV}$, where E_B^* is the energy of the *B* meson in the c.m. frame. The momentum of the *B* meson is calculated as: $\vec{p}_B^* = \vec{p}_{K\pi}^* + \frac{\vec{p}_Y^*}{|\vec{p}_Y^*|} \times (\sqrt{s}/2 - E_{K\pi}^*)$, to improve the resolution of $M_{\rm bc}$ and reduce the correlation between $M_{\rm bc}$ and ΔE variables.

The dominant source of background is from continuum events. Furthermore, a photon from the decay of a π^0 or η with high momentum can be misidentified as a photon from the signal *B* decay. There is also a small contribution from misreconstructed *B* meson decays. The high-energy photon candidate from the signal decay is paired with other photons in the event. Events having pairs consistent with a π^0 or η decay are vetoed using dedicated BDT classifiers, denoted as π^0/η veto. A separate BDT is employed to suppress the continuum background. The $B^- \to D^0[\to K^+\pi^-]\pi^-$ control channel was employed to assess the quality of the simulation and assign systematics for the BDT classifiers. Furthermore, the vertex quality selection criteria was validated using $B^- \to D^0[\to K^+\pi^-]\pi^-$ and $\overline{B}^0 \to D^+[\to K_S^0\pi^+]\pi^-$ control samples. These control channels exhibit a similar final state as the $B \to K^*\gamma$ transition, with relatively low background levels and significantly higher statistics.

The physics observables of $B \to K^* \gamma$ decay are obtained from an extended maximumlikelihood fit to unbinned M_{bc} and ΔE distributions. The results for the measurement performed for $B \to K^* \gamma$ decays using the Belle II dataset are as follows:

$$\mathcal{B}[B \to K^* \gamma] = (4.12 \pm 0.08 \pm 0.11) \times 10^{-5},\tag{4}$$

$$\mathcal{A}_{CP}[B \to K^* \gamma] = (-2.3 \pm 1.9 \pm 0.3)\%,$$
 (5)

$$\Delta \mathcal{R}_{CP} = (2.2 \pm 3.8 \pm 0.7)\% \tag{6}$$

$$\Delta_{0+} = (5.1 \pm 2.0 \pm 1.0 \pm 1.1)\%. \tag{7}$$

Here, the third uncertainty appearing for isospin asymmetry measurement is due to the ratio of the branching fraction of $\Upsilon(4S)$ to charged and neutral *B* meson pairs. The results are consistent with world-average values and SM expectations.

5 Exclusive measurement of $B \rightarrow \rho \gamma$ at Belle and Belle II

This section summarizes the most precise measurement of observables for exclusive $B \rightarrow \rho \gamma$ decays, based on a combined data sample of the Belle (711 fb⁻¹) and Belle II (362 fb⁻¹) experiments [45]. The exclusive radiative decay of *B* mesons to the $\rho\gamma$ final state allows for an independent search of BSM, complementary to the $B \rightarrow K^*\gamma$ decay. The $B \rightarrow \rho\gamma$ decay, being a $b \rightarrow d$ quark-level transition, exhibits a branching fraction which is an order

of magnitude smaller than the $b \to s\gamma$ transitions. Owing to a significant difference in the branching fractions of $b \to d\gamma$ and $b \to s\gamma$ transitions, one needs good particle identification detectors to cut down the charged kaon contamination from $B \to K^*\gamma$ decays. Akin to the $B \to K^*\gamma$ channel, the branching fraction for $B \to \rho\gamma$ give weak constraints on the BSM parameters, due to large uncertainties (around 20%) coming from the form factors [14]. Promising observables include $CP(\mathcal{R}_{CP})$ and isospin asymmetry (\mathcal{R}_I) , which are theoretically cleaner due to the cancellation of such effects. The precision measurement of the \mathcal{R}_I for $B \to \rho\gamma$ channel is particularly interesting since the current world average [35] is in slight tension with the SM [46].

Similar to the $B \to K^* \gamma$ decay, the $B \to \rho \gamma$ decay is reconstructed following a hierarchical approach, starting with the final-state particles. Hard photon candidates exhibiting a shower shape consistent with that of an isolated photon are selected within the energy range of 1.8 to 2.8 GeV. Tracks produced near the e^+e^- interaction point are selected based on the requirements $|d_r| < 0.5$ cm and $|d_z| < 2.0$ cm, where $d_r(d_z)$ denotes the track's transverse (longitudinal) impact parameters. A likelihood-based particle selector combining information from various detectors of Belle [47] or Belle II is used to identify charged tracks.

The π^0 candidates are reconstructed in the diphoton invariant-mass range of $119 < M_{\gamma\gamma} < 151 \text{ MeV}/c^2$. The photons are further required to satisfy various energy thresholds depending on the detector (Belle or Belle II) and the region of the ECL where the photon is detected. Subsequently, we reconstruct the ρ mesons via $\rho^0 \rightarrow \pi^+\pi^-$ and $\rho^+ \rightarrow \pi^+\pi^0$ modes with the selection 0.64 (0.65) $< M_{\pi\pi} < 0.89$ (0.90) GeV/ c^2 for Belle (Belle II). We reconstruct the *B* meson by combining a high-energy photon with the pion pair. Further selection criteria are applied to the variables $M_{\rm bc} > 5.2 \text{ GeV}/c^2$ and $|\Delta E| < 0.3 \text{ GeV}$. For the neutral mode, the momentum of the *B* meson in the center-of-mass frame is calculated as $\vec{p}_{B^0}^* = \vec{p}_{\rho^0}^* + \frac{\vec{p}_{\gamma}}{|\vec{p}_{\gamma}^*|} \times (\sqrt{s}/2 - E_{\rho^0}^*)$, to improve the resolution of $M_{\rm bc}$.

The sources of background, akin to the $B \to K^*\gamma$ channel, are hard photons from π^0/η decays, and combinatorial background from $e^+e^- \to q\bar{q}$ events. The background suppression strategy is similar to the $B \to K^*\gamma$ mode, dedicated BDT classifiers are employed to suppress each kind of background. $B^+ \to D^0[K^-\pi^+]\pi^+$, $B^0 \to D^-[K^+\pi^-\pi^-]\pi^+$, $B^0 \to K^{*0}[K^+\pi^-]\gamma$, and $B^+ \to K^{*+}[K^+\pi^0]\gamma$ control channels are studied to assess quality of the simulation and assign systematics for the BDT classifiers.

The physics observables of $B \to \rho \gamma$ decay are obtained from an extended maximumlikelihood fit to unbinned M_{bc} , ΔE , and $M_{K\pi}$ distributions, performed simultaneously for six independent datasets: B^+ , B^- , and B^0 in Belle and Belle II. Here, $M_{K\pi}$ is the invariant mass calculated assuming a π^+ to be a K^+ . Using $M_{K\pi}$ instead of $M_{\pi\pi}$ aids in better separation of the $B \to K^* \gamma$ background. The measured observables for $B \to \rho \gamma$ decays from combined Belle and Belle II datasets are as follows:

$$\mathcal{B}(B^+ \to \rho^+ \gamma) = (13.1^{+2.0+1.3}_{-1.9-1.2}) \times 10^{-7}, \tag{8}$$

$$\mathcal{B}(B^0 \to \rho^0 \gamma) = (7.5 \pm 1.3^{+1.0}_{-0.8}) \times 10^{-7}, \tag{9}$$

$$\mathcal{A}_{CP}(B \to \rho \gamma) = -8.2 \pm 15.2^{+1.6}_{-1.2}\%,$$
 (10)

$$\mathcal{A}_{I}(B \to \rho \gamma) = 10.9^{+11.2+6.8+3.8}_{-11.7-6.2-3.9}\%.$$
(11)

The third uncertainty appearing for isospin asymmetry measurement is due to the ratio of branching fraction of $\Upsilon(4S)$ to charged and neutral *B* meson pairs. These are the most precise measurements of $B \rightarrow \rho \gamma$ observables to date and supersede the previous measurements performed by Belle [48].

6 Search for double radiative $B \rightarrow \gamma \gamma$ decay using Belle and Belle II

The double radiative decay $B \rightarrow \gamma \gamma$ is the rarest decay measured using the combined data from the Belle and Belle II experiments [49]. This process is particularly challenging to study due to the presence of two photons in the final state, which leads to significant background interference. The Standard Model (SM) predicts an extremely low branching fraction for this decay, $\mathcal{B}(SM) = (1.4^{+1.4}_{-0.8}) \times 10^{-8}$ [4], making it highly suppressed, especially in comparison to $B_s \rightarrow \gamma \gamma$ due to CKM factors. The most stringent upper limit on the branching fraction for $B \rightarrow \gamma \gamma$ prior to Belle II came from the BaBar experiment [15], which reported a limit of 3.2×10^{-7} , still an order of magnitude higher than the SM expectation.

Candidate $B^0 \to \gamma \gamma$ decays are characterized by two nearly back-to-back highly energetic photons in the e^+e^- c.m. frame, as the B^0 mesons are produced almost at rest. Photons are selected from isolated clusters in the ECL that are not associated with tracks. We select events containing at least two photons with energies in the range 1.4 GeV < E_{γ}^* < 3.4 GEV. Selections applied to the ECL shower shape help select isolated photons, and a BDT trained on Zernike moments [43] separates high-energy photons from K_L^0 clusters. Selections on photon timing, akin to $B \to K^* \gamma$ decay, suppress contribution from the beam backgrounds.

Similar to the radiative *B* decays discussed earlier, the dominant sources of background for $B^0 \to \gamma \gamma$ include the misreconstructed continuum events and photons from π^0/η decays. Similar to $B \to K^* \gamma$, we use dedicated BDTs to suppress the contribution of these decays. In addition, the impact of $B^0 \to \pi^0 \pi^0$, $B^0 \to \eta \eta$, $B^0 \to \eta \pi^0$, and $B^0 \to \omega \gamma$ background events were checked through simulated samples. The largest contribution was found to be from $B^0 \to \pi^0 \pi^0$ decay, which constituted 0.03 events. Hence, it was concluded that these rare *B* decay backgrounds are negligible.

The signal yield was extracted by performing a three-dimensional extended unbinned maximum likelihood fit to M_{bc} , ΔE , and C'BDT simultaneously in the Belle and Belle II datasets. Here, C'BDT is the output of the continuum suppression BDT transformed using the probability integral transformation [50]. The branching fraction was found to be $(3.7^{+2.2}_{-1.8} \pm 0.5) \times 10^{-8}$. The resulting significance is 2.5 standard deviations, which includes the systematic uncertainties. As the significance of the signal yield is low, we calculate an upper limit (UL) on the B using a Bayesian approach with a flat prior. The UL on the branching fraction is determined by integrating the likelihood function, including the systematic uncertainty from zero to 90% of the area under the curve. The upper limit on the branching fraction obtained from the combined dataset is 6.4×10^{-8} , at 90% credibility level. This result supersedes the previous Belle measurement [51], and provides an UL that is five times more restrictive than the previous best limit from BABAR [15].

References

- [1] J. Iliopoulos, S. L. Glashow, and L. Maiani, Phys. Rev. D 2 (1970) 1285.
- [2] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.
- [3] M. Kobayashi, T. Maskawa, PTEP 49 (1973) 652.
- [4] Y.-L. Shen, Y.-M. Wang, and Y.-B. Wei, JHEP 12 (2020) 169.
- [5] S. Sahoo and R. Mohanta, Phys. Rev. D 91 (2015) 094019.
- [6] A. J. Buras et al., JHEP 02 (2015) 184.
- [7] D. Bečirević et al., Phys. Rev. D 98 (2018) 055003.
- [8] T. Ferber et al., JHEP 04 (2023) 131.
- [9] A. Filimonova, R. Schäfer, and S. Westhoff, Phys. Rev. D 101 (2020) 095006.
- [10] E. Kou et al. (Belle II Collaboration), PTEP 2019 (2019) 029201.

- [11] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. 71, 674.
- [12] A. Ali, B.D. Pecjak and C. Greub, Eur. Phys. J. C 55 (2008) 577.
- [13] A. Paul and D.M. Straub, JHEP 2017 (2017) 27.
- [14] M. Matsumori, A. I. Sanda, and Y.-Y. Keum, Phys. Rev. D 72 (2005) 014013.
- [15] P. del Amo Sanchez et al. (BaBar Collaboration), Phys. Rev. D 83 (2011) 032006.
- [16] A. Abashian et al. (Belle Collaboration), Nucl. Instrum. Meth. A 479 (2002) 117.
- [17] J. Brodzicka et al. (Belle Collaboration), PTEP 2012 (2012) 04D001.
- [18] S. Kurokawa and E. Kikutani, Nucl. Instrum. Meth. A 499 (2003) 1.
- [19] T. Abe et al., Prog. Theor. Exp. Phys. 2013 (2013) 03A001.
- [20] T. Abe et al. (Belle II Collaboration), arXiv:1011.0352462 [physics.ins-det].
- [21] K. Akai, K. Furukawa, and H. Koiso, Nucl. Instrum. Meth. A 907 (2018) 188.
- [22] K. Adamczyk et al. (Belle II SVD collaboration), JINST 17 (2022) P11042.
- [23] I. Adachi et al. (Belle II Collaboration), Phys. Rev. D 109 (2024) 112006.
- [24] W. G. Parrott, C. Bouchard, and C. T. H. Davies (HPQCD Collaboration), Phys. Rev. D 107, 014511 (2023); 107, 119903 (E) (2023).
- [25] W. Altmannshofer, A.J. Buras, D.M. Straub and M. Wick, JHEP 04 (2009) 022.
- [26] A.J. Buras, J. Girrbach-Noe, C. Niehoff and D.M. Straub, JHEP 02 (2015) 184.
- [27] D. Bečirević, G. Piazza and O. Sumensari, Eur. Phys. J. C 83 (2023) 252.
- [28] F. Abudinén et al. (Belle II Collaboration), Phys. Rev. Lett. 127 (2021) 181802.
- [29] O. Lutz et al. (Belle Collaboration), Phys. Rev. D 87 (2013) 111103.
- [30] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 87 (2013) 112005.
- [31] T. Keck et al., Comput. Software Big Sci. 3 (2019) 6.
- [32] T. Keck, Comput. Softw. Big Sci. 1 (2017) 2.
- [33] D. Martschei et al., J. Phys. Conf. Ser. 368, (2012) 012028.
- [34] J. P. Lees et al. (BaBar Collaboration), Phys. Rev. D 85 (2012) 112010.
- [35] R. L. Workman et al. (Particle Data Group), PTEP 2022 (2022) 083C01.
- [36] W. Altmannshofer and D.M. Straub, Eur. Phys. J. C 75 (2015) 382.
- [37] M. Matsumori, A.I. Sanda and Y.-Y. Keum, Phys. Rev. D 72 (2005) 014013.
- [38] A.L. Kagan and M. Neubert, Phys. Lett. B 539 (2002) 227.
- [39] M. Jung, X.-Q. Li and A. Pich, JHEP 10 (2012) 063.
- [40] C. Greub, H. Simma and D. Wyler, Nucl. Phys. B 434 (1995) 39483.
- [41] C. Dariescu and M.-A. Dariescu, arXiv:0710.3819.
- [42] Belle collaboration, Phys. Rev. Lett. 119 (2017) 191802.
- [43] A. Khotanzad and Y. Hong, IEEE Trans. Pattern Anal. Mach. Intell. 12 (1990) 489.
- [44] J.-F. Krohn et. al (Belle II Collaboration), Nucl. Instrum. Meth. A 976 (2020) 164269.
- [45] I. Adachi et al. (Belle II Collaboration), arXiv:2407.08984 [hep-ex].
- [46] J. Lyon and R. Zwicky, Phys. Rev. D 88 (2013) 09400.
- [47] E. Nakano, Nucl. Instrum. Meth. A 494 (2002) 402.
- [48] N. Taniguchi et al. (Belle Collaboration), Phys. Rev. Lett. 101 (2008) 111801.
- [49] I. Adachi et al. (Belle II Collaboration), arXiv:2405.19734 [hep-ex].
- [50] John E. Angus, SIAM Review 36 (1994) 652.
- [51] S. Villa et al. (Belle Collaboration), Phys. Rev. D 73 (2006) 051107.