Partial-wave analysis of $\tau^{\mp} \to \pi^{\mp}\pi^{\mp}\pi^{\pm}\nu_{\tau}$

Friday, December 8th, 2023

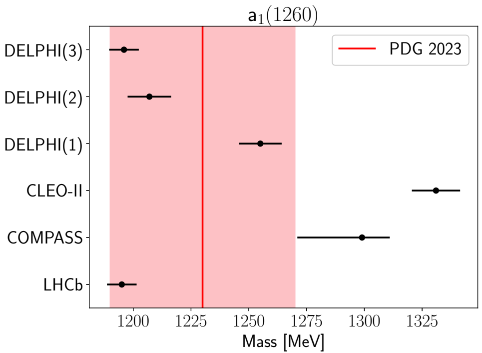
BELLE

τ2023

Motivation for $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$

Searches for new particles and interactions not part of the Standard Model: need to know various hadron form-factors

- Disagreement in measurements of $a_1(1260)$ parameters ($m = 1190$ MeV–1330 MeV)



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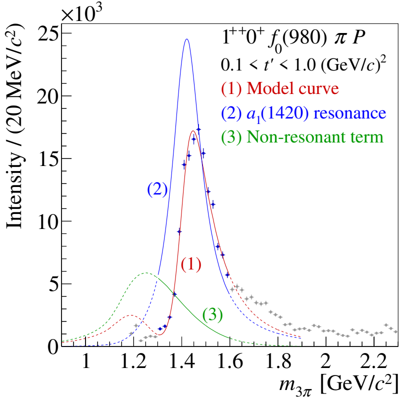
- COMPASS observed narrow peak $\mathrm{a}_1(1420)$ (PRL 115, 082001)
  - Isospin partner of $\mathrm{f}_1(1420)$?
  - $\mathrm{K}^*\mathrm{K}$ rescattering?

$\mathrm{a}_1(1420)$ observation at COMPASS



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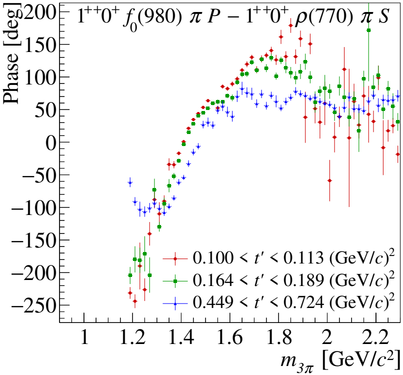

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Improve current model in event generators

Only amplitude analysis in $\tau \to 3\pi\nu_\tau$: CLEO-II PRD 61, 012002

Wave	Branching
$1^+[\rho(770)\pi]_S$	68.11%
$1^+[\sigma\pi]_P$	16.18%
$1^+[f_0(1370)\pi]_P$	4.29%
$1^+[\rho(1450)\pi]_D$	0.43%
$1^+[\rho(770)\pi]_D$	0.36%
$1^+[\rho(1450)\pi]_S$	0.30%
$1^+[f_2(1270)\pi]_P$	0.14%

Different channel ($\pi^\pm\pi^0\pi^0$)

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Same $a_1(1260)$ shape in all states

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No $1^+[f_0(980)\pi]_P$ wave discovered

Belle experiment

KEKB was asymmetric e^+e^- collider

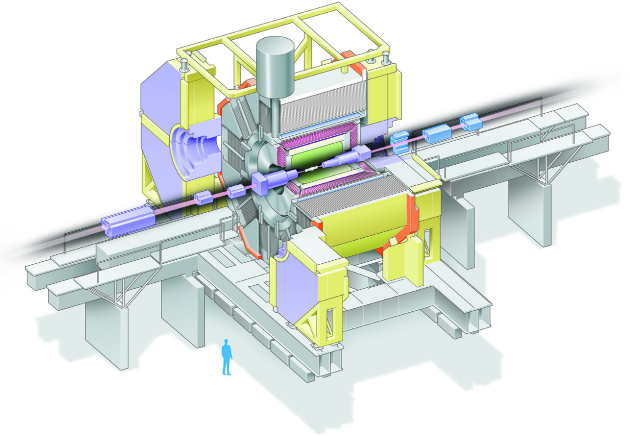
$E_{e^+} = 3.5$ GeV, $E_{e^-} = 8$ GeV

Total luminosity: 980 fb^{-1}

- $\Upsilon(4S)$: 711 fb^{-1}
- 0.9×10^9 tauon pairs produced

Tauon:

- $m = 1.777$ GeV
- $c\tau = 86$ µm
- $\mathcal{B}(\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu_\tau) = 9\%$



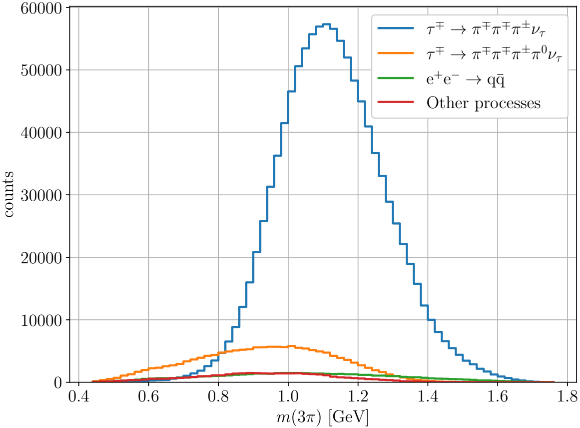
Belle detector

Event-selection criteria

	PRD 81 113007	Current
Efficiency	22 %	32 %
Purity	89 %	82 %
# of events	9×10^6	55×10^6
Tag	$\mu^\pm$ or $e^\pm$	1-prong

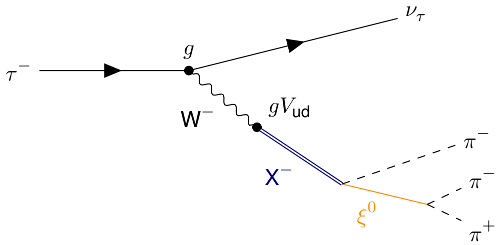
Main background components:

$\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\pi^0(\pi^0)\nu_\tau$	12.8 %
$e^+e^- \to q\bar{q}$	4 %
$\tau^\mp \to K^\mp\pi^\mp\pi^\pm\nu_\tau$	1 %



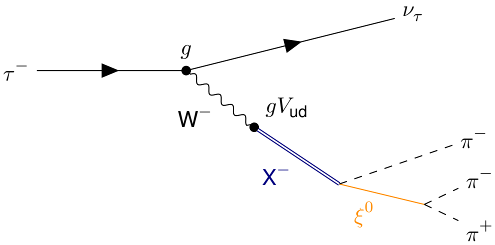
Simulated $m_{3\pi}$ spectrum

7D phase space



Decay rate

7D phase space



Average intensity over tauon azimuthal angle

Decay rate

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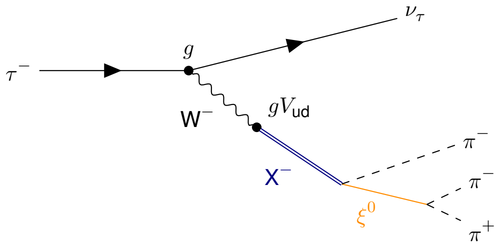
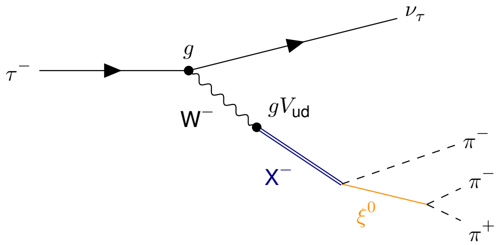


Average intensity over tauon azimuthal angle

Decompose hadron current J^{μ}_{had} into partial waves (EPJC 81 12, 1073)

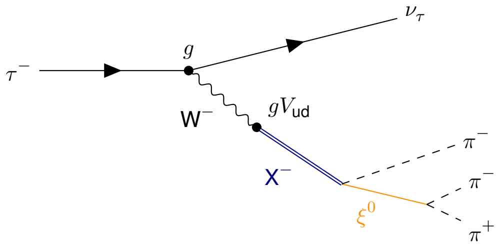
Decay rate

7D phase space



Average intensity over tauon azimuthal angle

Decompose hadron current J^μ_{had} into partial waves (EPJC 81 12, 1073): $J^\mu_{\text{had}} = \sum_w \mathcal{C}_w j^\mu_w$, 7D phase space



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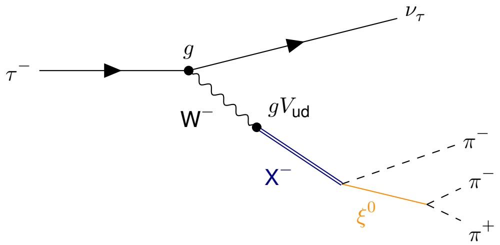
$\mathcal{C}_w$ partial-wave coefficients

Decay rate

7D phase space



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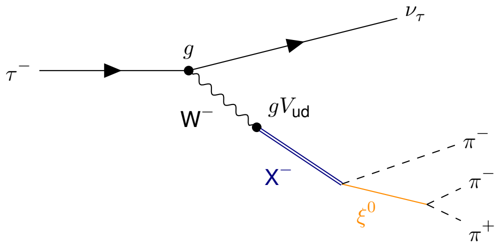
Decompose hadron current J^μ_{had} into partial waves (EPJC 81 12, 1073): $J^\mu_{\text{had}} = \sum_w \mathcal{C}_w j^\mu_w$,

j^μ_w are fixed, quantum numbers $J^P[\xi^0\pi]_L$, 16 partial waves

Decay rate

7D phase space



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$J^P[\xi^0\pi]_L$: L angular momentum between ξ^0 and bachelor π^-, $L = \text{S}, \text{P}, \text{D}$ or F

PWA strategy

Conventional PWA:

- Isobars shape is fixed
- Fit in bins of $m_{3\pi}$ (no a_1 shape assumptions)
- $2 \times N_{\mathrm{PW}} - 1$ free parameters ($\mathfrak{Re}$ and $\mathfrak{Im}$ of $\mathcal{C}_w$)
- Anchor wave: $\mathfrak{Im}\, \mathcal{C}_w = 0$ (usually $1^+[\rho(770)\pi]_S$ wave)

Intensity:

$$\mathcal{I} = \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v}$$

$$\mathcal{I}_{w\,v} = \overline{\mathcal{L}_{\mu\nu}} j_w^\mu \left(j_v^\nu\right)^*$$

Extended log likelihood function:

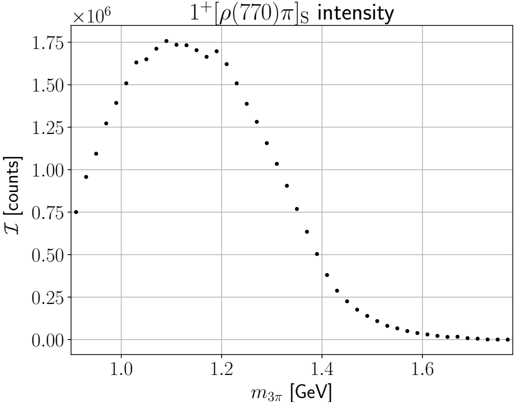
$$\ln \mathcal{L} = \sum_{\text{Data}} \ln \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v} - \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{N}_{w\,v}$$

Integral matrix:

$$\mathcal{N}_{wv} = \frac{\sum_{\text{Acc MC}} \mathcal{I}_{wv} / \left|\mathcal{M}_{\text{TAUOLA}}\right|^2}{\sum_{\text{Acc MC}} 1 / \left|\mathcal{M}_{\text{TAUOLA}}\right|^2}$$

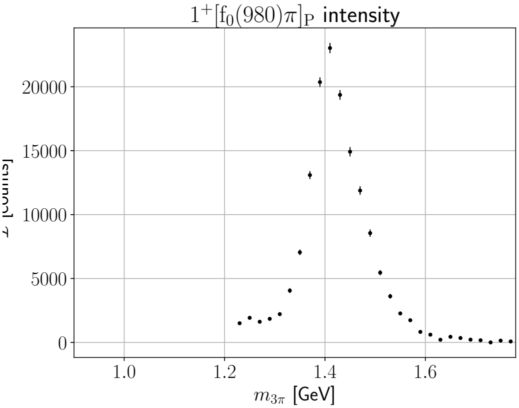
Decomposition results (statistical uncertainties only)

Partial wave	Fit fraction [%]
$1^{+}[\rho(770)\pi]_{S}$	97.37
$1^{+}[\sigma\pi]_{P}$	11.57
$1^{+}[\rho(770)\pi]_{D}$	2.48
$1^{+}[\rho(1450)\pi]_{S}$	1.77
$1^{+}[f_0(980)\pi]_{P}$	0.41
$1^{+}[f_2(1270)\pi]_{P}$	0.11
$1^{+}[f_0(1370)\pi]_{P}$	0.06
$1^{+}[f_2(1270)\pi]_{F}$	0.02
$1^{+}[\rho(1450)\pi]_{D}$	0.01
$1^{+}[f_0(1500)\pi]_{P}$	0.00
$0^{-}[\rho(770)\pi]_{P}$	0.36
$0^{-}[\sigma\pi]_{S}$	0.26
$0^{-}[f_2(1270)\pi]_{D}$	0.02
$0^{-}[f_0(980)\pi]_{S}$	0.01

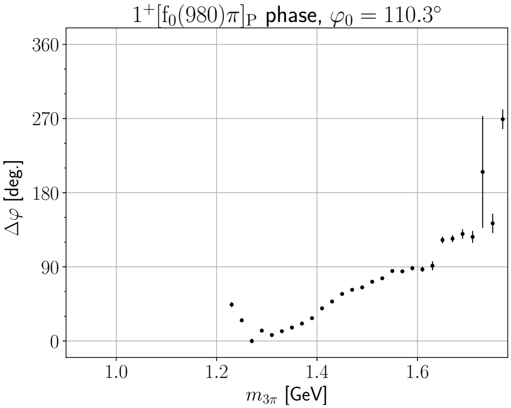


Broad $a_1(1260)$

$1^+[f_0(980)\pi]_P$-wave

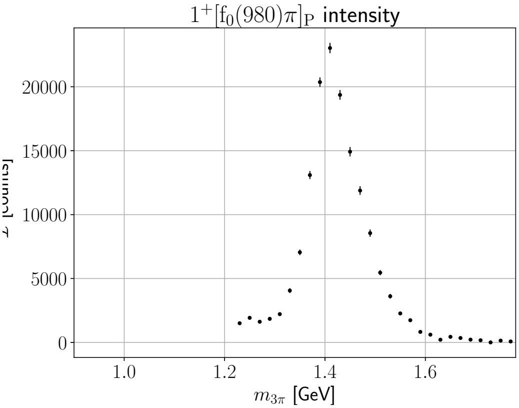


Narrow peak at 1.4 GeV



Phase rise at 1.4 GeV

$1^{+}[f_0(980)\pi]_P$-wave: $a_1(1420)$ observed!



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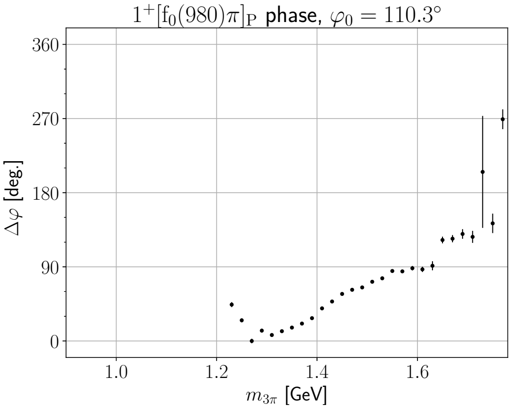


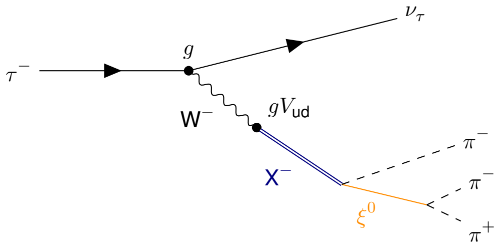
Phase rise at 1.4 GeV

Conventional PWA:

- Isobar's shape $\Delta(s)$ is fixed with a model
- One complex coefficient $\mathcal{C}_w$ per wave

Breit-Wigner (BW) parametrization (for example):

$$\Delta_\xi(s) = \mathrm{BW}_\xi(s) = \frac{m_\xi^2}{m_\xi^2 - s - i\sqrt{s}\Gamma(s)}$$



Freed-isobar PWA: look at two-pion spectrum

$$\Delta(s) = \sum_{w\,\text{freed}} \mathcal{C}_{w\,\text{freed}} \Theta_w(s)$$

$$\Theta_w(s) = \begin{cases} 1 & \text{if } s \text{ in the } m_{2\pi} \text{ bin } w \\ 0 & \text{otherwise} \end{cases}$$

1^{--} $m(2\pi)$ binning:

$$\begin{cases} 2\ \text{MeV} & [770, 792]\ \text{MeV} \\ 20\ \text{MeV} & [640, 920]\ \text{MeV} \\ 40\ \text{MeV} & \text{otherwise} \end{cases}$$

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0^{++} $m(2\pi)$ binning:

$$\begin{cases} 10 \text{ MeV} & [920, 1080] \text{ MeV} \\ 40 \text{ MeV} & \text{otherwise} \end{cases}$$

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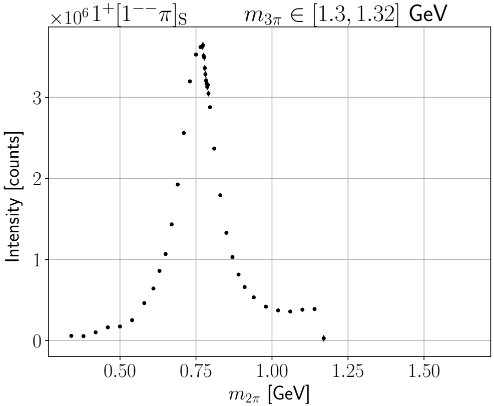
Mathematical ambiguities (zero modes)

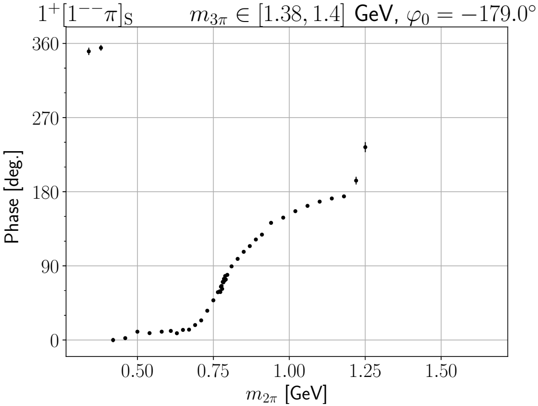
10.1103/PhysRevD.97.114008

$1^+[1^{--}\pi]_S$ wave $m_{3\pi} \in [1.3, 1.32]$ GeV

Clear $\rho(770)$ peak



Clear $\rho(770)$ phase rise



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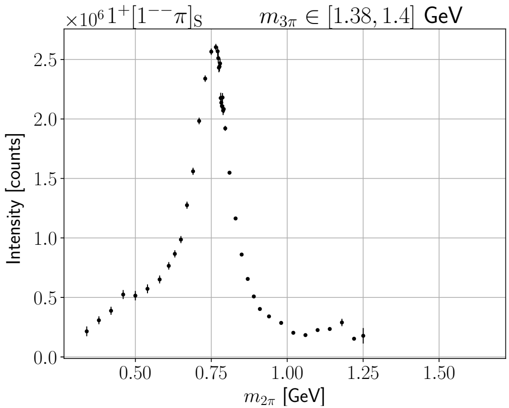

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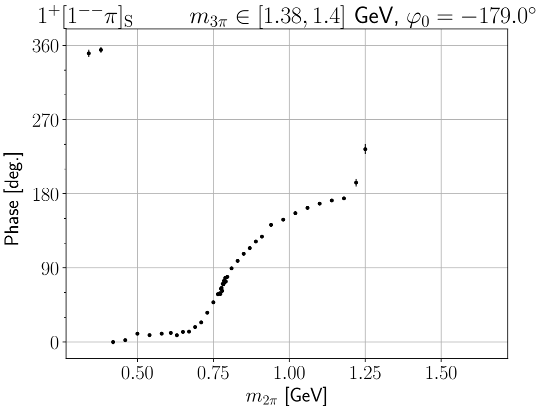
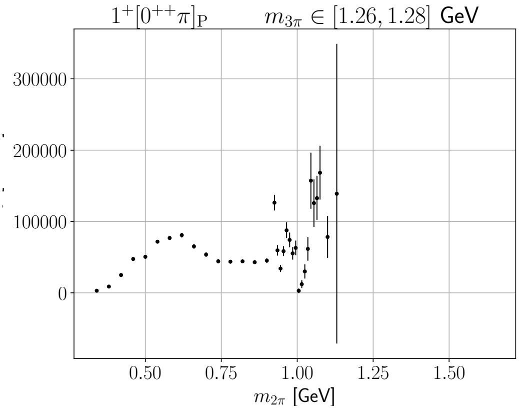


No $f_0(980)$ peak



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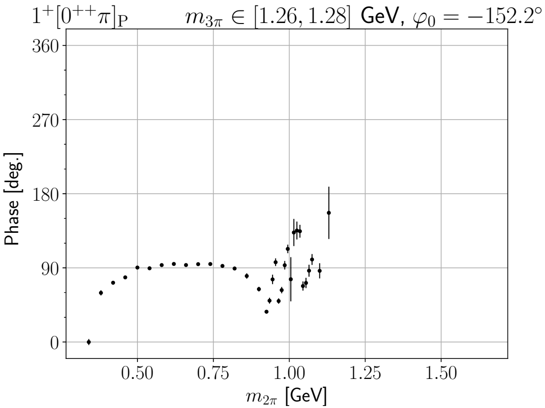
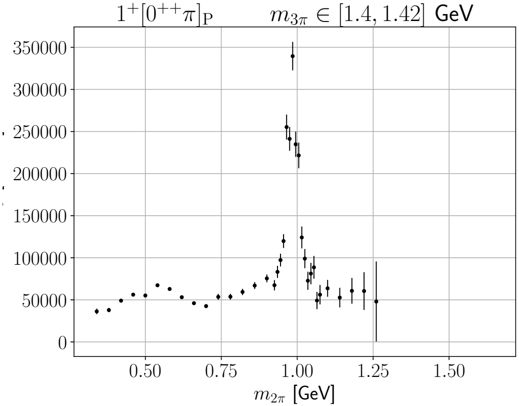
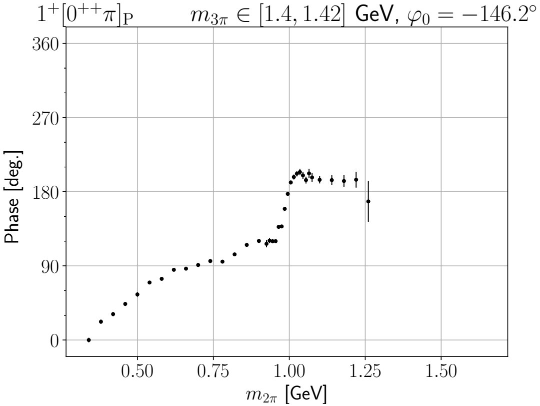


Clear narrow $f_0(980)$ peak

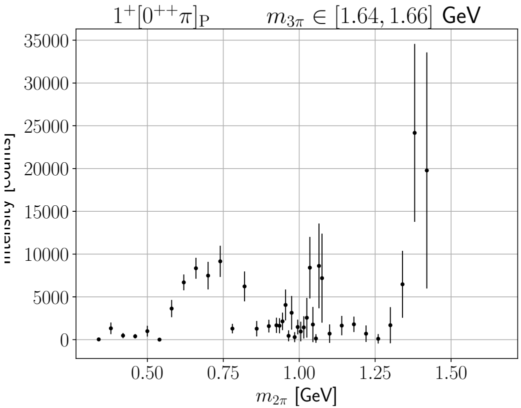


Clear $f_0(980)$ phase rise



$1^+[0^{++}\pi]_P$ wave $m_{3\pi} \in [1.64, 1.66]$ GeV

No $f_0(980)$ peak



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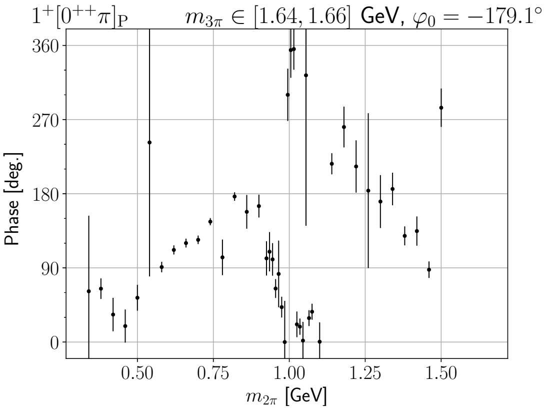

G-parity violation in $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$

Quantum number $G{\cdot}P{\cdot}(-1)^J = 1$ for first-class, -1 for second-class currents

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Not necessarily: $\omega(782) \to \pi^+\pi^-$ violates G-parity ($\mathcal{B} = 1.5\%$)

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Source of $\omega(782)$? $\tau \to \rho' \nu_\tau$

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Source of $\omega(782)$? $\tau \to \rho'\nu_\tau,\ \rho' \to \omega(782)\pi$

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$$\frac{\mathcal{B}(\tau^- \to \omega\pi^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)} = 22\%$$

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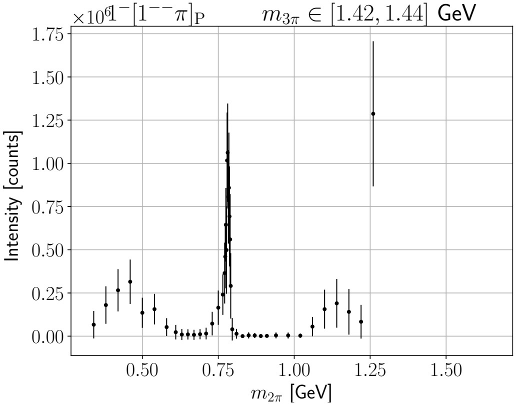
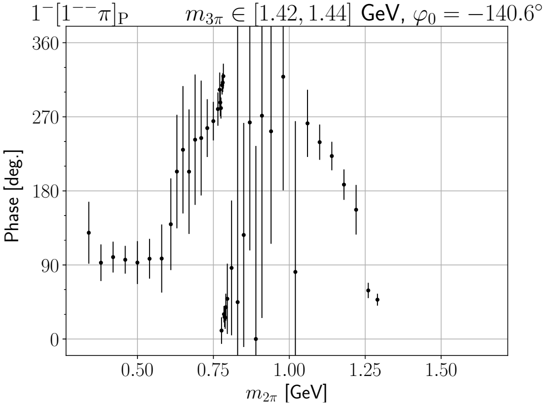
Expected fit fraction of $1^-[\omega(782)\pi]_{\mathrm{P}} = 0.3\%$

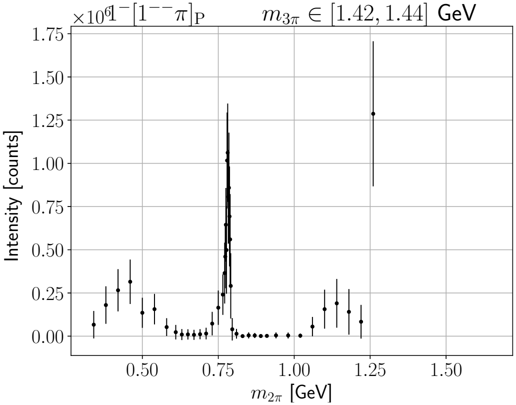
Clear $\omega(782)$ phase peak



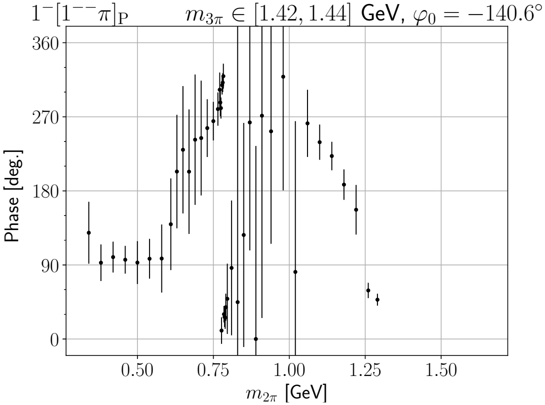
Clear $\omega(782)$ phase rise



Clear $\omega(782)$ phase peak



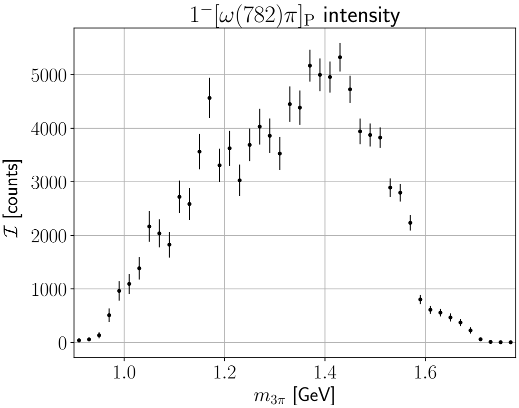
Clear $\omega(782)$ phase rise

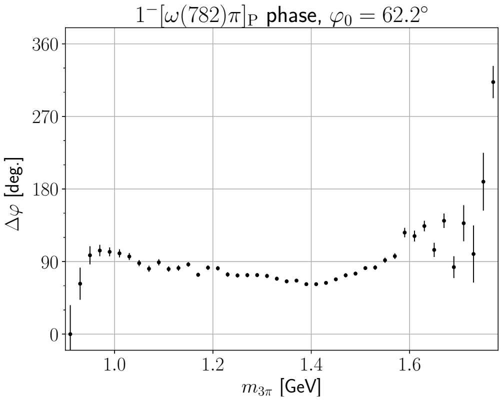


Add $1^-[\omega(782)\pi]_P$ wave to the wave set

Broad structure peaking at 1.4 GeV

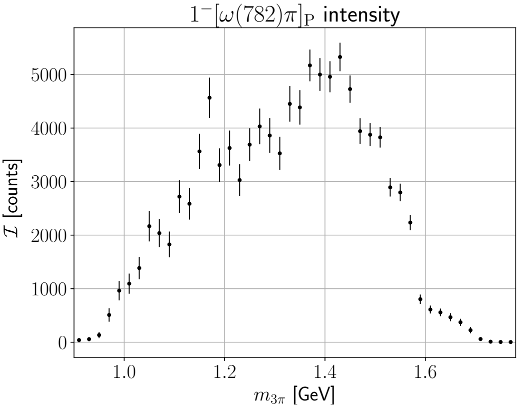
$$\phi(1^-[\omega(782)\pi]_P) - \phi(1^+[\rho(770)\pi]_S)$$



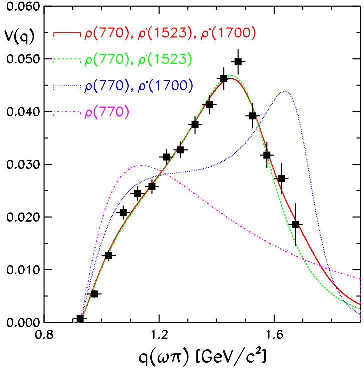


$1^{-}[\omega(782)\pi]_{\mathrm{P}}$ wave

This analysis



$m_{\omega\pi^-}$ for $\tau^- \to \omega\pi^-\nu_\tau$ (PRD 61 072003)



Summary

Current status:

- Selection criteria
- Partial-wave decomposition
  - Fixed-isobar
  - Freed-isobar
- Resonance-model and isobar fits
- Systematic uncertainties:
  - Model
  - Background
  - Acceptance
  - Resolution

Results:

- $\mathrm{a}_1(1420)$ discovered
- $1^-[\omega(782)\pi]_\mathrm{P}$ wave observed

Remaining systematic studies:

- Trigger
- PID

Future projects for $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$

- G-parity violation in $J^P = 1^-$ w/o $\omega(782)$
- CP-asymmetry

Thank you for your attention!

Backup

Event-selection criteria

Event-class selection

- Standard Belle selection for tauon pairs
- Topology: 3–1
- Boosted Decision Tree

Signal hemisphere

- Tracks identification:
  - Veto signal-side particles being electrons or muons
  - Veto the like-sign signal-side particles being kaons
- Veto pions coming from K_0: $\left|m_{2\pi} - m_{K_S}\right| < 12$ MeV
- Veto π^0s in signal hemisphere: $\sum E_\gamma < 480$ MeV

List of selections

Good tracks selection:

- $|\Delta r| < 0.5$ cm
- $|\Delta z| < 2.5$ cm
- $p_\perp > 0.1$ GeV

Good photons selection:

- $E_\gamma > 0.04$ GeV
- $w > 0.5$ cm
- $N_{\text{hits}} > 2$
- $E_{\text{seed}}/E_{\text{cluster}} < 0.95$

Preselection: four good tracks with sum charge zero

Skimming:

- `tau_skimB` or `HadronBJ`
- BDT response $b_{\tau\tau} > 0$

Topology: $3 + 1$

Loose π^0-veto in signal hemisphere:
$\sum E_\gamma < 0.48$ GeV

Signal tracks PID:

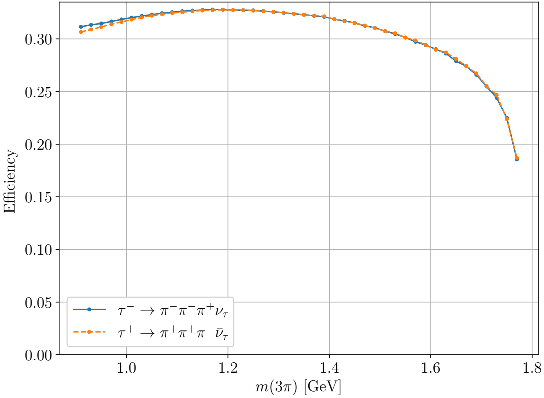
- three tracks: e-veto, μ-veto
- two tracks same charge: K-veto

K_S-veto in signal hemisphere:
$|m_{2\pi} - m_{K_S}| < 0.012$ GeV

Efficiency and purity

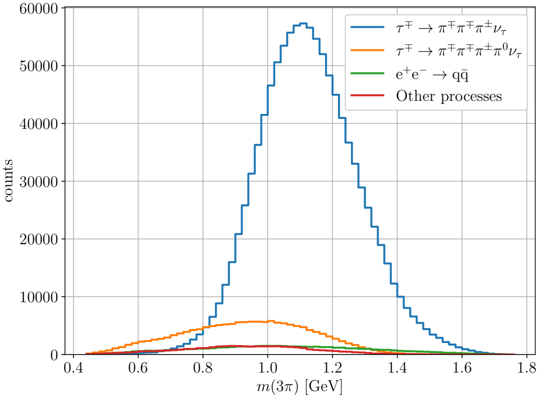
Sequential efficiencies and purities:

Criterion	purity [%]	efficiency [%]
baseline	22.1	44.4
trig	23.1	43.8
skim	24.5	43.1
BDT	50.6	39.9
LID	54.0	37.8
HID	57.4	36.7
PHS	57.7	36.7
ISR	58.2	35.9
KS_veto	60.7	34.3
pi0_veto	81.6	32.3

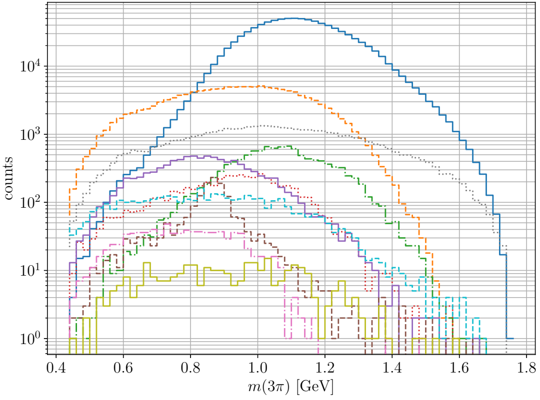


$m(3\pi)$ efficiency

Efficiency and purity

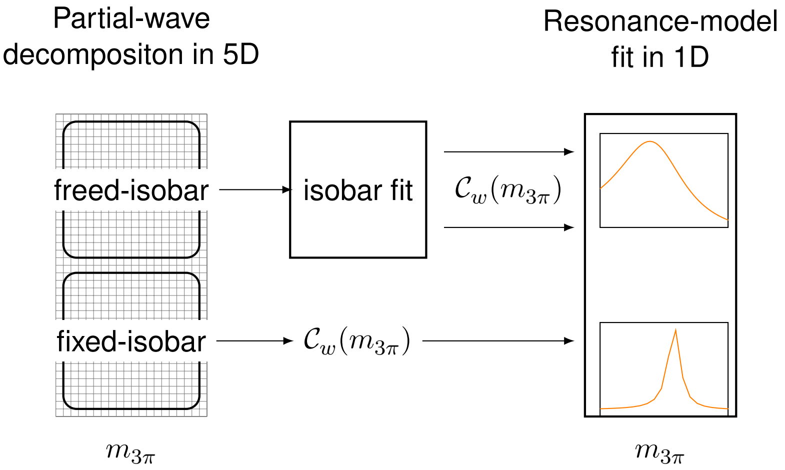


$m(3\pi)$ spectrum in MC



$m(3\pi)$ spectrum in MC, $\log\mathrm{OY}$

Fitting strategies



Fitting strategies

Conventional PWA:

- Isobars shape is fixed
- Fit in bins of $m_{3\pi}$ (no a_1 shape assumptions)
- $2 \times N_{\text{PW}} - 1$ free parameters: partial-waves complex coefficients $\mathcal{C}_w$ (one phase is fixed)

Intensity:

$$\mathcal{I} = \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v}$$

$$\mathcal{I}_{w\,v} = \overline{\mathcal{L}_{\mu\nu}} j_w^{\mu} \left(j_v^{\nu}\right)^*$$

Extended log likelihood function:

$$\ln \mathcal{L} = \sum_{\text{Data}} \ln \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v} - \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{N}_{w\,v}$$

Integral matrix:

$$\mathcal{N}_{wv} = \frac{\sum_{\text{Acc MC}} \mathcal{I}_{wv} / \left|\mathcal{M}_{\text{MC Generator}}\right|^2}{\sum_{\text{Acc MC}} 1 / \left|\mathcal{M}_{\text{MC Generator}}\right|^2}$$

$\left|\mathcal{M}_{\text{MC Generator}}\right|^2$ `TAUOLA` intensity (CLEO current for $\tau \to 3\pi\nu_\tau$)

Partial waves

* 2305.11711 [hep-ph]

Partial wave	$m(\xi)$ [GeV]	$\Gamma(\xi)$ [GeV]	Threshold [GeV]
$1^+[\sigma\pi]_\mathrm{P}$	Broad $[\pi\pi]_\mathrm{S}$-wave component*		
$1^+[f_0(980)\pi]_\mathrm{P}$	0.990	0.07	1.22
$1^+[f_0(1500)\pi]_\mathrm{P}$	1.504	0.109	1.64
$1^+[\rho(770)\pi]_\mathrm{S}$	0.769	0.1509	—
$1^+[\rho(770)\pi]_\mathrm{D}$			—
$1^+[\rho(1450)\pi]_\mathrm{S}$	1.465	0.40	1.18
$1^+[\rho(1450)\pi]_\mathrm{D}$			1.34
$1^+[f_2(1270)\pi]_\mathrm{P}$	1.2755	0.1867	1.4
$1^+[f_2(1270)\pi]_\mathrm{F}$			1.44
$0^-[\sigma\pi]_\mathrm{S}$			—
$0^-[f_0(980)\pi]_\mathrm{S}$			1.14
$0^-[\rho(770)\pi]_\mathrm{P}$			—
$0^-[f_2(1270)\pi]_\mathrm{D}$			1.0

Partial wave	Threshold [GeV]
$1^-[\omega(782)\pi]_\mathrm{P}$	—
$1^-[\rho(770)\pi]_\mathrm{P}$	0.96
$1^-[f_2(1270)\pi]_\mathrm{D}$	—

G-parity violating waves

$$G = (-1)^I C = (-1)^{J+L+I}$$

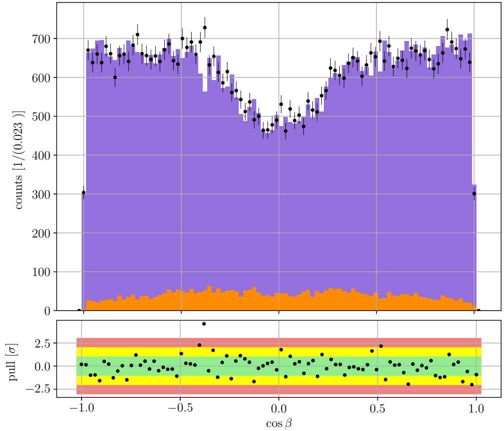
Comparison of simulation with data

Slice of $m_{3\pi} \in [1.50, 1.52]$ GeV

Fit to $20\% L_{\text{int}}$ data and simulate $\tau^{\mp} \to \pi^{\mp}\pi^{\mp}\pi^{\pm}\nu_{\tau}$

Plot legend:

- Black dots: data
- Blue hist: Fit prediction with `TAUOLA-m`
- Orange hist: background



$\cos\beta$

Comparison of simulation with data

Slice of $m_{3\pi} \in [1.50, 1.52]$ GeV

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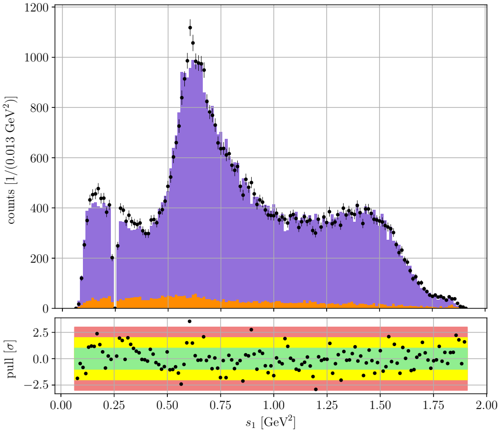
Data overshoots simulation at the $\rho(770)$ peak position



s_1 [GeV^2]

G-parity violating waves

Partial wave	$\mathbb{B}$ [%]
$1^{-}[f_2(1270)\pi]_D$	0.46
$1^{-}[\omega(782)\pi]_P$	0.30
$1^{-}[\rho(770)\pi]_P$	0.12

Blatt-Weisskopf centrifugal-barrier factor F

F takes into the finite size of a meson

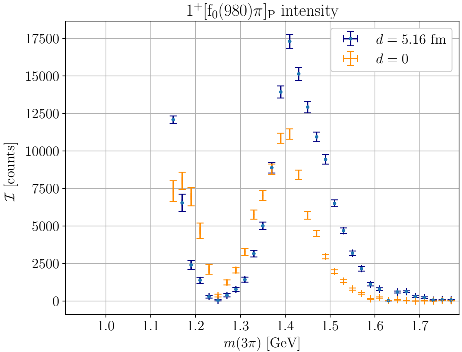
Each partial wave is multiplied by two Fs corresponding to either X^- or ξ^0

F_S for ξ^0 depends on the break-up momentum in the rest frame of ξ^0

F_L for X^- depends on the break-up momentum in the rest frame of X^-

There are two alternative parametrizations for F, I use the relativistic one, for example for a P wave

$$F_P(x) = \sqrt{\frac{1-x_0}{1-x}},\ x = (pd)^2$$



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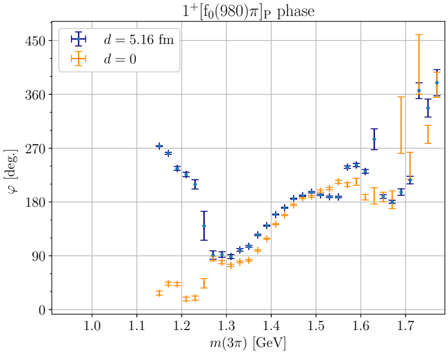


Blatt-Weisskopf centrifugal-barrier factor F

F takes into the finite size of a meson

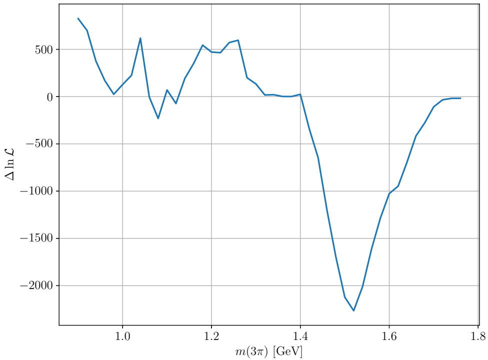
Each partial wave is multiplied by two Fs corresponding to either X^{-} or ξ^0

F_S for ξ^0 depends on the break-up momentum in the rest frame of ξ^0

F_L for X^{-} depends on the break-up momentum in the rest frame of X^{-}

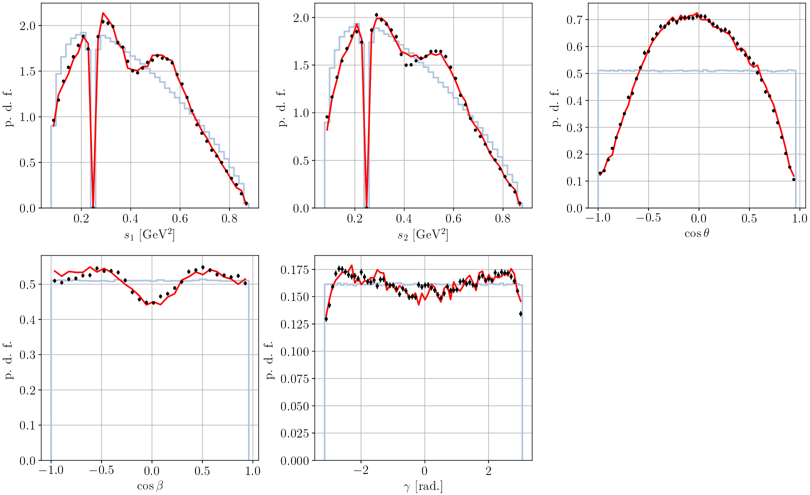
There are two alternative parametrizations for F, I use the relativistic one, for example for a P wave

$$F_P(x) = \sqrt{\frac{1 - x_0}{1 - x}}, \; x = (pd)^2$$



Difference between $\ln \mathcal{L}$ of the fit.

1D projections



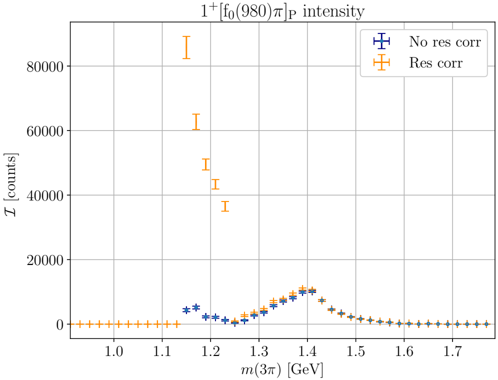
Detector resolution

$$\mathcal{I}_{w\,v}(\Phi) \to \int \mathcal{I}_{w\,v}(\Phi')\varepsilon(\Phi, \Phi')\mathrm{d}\Phi',$$

Φ — reconstructed phase space variables, Φ' — generated phase space variables

Requires MC sampling for each event

Unknown $\varepsilon(\Phi, \Phi')$



Intensity of the $1^+[f_0(980)\pi]_P$ wave.

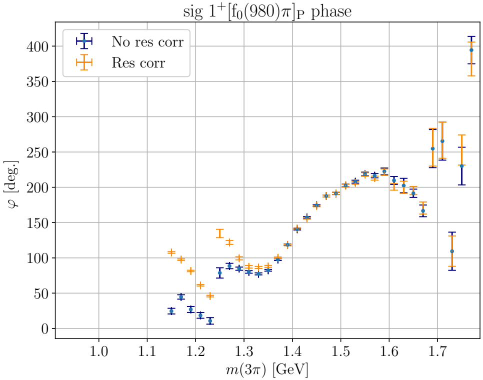
Detector resolution

$$\mathcal{I}_{w\,v}(\Phi) \to \int \mathcal{I}_{w\,v}(\Phi')\varepsilon(\Phi, \Phi')\mathrm{d}\Phi',$$

Φ — reconstructed phase space variables, Φ' — generated phase space variables

Requires MC sampling for each event

Unknown $\varepsilon(\Phi, \Phi')$



Phase of the $1^+[f_0(980)\pi]_P$ wave.

Systematic effects: overview

Four major sources:

- Model:
  - Isobar parametrization
  - Model selection
- Background
  - Model in simulations
  - Neural network parametrization
- Acceptance
  - Stat. uncertainty of $\mathcal{N}_{wv}$
  - Momentum correction
- Detector resolution

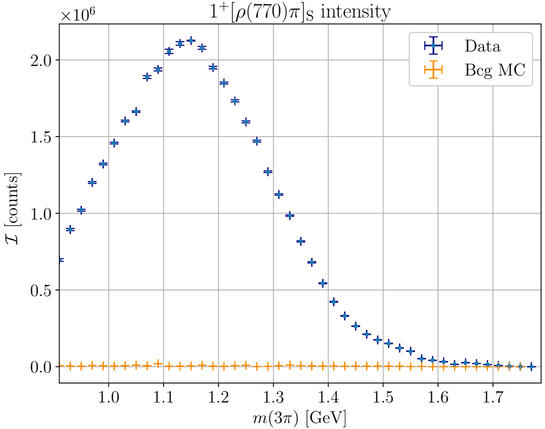
Background

Neural network* trained on simulated data

Neural network shape fixed in PWA

Test background leakage on simulated data

Background still leaks to signal



Background

* JINST 16 (2021) 06, P06016

Neural network* trained on simulated data

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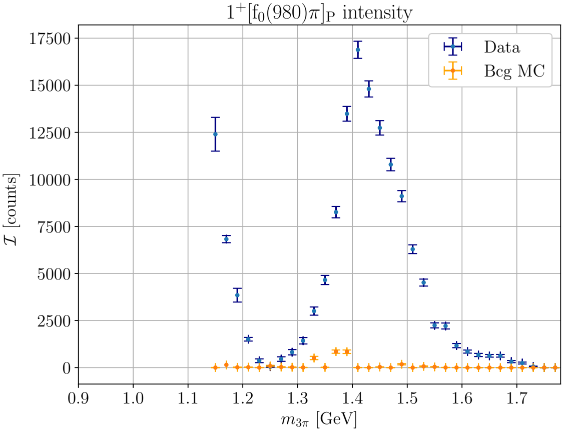


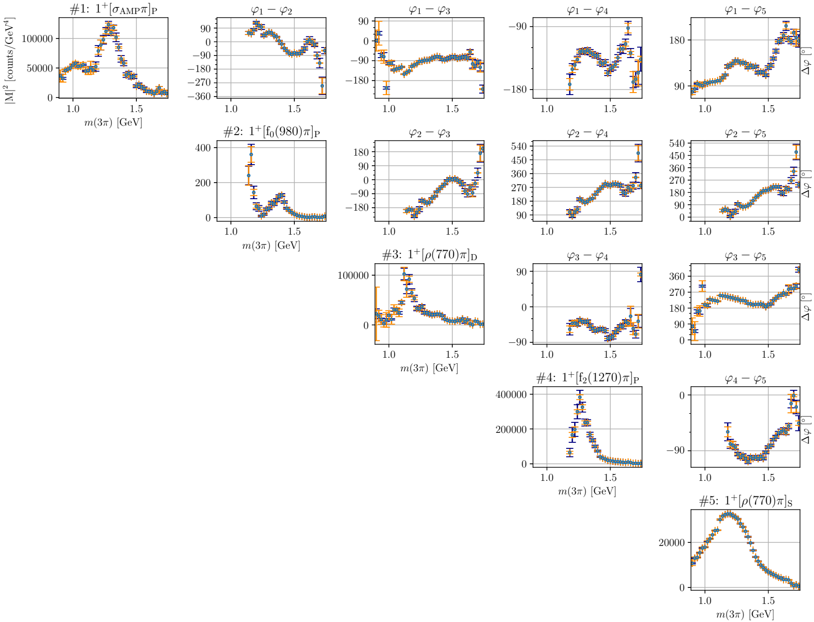
Neural network uncertainty:

Plots' legend:

- Stat. uncertainty
- Syst. uncertainty

Propagate uncertainties by varying neural network parameters before PWA

25–120 networks/bin



Integrals uncertainty:

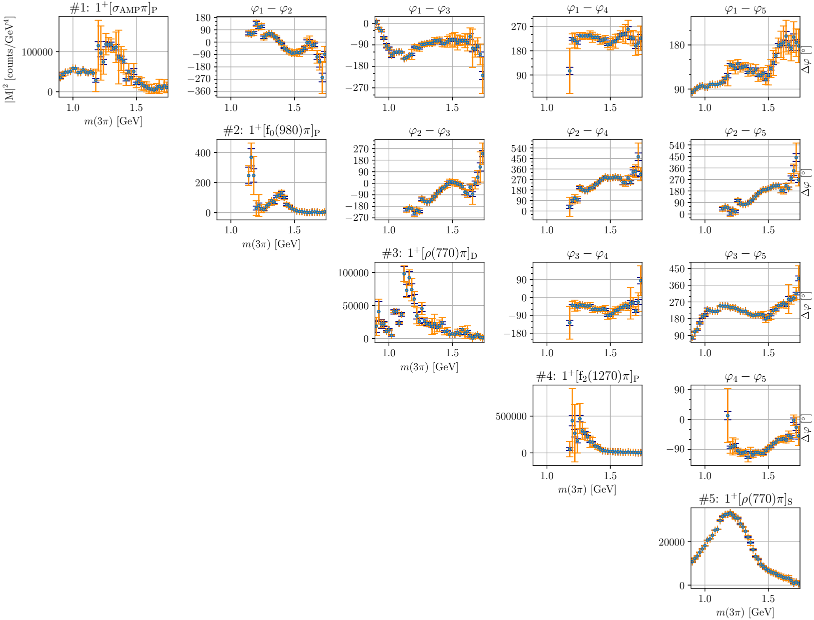
Plots' legend:

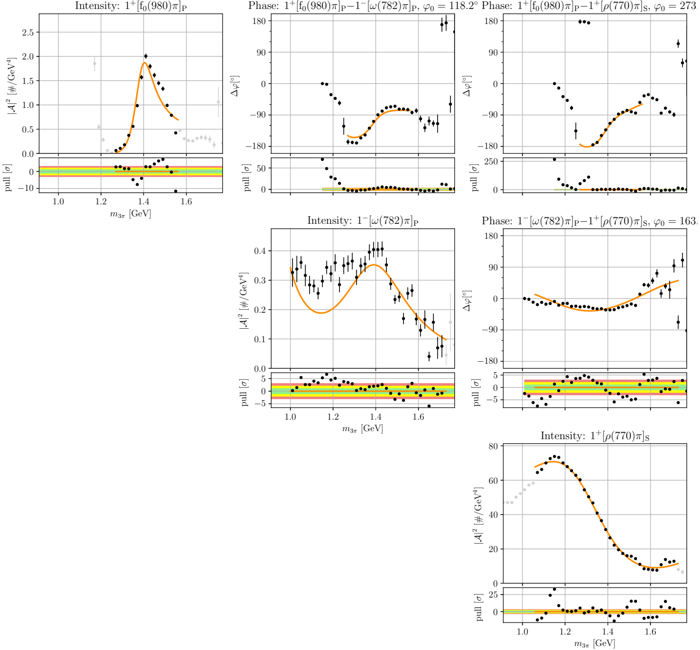
- Stat. uncertainty
- Syst. uncertainty

Propagate statistical uncertainties of integrals by varying integrals within their statistical uncertainties before PWA

Correlations not taken into account

$100\ \mathcal{N}_{wv}$/bin




Intensity: $1^{+}[f_0(980)\pi]_P$
Phase: $1^{+}[f_0(980)\pi]_P$–$1^{-}[\omega(782)\pi]_P$, $\varphi_0 = 118.2°$
Phase: $1^{+}[f_0(980)\pi]_P$–$1^{+}[\rho(770)\pi]_S$, $\varphi_0 = 273.$
Intensity: $1^{-}[\omega(782)\pi]_P$
Phase: $1^{-}[\omega(782)\pi]_P$–$1^{+}[\rho(770)\pi]_S$, $\varphi_0 = 163.$
Intensity: $1^{+}[\rho(770)\pi]_S$
$|\mathcal{A}|^2$ [#/GeV^4]
$\Delta\varphi$ [°]
pull [σ]
$m_{3\pi}$ [GeV]