

# **Recent Belle II results on hadronic B decays**

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We present recent measurements of hadronic *B* meson decays produced in electron-positron collisions at the  $\Upsilon(4S)$  resonance collected between 2019 and 2022 with the Belle II detector. We show measurements of  $B \to D^{(*)}KK_S^0$  decays including the first observation of three modes. Furthermore, we report on new results to determine the quark-mixing parameter  $\Phi_3/\gamma$ , performed on a combination of Belle and Belle II data. Measurements of  $B \to \rho\rho$  and  $B \to \pi\pi$  are presented which serve as input for the determination of  $\Phi_2/\alpha$ . Finally, determinations of branching fractions and direct *CP*-violating asymmetries of  $B \to K\pi$  modes are shown to perform a direct test of the standard-model via the  $K\pi$  isospin sum rule.

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#### 1. Introduction

Hadronic *B* decays are sensitive to all angles of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix and provide sensitive probes of physics beyond the Standard Model (SM). To this end, he Belle II experiment [1] targets the measurement of decay products of asymmetric  $e^+e^-$  collisions at the  $\Upsilon(4S)$  resonance from the SuperKEKB collider [2].

Belle II is a  $4\pi$  particle spectrometer aragned cylindrically around the beam pipe. It's innermost part consists of a silicon based vertex detector, followed by a 56-layer drift chamber. In combination with a 1.5 T magnetic field they reconstruct tracks of charged particles. Particle identification is performed by a time-of-propagation Cherenkov detector in the barrel region and an areogel ring-imaging Cherenkov detector in the forward region. The energy of electrons and photons is reconstructed in a CsI(Tl)-crystal electromagnetic calorimeter. The outermost sub-detector consists of alternating layers of iron plates and active detector elements (plastic scintillators and resistiveplate chambers) to reconstruct muons and  $K_I^0$  s.

Belle IIs excellent neutral reconstruction and the precise knowledge of the initial state opens the door for decays including one or multiple neutral final state particles.

Belle II started data taking in 2019 and has collected an integrated luminosity of  $362 \text{ ab}^{-1}$ , aiming to collect 50 ab<sup>-1</sup> in the next decade.

The Belle II hadronic physics program covers a wide range of measurements including the fundamental determinations of branching fractions and CP-violating asymmetries, the measurement of all CKM angles and direct tests of the SM via isospin sum rules. The CKM angle  $\phi_3 = arg(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*})$ can be determined via tree-level only *B*-meson decays and hence exhibits negligible theoretical uncertainties.  $\phi_1 = arg(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*})$  is the least-well measured CKM angle and can be determined by exploiting isospin relations in  $B \to \pi\pi$  and  $B \to \rho\rho$  decays. The isospin sum rule in  $B \to K\pi$ decays combines branching fractions and CP-violating asymmetries and provides a stringent null test of the SM [3].

Hadronic *B*-meson decays are typically polluted with large backgrounds coming from  $e^+e^- \rightarrow q\bar{q}$ ,  $q \in \{u, d, c, s\}$ , called "continuum" events. Due to their lower masses, compared to the *b*-quark, their event topology is jet like, while the *B*-mesons decay isotropically. This can be exploited in binary classifiers *C* based on boosted decision trees that we use to suppress continuum events. The extraction of relevant signal yields is based on observables offering distinguishing power between signal events and remaining backgrounds. We use the difference between the reconstructed energy of the *B*-meson and the beam-energy,  $\Delta E = E_B^* - E_{beam}^*$ , and the the beam constrained mass  $M_{bc} = \sqrt{E_{beam}^{*2} - |\vec{p}_{B^0}|^2}$ , where \* indicates that the quantity is evaluated in the center of mass system.

# 2. $B \rightarrow D^{(*)}KK^0_S$ decays

 $B \to D^{(*)}KK_S^0$  decays make up a few percent of the overall hadronic branching fraction, however only a small amount of them are well measured. They serve as inputs for simulations and tagging techniques. Hence, improving our knowledge on them will significantly improve numerous other measurements. We report a preliminary measurement of the  $B^- \to D^0 K^- K_S^0$  branching fraction and the first observation of  $\overline{B}^0 \to D^+ K^- K_S^0$ ,  $B^- \to D^{*0} K^- K_S^0$  and  $\overline{B}^0 \to D^{*+} K^- K_S^0$ :

$$\begin{split} \mathcal{B}(B^- \to D^0 K^- K_S^0) &= (1.89 \pm 0.16 \pm 0.10) \times 10^{-4}, \\ \mathcal{B}(\overline{B}^0 \to D^+ K^- K_S^0) &= (0.85 \pm 0.11 \pm 0.05) \times 10^{-4}, \\ \mathcal{B}(B^- \to D^{*0} K^- K_S^0) &= (1.57 \pm 0.27 \pm 0.12) \times 10^{-4}, \\ \mathcal{B}(\overline{B}^0 \to D^{*+} K^- K_S^0) &= (0.96 \pm 0.18 \pm 0.06) \times 10^{-4} \end{split}$$

where the first uncertainties are statistical and the second are systematic [4]. The branching fractions are determined in likelihood fits to the unbinned distributions of the energy difference  $\Delta E$ . Furthermore, we extract sweights [5] to study the "background" free invariant mass distributions of the  $KK_S^0$  system. In the low-mass region, we observe structures not compatible with a three-body phase space distribution in all four channels. Further studies are performed to understand their natures.



**Figure 1:**  $\Delta E$  distribution (left),  $m(K^-K_S^0)$  invariant mass distribution (middle) and Dalitz plot (right) of  $B^- \rightarrow D^0 K^- K_S^0$ 

# **3.** Measurements of the CKM angle $\Phi_3$

The CKM angle  $\Phi_3$  describes the weak phase between  $b \to c$  and  $b \to u$  tree transitions. Its current world average  $\Phi_3 = 65.9^{+3.3}_{-3.5}$  [6] is dominated by LHCb determinations. Experimentally,  $\Phi_3$  can be measured in the interference of favored  $B^- \to D^0 K^-$  and Cabibbo-suppressed  $B^- \to \overline{D}^0 K^-$  transitions. Reconstructing the  $D^0$  and  $\overline{D}^0$  from a common final state allows the extraction of the weak phase. Different methods are proposed depending on the chosen final states. We present two determinations using Cabibbo-suppressed modes and CP eigenstates. Both methods extract the relevant signal yields using likelihood fits to the unbinned  $\Delta E$  and continuum-suppression classifier output *C*. Both determinations are statistically limited.

#### 3.1 Cabibbo-suppressed modes

The GLS method uses Cabibbo-suppressed decays of *D*-mesons,  $B^{\pm} \to D[\to K_S^0 K^{\pm} \pi^{\mp}] h^{\pm}$ , where *D* is a superposition of  $D^0$  and  $\overline{D}^0$  to measure  $\Phi_3$  [7]. The kaon originating from the *D*meson can either have the same-sign (SS) or the opposite-sign (OS) as the *B*-meson. The amplitude and strong phase of the *D*-meson decay are taken as an external input from CLEO [8].

We measure four *CP*-violating asymmetries and three ratios of branching fractions [9] in the full D phase space and in the  $K^*(892)$  region of the  $m(KK_S^0)$  invariant mass, where a large strong phase difference enhances the sensitivity on  $\Phi_3$ . The measurement is performed on a combined Belle (711 fb<sup>-1</sup>) and Belle II (362 fb<sup>-1</sup>) dataset and is consistent, although less precise, with LHCb results [10]. The results constraint the value of  $\Phi_3$  by combining them with other measurements.

	Full D phase space	K <sup>*</sup> region
$\mathcal{A}_{SS}^{DK}$	$-0.089 \pm 0.091 \pm 0.011$	$0.055 \pm 0.119 \pm 0.020$
$\mathcal{A}_{OS}^{\widetilde{DK}}$	$0.109 \pm 0.133 \pm 0.013$	$0.231 \pm 0.184 \pm 0.014$
$\mathcal{A}_{SS}^{\tilde{D}\pi}$	$0.018 \pm 0.026 \pm 0.009$	$0.046 \pm 0.029 \pm 0.016$
$\mathcal{A}_{OS}^{\widetilde{D}\pi}$	$-0.028 \pm 0.031 \pm 0.009$	$0.009 \pm 0.046 \pm 0.009$
$\mathcal{R}_{SS}^{\tilde{D}\tilde{K}/D\pi}$	$0.122 \pm 0.012 \pm 0.004$	$0.093 \pm 0.012 \pm 0.005$
$\mathcal{R}_{OS}^{\widetilde{DK}/D\pi}$	$0.093 \pm 0.013 \pm 0.003$	$0.103 \pm 0.020 \pm 0.006$
$\mathcal{R}_{SS/OS}^{D\pi}$	$1.428 \pm 0.057 \pm 0.002$	$2.412 \pm 0.132 \pm 0.019$

**Table 1:** *CP*-violating charge asymmetries ( $\mathcal{A}$ ) and ratios of branching fractions ( $\mathcal{R}$ ) of  $B^{\pm} \rightarrow D[\rightarrow K_S^0 K^{\pm} \pi^{\mp}] h^{\pm}$  decays. The results are extracted from a combined Belle (711 fb<sup>-1</sup>) and Belle II (362 fb<sup>-1</sup>) dataset in the full *D* phase space and in the  $K^*$  region. The first uncertainties are statistical and the second are systematic.

#### 3.2 CP eigenstates

The GLW method uses *D*-meson decays to *CP* eigenstates to extract  $\Phi_3$  [11, 12]. We are reconstructing the *D*-meson from the *CP*-even final state  $K^+K^-$  and the *CP*-odd final state  $K_S^0\pi^0$ , where the latter is uniquely accessible to Belle II due to its neutral final state particles. The measurement is performed on a combined Belle (711 fb<sup>-1</sup>) and Belle II (362 fb<sup>-1</sup>) dataset and is consistent but not competitive with results from BaBar [13] and LHCb [14]. We report two ratios of branching fractions ( $\mathcal{R}_{CP^{\pm}}$ ) and two *CP*-violating asymmetries  $\mathcal{A}_{CP^{\pm}}$ 

 $\begin{aligned} \mathcal{R}_{CP^+} &= 1.164 \pm 0.081 \pm 0.036, \\ \mathcal{R}_{CP^-} &= 1.151 \pm 0.074 \pm 0.019, \\ \mathcal{A}_{CP^+} &= 0.125 \pm 0.058 \pm 0.014, \\ \mathcal{R}_{CP^-} &= -0.167 \pm 0.057 \pm 0.060, \end{aligned}$ 

where the first uncertainties are statistical and the second are systematic [15]. We observe evidence for different sign of  $\mathcal{A}_{CP^+}$  and  $\mathcal{A}_{CP^-}$ . The results constrain  $\Phi_3$  in combination with other measurements.

#### 4. Towards the CKM angle $\Phi_2$

In the absence of loop contributions  $sin(2\Phi_2)$  is proportional to the time-dependent *CP*violation amplitude in  $b \rightarrow u\bar{u}d$  transitions. However, non-negligible loop contributions introduce a large non-trivial shift. The tree and loop contributions can be disentangled exploiting isospin relations in  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ . Belle II has the unique capability to measure all modes in a consistent way.

#### 4.1 $B \rightarrow \rho \rho$ decays

Since the  $\rho$ -meson is a vector-meson the analysis requires an angular analysis to extract the polarization states. The fit is six-dimensional in  $M_{bc}$ ,  $\Delta E$ , the dipion masses and the helicity angles of the  $\rho$  candidates. We report preliminary Belle II results, using 189 fb<sup>-1</sup> of data, of  $B^0 \rightarrow \rho^+ \rho^-$  and  $B^+ \rightarrow \rho^+ \rho^0$  [16, 17]. The measurements are on par with previous Belle [18, 19] and BaBar [20, 21] determinations. The results are shown in Table 2.

#### **4.2** $B \rightarrow \pi \pi$ decays

We present a measurement of  $B^0 \to \pi^0 \pi^0$  using 189 fb<sup>-1</sup> Belle II data [22]. The final state, consisting of four photons only, as well as the CKM- and colour-suppressed nature of this decay poses experimental challenges. The fit to  $M_{\rm bc}$ ,  $\Delta E$  and C achieves a similar precision as Belle, despite using a dataset only one fourth of the size, due to improved  $\pi^0$  and continuum-suppression selection criteria.

Furthermore, we report measurements of  $B^0 \to \pi^+\pi^-$  and  $B^+ \to \pi^+\pi^0$  using 362 fb<sup>-1</sup> of Belle II data [23]. The branching ratio of  $B^0 \to \pi^+\pi^-$  is the worlds most precise determination. The results are shown in Table 2.

	$\mathcal{B}[10^{-6}]$	$\mathcal{A}_{CP}$	$f_L$
$B^0 \to \rho^+ \rho^-$	$26.7 \pm 2.8 \pm 2.8$	-	$0.956 \pm 0.035 \pm 0.033$
$B^+ \to \rho^+ \rho^0$	$23.2^{+2.2}_{-2.1} \pm 2.7$	$-0.069 \pm 0.068 \pm 0.060$	$0.943^{+0.035}_{-0.033} \pm 0.027$
$B^0 \to \pi^0 \pi^0$	$5.83 \pm 0.22 \pm 0.17$	-	-
$B^0 \rightarrow \pi^+ \pi^-$	$5.02 \pm 0.28 \pm 0.31$	$-0.082 \pm 0.054 \pm 0.008$	-
$B^+ \to \pi^+ \pi^0$	$1.38 \pm 0.27 \pm 0.22$	$0.14 \pm 0.46 \pm 0.07$	-

**Table 2:**  $B \to \rho \rho$  and  $B \to \pi \pi$  results for the branching ratios ( $\mathcal{B}$ ), the direct *CP* violation ( $\mathcal{A}$ ) and the longitudinal polarization fraction ( $f_L$ ). The first uncertainties are statistical and the second are systematic.

# 5. $K\pi$ isospin sum rule

The  $K\pi$  isospin sum rule combines the branching fractions and direct *CP* violation parameters of the four isospin related  $B \rightarrow K\pi$  modes and is defined as

$$I_{K\pi} = \mathcal{A}_{CP}^{K^{+}\pi^{-}} + \mathcal{A}_{CP}^{K^{0}\pi^{+}} \frac{\mathcal{B}_{K^{0}\pi^{+}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\tau_{B^{0}}}{\tau_{B^{+}}} - 2\mathcal{A}_{CP}^{K^{+}\pi^{0}} \frac{\mathcal{B}_{K^{+}\pi^{0}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\tau_{B^{0}}}{\tau_{B^{+}}} - 2\mathcal{A}_{CP}^{K^{0}\pi^{0}} \frac{\mathcal{B}_{K^{0}\pi^{0}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\mathcal{B}_{K^{0}\pi^{-}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\mathcal{B}_{K^{0}\pi^{-}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\mathcal{B}_{K^{0}\pi^{-}}}{\mathcal{B}_{K^{+}\pi^{-}}} \frac{\mathcal{B}_{K^{0}\pi^{-}}}{\mathcal{B}_{K^{0}\pi^{-}}} \frac{\mathcal{B}_$$

where  $\tau_{B^0}$  and  $\tau_{B^+}$  are the lifetimes of the neutral and charged *B*-mesons.  $I_{K\pi}$  is predicted to be zero within an uncertainty of 1% in the SM [24]. Large deviations from the SM prediction could

indicate anomalously enhanced amplitudes or non-SM physics. The current experimental precision is about 11% and is limited by  $\mathcal{A}_{CP}^{K^0\pi^0}$  with an uncertainty of 13% [25].

We report measurements of all involved branching fractions and *CP* violation parameters using Belle II data of 362 fb<sup>-1</sup>. Belle II has the unique capability to measure all involved final states in a consistent way allowing for the cancellation of related systematic uncertainties. The relevant signal yields are extracted in fits to the unbinned  $\Delta E$  and *C* distributions. Fit projections in  $\Delta E$  are shown in Figure 2. The results are summarized in Table 3. They are in agreement and competitive with



**Figure 2:**  $\Delta E$  distributions of  $B \to K^+\pi^-$  (top left),  $B \to K^+\pi^0$  (top right),  $B \to K_S^0\pi^+$  (bottom left) and  $B \to K_S^0\pi^0$  (bottom right) candidates reconstructed in Belle II data. The fit projections are overlaid as solid black line.

world averages [25]. By combining the reported time-integrated measurement of  $B^0 \to K_S^0 \pi^0$  with

	$\mathcal{B}[10^{-6}]$	$\mathcal{A}_{CP}$	
$B^0 \rightarrow K^+ \pi^-$	$20.67 \pm 0.37 \pm 0.62$	$-0.072 \pm 0.019 \pm 0.007$	
$B^+ \to K^+ \pi^0$	$13.93 \pm 0.38 \pm 0.71$	$0.013 \pm 0.027 \pm 0.005$	
$B^+ \to K^0 \pi^+$	$24.37 \pm 0.71 \pm 0.86$	$0.046 \pm 0.029 \pm 0.007$	
$B^0 \to K^0 \pi^0$	$10.40 \pm 0.66 \pm 0.60$	$-0.06 \pm 0.15 \pm 0.04$	
$I_{K\pi}$	$-0.03 \pm 0.13 \pm 0.04$		

**Table 3:**  $B \to K\pi$  results using 362 fb<sup>-1</sup> of Belle II data. The first uncertainties are statistical and the second are systematic.

a second, time-dependent measurement [26], we achieve the worlds most precise determination of  $\mathcal{R}_{K^0\pi^0}$ .

### 6. Summary

We present several new measurements of hadronic *B* decays at Belle II. We observe three  $B \rightarrow D^{(*)}KK_S^0$  modes for the first time. The CKM angle  $\Phi_3$  is measured via the GLS and GLW method on a combined Belle and Belle II dataset. We measure  $B \rightarrow \rho\rho$  and  $B \rightarrow \pi\pi$  modes needed for the determination of  $\Phi_2$ . Finally, we report a measurement of the  $K\pi$  isospin sum rule on par with the worlds best result.

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