Measurements of $|V_{cb}|$ and $|V_{ub}|$ from Belle (II)

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Measurements covered in this talk:

**Exclusive $|V_{cb}|$:**
- Had. tagged $B^0 \rightarrow D^* \ell \nu$
- Had. tagged $B \rightarrow D^* \ell \nu$ and shapes of key kinematic variables

**Exclusive $|V_{ub}|$:**
- Untagged $B^0 \rightarrow \pi^- \ell \nu$

**Inclusive $|V_{ub}|$:**
- Partial & differential branching fractions of $B \rightarrow X_u \ell \nu$

**Combined measurements:**
- Excl. $|V_{ub}|$ / incl. $|V_{ub}|$
- Incl. $|V_{ub}|$ / incl. $|V_{cb}|$
Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

- Decay chain: $B^0 \rightarrow D^{*+} \ell \nu$, $D^{*+} \rightarrow D^0 \pi^*_{\text{slow}}$, $D^0 \rightarrow K^- \pi^+$
- Data set of 189.3 fb$^{-1}$ with untagged strategy (higher efficiency than tagged)
- Select events with energetic lepton $p_{\text{CM}} > 1.2$ GeV, and $\Delta M = M(D^{*+}) - M(D^0) = [0.141, 0.156]$ GeV, $\cos \theta_{BY} = [-4, 2]$
- 2D binned likelihood fit on $(\cos \theta_{\ell}, \Delta M)$ for each bin of kinematic variables: $w, \cos \theta_{\ell}, \cos \theta_{\nu}, \chi$
- Systematic shape variations incorporated as bin-wise Nuisance para. for each fit template
Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

- Unfold signal yields using **singular-value-decomposition (SVD)**
- Full post-unfolding stat. & syst. covariance propagated into partial decay rate

\[
\Delta \Gamma_i = \frac{\epsilon_i N_{B^0} B(D^+ \rightarrow D^0 \pi^+) B(D^0 \rightarrow K^- \pi^+) \tau_{B^0}}{\text{reco. eff & acc.}} \cdot y_i^{\text{unfolded}}
\]

\[
\Gamma = \left( \sum_{i=1}^{10} \Delta \Gamma_i^w + \sum_{i=1}^{8} \Delta \Gamma_i^{\cos \theta_L} + \sum_{i=1}^{10} \Delta \Gamma_i^{\cos \theta_V} + \sum_{i=1}^{10} \Delta \Gamma_i^\chi \right) / 4
\]

Branching fraction extracted by the total rate summing over partial decay rates and averaging all kin. variables

- $e$ mode: $B(B^0 \rightarrow D^+ e^- \bar{\nu}_e) = (4.94 \pm 0.03 \pm 0.22)\%$
- $\mu$ mode: $B(B^0 \rightarrow D^+ \mu^- \bar{\nu}_\mu) = (4.94 \pm 0.03 \pm 0.24)\%$
- Average: $B(B^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell) = (4.94 \pm 0.02 \pm 0.22)\%$
Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

- Include all measured $w, \cos\theta_\ell, \cos\theta_\nu, \chi$ to extract form factor & $|V_{cb}|$
- BGL truncation based on nested hypothesis test

$$|V_{cb}|_{\eta_{EW}} F(1) = \frac{1}{\sqrt{m_{B}m_{D^*}}} \left( \frac{|b_0|}{P_f(0)\phi_f(0)} \right)$$

$$|V_{cb}|_{\text{BGL}} = (40.9 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

$$|V_{cb}|_{\text{CLN}} = (40.4 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

Slow pion eff. plays leading role in syst. Input from LQCD at zero-recoil $F(1)$
Branching Fraction of $B^0 \to D^* \ell \nu$ and $|V_{cb}|$

- Include all measured $w$, $\cos \theta_\ell$, $\cos \theta_\nu$, $\chi$ to extract form factor & $|V_{cb}|$


- BGL truncation based on nested hypothesis test [Phys. Rev. D100, 013005]


| $|V_{cb}|$ shifts when include LQCD full constraints |

Similar tension seen in recent Belle (2023) measurement [arXiv:2301.07529] ⇒ Both found large disagreements wrt LQCD results on $R_2$
Branching Fraction of $B^0 \to D^* \ell \nu$ and $|V_{cb}|$

- Lepton-flavor-universality tested with separate results on e- & mu-mode
- All in good agreement with SM expectations

Test on branching fraction ratio: $R_{e/\mu} = 1.001 \pm 0.009 \pm 0.021$

Test on forward-backward asymmetry:

$$A_{FB} = \int_0^1 \frac{d \cos \theta_c d \Gamma / d \cos \theta_c - \int_{-1}^0 d \cos \theta_c d \Gamma / d \cos \theta_c}{\int_0^1 d \cos \theta_c d \Gamma / d \cos \theta_c + \int_{-1}^0 d \cos \theta_c d \Gamma / d \cos \theta_c}$$

$$\Delta A_{FB} = A_{FB}^\mu - A_{FB}^e$$

$A_{FB}^e = 0.219 \pm 0.011 \pm 0.020$,

$A_{FB}^\mu = 0.215 \pm 0.011 \pm 0.022$,

$\Delta A_{FB} = (-4 \pm 16 \pm 18) \times 10^{-3}$

Test on $D^*$ longitudinal polarization fraction:

$$\frac{1}{\Gamma} \frac{d \Gamma}{d \cos \theta_V} = \frac{3}{2} \left( F_L \cos^2 \theta_V + \frac{1 - F_L}{2} \sin^2 \theta_V \right)$$

$$\Delta F_L = F_L^\mu - F_L^e$$

$F_L^e = 0.521 \pm 0.005 \pm 0.007$

$F_L^\mu = 0.534 \pm 0.005 \pm 0.006$

$\Delta F_L = 0.013 \pm 0.007 \pm 0.007$
$|V_{cb}|$ & Differential Shapes of $B \rightarrow D^* \ell \nu$

- Full Belle data set of 711 fb$^{-1}$ for $B^{\pm,0}, \ell = e, \mu$
- **Hadronic tagging** using Belle II tool (Full Event Interpretation [Comp. Soft. Big Sci 3 (2019) 6])
- Background subtracted via fitting $M^2_{\text{miss}}$ for bins of $w$, $\cos \theta_\ell$, $\cos \theta_\nu$, $\chi$ in each decay mode independently
- Combined **all kin. shapes** to extract $|V_{cb}|$ in BGL/CLN with external constraints on branching fractions (HFLAV) and LQCD (FNAL/MILC)

**Fitted Shapes**

![Graph showing fitted shapes](Graph.png)
In $|V_{cb}|$ extraction, tested different BGL truncations, LQCD constraining scenario (at or beyond zero-recoil).

- **Forward-backward asymmetry $A_{FB}$** and $D^*$ longitudinal polarization fraction $F_{L}^{D^*}$ and their differences between $e, \mu$ also derived. **No significant LFUV found.**
|\textbf{V}_{\text{ub}}| \text{ in } B^0 \rightarrow \pi^- \ell^+ \nu \text{ Decay}

- Data set of 189.3 fb$^{-1}$ with untagged analysis strategy
- Extract signal in beam-constrained mass $M_{bc}$ and energy difference $\Delta E$ for each bin of $q^2$

\[
\Delta E = E_B^* - E_{\text{beam}}^* = E_B^* - \frac{\sqrt{s}}{2}
\]

\[
M_{bc} = \sqrt{E_{\text{beam}}^* - |\vec{p}_B|^2} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - |\vec{p}_B|^2}
\]
$|V_{ub}|$ in $B^0 \rightarrow \pi^- \ell^+ \nu$ Decay

- Data set of 189.3 fb$^{-1}$ with untagged analysis strategy
- Extract signal in beam-constrained mass $M_{bc}$ and energy difference $\Delta E$ for each bin of $q^2$
- $|V_{ub}|$ fitted with Bourrely-Caprini-Lellouch (BCL) [Phys.Rev.D79, 013008] expansion including LQCD constraints (FNAL/MILC [Phys. Rev. D92, 014024])

\[
\mathcal{B} = (1.426 \pm 0.056_{\text{stat}} \pm 0.125_{\text{syst}}) \times 10^{-4}
\]

\[
|V_{ub}| = (3.55 \pm 0.12_{\text{stat}} \pm 0.13_{\text{syst}} \pm 0.17_{\text{theo}}) \times 10^{-3}
\]

**Belle II Preliminary**

\[
\int \mathcal{L} dt = 189 \text{ fb}^{-1}
\]

- $d\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) / dq^2$ [GeV$^{-2}$]
- $q^2$ [GeV$^2$]

- Data set of 189.3 fb$^{-1}$ with untagged analysis strategy
- Extract signal in beam-constrained mass $M_{bc}$ and energy difference $\Delta E$ for each bin of $q^2$
- $|V_{ub}|$ fitted with Bourrely-Caprini-Lellouch (BCL) [Phys.Rev.D79, 013008] expansion including LQCD constraints (FNAL/MILC [Phys. Rev. D92, 014024])

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\[
|V_{ub}| = (3.55 \pm 0.12_{\text{stat}} \pm 0.13_{\text{syst}} \pm 0.17_{\text{theo}}) \times 10^{-3}
\]

- dominated by background modelling (continuum, $B \rightarrow \rho \ell \nu$)
Recent Belle II Results on Exclusive $|V_{xb}|$

| $B^0 \rightarrow D^*-\ell^+\nu$, untagged | $|V_{cb}| \times 10^3$ | References |
|-------------------------------------------|----------------|----------------|
| $B^0 \rightarrow D^*-\ell^+\nu$, tagged   | 37.9 ± 2.7 (CLN) | arXiv:2301.04716 |
| $B \rightarrow D\ell\nu$, untagged       | 38.28 ± 1.16 (BGL) | arXiv:2210.13143 |

| $|V_{ub}| \times 10^3$ | References |
|-----------------------|----------------|
| $B \rightarrow \pi\ell\nu$, tagged   | 3.88 ± 0.45   | arXiv:2206.08102 |
| $B \rightarrow \pi\ell\nu$, untagged | 3.55 ± 0.25   | arXiv:2210.04224 |

$|V_{cb}|_{excl} = (39.10 \pm 0.50) \times 10^{-3}$

$|V_{ub}|_{excl} = (3.51 \pm 0.12) \times 10^{-3}$

HFLAV 2023
Inclusive $B \rightarrow X_u \ell \nu$ and $|V_{ub}|$

- Full Belle data set of 711 fb$^{-1}$ with **Hadronic tagging**
- Use **machine learning (BDT)** to suppress backgrounds with 11 training features, e.g. $M^2$, $\#K^\pm$, $\#K_s$, etc.

\[ (4.10 \pm 0.09)_{\text{stat}} \pm 0.22_{\text{sys}} \pm 0.15_{\text{theo}} \times 10^{-3} \]

$|V_{ub}|$ derived according to

\[ \Delta \mathcal{B}(E^\ell_B > 1 \text{ GeV}) = (1.59 \pm 0.07 \pm 0.16) \times 10^{-3} \]

Arithmetic avr. $|V_{ub}|$ based on various theo. decay rate:

\[
(4.10 \pm 0.09_{\text{stat}} \pm 0.22_{\text{sys}} \pm 0.15_{\text{theo}}) \times 10^{-3}
\]

compatible with excl. and CKM expectation within $1.3\sigma$ and $1.6\sigma$, respectively

- Extract signal using binned likelihood in **3 phase space (PS) regions:**
  - $E^\ell_B > 1 \text{ GeV}$ (covers 86% of available signal PS)
  - $E^\ell_B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}$ (50%)
  - $E^\ell_B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}, q^2 > 8 \text{ GeV}^2$ (31%)

- Partial BF and inclusive $|V_{ub}|$ derived in each PS

$|V_{ub}| = \sqrt{\frac{\Delta \mathcal{B}(B \rightarrow X_u \ell \nu)}{\tau_B \cdot \Delta \Gamma(B \rightarrow X_u \ell \nu)}}$
First Measurement of Differential Spectra of $B \to X_u \ell \nu$

- Inherit same analysis strategy in the partial BF measurement [PRD 104, 012008 (2021)]
- Additional selections on $|E_{\text{miss}} - P_{\text{miss}}| < 0.1$ GeV & $M_x < 2.4$ GeV to improve resolution and significance
- Background subtracted via $M_x$ fit, further corrected for efficiency & acceptance effects (phase space: $E_\ell^B > 1$ GeV)
- Necessary input for future model-independent determinations of $|V_{ub}|$ (e.g. NNVub, SIMBA)

![Graphs showing differential spectra](image)

$P_+$

$P_-$

$E_\ell^B$

$q^2$

$M_x$

$M_x^2$
First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

- Inherit **same analysis strategy** in the partial BF measurement [PRD 104, 012008 (2021)]
- Extract signal in $q^2 : N_{\pi^\pm}$ for $B \to \pi \ell \nu$ and $B \to X_u \ell \nu$ simultaneously
- Fitter corporates experimental observation of templates’ **normalisations** and $B \to \pi \ell \nu$ **form factor** ($q^2$ shape)

Fit results provide all $\mathcal{B}$ and $B \to \pi \ell \nu$ FF (decay rate)

$\Delta \mathcal{B}(B \to X_u \ell \nu) = \mathcal{B}(B \to X_u \ell \nu) \cdot \epsilon_{\Delta \mathcal{B}(B \to X_u \ell \nu)}$

$|V_{ub}^{\text{incl.}}| = \sqrt{\frac{\Delta \mathcal{B}(B \to X_u \ell \nu)}{\tau_B \cdot \Delta \Gamma_{\text{GGOU}}}}$

$|V_{ub}^{\text{excl.}}| = \sqrt{\frac{\mathcal{B}(B \to \pi \ell \nu)}{\tau_B \cdot \Gamma_{\text{FF}}}}$

Theoretical decay rate based on GGOU prediction [Gambino-Giordano-Ossola-Uraltsev, JHEP 10 (2007) 058]
First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

- Various fit scenarios applied:
  - **Combined** or separate $B \to \pi^+ \ell \nu$, $B \to \pi^0 \ell \nu$ (iso-spin relation)
  - Input BCL constraint: LQCD + exp. or only LQCD [FLAG: Eur. Phys. J. C 82, 869 (2022)]

$$|V_{ub}|$$ in **combined** scenario with LQCD+exp const.:
- **Excl.** $(3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3}$
- **Incl.** $(3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3}$
- Ratio $0.97 \pm 0.12$ ($\rho = 0.10$)

Weighted average of excl. & incl.:
- $(3.85 \pm 0.26) \times 10^{-3}$

**CKM global fit** (w/o $|V_{ub}|$): $(3.64 \pm 0.07) \times 10^{-3}$, compatible within 0.8σ
Ratio of Inclusive $\Delta \mathcal{B}(B \rightarrow X_u \ell \nu)$ and $\Delta \mathcal{B}(B \rightarrow X_c \ell \nu)$

- Full Belle data set of 711 fb$^{-1}$ with **Hadronic tagging** using Belle II tool (Full Event Interpretation)
- **Modified $B \rightarrow X_c \ell \nu$ modeling** using sideband data
- $B \rightarrow X_u \ell \nu$ yields extracted in $q^2 : p^B_\ell$; $B \rightarrow X_c \ell \nu$ yields obtained by subtracting other contributions in total $B \rightarrow X \ell \nu$
- Measured partial phase space region of $p^B_\ell > 1$ GeV with fractions of $\epsilon^u_\Delta = 86\%$, $\epsilon^c_\Delta = 79\%$

\[
\frac{\Delta \mathcal{B}(B \rightarrow X_u \ell \nu)}{\Delta \mathcal{B}(B \rightarrow X_c \ell \nu)} = 1.95(1 \pm 8.4\%_{\text{stat}} \pm 7.8\%_{\text{syst}}) \times 10^{-2}
\]

Based on this, one could try the following two quick and naive conversions

\[
|V_{ub}| = \sqrt{\frac{1}{\tau_B \Delta \Gamma(B \rightarrow X_u \ell \nu) \Delta \mathcal{B}(B \rightarrow X_c \ell \nu)}}
\]

**Consistent with recent Belle result** PRD 104, 012008 (2021)
**Ratio of Inclusive $\Delta \mathcal{B}(B \to X_u \ell \nu)$ and $\Delta \mathcal{B}(B \to X_c \ell \nu)$**

- Full Belle data set of 711 fb$^{-1}$ with *Hadronic tagging* using Belle II tool (Full Event Interpretation)
- **Modified $B \to X_c \ell \nu$ modeling** using sideband data
  
  - $B \to X_u \ell \nu$ yields extracted in $q^2 : p_\ell^B$; $B \to X_c \ell \nu$ yields obtained by subtracting other contributions in total $B \to X \ell \nu$
  
  - Measured partial phase space region of $p_\ell^B > 1$ GeV with fractions of $\epsilon_\Delta^u = 86\%$, $\epsilon_\Delta^c = 79\%$

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\]

\[
\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta \mathcal{B}(B \to X_u \ell \nu) \Delta \Gamma(B \to X_c \ell \nu)}{\Delta \mathcal{B}(B \to X_c \ell \nu) \Delta \Gamma(B \to X_u \ell \nu)}}
\]

**Theo. decay rates:**

- $\Delta \Gamma^{\text{GOU}}(B \to X_u \ell \nu) = 58.5 \pm 2.7$ ps$^{-1}$
- $\Delta \Gamma^{\text{Kin}}(B \to X_c \ell \nu) = 29.9 \pm 1.2$ ps$^{-1}$

*Preliminary*
Summary

- Many new results are me seared recently and will be very helpful to examine the long-standing $|V_{xb}|$ puzzle

- Continuous efforts from **experiment** and **theory** are still needed
  
  - Seen discrepancies in LQCD vs. Exp. for $B \rightarrow D^* \ell \nu$ need to be investigated
  
  - BGL & CLN resulted in consistent $|V_{cb}|$ (no dependence on parameterizations)
  
  - Higher precision expected at Belle II for simultaneous excl. & incl. $|V_{ub}|$ and inclusive $|V_{ub}|/|V_{cb}|$ ratio

- Beyond these important results, the accumulated knowledge on MC modeling, analysis techniques, etc. will be beneficial for future measurements by e.g. **Belle II** or **LHCb**
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THANK YOU
Backup: Tagging vs. Untagging

- **Untagged**
  - Loose constraints on signal
  - Very large statistics, but also very large background
  - Efficiency $\epsilon \approx \mathcal{O}(100\%)$

- **Semileptonic tag**
  - Mid-range reconstruction efficiency
  - Due to multiple neutrinos, less information about $B_{\text{tag}}$

- **Hadronic tag**
  - Cleaner sample
  - Knowledge of $p(B_{\text{sig}})$
  - Low tag-side efficiency $\epsilon \approx \mathcal{O}(0.1\%)$
Fitter corporates experimental observation of templates’ normalisations and $B \rightarrow \pi \ell \nu$ form factor.

Systematic uncertainties included via Nuisance parameters for both of additives and multiplicative impacts.

Dominant syst. are non-resonant $B \rightarrow X_u \ell \nu$ modelling, fragmentation and reconstruction efficiency (stat. limits $B \rightarrow \pi \ell \nu$).

\[-2 \log \mathcal{L} = -2 \log \prod_i \text{Poisson} \left( \eta_{\text{obs}} \cdot \eta_{\text{pred}} \cdot (1 + \epsilon \cdot \theta) \right) + \theta \rho^{-1} \theta^T + \chi^2_{\text{FF}}\]

Constraints on BCL parameters, input taken from LQCD / LQCD+exp fits in FLAG Review 2021.

Normalizations can be linked with isospin relation, or floating separately (nominal: linked).

Forward-folding $q^2$.
Backup: First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

Decay rate as a function of $q^2$ with fitted BCL para.

Results with various input of inclusive decay rates
Backup: Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

- Nested hypothesis test included with LQCD beyond-recoil constraints
• Signal shapes corrected for resolution, reco. efficiency and acceptance effects
• Combined all kinematic shapes to extract $|V_{cb}|$ in BGL/CLN with external constraints on branching fractions (HFLAV) and LQCD results (FNAL/MILC)

$$
\chi^2 = \left( \frac{\Delta \Gamma_{\text{m}}}{\Gamma_{\text{m}}} - \frac{\Delta \Gamma_{\text{p}}(\overline{x})}{\Gamma_{\text{p}}(\overline{x})} \right) C_{\text{exp}}^{-1} \left( \frac{\Delta \Gamma_{\text{m}}}{\Gamma_{\text{m}}} - \frac{\Delta \Gamma_{\text{p}}(\overline{x})}{\Gamma_{\text{p}}(\overline{x})} \right)^T + \frac{(\Gamma_{\text{ext}} - \Gamma_{\text{p}}(\overline{x}))^2}{\sigma(\Gamma_{\text{ext}})^2} + (h_X - h_X^{\text{LQCD}})C_{\text{LQCD}}^{-1}(h_X - h_X^{\text{LQCD}})
$$

Backup: $|V_{cb}|$ & Differential Shapes of $B \to D^* \ell \nu$

arXiv: 2301.07529
accepted by PRD
• Nested hypothesis test w/o & w/ LQCD beyond-recoil constraints