$B \to K^{(*)}\nu\bar{\nu}$ SM predictions

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This is a numerical update of [1].

1 Overview

The branching ratios of $B^+ \to K^+\nu\bar{\nu}$ and $B^0 \to K^{*0}\nu\bar{\nu}$ can be written in the SM as

$$\text{BR}(B^+ \to K^+\nu\bar{\nu})_{\text{SM}} = \tau_{B^+} 3|N|^2 \frac{X_t^2}{s_w^4} \langle \rho_K \rangle \,, \tag{1}$$

$$\text{BR}(B^0 \to K^{*0}\nu\bar{\nu})_{\text{SM}} = \tau_{B^0} 3|N|^2 \frac{X_t^2}{s_w^4} . \langle \rho_{A_1} + \rho_{A_{12}} + \rho_V \rangle \,. \tag{2}$$

These expressions contain

- The lifetimes. There is no isospin asymmetry, so the B^0 vs. B^+ branching ratios can be trivially obtained by rescaling with the appropriate lifetimes.
- A factor of 3 for the three light neutrino flavours.
- The numerical factor
$$N = V_{tb} V_{ts}^* \frac{G_F \alpha}{16\pi^2} \sqrt{\frac{m_B}{3\pi}} \,, \tag{3}$$
containing in particular the CKM elements.
- The Wilson coefficient X_t of the single contributing operator
$$\mathcal{H}_{\text{eff}} = \frac{4\,G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_w^2} V_{tb} V_{ts}^* X_t (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_L \gamma^\mu \nu_L) \;+\; \text{h.c.} \,. \tag{4}$$
- Rescaled form factors (as defined in [1]) integrated over q^2,
$$\langle \rho_i \rangle = \int dq^2 \rho_i(q^2) \tag{5}$$

The only non-trivial pieces are the Wilson coefficient and the form factors and will be discussed in turn.

s_w^2	0.23126(5)	[6, 5]	τ_{B^0}	1.519(5) ps	[6]
α	127.925(16)	[6, 5]	τ_{B^+}	1.638(4) ps	[6]

Table 1: Input parameters used for the SM predictions.

2 Wilson coefficient

Including NLO QCD corrections [2, 3, 4] and two-loop electroweak corrections [5], one gets

$$X_t = 1.469 \pm 0.017 \pm 0.002 \,. \tag{6}$$

where the first and second error are due to higher-order QCD and electroweak corrections, respectively. This is computed in an on-shell scheme for the masses, but $\overline{\text{MS}}$ scheme for α and s_w^2. The appropriate values are listed in table 1.

3 CKM elements

Assuming the SM, global CKM fits can be used to extract the CKM combination $\lambda_t = V_{tb}V_{ts}^*$. One finds

$$|\lambda_t|^{\text{CKMfitter}} = (4.104^{+0.031}_{-0.067}) \times 10^{-2} \,, \qquad |\lambda_t|^{\text{UTfit}} = (4.088 \pm 0.055) \times 10^{-2} \,. \tag{7}$$

However, to probe new physics it is better to use CKM elements from tree-level observables, i.e. $|V_{ub}|$, V_{cb}, V_{us}, and γ. One obtains

$$|\lambda_t| = V_{cb}\left(1 - \frac{V_{us}^2}{2} + V_{us}\cos\gamma\,\frac{|V_{ub}|}{V_{cb}} + O(\lambda^4)\right) \tag{8}$$

$$= (0.983 \pm 0.003)\, V_{cb} \,. \tag{9}$$

The value of λ_t now depends on whether one uses the inclusive or the exclusive determination of V_{cb},

$$|\lambda_t|^{\text{incl}} = (4.17 \pm 0.08) \times 10^{-2} \,, \qquad |\lambda_t|^{\text{excl}} = (3.83 \pm 0.12) \times 10^{-2} \,. \tag{10}$$

An average of the two using the PDG prescription leads to

$$|\lambda_t| = 4.06 \pm 0.16, \tag{11}$$

perfectly compatible with the global fits but with much larger uncertainty.

4 Form factors

4.1 $B \to K^*$

A combined fit to lattice and LCSR $B \to K^*$ form factors gives

$$\langle \rho_{A_1} + \rho_{A_{12}} + \rho_V \rangle = (110 \pm 9)\,\text{GeV}^2 \,. \tag{12}$$

Considering lattice and LCSR results separately instead and using them for q^2 above and below $12\,\mathrm{GeV}^2$, respectively, where they are reliable, one finds

$$\langle\rho_{A_1}+\rho_{A_{12}}+\rho_V\rangle^{\mathrm{LCSR}}_{q^2<12\,\mathrm{GeV}^2}=(71\pm 9)\,\mathrm{GeV}^2\,, \tag{13}$$
$$\langle\rho_{A_1}+\rho_{A_{12}}+\rho_V\rangle^{\mathrm{LQCD}}_{q^2>12\,\mathrm{GeV}^2}=(46\pm 6)\,\mathrm{GeV}^2\,. \tag{14}$$

4.2 $B\to K$

For the $B\to K$ form factors more precise lattice form factors exist and have been extrapolated to the full q^2 range using a third-order z-expansion. The results are compatible with low-q^2 predictions from LCSR. One finds

$$\langle\rho_K\rangle=(50.4\pm 5.5)\,\mathrm{GeV}^2. \tag{15}$$

5 Numerics

Using (6), (11), (12), and (15), one obtains

$$\mathrm{BR}(B^+\to K^+\nu\bar\nu)_{\mathrm{SM}}=(4.68\pm 0.64)\times 10^{-6}, \tag{16}$$
$$\mathrm{BR}(B^0\to K^{*0}\nu\bar\nu)_{\mathrm{SM}}=(9.48\pm 1.10)\times 10^{-6}. \tag{17}$$

The uncertainties are dominated by the CKM and form factor uncertainties, which are comparable when using (11).

References

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