This is a numerical update of [1].

1 Overview

The branching ratios of $B^+ \to K^+ \nu \bar{\nu}$ and $B^0 \to K^{*0} \nu \bar{\nu}$ can be written in the SM as

$$\text{BR}(B^+ \to K^+ \nu \bar{\nu})_{\text{SM}} = \tau_{B^+} 3 |N|^2 \frac{X_t^2}{s_w^4} \langle \rho_K \rangle,$$  \hspace{1cm} (1)

$$\text{BR}(B^0 \to K^{*0} \nu \bar{\nu})_{\text{SM}} = \tau_{B^0} 3 |N|^2 \frac{X_t^2}{s_w^4} \langle \rho_{A_1} + \rho_{A_{12}} + \rho_{\nu} \rangle. \quad (2)$$

These expressions contain

- The lifetimes. There is no isospin asymmetry, so the $B^0$ vs. $B^+$ branching ratios can be trivially obtained by rescaling with the appropriate lifetimes.
- A factor of 3 for the three light neutrino flavours.
- The numerical factor
  $$N = \frac{V_{tb} V_{ts}}{\sqrt{2}} \frac{G_F \alpha \sqrt{m_B}}{16\pi^2 \sqrt{3\pi}}, \quad \text{(3)}$$
  containing in particular the CKM elements.
- The Wilson coefficient $X_t$ of the single contributing operator
  $$\mathcal{H}_{\text{eff}} = \frac{4 G_F \alpha}{\sqrt{2} 2\pi s_w^2} V_{tb} V_{ts}^* X_t (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_L \gamma^\mu \nu_L) + \text{h.c.}. \quad \text{(4)}$$

- Rescaled form factors (as defined in [1]) integrated over $q^2$,
  $$\langle \rho_i \rangle = \int dq^2 \rho_i(q^2) \quad \text{(5)}$$

The only non-trivial pieces are the Wilson coefficient and the form factors and will be discussed in turn.
Table 1: Input parameters used for the SM predictions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_w^2$</td>
<td>0.23126(5)</td>
<td>[6, 5]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>127.925(16)</td>
<td>[6, 5]</td>
</tr>
<tr>
<td>$\tau_{B^0}$</td>
<td>1.519(5) ps</td>
<td>[6]</td>
</tr>
<tr>
<td>$\tau_{B^+}$</td>
<td>1.638(4) ps</td>
<td>[6]</td>
</tr>
</tbody>
</table>

2 Wilson coefficient

Including NLO QCD corrections [2, 3, 4] and two-loop electroweak corrections [5], one gets

$$X_t = 1.469 \pm 0.017 \pm 0.002.$$  \hfill (6)

where the first and second error are due to higher-order QCD and electroweak corrections, respectively. This is computed in an on-shell scheme for the masses, but $\overline{\text{MS}}$ scheme for $\alpha$ and $s_w^2$. The appropriate values are listed in table 1.

3 CKM elements

Assuming the SM, global CKM fits can be used to extract the CKM combination $\lambda_t = V_{tb}V_{ts}^*$. One finds

$$|\lambda_t|^{\text{CKMfitter}} = (4.104^{+0.031}_{-0.067}) \times 10^{-2}, \quad |\lambda_t|^{\text{UTfit}} = (4.088 \pm 0.055) \times 10^{-2}.$$  \hfill (7)

However, to probe new physics it is better to use CKM elements from tree-level observables, i.e. $|V_{ub}|, V_{cb}, V_{us}$, and $\gamma$. One obtains

$$|\lambda_t| = V_{cb} \left( 1 - \frac{V_{us}^2}{2} + V_{us} \cos \gamma \frac{|V_{ub}|}{V_{cb}} + O(\lambda^4) \right)$$  \hfill (8)

$$= (0.983 \pm 0.003) V_{cb}. \hfill (9)$$

The value of $\lambda_t$ now depends on whether one uses the inclusive or the exclusive determination of $V_{cb}$,

$$|\lambda_t|^{\text{incl}} = (4.17 \pm 0.08) \times 10^{-2}, \quad |\lambda_t|^{\text{excl}} = (3.83 \pm 0.12) \times 10^{-2}.$$  \hfill (10)

An average of the two using the PDG prescription leads to

$$|\lambda_t| = 4.06 \pm 0.16,$$  \hfill (11)

perfectly compatible with the global fits but with much larger uncertainty.

4 Form factors

4.1 $B \to K^*$

A combined fit to lattice and LCSR $B \to K^*$ form factors gives

$$\langle \rho_{A_1} + \rho_{A_{12}} + \rho_V \rangle = (110 \pm 9) \text{ GeV}^2.$$  \hfill (12)
Considering lattice and LCSR results separately instead and using them for $q^2$ above and below 12 GeV$^2$, respectively, where they are reliable, one finds

$$\langle \rho_{A_1} + \rho_{A_{12}} + \rho_V \rangle_{LCSR}^{q^2 < 12 \text{ GeV}^2} = (71 \pm 9) \text{ GeV}^2, \quad (13)$$

$$\langle \rho_{A_1} + \rho_{A_{12}} + \rho_V \rangle_{LQCD}^{q^2 > 12 \text{ GeV}^2} = (46 \pm 6) \text{ GeV}^2. \quad (14)$$

### 4.2 $B \to K$

For the $B \to K$ form factors more precise lattice form factors exist and have been extrapolated to the full $q^2$ range using a third-order $z$-expansion. The results are compatible with low-$q^2$ predictions from LCSR. One finds

$$\langle \rho_K \rangle = (50.4 \pm 5.5) \text{ GeV}^2. \quad (15)$$

### 5 Numerics

Using (6), (11), (12), and (15), one obtains

$$\text{BR}(B^+ \to K^+ \nu \bar{\nu})_{SM} = (4.68 \pm 0.64) \times 10^{-6}, \quad (16)$$

$$\text{BR}(B^0 \to K^{*0} \nu \bar{\nu})_{SM} = (9.48 \pm 1.10) \times 10^{-6}. \quad (17)$$

The uncertainties are dominated by the CKM and form factor uncertainties, which are comparable when using (11).

### References


