Hadronic molecules from $X$ to $Z$

Feng-Kun Guo$^{1,*}$

$^1$Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

June 17, 2015

Abstract

This summarizes briefly my talk “Hadronic molecules from $X$ to $Z$” presented at the 2nd B2TiP Workshop held from 27th to 29th April 2015 in Krakow.

Since the discovery of the $X(3872)$ by the Belle Collaboration in 2003, lots of new structures, called $XYZ$ states, were discovered in the mass regime of heavy quarkonia. In particular, most of the new charmonium-like structures do not fit in the expectations from quark models for mesons, such as the famous Godfrey–Isgur quark model [1] which was quite successful in describing heavy quarkonium states below open-flavor thresholds. Therefore, many of these $XYZ$ states were suggested to be candidates of exotic hadrons, a concept based on quark model, which include all configurations beyond the simple quark–antiquark picture for mesons such as glueballs, hybrid states, multiquarks, hadro-quarkonia and hadronic molecules. Among these various configurations, hadronic molecule is special in the sense that it is a state composed of hadrons, analogous to the deuteron as a bound state of the neutron and proton. It has a mass close to the threshold of the constituent hadrons, and thus its size, $\sim R = 1/\sqrt{2\mu E_B}$ with $\mu$ the reduced mass and $E_B = m_1 + m_2 - M$ the binding energy, is large in comparison with the typical size of a hadron.

There are several reasons for studying hadronic molecules:

- Hadronic molecules present one possible realization of color-neutral objects. Being analogues of atomic nuclei, it is natural to expect them, at least in some systems, to exist.
- They can provide important information on hadron-hadron interactions, and hence to low-energy QCD.
- Some of the $XYZ$ states, such as the $X(3872)$, are nice candidates of hadronic molecules.
- Model-independent statements can be made, see the following.

One needs to notice that only those hadrons with a small width can be considered as constituents of hadronic molecules [2, 3]. This can be easily understood: if a hadron has a large width, its life time is too short for it to interact with the other hadron to form a hadronic molecule. Thus, the hadron width should be much smaller than the inverse of the range of forces [3]. Next, I will discuss two methods towards identifying hadronic molecules.

The first method is model-independent for $S$-wave loosely bound states. Because of the small binding energy, such states can be described in nonrelativistic quantum mechanics. For $S$-wave, one can derive a relation between the effective coupling of the physical state to the constituents and the compositeness, $1 - Z$, which measures the probability of finding the physical state in the two-body continuum. The relation reads [4, 5]

$$g^2_{NR} \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}.$$  (1)
Furthermore, the scattering length and effective range can also be approximated in terms of the $Z$ factor [4]

$$a = \frac{2R(1-Z)}{2-Z} \left[ 1 + \mathcal{O}\left( \frac{r}{R} \right) \right], \quad r_e = \frac{RZ}{1-Z} \left[ 1 + \mathcal{O}\left( \frac{r}{R} \right) \right],$$

(2)

where $r$ is the range of forces. This method was used by S. Weinberg to show that the deuteron is a proton-neutron bound state instead of an elementary particle half a century ago [4]. Indeed, one may apply the above relations to the case of the $X(3872)$. There have been lattice calculations for the $X(3872)$. In Ref. [6], the authors found evidence for the $X(3872)$ on a small lattice with $L \approx 2$ fm, and reported values for the scattering length and effective range for the $D\bar{D}^*$ $S$-wave scattering: $a_{D\bar{D}^*} = (-1.7 \pm 0.4)$ fm and $r_{D\bar{D}^*} = (0.5 \pm 0.1)$ fm. These values correspond to a large compositeness $1 - Z \gtrsim 0.7$. I regard this as a lattice evidence for the $X(3872)$ being dominantly a $D\bar{D}^*$ hadronic molecule.

The second method is the use of heavy quark spin symmetry (HQSS) to predict spin partners of hadronic molecules and their properties. For hadronic molecules formed by a heavy quarkonium and a light hadron, the leading order (LO) interaction is independent of the heavy quarkonium spin. Being color singlets, the two hadrons need to exchange at least two gluons which are chromo-electric at LO. Thus, such hadronic molecules are organized into spin multiplets with the same constituents are related to each other. A nice example is provided by the production of heavy quarkonium at LO [7]. A nice example is provided by the core heavy quarkonium being spin partners of each another, and the mass splitting is the same as that for the core heavy quarkonium states at LO [7]. A nice example is provided by the $Y(4660)$ which was proposed to be a $\psi^\prime f_0(980)$ hadronic molecules in Ref. [8] based on an analysis using the nonrelativistic relations discussed above. HQSS implies that it has a spin partner, $\eta_c f_0(980)$ hadronic molecule, with a mass and width of [7]

$$M \approx M_{Y(4660)} - (M_{\psi^\prime} - M_{f_0}) \approx 4616 \text{ MeV}, \quad \Gamma = (60 \pm 30) \text{ MeV},$$

(3)

respectively. Such a state may be searched for in the processes $B^{\pm} \to K^{\pm} \eta_c^* \pi^+ \pi^-$. For hadronic molecules composed of a pair of open-flavor heavy mesons, one can use an effective Lagrangian to study the consequences of HQSS. The LO Lagrangian consists of four contact constant terms [9]. On one hand, one finds that the LO interaction between $D$ and $D^*$ with $J^{PC} = 1^{++}$ is the same as the one for the $D^*\bar{D}$ system with $J^{PC} = 2^{++}$, see, e.g. Ref. [10, 11]. One thus expects the existence of $X_2$ as the spin partner of the $X(3872)$, and $M_{X_2} - M_{X(3872)} \approx M_{D^*} - M_D$. This state is expected to decay dominantly into the open-charm channels $D\bar{D}$ and $D\bar{D}^*$ in a $D$-wave. Assuming the one-pion exchange can be dealt with perturbatively, we estimated the decay width of the $X_2$, and found that it is of the order of a few MeV [12]. On the other hand, the interaction between the $D\bar{D}$ pair depends on a different linear combination of the contact terms. This means that one cannot derive a $D\bar{D}$ hadronic molecule solely based on the $X(3872)$. Yet, if such a state exists close to the $D\bar{D}$ threshold, it would have a large impact on the partial decay width of $X(3872) \to D^0 \bar{D}^0 \pi^0$ [13] which is supposed to be sensitive to the long-distance structure of the $X(3872)$ (see, e.g. [14, 15]).

It is important to notice that the productions and decays of different hadronic molecules with the same constituents are related to each other. A nice example is provided by the production of the $X(3872)$ and $Z_c(3900)$ in decays of the $Y(4260)$, shown in Fig. 1. Here we assume the dominant components for the $X(3872)$, $Y(4260)$, and $Z_c(3900)$ are $D\bar{D}^*(1^{++})$ [16], $\bar{D}D_1(1^{--})$ [17] and $D\bar{D}^*(1^{+-})$ [17, 11], respectively. It was predicted in Ref. [18] that the $X(3872)$ can be easily produced in the radiative decays of the $Y(4260)$ because of an enhancement of the triangle loop

![Figure 1: Decays of the Y(4260) into γX(3872) and πZ(3900) in the hadronic molecule picture.](image-url)
diagram following a nonrelativistic velocity power counting. The prediction was later on confirmed by the observation of such a process by the BESIII Collaboration [19]. In the same way, one can expect that the $Z_c(3900)$ can be easily produced in the pionic decays of the $Y(4260)$ [17].

It is also worth mentioning that both the $X(3872)$ and $X_2$ are expected to have bottom analogues. Assuming the LO interaction between a pair of bottom mesons is the same as that for the charm ones, the mass of the $X_b$ with $1^{++}$ was predicted to be around 10.58 GeV [11], and the mass of the $X_{b2}$ with $2^{++}$ is higher by $M_{B^*} - M_B$ approximately. Both of them are isoscalar states. Because the isospin splitting between the charged and neutral bottom mesons is one order of magnitude smaller than the one for the charm mesons, and the mass difference between $X_b$ and $\Upsilon(1S)$ is much larger than the $\rho$ and $\omega$ masses, the isospin violated decay $X_b \rightarrow \Upsilon(1S)\pi\pi$ should be highly suppressed with a branching fraction $\lesssim 10^{-2}$. Thus, the $X_b$ state should be searched for in the channels $\Upsilon(1S)\pi^+\pi^-\pi^0$, instead of $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(nS)\gamma$ [11, 20]. In the hadronic molecule picture, it is also natural to explain the $Z_b(10610, 10650)$, as $B\bar{B}$ and $B^*\bar{B}^*$ hadronic molecules [21] and $Z_c(3900, 4020)$ are flavor partners [11]. Some relations for the decays of the $Z_b$ states are discussed in Ref. [22]. There could be more isovector hadronic molecules in the bottomonium sector, see discussions in Refs. [23, 24].

Belle-II can do a lot for the study of hadronic molecules. One obvious task is to search for the spin and flavor partners of the discovered hadronic molecule candidates, such as the spin partner of the $Y(4660)$ with $0^{-+}$ and a mass around 4.62 GeV in $\eta_c\pi^+\pi^-$, the $X_2$ around the $D^*\bar{D}^*$ threshold with $2^{++}$ decaying into $D\bar{D}$ and $D\bar{D}^*$, the $X_b$ and $X_{b2}$ with $1^{++}$ and $2^{++}$, respectively, in $\Upsilon\pi\pi$, $\chi_{bJ}\pi\pi$ and $\Upsilon\gamma$. At last, I want to mention that not all processes are sensitive to the hadronic molecule structure, which is long-distance because of the small binding energy. For instance, the radiative decays of the $X(3872)$ into $J/\psi(\psi')$ depend on the short-distance physics and are insensitive to the hadronic molecule structure of the $X(3872)$ [25, 26]. Thus, it is important to make precise measurements on the open-flavor decays of the $X(3872)$, $Z_c$ and $Z_b$ states, which are sensitive to the long-distance structure, in order to make decisive statements on their dominant components. More suggestions for hadronic physics at Belle-II can be found in Ref. [27].

References


