Recent results and discussions on the charmoniumlike states

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In this proceedings, we review and discuss the recent studies on the charmoniumlike states from experiments, mainly the e^+e^- annihilation experiments BESIII and Belle, and lattice QCD studies.

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1 Introduction

Since its discovery, charmonium has been an excellent tool for probing QCD in the non-perturbative regime, and charmonium states remain of high interest both experimentally and theoretically. All of the charmonium states with masses that are below the open-charm have been firmly established [1]. However, the status of the states that are above the open charm threshold remains unsettled. During the past decade, a number of new charmoniumlike states were discovered, such as the X(3872) [2], the $Y(4260)$ [3, 4] and the Zc(3900) [4, 5]. These states provide a strong evidence for the existence of exotic QCD states. Exotic hadrons include glueballs, hybrids, multi-quark states and hadron molecules. Although many candidates were proposed, no solid conclusion was reached. Thus, a more complete understanding of the charmonium(like) spectroscopy is necessary and urgent.

So far lattice QCD found evidence for most of the observed flavour non-exotic states: those include for example all charmonia below open charm threshold, shallow bound states $X(3872)$ with $I = 0$, $D_{s0}^*(2317)$, $D_{s1}(2460)$, and meson resonances ρ, a_1, b_1, $K^*(892)$, $K_0^*(1430)$, $D_0^*(2400)$, $D_1(2430)$. All these manifest themselves via an additional energy level in the discrete spectrum, as discussed bellow.

On the other hand, lattice QCD has not found yet reliable evidence for the mesons with manifestly exotic flavour, for example $X(3872)$ with $I = 1$, $Z_c^+ \simeq \bar{c}c\bar{d}u$ or $cc\bar{d}\bar{u}$. This may indicate that the observed Z_c^+ structures may have different origin than flavour non-exotic states. Given the absence of the exotics at current lattice QCD simulations it is difficult to predict which channels and energies could be relevant for future discoveries of new states at BelleII. This may change if the near-future lattice simulations find out the reasons for structures like Z_c^+ observed in experiment.

In this BelleII Theory interface Platform (B2TiP) workshop, we reviewed some recent results on the charmoniumlike spectroscopy from experimental and lattice QCD results, showed some puzzles and discussed possible developments in the future.

2 Recent experimental results on Charmoniumlike states

After the initial observation of the $Y(4260)$ by BaBar Collaboration, an ISR analysis by the Belle experiment with 548 fb^{-1} of data showed a significant $Y(4260)$ signal as well as an excess of $\pi^+\pi^- J/\psi$ event production near 4 GeV that could be described by a broad Breit-Wigner (BW) parameterization — the so-called $Y(4008)$ [6]. Recently, the BaBar Collaboration reported an updated ISR analysis with 454 fb^{-1} of data and a modified approach for the background description [7]; the $Y(4260)$ state was observed with improved significance, but the $Y(4008)$ structure was not confirmed, as shown in Fig. 1(a). Instead, they attributed the structure below the $Y(4260)$ to exponentially falling non-resonant $\pi^+\pi^-J/\psi$ production. Belle again updated the process $e^+e^- \to \pi^+\pi^-J/\psi$ to check structures in the $\pi^+\pi^-J/\psi$, $\pi^\pm J/\psi$, and $\pi^+\pi^-$ systems with an integrated luminosity of 967 fb^{-1} [4]. The obtained $\pi^+\pi^-J/\psi$ mass distribution was shown in Fig. 1(b). Belle confirmed their previous published results with the $Y(4008)$ and $Y(4260)$ signals observed clearly. So the measured cross sections for $e^+e^- \to \pi^+\pi^-J/\psi$ between 3.8 and 4.1 GeV are obvious different from updated Belle and BaBar measurements. A question on the existence of the $Y(4008)$ needs to be answered. Or are there many more structures in this mass range ?

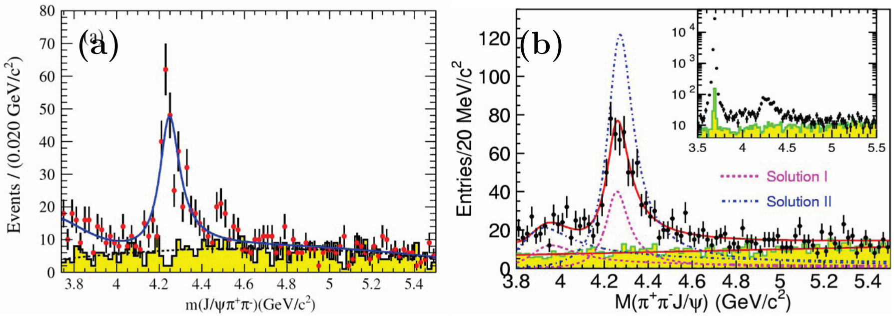

Figure 1: Invariant mass distributions of $\pi^+\pi^-J/\psi$ via ISR process $e^+e^- \to \pi^+\pi^-J/\psi$ from (a) BaBar [3] and (b) Belle [4]. Points with error bars are data, and the shaded histograms are the normalized J/ψ mass sidebands. The solid curves show the total best fit.

For events in the $Y(4260)$ signal region, Belle observed structures in the $\pi^+\pi^-$ and π^+J/ψ systems. Figure 2(a) shows the $M_{\max}(\pi^\pm J/\psi)$ distribution from Belle measurement [4]. At the same time, BESIII also analyzed the process $e^+e^- \to \pi^+\pi^-J/\psi$ at a center-of-mass energy of 4.260 GeV using a 525 pb^{-1} data sample. Figure 2(b) shows the $M_{\max}(\pi^\pm J/\psi)$ distribution from BESIII measurement [5]. Unbinned maximum likelihood fits are applied to the distributions of $M_{\max}(\pi^\pm J/\psi)$. Figure 2 shows the fit results. The measured masses are $(3899.0 \pm 3.6 \pm 4.9)$ MeV/c^2 and $(3894.5 \pm 6.6 \pm 4.5)$ MeV/c^2 and the measured widths are $(46 \pm 10 \pm 20)$ MeV/c^2 and $(63 \pm 24 \pm 26)$ MeV/c^2 from Belle and BESIII experiments, respectively. They are consistent with each other within the errors. The signal significance is greater than 5σ in both of the measurements. This state denoted as $Z_c(3900)$ is close to the $D\overline{D}^*$ mass threshold. As the $Z_c(3900)$ state has a strong coupling to charmonium and is charged, it cannot be a conventional $c\bar{c}$ state. There have been a number of different interpretations, including tetraquark state, hadronic molecule, hadron-charmonium state and so on. Since the observation of the $Z_c(3900)$, a series of charged Z_c states were found including $Z_c(3885)^\pm$ in $(D\overline{D}^*)^\pm$ [8], $Z_c(4020)^\pm$ in $\pi^\pm h_c$ [9], $Z_c(4025)^\pm$ in $(D^*\overline{D}^*)^\pm$ [10], $Z_c(4200)^\pm$ in $\pi^\pm J/\psi$ from $\overline{B}^0 \to J/\psi K^-\pi^+$ [11], $Z_c(4050)$ in $\pi^\pm\psi(2S)$ [12], etc. Belle also tried to search for a strange partner of the $Z_c(3900)^\pm$, called Z_{cs}, in $K^\pm J/\psi$

system in the process $e^+e^- \to K^+K^-J/\psi$ [13]. No obvious structures are observed in the $K^\pm J/\psi$ system. The above observations indicate one kind of the exotic states has been observed. The nature of these states have been discussed in many proposals, but no solid conclusion can be drawn. Searches for new decay modes and measuring their quantum numbers may provide information that is useful for understanding the nature of them. There are also many open questions. For example, may we observe more excited Z_c states ? Do the strange partners of the Z_c states exist ?

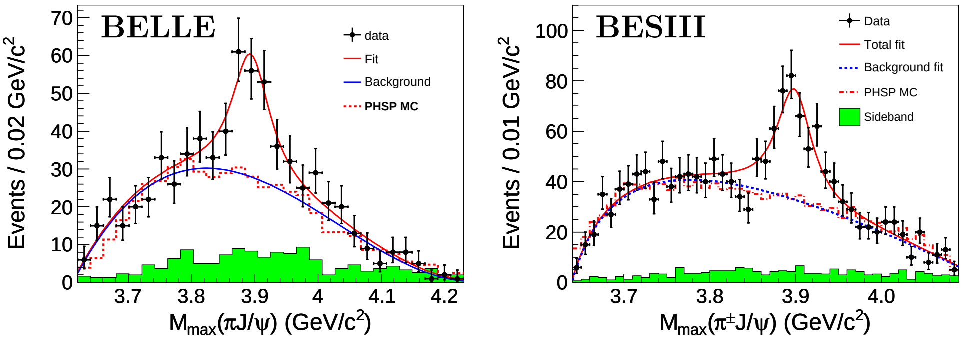

Figure 2: Fits to the distributions of the $M_{\max}(\pi J/\psi)$ from Belle [4] and BESIII [5] experimental data. The solid curves are the best fits, the dashed histograms represent the results of phase space distribution and the shaded histograms are J/ψ sidebands.

Recently BESIII accumulate large data samples between 4.0 and 4.6 GeV for the study of the charmonium and charmoniumlike states. Some results to the cross section measurements for some processes are very interesting. For examples: (1). BESIII measured $e^+e^- \to \pi^+\pi^-h_c$ cross sections [9] at center-of-mass energies between 3.90 and 4.42 GeV as shown in Fig. 3(a) together with the comparisons of the $\pi^+\pi^-h_c$ cross sections from CLEO-c [14] and $\pi^+\pi^-J/\psi$ cross sections from Belle [4]. From this plot, we found the cross sections of $\pi^+\pi^-h_c$ are of the same order of magnitude as those of the $e^+e^- \to \pi^+\pi^-J/\psi$, but with a different line shape. There is a broad structure at high energy with a possible local maximum at around 4.23 GeV. (2). The first observed XYZ state, $X(3872)$, was observed by BESIII in $e^+e^- \to \gamma X(3872)$ [15]. Figure 3(b) shows the energy-dependent cross section together with the fitted results with a $Y(4260)$ resonance, a linear continuum, or a $E1$-transition phase space term. The $Y(4260)$ resonance describes the data better than the other two options, which strongly supports the existence of the radiative transition process $Y(4260) \to \gamma X(3872)$. (3). Based on data samples collected at 9 center-of-mass energies from 4.21 to 4.42 GeV, BESIII searched for the production of $e^+e^- \to \omega\chi_{c0}$ [16], where ω is reconstructed with $\pi^+\pi^-\pi^0$ decay mode, χ_{c0} is reconstructed with $\pi^+\pi^-$ and K^+K^- decay modes. Figure 3(c) shows the Born cross sections of $e^+e^- \to \omega\chi_{c0}$ in the full energy region. The cross sections were fitted by using a $Y(4260)$ resonance and the fit gives $\chi^2/ndf = 40.2/8$, so the $Y(4260)$ resonance can not describe the data well. It indicates that the $\omega\chi_{c0}$ signals are very unlikely from the $Y(4260)$ decays. By assuming the $\omega\chi_{c0}$ signals come from a resonance, the fitting results are $\Gamma_{ee}\mathcal{B}(\omega\chi_{c0}) = (2.7 \pm 0.7)$ eV, $M = (4229 \pm 11)$ MeV/c^2, and $\Gamma_t = (40 \pm 14)$ MeV/c^2, which are consistent with the $Y(4220)$ state observed in the cross section of $e^+e^- \to \pi^+\pi^- h_c$ [17]. The data may suggest that the $Y(4260)$ signals observed in $e^+e^- \to \pi^+\pi^- J/\psi$ [4] have fine structures as observed in $e^+e^- \to \pi^+\pi^- h_c$ [9], and the lower mass structure at about 4230 MeV/c^2 has a sizable coupling to the $\omega\chi_{c0}$ channel as predicted in Ref. [18]. (4). BESIII searches for production of the ψ_2 state, called $X(3823)$, via the process $e^+e^- \to \pi^+\pi^-\psi_2$. The ψ_2 candidates are reconstructed in their $\gamma\chi_{c1}$ and $\gamma\chi_{c2}$ decay modes, with $\chi_{c1,c2} \to \gamma J/\psi$. The measured energy-dependent cross sections of $e^+e^- \to \pi^+\pi^- X(3823)$ are shown in Fig. 3(c), which are fitted with a $Y(4360)$ shape or a $\psi(4415)$ shape (with their resonant parameters fixed to the PDG values [1]). Both fits can describe the data well due to the large statistical errors.

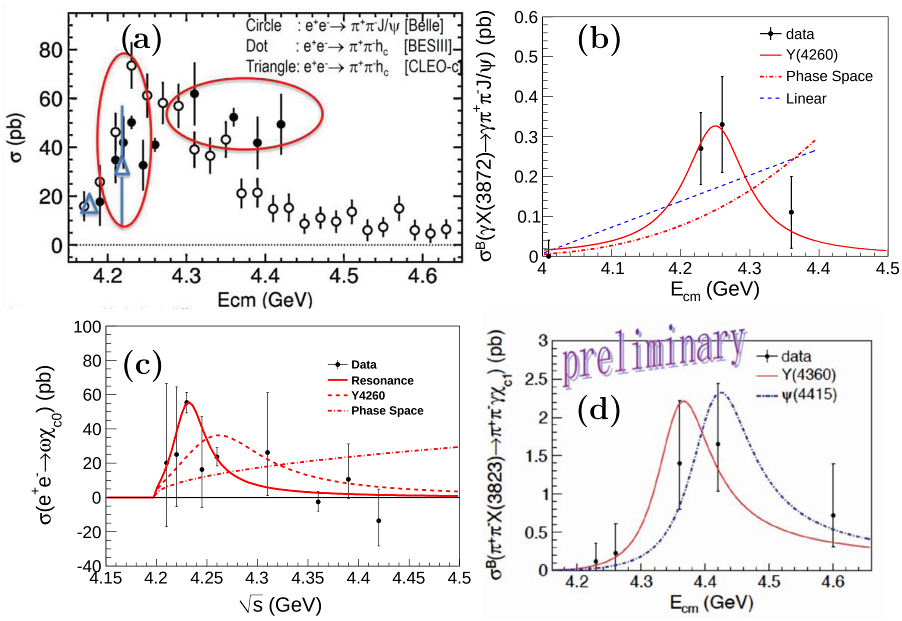

Figure 3: (a) The distributions of $\sigma(e^+e^- \to \pi^+\pi^- J/\psi)$ from Belle [4], $\sigma(e^+e^- \to \pi^+\pi^- h_c)$ from BESIII [9] and CLEO-c [14] experiments. Fits to (b) $\sigma[e^+e^- \to \gamma X(3872)] \times \mathcal{B}[X(3872) \to \pi^+\pi^- J/\psi]$ [15], (c) $\sigma(e^+e^- \to \omega\chi_{c0})$ with a resonance (solid curve), the $Y(4260)$ (dashed curve), or a phase space term (dot-dashed curve) [16] and (d) $\sigma(e^+e^- \to \pi^+\pi^- X(3823))$ with a $Y(4360)$ or a $\psi(4415)$ signal (preliminary results) from BESIII measurements.

There have been lots of charmoniumlike states observed in charmonium mass region but many of them show properties different from the naive expectation of conventional charmonium states. Even so, recently Belle and especially BESIII are producing more results on these XYZ states. With more discoveries, we found we have more questions to answer. For examples:

- The $X(3872)$ and $X(3823)$ signals were observed by BESIII via processes $e^+e^- \to \gamma X(3872)$ and $\pi^+\pi^- X(3823)$. Where are the signals from, resonances decays or continuum productions ? May other similar X states could be observed in similar processes ?

- Although the $Y(4260)$ was established by Belle and BaBar experiments, there is a possible local maximum at around 4.23 GeV in $e^+e^- \to \pi^+\pi^- J/\psi$ cross sections. Is the Y(4260) a single resonance? The lower broad bump $Y(4008)$ discovered by Belle was not confirmed by the updated BaBar results. Is the $Y(4008)$ a real structure?

- In the updated Belle $e^+e^- \to \pi^+\pi^-\psi(2S)$ analysis, it seems there are a few events accumulating at around 4.26 GeV. With the $Y(4260)$ signal included, the signal significance is less than 3σ under current statistic. Does the $Y(4260)$ decay to $\pi^+\pi^-\psi(2S)$? Do the $Y(4360)$ and $Y(4660)$ decay only to $\pi^+\pi^-\psi(2S)$ mode ? Are there any other decay modes, like $\eta J/\psi$?

- The cross sections of $\pi^+\pi^- h_c$ are of the same order of magnitude as those of the $e^+e^- \to \pi^+\pi^- J/\psi$, but with a different line shape. There is a broad structure at high energy with a possible local maximum at around 4.23 GeV. How many structures are there ?

- Many charged Z_c states including neutral $Z_c(3900)$ and $Z_c(4020)$ decaying to the charmonium plus light hadrons are observed. At the same time, in the charm meson pairs invariant mass distributions, like $(D\overline{D}^*)^\pm$, $(D^*\overline{D}^*)^\pm$, some similar Z_c states are observed. Some of them are the same states ? What are the correlations between the charm productions and charmonium transitions?

- The latest discovered Z_c state is $Z_c(4050)^\pm$ decaying to $\pi^\pm\psi(2S)$. However, the signal significance is less than 5σ. It is only a evidence. Is it a real resonance ? Is it a excited state of $Z_c(3900)$?

- As we mentioned above, BESIII recently measured cross sections for some processes including $\omega\chi_{cJ}$, $\pi^+\pi^- X(3823)$, $\pi^+\pi^- X(3872)$, $\eta J/\psi$, and so on. BESIII also is doing more channels, like $\eta' J/\psi$, $\eta\psi(2S)$, ηh_c, $\phi\chi_{cJ}$, etc. May we observe some new resonances ? What are the real line shapes for all these channels ? Some higher missing charmonium states, for example $\psi(4S)$, η_{c2}, can be observed in some of these decays ?

- The Z_{cs} state, a strange partner of the $Z_c(3900)^{\pm}$, was predicted in some theoretical models. With full of the Belle data sample, no any structure could be observed in $K^{\pm}J/\psi$ system. Does the Z_{cs} state exist ? Are there any excited Z_{cs} states ?

- Many Z_c states were observed decaying to charmonium or charm meson pairs. Do they have decay modes to light hadrons ? Are the Z_c states from resonance decays or continuum productions ?

- Why some Z_c states are observed in resonance decays or continuum productions, while some Z_c states are observed in B decays only ? What are the nature of them ? How to separate the tetraquark state or molecule state ?

For most of the above questions, Belle has no ability to give satisfied answeres due to the limited statistic. Figure 4 gives the ISR effective luminosity distributions in the mass range from 3 to 5 GeV/c^2 in 10 MeV/c^2 each bin under the conditions of total integrated luminosity of 1 ab^{-1}, 10 ab^{-1} and 50 ab^{-1}, which was calculated using the probability density function for ISR photon emission [19]. From it, we see with the current total luminosity of 1 ab^{-1} at Belle the effective luminosity is only at a few tens of pb^{-1}. For some channels, like $\pi^+\pi^-h_c$, $\pi^+\pi^-h_c(2P)$, $\omega\chi_{cJ}$, can not be done at the moment. But at BelleII, when we have 10 ab^{-1} luminosity, we can compete with the current BESIII. And the advantage is we can observe the whole line shape, while BESIII can only measure the cross sections at some fixed energy points. When we have 50 ab^{-1} data, all of the above discussed channels can be done. The physics behind them should be clear at that time. For some channels, we can even do partial wave analysis or amplitude analysis

3 Lattice QCD studies of quarkoniumlike states

3.1 Search for Z_c^+

All lattice searches for Z_c^+ considered $I^G(J^{PC}) = 1^+(1^{+-})$ channel, which is the most relevant experimentally. Simulation have not found reliable Z_c^+ candidates yet. This does not necessarily imply that Z_c^+ do not exist, but it may point to a different nature than flavour non-exotic hadrons.

There is one exception - the preliminary results of the HALQCD method may perhaps hint to an increase of the cross-section which resembles the experimental peak related to $Z_c^+(3900)$. These preliminary results were first presented at the KEK Nov2014 meeting.

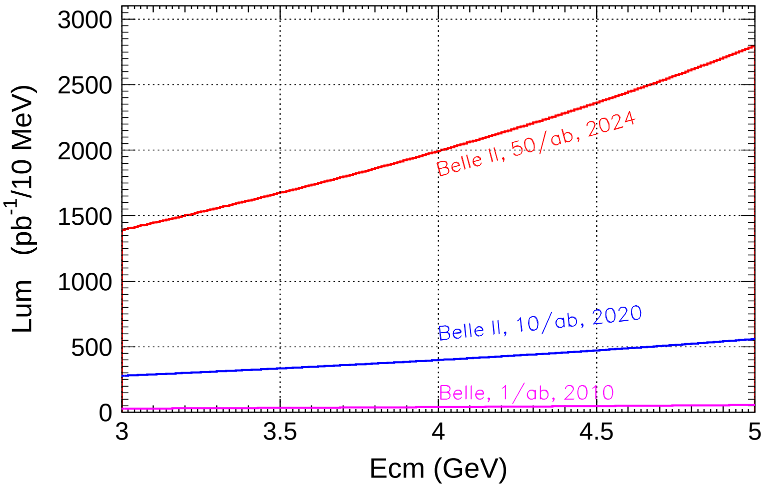

Figure 4: Effective luminosity distributions in each 10 MeV bin under the conditions of total integrated luminosity of 1 ab^{-1}, 10 ab^{-1} and 50 ab^{-1}.

3.1.1 Three-coupled channels with HALQCD method

The coupled channels $D\overline{D}^*$, $J/\psi\pi$ and $\eta_c\rho$ are simulated in lattice QCD using HALQCD method. The potential $V(r)$ between the two mesons (for example $D\overline{D}^*$) and the potential between pairs of channels is determined from the two-hadron wave functions on the lattice. The potential renders the scattering matrix for three-coupled channels and sizable coupling between channels $D\overline{D}^*$ and $J/\psi\pi$ is observed. This seems to be responsible for a peak in the $J/\psi\pi$ cross-section above $D\overline{D}^*$ threshold that resembles $Z_c^+(3900)$. The HALQCD collaboration is currently looking for poles in the complex plane of the resulting scattering matrix. The results of this study are not yet available in a form of a paper or a preprint, and were first reported by **T. Hatsuda** at the KEK Nov2014 meeting.

3.1.2 $D\overline{D}^*$ scattering phase shifts with Lüscher method

The $D\overline{D}^*$ scattering was simulated omitting the $J/\psi\pi$ channel and other channels by CLQCD [20]. The s-wave and p-wave phase shift near $D\overline{D}^*$ threshold where determined using Lüscher-type equation. The authors conclude that no evidence for $Z_c^+(3900)$ is found. I would like to caution that $D\overline{D}^*$ correlators relax to the true ground state $e^{-(m_{J/\psi}+m_\pi)t}$ at large t in [21], so the energies in [20] may not be reliable since only $D\overline{D}^*$ interpolating fields are employed.

3.1.3 Searching for an additional energy eigenstate

A resonance or a bound state in a two-hadron channel H_1H_2 manifests itself as an additional energy eigenstate in the spectrum - i.e. a state in addition to the "two-hadron" discrete energy eigenstates discussed in the next paragraph. Most of the flavour non-exotic hadrons manifest themselves as an additional energy level in the spectrum: the examples in one-channel case are the resonances ρ [22], $K^*(892)$ [23], $D_0^*(2400)$ [24], and the bound state $D_{s0}^*(2317)$ [25]. Additional levels related to $K_0^*(1430)$ [26] and $X(3872)$ [27] have been found in the simulations of two coupled channels. The evidence for flavour non-exotic hadrons was reported by **S. Prelovsek** at KEK Nov2014 meeting.

Let me briefly describe the notion of "two-hadron" states which make QCD studies above thresholds challenging. When only one two-hadron channel H_1H_2 is open, the spectrum consists of two-hadron energy eigenstates $H_1(\vec{p})H_2(-\vec{p})$ with $|\vec{p}| \simeq 2\pi|\vec{n}|/L$ as a consequence of periodic boundary condition on a finite box of extent L. In experiment there two hadron states correspond to a two-hadron decay products with continuous energy spectrum. A relatively narrow resonance or a bound state renders an extra energy level in addition to tower of $H_1(\vec{p})H_2(-\vec{p})$.

The challenge with Z_c channel $I^G(J^{PC}) = 1^+(1^{+-})$ is that three channels $D\overline{D}^*$, $J/\psi\pi$ and $\eta_c\rho$ are coupled in the vicinity of $Z_c(3900)$ and further channels $\psi_{2S}\pi$, $\psi_{1D}\pi$, $D^*\overline{D}^*$, $h_c\pi$ open by $E \simeq 4.2$ GeV. At present it is impossible to extract scattering matrix of all these open channels on the lattice using rigorous Lüsher's approach. A more simplistic approach is to search for an extra eigenstate in the spectrum, i.e. a state in addition to expected possible two-hadron states.

The first lattice search considered $J/\psi\,\pi$ and $D\,\overline{D}^*$ scattering. Only two-meson states $J/\psi\,\pi$ and $D\,\overline{D}^*$ were found, but no additional candidate for $Z_c^+(3900)$ [21].

The most extensive lattice search for Z_c^+ with mass below 4.2 GeV was performed in [28]. Thirteen two-meson states are expected on the lattice with $L \simeq 2$ fm and $m_\pi \simeq 266$ MeV in the energy region below 4.3 GeV. The lattice spectrum of eigenstates is extracted using a number of meson-meson and $[\bar{c}\bar{d}]_{3_c}[cu]_{\bar{3}_c}$ interpolating fields. No additional state that could represent Z_c^+ is found below 4.2 GeV [28]. This was reported by **S. Prelovsek** at the KEK Nov2014 meeting.

The lattice simulation of the same channel based on the HISQ action also considered a number of coupled two-meson channels [29]. Two-meson states are seen and no additional state that could represent Z_c^+ is found. This was reported by **C. DeTar** at the KEK Nov2014 meeting.

3.2 Search for $Y(4140)$ in $J/\pi\phi$ scattering

The s-wave and p-wave $J/\psi\ \phi$ scattering phase shift was extracted in a $N_f = 2+1$ simulation [30]. The $\bar{s}s$ annihilation contribution was omitted. The phase shifts do not support a resonant structure at the position where $Y(4140)$ was found experimentally. The results were reported by **S. Ozaki**.

3.3 Evidence for $X(3872)$ from lattice QCD

The evidence for the neutral $X(3872)$ with $J^{PC} = 1^{++}$ and $I = 0$ was found in two simulations [27, 29], both performed in the isospin limit $m_u = m_d$. One of the signatures for $X(3872)$ is an extra energy eigenstate in the spectrum. The other signature is the presence of the pole in the scattering matrix 11 ± 7 MeV below the $D\overline{D}^*$, which indicated it is a shallow bound state. The established $X(3872)$ has a large overlap with $\bar{c}c$ as well as $D\overline{D}^*$ interpolating fields; it is absent if one or the other type of interpolators are omitted. This was reported by **S. Prelovsek** and **C. DeTar** at the KEK Nov2014 meeting.

The charged partner with $I = 1$ was not found in these simulations, which agrees with the absence of the experimental candidate for charged X. The $I = 1$ component of the neutral state arises in various scenarios via isospin breaking in composition or decay, but this would be absent in simulations with exact isospin.

One improvement which can be expected by the time of BelleII is a simulation of this system with isospin breaking, which is important in this channel.

3.4 Charmonia in the single-hadron approximation

The most extensive spectrum of the excited charmonia for a number of J^{PC} was extracted by the Hadron Spectrum Collaboration at $M_\pi \simeq 400$ MeV some time ago [31]. Full multiplets related to hybrids were also found, which span over conventional and exotic J^{PC}. This spectrum, presented by **Liuming Liu**, represents a valuable reference point for the charmonium(like) states.

One has to be aware of the serious assumptions that go into this impressive spectrum, which is obtained using only $\bar{c}c$ interpolating fields. It neglects strong decays of resonances and effect of threshold for near-threshold states. Such an approach can not reliably provide evidence for possible exotic states near thresholds, since it is assumed that none of the lattice levels is a two-meson state.

3.5 Bottomonia and their radiative transitions

An extensive spectrum of $\bar{b}b$ with various J^{PC} below $B\overline{B}$ threshold was extracted using NRQCD in [32]. The results for the leptonic decay widths of Υ and Υ' were compared to HPQCD and experiment. The radiative $M1$ transitions between bottomonia $\Upsilon(nP) \to \eta_b(n'S)$ and $\eta_b(nS) \to \Upsilon(n'P)$ were calculated in [33, 34], and compared to experiment where available. The results were reported by **R. Lewis**.

3.6 Lattice QCD prospects by the time of BelleII

Lattice simulations of charmonia (bottomonia) below $D\overline{D}$ ($B\overline{B}$) threshold are precise and under control. The masses have been extracted from a number of simulations and necessary extrapolations/interpolations in a, L and m_q have been done in many cases. The decay constants and radiative transition rates between some of these states have also been determined. The radiative transitions become technically more involved for higher excitations but it is in principle known how to attack the problem for states below $D\overline{D}$ ($B\overline{B}$) threshold and more results along these lines (with various degree of accuracy) can be expected.

There is no uncontrolled approximations involved for states below $D\overline{D}$ ($B\overline{B}$) threshold except for the neglect of charm (bottom) annihilation Wick contractions. This is omitted also in all simulations of exotic channels with closed charm or bottom flavour. This approximation represents the main uncertainty for states below thresholds at the moment. Going beyond this approximation represents a major challenge, since these contractions invoke strong decays of quarkonia to a number of light hadrons.

The states that reside in a single scattering channel can be treated rigorously by simulating the scattering in this channel. Same applies for states that are situated slightly below the corresponding threshold. Reasonable results can be obtained also for states with two channels where one channel is approximately decoupled. This, for example, applies for $\psi(3770)$, $\chi_{c0}(2P)$, $X(3872)$ in $D\overline{D}^{(*)}$ scattering. Many results with approved accuracy along these lines are expected by the time of BelleII.

The states that can decay hadronicaly into two different two-meson final states are also manageable. The proof of principle has been shown to work by HSC in the first simulation of this kind, that considered $K\pi$, $K\eta$ coupled system [26, 35]. This is a very challenging problem, but pioneering results for other channels relevant for charmonium(like) spectroscopy at BelleII can be expected by the time it starts operating. Reasonable results can be obtained also for states with three channels where one channel can be treated as approximately decoupled from the system.

BelleII will aim also the quarkonium and quarkonium-like states that are (unfortunately) located above multiple thresholds. Every additional two-hadron state with given J^{PC} that appears below the state of interest represents significant increase of effort and challenge for lattice QCD.

Therefore lattice simulations of states situated above three or more thresholds will necessarily involve simplifying working assumptions, even by the time when BelleII starts operating. An example is the search for Z_c^+ candidates above 4 GeV, which lie above more than 3 thresholds. The current working assumption in the search for these states was, for example, looking for an additional energy eigenstates. A significant amount of analytical work and numerical studies will be needed to understand to what extent these assumptions could be reliable, and how to overcome them.

4 Summary and outlook

In the near future, BESIII experiment will accumulate more data between 4.0 and 4.6 GeV for further study; the BelleII experiment under construction, with about 50 ab^{-1} data accumulated, will surely improve our understanding of all quarkonium(like) states.

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