Lepton flavor universality at Belle and Belle II

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on behalf of the Belle & Belle II collaborations

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Belle experiment is located at the KEKB accelerator in Tsukuba, Japan.

Asymmetric $e^+e^-$ collider with center-of-mass (CM) energy at $B\bar{B}$ threshold, 10.58 GeV.

Data taking from 1999 to 2010.

Data collected: $711 \text{ fb}^{-1} = 772 \text{ million } B\bar{B}$ pairs.

$$e^+e^- \rightarrow \gamma(4S) \rightarrow B\bar{B}$$
SuperKEKB Accelerator

- Located at Tsukuba, Japan.
- Asymmetric $e^+ (4 \text{ GeV}) - e^- (7 \text{ GeV})$ collider with CM energy at $B\bar{B}$ threshold, 10.58 GeV.
- Aims to collect 50 ab$^{-1}$ ($50 \times$ Belle) of data sample.
- Plan to deliver collision at a peak luminosity of $6.5 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ (30 times that of KEKB) by:
  - reducing beam size by 20 times.
  - increasing beam current by 1.5 times.

<table>
<thead>
<tr>
<th></th>
<th>KEKB Achieved</th>
<th>SuperKEKB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (GeV)</td>
<td>LER/HER</td>
<td>LER/HER</td>
</tr>
<tr>
<td>$\beta^*$ (mm)</td>
<td>5.9/5.9</td>
<td>0.27/0.3</td>
</tr>
<tr>
<td>$\epsilon_\gamma$ (nm)</td>
<td>18/24</td>
<td>3.2/2.4</td>
</tr>
<tr>
<td>$\epsilon_e$ (nm)</td>
<td>0.13/0.09</td>
<td>0.09/0.09</td>
</tr>
<tr>
<td>$\varphi$ (mrad)</td>
<td>11</td>
<td>41.5</td>
</tr>
<tr>
<td>$I_{\text{beam}}$ (A)</td>
<td>1.6/1.2</td>
<td>2.8/2.0</td>
</tr>
<tr>
<td>Luminosity (cm$^2$s$^{-1}$) $\times 10^{34}$</td>
<td>2.11</td>
<td>65</td>
</tr>
</tbody>
</table>

**Factor ~30 in the luminosity**
Belle II detector

- Designed to operate with a performance similar or better than Belle.
- New detector (only the structure, the superconducting magnets, the crystals of the calorimeter, and KLM RPCs are re-utilized).
- Expect increased beam background ($\times 10 - 20$) at design luminosity
  - Upgraded trigger system and sub-detectors.

- MC simulation improved vertex resolution ($2 \times$ Belle) and $K_S^0$ reconstruction efficiency
- enhanced $K/\pi$ separation
- new trigger lines for dark sector searches
- more efficient reconstruction and analysis tools
SuperKEKB: new intensity frontier machine

- Recorded integrated luminosity: 424 fb$^{-1}$, data taken between 2019–2022
  - $\sim 362$ fb$^{-1}$ taken at a CM energy of 10.58 GeV, corresponding to the mass of the $\Upsilon(4S)$
  - $\sim 42$ fb$^{-1}$ taken 60 MeV below the $\Upsilon(4S)$ peak, continuum background
  - $\sim 19$ fb$^{-1}$ taken around 10.75 GeV for exotic hadron searches
- World record instantaneous luminosity of $L = 4.71 \times 10^{34}$ cm$^{-2}$s$^{-1}$ ($> 2 \times$ KEKB record) on 22 June, 2022: Entering the regime of a “Super $B$ factory”.

- Shutdown (LS1) from summer 2022 for 15 months: improvement of machine and detector (beam pipe, pixel vertex detector, Time-Of-Propagation PMT)
Belle

- LFU in $B^0 \rightarrow D^{* -} \ell^+ \nu_\ell$
- $\mathcal{R}(D)$ & $\mathcal{R}(D^*)$
- $R_K$ & $R_{K^*}$
- LFU in $\Omega^0_c \rightarrow \Omega^- \ell^+ \nu_\ell$

Belle II

- Inclusive measurement of $R(Xe/\mu)$
- Measurement of $\mathcal{B}(B \rightarrow K^* \ell\ell)$
- $R_K(J/\psi)$

Results shown are with 711 fb$^{-1}$ (920 fb$^{-1}$ for $\Omega^0_c \rightarrow \Omega^- \ell^+ \nu_\ell$) and 190 fb$^{-1}$ data sample of Belle and Belle II, respectively.
LFU in $b \to c\tau\nu$

- $B$ decays with $b \to c$ tree-level transition (thus, assumed to be unaffected by NP) are an important probe to test LFU

- Ratio of exclusive decays with $\tau$ lepton can be used to test SM

$$R_{D^(*)} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

- Experimentally clean final state due to the entire decay chain being reconstructed


- Tension has decreased to 3.1$\sigma$ from 3.8$\sigma$ after recent measurement from Belle [PRL 124 161803 (2020)]
Measurement of LFU using exclusive semileptonic $B$ decay, $B^0 \rightarrow D^*^- (\bar{D}^0 (K^- \pi^+) \pi^-) \ell \nu_{\ell}$

Untagged approach (high efficiency but sizable background)

$D^*$ momentum in the CM frame ($p^* (D^*) < 2.45$ GeV/$c$ to suppress fake $D^*$

Signal yield is extracted with a 3-dimensional binned ML fit to

\[ \cos \theta_{B,D^* \ell} \left( \frac{2E_{B}E_{D^* \ell} - m_B^2 - m_{D^* \ell}^2}{2|p_B||p_{D^* \ell}|} \right), \]

$\Delta M = M(D^* - D^0)$, and lepton momentum ($p_{\ell}$)

Ratio of the branching fractions of modes with electrons and muons provides a test of LFU

\[ \frac{\mathcal{B}(B^0 \rightarrow D^*^- e^+ \nu_e)}{\mathcal{B}(B^0 \rightarrow D^*^- \mu^+ \nu_{\mu})} = 1.01 \pm 0.01 \pm 0.03 \]

Result is consistent with unity within the uncertainty

Stringent test of LFU in $B$ decays to date
Simultaneous measurement of $R(D)$ & $R(D^*)$ at Belle with semileptonic tagging method

- Exclusive semileptonic $B$ decay analysis
  - tag-side: $B^0/\pm \rightarrow D(\pm)\ell^-\nu$
  - signal side: signal channel $B^0/\pm \rightarrow D(\pm)\tau^- (\ell^- \bar{\nu}\nu)\nu$ and normalization channel $B^0/\pm \rightarrow D(\pm)\ell^-\nu$

- High signal purity at the cost of lower signal reconstruction efficiency.

- Signal is extracted by 2-dimensional binned extended ML fit to $E_{ECL}$ and $O_{cls}$
  - $E_{ECL}$: Sum of energies of neutral clusters in the ECL, not associated with any reconstructed particles
  - $O_{cls}$: Background suppression classifier output

- Most precise measurements of $R(D)$ & $R(D^*)$

- Results are in agreement with the SM within 0.2$\sigma$ and 1.1$\sigma$ for $R(D)$ & $R(D^*)$

- Combination of $R(D)$ & $R(D^*)$ has a deviation of 0.8$\sigma$
LFU in $b \to c \tau \nu$

- $R(X) = \frac{\mathcal{B}(B \to X\tau\nu)}{\mathcal{B}(B \to X\ell\nu)}$ is the complimentary measurement to the $R_{D(*)}$ via inclusive reconstruction for $b \to c$ transition.
- In the SM, $R(X) = 0.223 \pm 0.004$ [PRD 92, 054018 (2015)]
- More challenging due to larger background from less constrained $X$ system

\[ R_{\text{SM}}(X_e/\mu) = 1.006 \pm 0.001 \] [arXiv:2207.03432]

- Inclusive decay with $\tau$ lepton is challenging
- World average from LEP experiments, $\mathcal{B}(B \to X\tau\nu) = (2.41 \pm 0.23)\%$ is consistent with the SM $(2.45 \pm 0.10)\%$
- LFU can also be tested using light leptons
  \[ R(X_{e/\mu}) = \frac{\mathcal{B}(B \to Xe\nu)}{\mathcal{B}(B \to X\mu\nu)} \]
- $R(X_{e/\mu}) \quad \text{(SM)} = 1.006 \pm 0.001$ [arXiv:2207.03432]
Measurement of $R(X_{e/\mu})$ at Belle II

- Test LFU with inclusive measurement of $R(X_{e/\mu})$
- Analysis is performed on Belle II data sample of 189/fb with hadronic-$B$ tagging (FEI)
- Constrain flavor and kinematics of the signal $B$ meson by tagging the other $B$ in its fully hadronic decays, i.e., good purity at the cost of lower signal reconstruction efficiency.
- $X$ system in signal side contains a large variety of different charged and neutral final-state particles
  
  **Y(4S) Reconstruction**
  - Select best candidate! $p_{\text{had.FEI}} > 0.1$
    $M_{bc} \in [5.2725, 5.285]$
    $\Delta E \in [-0.15, 0.1]$
  - Reconstruct $Y(4S) \rightarrow B^- \tau^+ (\rightarrow \ell^+)$, $Y(4S) \rightarrow B^0 \tau^- (\rightarrow \ell^-)$
  - Cut on continuum suppression BDT

- Lepton momentum in the CM frame of the signal $B$ meson, $p_{\ell}^*$ is used to extract signal yield
- Require lepton to have high probability to be an electron or muon and $p_{\ell}^* > 1.3$ GeV/$c$ to suppress backgrounds from hadron faking leptons and secondary leptons from $b \rightarrow c \rightarrow (\ell, s)$ cascades and $B \rightarrow X\tau\nu$

Seema Choudhury
Lepton flavor universality at Belle and Belle II
August 29, 2022 11/20
Signal yields for $B \rightarrow X\mu\nu$ and $B \rightarrow Xe\nu$ are extracted in 10 bins of $p_{\ell}^*$.

Simultaneous fit for $\mu$ and $e$ channel: one-dimensional binned ML fit

- 48034 ± 286 and 58569 ± 429 signal events for $B \rightarrow Xe\nu$ and $B \rightarrow X\mu\nu$ channels.

$$R(X_e/\mu) = 1.033 \pm 0.010 \pm 0.020 \text{ for } p_{\ell}^* > 1.3 \text{ GeV/c}$$

First inclusive test of $(e, \mu)$ lepton flavor universality in semileptonic $B \rightarrow X\ell\nu$ decays

Measurement in agreement with unity within 1.5$\sigma$

World leading precision (2.2% combined uncertainty)

Paved the path for inclusive $R(X_{\tau}/\ell) = R(X)$ measurement
LFU in $b \to s \ell \ell$

- $B$ decays with rare $b \to s$ loop-level transitions are an important probe to test LFU.

- LFU ratio is named as $R_K$ and $R_{K^*}$ for $B \to K \ell \ell$ and $B \to K^* \ell \ell$

$$R_{K(*)} = \frac{\mathcal{B}(B \to K(*) \mu \mu)}{\mathcal{B}(B \to K(*) ee)}$$

- According to SM this ratio should be 1 [EPJC 76, 440 (2016)], as the coupling of lepton to gauge boson is independent of flavor.

LHCb [arXiv:2103.11769] $R_{K^+} = 0.846^{+0.044}_{-0.041}$ for $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$ has a deviation of $3.1\sigma$ from SM prediction with $9 \text{ fb}^{-1}$ data sample, where $q^2 = M_{\ell \ell}^2$.

- $R_{K^*+}$ and $R_{K^0_S}$ results from LHCb [PRL 128, 191802 (2022)] are individually consistent with the SM at the $1.4\sigma$ and $1.5\sigma$ level.

- $B \to J/\psi K(*)$ can be used to cross-check the ratio, which is compatible with the SM prediction of unity.
Measurement of $R_{K^*}$ at Belle

- $R_{K^*}$ tests the lepton-flavor-universality in $B \to K^* \ell^+ \ell^-$.  
- Reconstructed 4 decay modes;  $B^+ \to K^{*+}(K^+ \pi^0, K_S^0 \pi^+) \ell^+ \ell^-$, $B^0 \to K^{*0}(K^+ \pi^-, K_S^0 \pi^0) \ell^+ \ell^-$.  
- Kinematic variables to distinguish signal from background;  
  \[ M_{bc} = \sqrt{E_{\text{beam}}^2/c^4 - |p_B|^2/c^2} \]  
  \[ \Delta E = E_B - E_{\text{beam}} \]  
- Continuum and $B\bar{B}$ backgrounds are suppressed using Neural Networks.  
- Performed 1D unbinned extended ML fit to extract the signal yield.  
- $103^{+13.4}_{-12.7}$ and $139.9^{+16.0}_{-15.4}$ events for electron and muon modes.  
- $R_{K^{*+}}$, $R_{K^{*0}}$ and $R_{K^*}$ are measured for both low and high $q^2$ bins.  
- Results consistent with the SM predictions.  
- First result for $R_{K^{*+}}$ measurement.
Measurement of $R_K$ at Belle

- $R_K$ tests the lepton-flavor-universality in $B \to K \ell \ell$.

- Decay modes reconstructed, $B^+ \to K^+ \ell \ell$ and $B^0 \to K^0_S \ell \ell$.

- Background from continuum and $B\bar{B}$ are suppressed using a Neural Network having event shape, vertex quality, and kinematic variables.

- Performed 3D unbinned ML fit in $M_{bc}$, $\Delta E$, and modified NN output ($O'$) to extract the signal yield.

- Control mode is consistent with expectation;
  
  \[
  R_{K^+}(J/\psi) = 0.994 \pm 0.011 \pm 0.010 \\
  R_{K^0}(J/\psi) = 0.993 \pm 0.015 \pm 0.010
  \]

- $137 \pm 14, 138 \pm 15, 27.3^{+6.6}_{-5.8}$, and $21.8^{+7.0}_{-6.1}$ signal events for $B^+ \to K^+ \mu \mu$, $B^+ \to K^+ ee$, $B^0 \to K^0_S \mu \mu$, and $B^0 \to K^0_S ee$.

- $R_{K^+}, R_{K^0}, R_K$ are measured in different $q^2$ bins.

- $q^2 \in [0.1, 4.0], [4.0, 8.12], [1.0, 6.0], [10.2, 12.8], > 14.18$, and $> 0.1$ GeV$^2$/c$^4$

- $R_K$ values for various $q^2$ bins agree with the SM prediction.
Measurement of $B(B \rightarrow K^* \ell \ell)$ at Belle II

- Decay modes reconstructed: $B^0 \rightarrow K^{*0}(K^+\pi^-)\ell\ell$ and $B^+ \rightarrow K^{*+}(K^0\pi^0, K_S^0\pi^+\pi^-)\ell\ell$

- Background from continuum and $B \bar{B}$ is suppressed using a BDT having event shape, vertex quality, and kinematic variables.

- Performed 2D unbinned ML fit in $M_{bc}$ and $\Delta E$ to extract the signal yield.

Branching fraction are measured for the entire range of the dilepton mass, excluding the very low mass region to suppress the $B \rightarrow K^* \gamma(\rightarrow e^+e^-)$ background and regions compatible with decays of charmonium resonances

$$B(B \rightarrow K^*(892)\mu^+\mu^-) = (1.19 \pm 0.31^{+0.08}_{-0.07}) \times 10^{-6},$$

$$B(B \rightarrow K^*(892)e^+e^-) = (1.42 \pm 0.48 \pm 0.09) \times 10^{-6},$$

$$B(B \rightarrow K^*(892)\ell^+\ell^-) = (1.25 \pm 0.30^{+0.08}_{-0.07}) \times 10^{-6},$$

- Results are compatible with world averages within the uncertainties.

- Observation of these decays is the first step towards LFU test ($R_{K^*}$).
- Decay channels: $B^+ \rightarrow J/\psi(\ell\ell)K^+$ and $B^0 \rightarrow J/\psi(\ell\ell)K^0$

- Important channels to test our analysis method

- Signal yield is extracted by a 2D unbinned ML fit in $M_{bc}$ and $\Delta E$.

- Signal purity is 90 – 95%.

- Results are statistically dominated and in agreement with results from Belle and LHCb.

- Systematics uncertainties have been reduced compared to most precise measurements from Belle [JHEP 03, 105 (2021)].
**Measurement of $R(\Omega)$ at Belle**

- LFU in $\Omega_c^0$ is probed for the first time with $\Omega_c^0 \to \Omega^- \ell^+ \nu_\ell$
- $\Omega_c^0$ are reconstructed in the process; $e^+ e^- \to c\bar{c} \to \Omega_c^0 +$ anything
- Used 89.5, 711, and 121.1 fb$^{-1}$ data collected at the CM energies of 10.52, 10.58, and 10.86 GeV.
- $\Omega_c^0$ signals are extracted by binned ML fits to the invariant mass ($M_{\Omega\ell}$) spectra.

Significances of the $\Omega_c^0 \to \Omega^- \pi^+$ are both larger than 10$\sigma$, $\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu$ decay is seen for first time in Belle.

- 865.3 ± 35.3, 707.6 ± 37.7, and 367.9 ± 31.4 signal events for $\Omega_c^0 \to \Omega^- \pi^+$, $\Omega_c^0 \to \Omega^- e^+ \nu_e$, $\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu$
- $\Omega_c^0$ semileptonic decay branching fraction ratio;

$$R(\Omega) = \frac{B(\Omega_c^0 \to \Omega^- e^+ \nu_e)}{B(\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu)} = 1.02 \pm 0.10 \pm 0.02$$

- $B(\Omega_c^0 \to \Omega^- e^+ \nu_e)/B(\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu)$ agrees with the expected LFU value 1.03 ± 0.06 [arXiv:2108.06110].
some LFU prospects at Belle II

- $R(X)$ (and in general inclusive processes) is unique to Belle II
- Currently estimated precision on $R(X)$ to be $\sim 17\%$ (stat+syst)
- Few $ab^{-1}$ of data will be sufficient to clarify whether the anomaly on $R(D) - R(D^*)$ has a statistical or systematic origin

- Belle II can perform $R_K$ and $R_{K^*}$ measurements for low as well as high $q^2$ bins.
- Belle II will provide an independent measurement to confirm the tension with few $ab^{-1}$ of data
- $R_{K^+}$ and $R_{K^*}$ statistical sensitivity will be $< 2\%$ for entire $q^2$ region and $\sim 3\%$ for $q^2 \in [1 - 6] \text{ GeV}^2/c^4$
Belle II has now collected $\sim 424 \text{ fb}^{-1}$ of data sample (comparable to size of that of BaBar) and can be combined with that of Belle ($711 \text{ fb}^{-1}$)

Flavor physics in $e^+e^-$ collisions offers an extremely rich physics program with many opportunities to probe New Physics

- Access to charged and neutral $B$ with equal efficiency
- Equal sensitivity for muon and electron channels
- Access to inclusive decay modes in addition to exclusive modes
- Untagged (high statistics) vs tagged (high purity) analysis
- Long way to go! A beginning has been made!

No sign of LFU violation so far from Belle or Belle II

An exciting era of discoveries and precision measurements !!!
TABLE I. Systematic uncertainties contributing to the $\mathcal{R}(D^{(*)})$ results, together with their correlation.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta \mathcal{R}(D)$(%)</th>
<th>$\Delta \mathcal{R}(D^*)$(%)</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{**}$ composition</td>
<td>0.76</td>
<td>1.41</td>
<td>−0.41</td>
</tr>
<tr>
<td>PDF shapes</td>
<td>4.39</td>
<td>2.25</td>
<td>−0.55</td>
</tr>
<tr>
<td>Feed-down factors</td>
<td>1.69</td>
<td>0.44</td>
<td>0.53</td>
</tr>
<tr>
<td>Efficiency factors</td>
<td>1.93</td>
<td>4.12</td>
<td>−0.57</td>
</tr>
<tr>
<td>Fake $D^{(*)}$ calibration</td>
<td>0.19</td>
<td>0.11</td>
<td>−0.76</td>
</tr>
<tr>
<td>$B_{\text{tag}}$ calibration</td>
<td>0.07</td>
<td>0.05</td>
<td>−0.76</td>
</tr>
<tr>
<td>Lepton efficiency and fake rate</td>
<td>0.36</td>
<td>0.33</td>
<td>−0.83</td>
</tr>
<tr>
<td>Slow pion efficiency</td>
<td>0.08</td>
<td>0.08</td>
<td>−0.98</td>
</tr>
<tr>
<td>$B$ decay form factors</td>
<td>0.55</td>
<td>0.28</td>
<td>−0.60</td>
</tr>
<tr>
<td>Luminosity, $f^{++, -}$, $f^{00}$, and $B(\Upsilon(4S))$</td>
<td>0.10</td>
<td>0.04</td>
<td>−0.58</td>
</tr>
<tr>
<td>$\mathcal{B}(B \rightarrow D^{(*)}\ell\nu)$</td>
<td>0.05</td>
<td>0.02</td>
<td>−0.69</td>
</tr>
<tr>
<td>$\mathcal{B}(D)$</td>
<td>0.35</td>
<td>0.13</td>
<td>−0.65</td>
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<td>$\mathcal{B}(D^*)$</td>
<td>0.04</td>
<td>0.02</td>
<td>−0.51</td>
</tr>
<tr>
<td>$\mathcal{B}(\tau^- \rightarrow \ell^-\bar{\nu}<em>{\ell}\nu</em>{\tau})$</td>
<td>0.15</td>
<td>0.14</td>
<td>−0.11</td>
</tr>
<tr>
<td>Total</td>
<td>5.21</td>
<td>4.94</td>
<td>−0.52</td>
</tr>
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</table>
### $R_K$ systematics

<table>
<thead>
<tr>
<th>Sources</th>
<th>$B^+ \rightarrow J/\psi K^+$</th>
<th>$B^0 \rightarrow J/\psi K_S^0$</th>
<th>$R_K^+ (J/\psi)$</th>
<th>$R_K^0 (J/\psi)$</th>
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<tbody>
<tr>
<td>Lepton identification</td>
<td>±0.68</td>
<td>±0.68</td>
<td>±0.97</td>
<td>±0.97</td>
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<tr>
<td>Kaon identification</td>
<td>±0.80</td>
<td>−</td>
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<tr>
<td>$K_S^0$ identification</td>
<td>–</td>
<td>±1.57</td>
<td>–</td>
<td>−</td>
</tr>
<tr>
<td>Track reconstruction</td>
<td>±1.05</td>
<td>±1.40</td>
<td>−</td>
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<tr>
<td>Efficiency calculation</td>
<td>±0.14</td>
<td>±0.18</td>
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<td>±0.25</td>
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<tr>
<td>Number of $B\bar{B}$ pairs</td>
<td>±1.40</td>
<td>±1.40</td>
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</tr>
<tr>
<td>$f^{+-}(00)$</td>
<td>±1.20</td>
<td>±1.20</td>
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<tr>
<td>$\mathcal{O}_{\text{min}}$</td>
<td>±0.16</td>
<td>±0.28</td>
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<td>+0.05</td>
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<td></td>
<td>−0.20</td>
<td>−0.10</td>
<td>−0.31</td>
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<tr>
<td>Total</td>
<td>±2.38</td>
<td>±2.90</td>
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<table>
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<tr>
<th>Source</th>
<th>$B (B \rightarrow K J/\psi)$</th>
<th>$R_K K^+$</th>
<th>$K^+$</th>
<th>$K_S^0$</th>
<th>$K_S^0$</th>
<th>$K_S^0$</th>
<th>$K_S^0$</th>
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<tr>
<td>Tracking efficiency</td>
<td>0.9</td>
<td>0.9</td>
<td>1.2</td>
<td>1.2</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<tr>
<td>Simulation sample size</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
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</tr>
<tr>
<td>$\Upsilon(4S)$ branching fraction</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$(\tau_{B^+}/\tau_{B^0})$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3.2</td>
<td>3.2</td>
<td>4.4</td>
<td>4.4</td>
<td>0.8</td>
<td>0.8</td>
<td>–</td>
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</tr>
</tbody>
</table>
**$R(X_e/\mu)$ systematics**

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon$ channel</th>
<th>$\mu$ channel</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_X$ scaling</td>
<td>7.8% (21.2%)</td>
<td>12.5% (20.5%)</td>
<td>8.7% (26.8%)</td>
</tr>
<tr>
<td>PID</td>
<td>1.8% (1.2%)</td>
<td>7.1% (6.6%)</td>
<td>2.1% (1.6%)</td>
</tr>
<tr>
<td>Tracking eff.</td>
<td>2.9% (2.8%)</td>
<td>5.1% (3.4%)</td>
<td>3.4% (4.0%)</td>
</tr>
<tr>
<td>$X_c\ell\nu$ BRs</td>
<td>6.6% (15.2%)</td>
<td>11.1% (15.9%)</td>
<td>7.5% (19.9%)</td>
</tr>
<tr>
<td>$X_c\ell\nu$ FFs</td>
<td>4.5% (7.1%)</td>
<td>7.2% (6.8%)</td>
<td>5.0% (8.9%)</td>
</tr>
<tr>
<td>Statistical</td>
<td>10.8% (40.3%)</td>
<td>19.4% (48.9%)</td>
<td>9.4% (31.3%)</td>
</tr>
<tr>
<td>Total</td>
<td>17.0% (100%)</td>
<td>27.7% (100%)</td>
<td>16.8% (100%)</td>
</tr>
</tbody>
</table>

- Dominate systematic comes from $X_c\ell\nu$ BRs because of discrepancy between the inclusive semileptonic $B$ meson width and sum of exclusively measured BRs. This difference is usually filled by $D^*\pi\pi$ and $D^*\eta$ modes and are scaled to inclusive $B$ meson width. As this is speculative, they are assigned with 100% uncertainty, this become one of the leading systematic uncertainty.