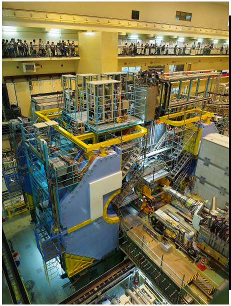
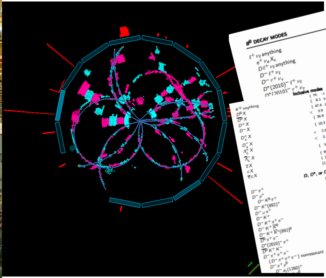
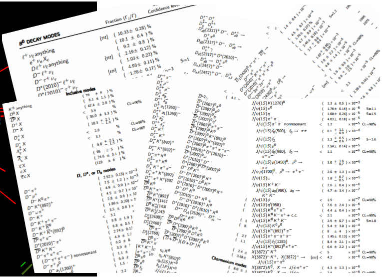
Semileptonic B decays at Belle II

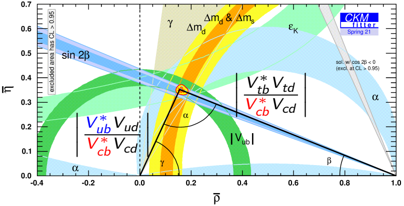




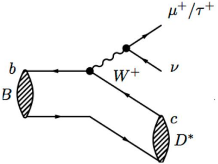


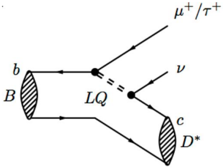
Why are semileptonic B decays important?

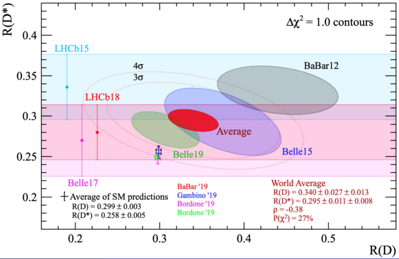
- Precision measurements of the SM:
  - Semileptonic B decays used to extract the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$,
- Potential probes of new physics
  - Longstanding anomaly observed in $R(D^*)$ measurements.



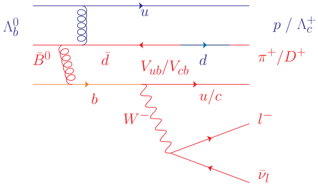
$$R\left(D^{(*)}\right) = \frac{\mathcal{B}(B \to D^{(*)}\tau^-\bar\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell^-\bar\nu_\ell)}$$

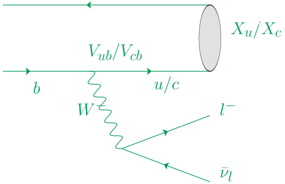


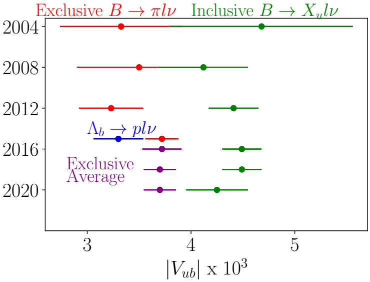


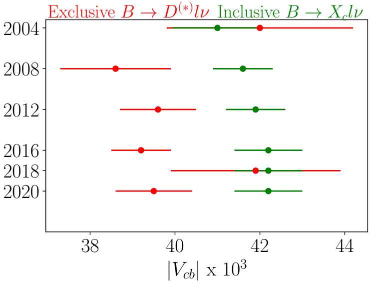


The status of $|V_{ub}|$ and $|V_{cb}|$ determinations.





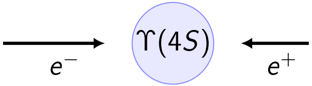




- There exists a longstanding discrepancy between inclusive and exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$

Tag-side reconstruction at Belle II

- Collide e^+e^- to make $\Upsilon(4S)$ particles.



- FEI: employs over 200 BDTs to reconstruct 10000 B decay chains.
- $\epsilon_{B^+} \sim 0.5\%$, $\epsilon_{B^0} \sim 0.3\%$ at low purity.

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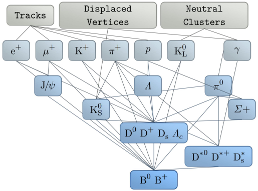

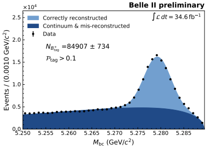

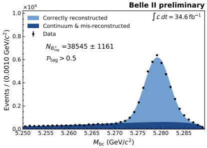

$$M_{bc} = \sqrt{E_{beam}^2/4 - (p_{B_{\rm tag}}^{\rm cm})^2} > 5.27\ \mathrm{GeV}/c^2$$

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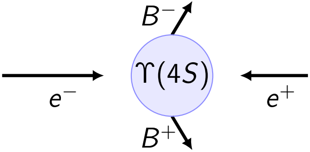

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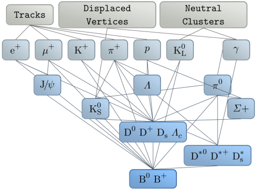

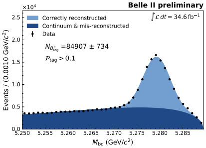

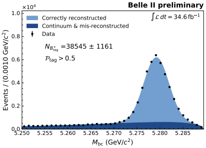

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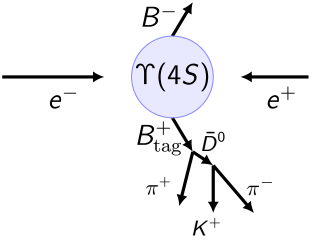

- Reconstruct tag-side ($B_{\rm tag}$).

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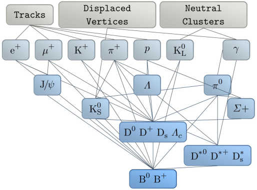

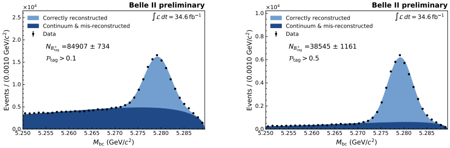

$$M_{bc} = \sqrt{E^2_{beam}/4 - (p^{\rm cm}_{B_{\rm tag}})^2} > 5.27\ \mathrm{GeV}/c^2$$

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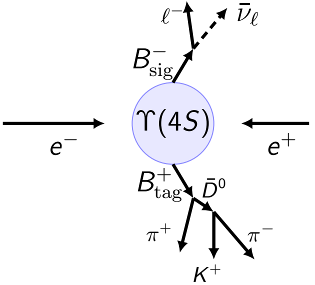

- Reconstruct tag-side (B_{tag}).
- Study remaining B as signal (B_{sig}).

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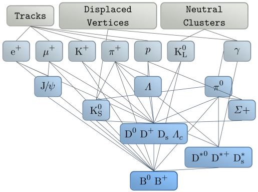

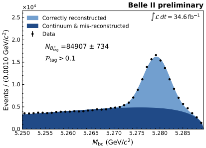

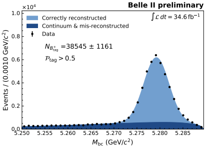

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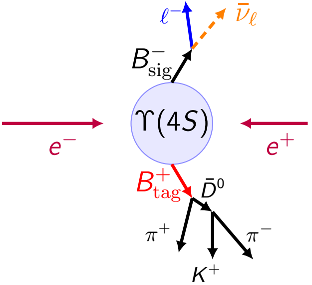

- Reconstruct tag-side (B_{tag}).
- Study remaining B as signal (B_{sig}).
- Flavour constraints: $B^+_{\text{tag}} \implies B^-_{\text{sig}}$
  Kinematic constraints:
  $p_\nu = p_{e^+e^-} - p_{\ell^-} - p_{B^+}$

- FEI: employs over 200 BDTs to reconstruct 10000 B decay chains.
- $\epsilon_{B^+} \sim 0.5\%$, $\epsilon_{B^0} \sim 0.3\%$ at low purity.

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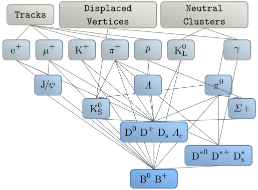
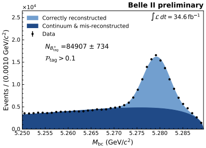
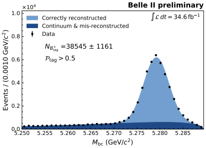
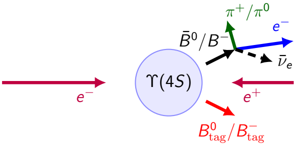


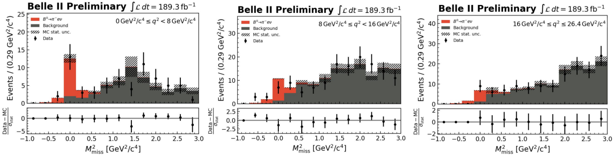


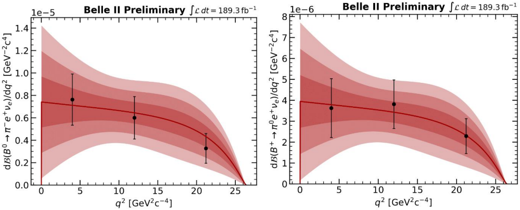


$$M_{bc} = \sqrt{E^2_{beam}/4 - (p^{\text{cm}}_{B_{\text{tag}}})^2} > 5.27\ \text{GeV}/c^2$$

Measuring $|V_{ub}|$ from $B \to \pi e \nu_e$







- $\epsilon_{\pi^0 e^+ \nu} \sim 0.2\%$, $\epsilon_{\pi^- e^+ \nu} \sim 0.4\%$
- Fit $p_\nu^2 = M^2_{\text{miss}}$ in 3 bins of q^2.

$$M^2_{\text{miss}} = (p_{e^+e^-} - p_{B_{\text{tag}}} - p_e - p_\pi)^2$$

- q^2 momentum transfer squared

$$q^2 = m^2_{\ell\nu} = (p_{e^+e^-} - p_{B_{\text{tag}}} - p_\pi)^2$$

- Measure:

$$\frac{d\mathcal{B}}{dq^2}(B \to \pi \ell \nu) \propto |V_{ub}|^2 f_+^2(q^2)$$

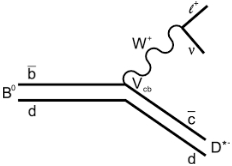
Decay mode	Fitted $\lvert V_{\text{ub}}\rvert$
$B^0 \to \pi^- e^+ \nu_e$	$(3.71 \pm 0.55) \times 10^{-3}$
$B^+ \to \pi^0 e^+ \nu_e$	$(4.21 \pm 0.63) \times 10^{-3}$
Combined fit	$(3.88 \pm 0.45) \times 10^{-3}$

LQCD theory inputs from:

Fermi MILC, Phys.Rev.D 92 (2015) 1 014024, arXiv:1503.07839.

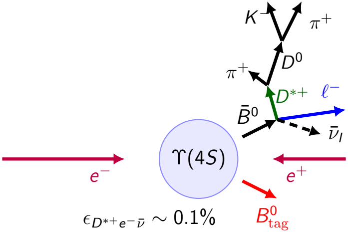
Measuring $|V_{cb}|$ from $B^0 \to D^{*-}\ell^+\nu_\ell$ ($\ell = e, \mu$)

$$\frac{d\Gamma}{dw} \propto \mathcal{F}^2(w)|V_{cb}|^2\eta_{\mathrm{EW}}^2$$



$$w = \frac{(m_B^2 + m_{D^{(*)}}^2 - q^2)}{2m_B m_{D^{(*)}}}$$

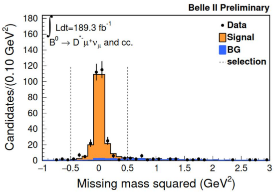
- Under HQET a single form factor $\mathcal{F}^2(w)$
- $\mathcal{F}^2(w)$ parametrised by $F(1)$, ρ^2, $R_1(1)$ and $R_2(1)$
CLN param. [Nucl. Phys. B530, 153 (1998)]

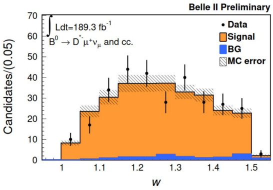


- measure $\eta_{\mathrm{EW}}F(1)|V_{cb}|$ and ρ^2.

$$M_{\mathrm{miss}}^2 = (p_{e^+e^-} - p_{B_{\mathrm{tag}}} - p_\ell - p_{D^*})^2$$

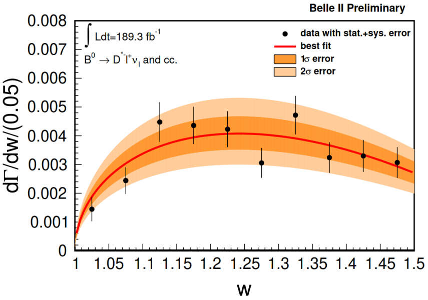
$$q^2 = (p_{e^+e^-} - p_{B_{\mathrm{tag}}} - p_{D^*})^2$$

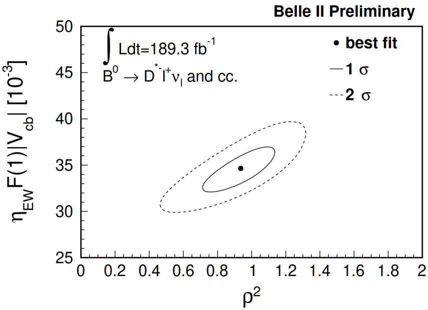




Measuring $|V_{cb}|$ from $B^0 \to D^{*-}\ell^+\nu_\ell$ ($\ell = e, \mu$)

$$\mathcal{B}(B^0 \to D^{*+}\ell\nu) = 0.0527 \pm 0.0022(stat) \pm 0.0038(sys)$$



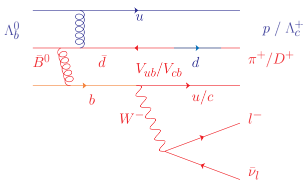


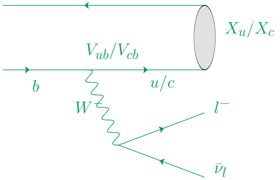
$$\eta_{\mathrm{EW}} F(1)|V_{cb}| = (34.6 \pm 2.5) \times 10^{-3}$$

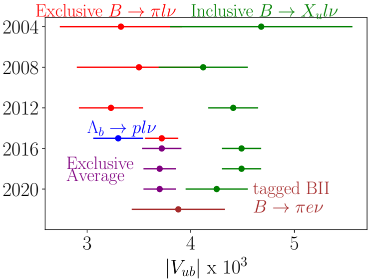
$$\rho^2 = 0.94 \pm 0.21$$

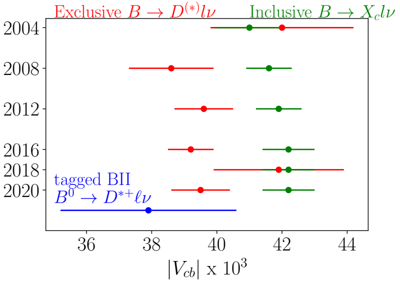
$$|V_{cb}| = (37.9 \pm 2.7) \times 10^{-3}$$

First Belle II $|V_{ub}|$ and $|V_{cb}|$ values









- These first Belle II tagged determinations of $|V_{ub}|$ and $|V_{cb}|$ are statistically limited.
- Expect soon higher precision untagged determinations as $\epsilon_{\text{untagged}} \sim$ 20-30%.

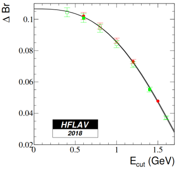
$|V_{cb}|$ from inclusive $B \to X_c l\nu$ decays

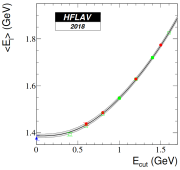
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left(1 + \frac{c_5(\mu)O_5(\mu)}{m_b^2} + \frac{c_6(\mu)O_6(\mu)}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right)\right)$$

- Operator Product Expansion (OPE):
  - O_i: hadronic matrix elements (non-perturbative)
  - c_i: coefficients (perturbative)

Order	HQE parameters
$\mathcal{O}(1)$	m_b, m_c
$\mathcal{O}(1/m_b^2)$	μ_π^2, μ_G^2
$\mathcal{O}(1/m_b^3)$	ρ_D^3, ρ_{LS}^3

- Moments of E_l, M_X can be expressed with same OPE formulation.
- $\implies$ Measure moments to constrain expansion parameters.

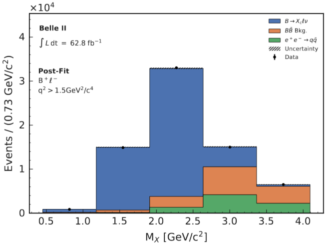




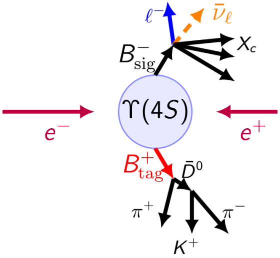
	$\lvert V_{cb}\rvert$ $[10^{-3}]$	m_b^{kin} [GeV]	$m_c^{\overline{MS}}$ [GeV]	μ_π^2 [GeV^2]	ρ_D^3 [GeV^3]	μ_G^2 [GeV^2]	ρ_{LS}^3 [GeV^3]
value	42.19	4.554	0.987	0.464	0.169	0.333	−0.153
error	0.78	0.018	0.015	0.076	0.043	0.053	0.096
$\lvert V_{cb}\rvert$	1.000	−0.257	−0.078	0.354	0.289	−0.080	−0.051
m_b^{kin}		1.000	0.769	−0.054	0.097	0.360	−0.087
$m_c^{\overline{MS}}$			1.000	−0.021	0.027	0.059	−0.013
μ_π^2				1.000	0.732	0.012	0.020
ρ_D^3					1.000	−0.173	−0.123
μ_G^2						1.000	0.066
ρ_{LS}^3							1.000

q^2 moments from $B \to X_c \ell \nu$ decays

- Novel idea to use $q^2 = m^2_{\ell\nu}$ moments due to less HQE parameters. [arXiv:1812.07472]
- At $\mathcal{O}(1/m_b^4)$ 8 parameters rather than 13.
- New Belle II measurement $\langle (q^2)^n \rangle$ of $n = $ 1-4



- Fit M_X to determine background normalisation.



- nth moment computed with a weighted average:

$$\left\langle (q^2)^n \right\rangle = \frac{\sum_i w(q_i^2)(q^2)^n_{i,calib}}{\sum_j w(q_j^2)} \cdot \mathcal{C}_{\text{calib}} \cdot \mathcal{C}_{\text{true}}$$

- $(q^2)^n_{i,calib} = ((q^2)^n_{i,reco} - c)/m$
- Remaining bias corrections: $\mathcal{C}_{\text{calib}}$, $\mathcal{C}_{\text{true}}$

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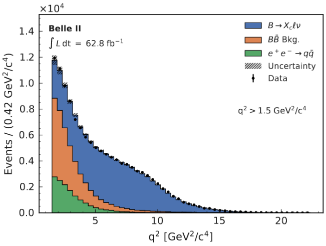
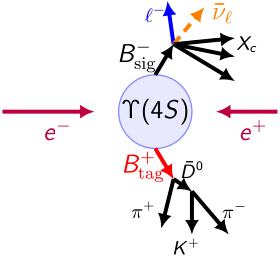


- Data MC comparison of q^2 spectrum.



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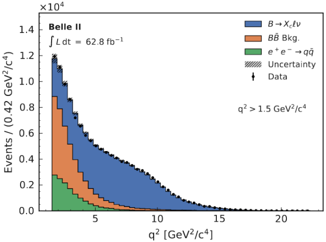
- Remaining bias corrections: $\mathcal{C}_{\mathrm{calib}}$, $\mathcal{C}_{\mathrm{true}}$

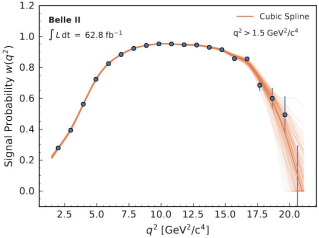
q^2 moments from $B \to X_c \ell \nu$ decays

- Novel idea to use $q^2 = m^2_{\ell\nu}$ moments due to less HQE parameters. [arXiv:1812.07472]
- At $\mathcal{O}(1/m_b^4)$ 8 parameters rather than 13.
- New Belle II measurement $\langle (q^2)^n \rangle$ of $n = $ 1-4



- Data MC comparison of q^2 spectrum.

- Determine signal probability, $w(q^2)$



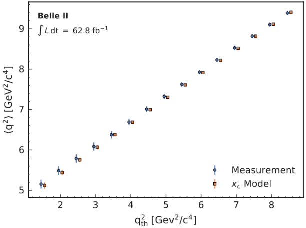
- nth moment computed with a weighted average:

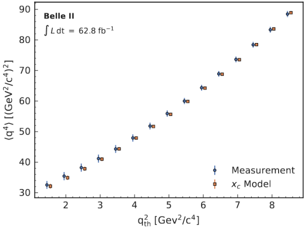
$$\left\langle (q^2)^n \right\rangle = \frac{\sum_i w(q_i^2)(q^2)^n_{i,calib}}{\sum_j w(q_j^2)} \cdot \mathcal{C}_{\mathrm{calib}} \cdot \mathcal{C}_{\mathrm{true}}$$

- $(q^2)^n_{i,calib} = ((q^2)^n_{i,reco} - c)/m$
- Remaining bias corrections: $\mathcal{C}_{\mathrm{calib}}$, $\mathcal{C}_{\mathrm{true}}$

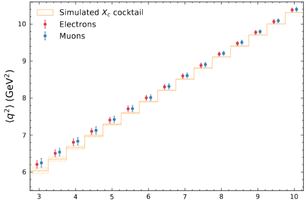
q^2 moments from $B \to X_c \ell \nu$ decays

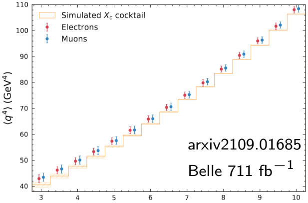
- Here compare $n = 1, 2$ q^2 moments as a function of lower q^2 threshold between Belle II and Belle.





Paper soon!

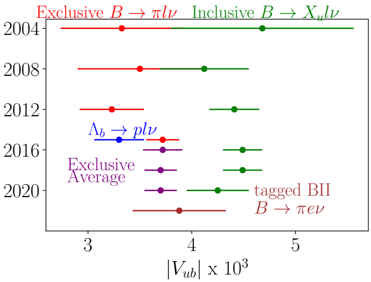


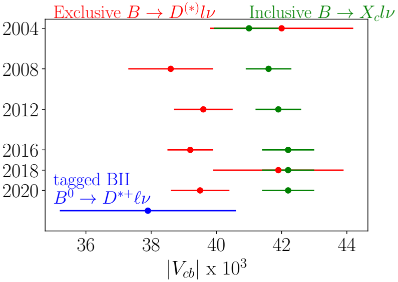


- Expect global fits for inclusive $|V_{cb}|$ using the moments in the near future.

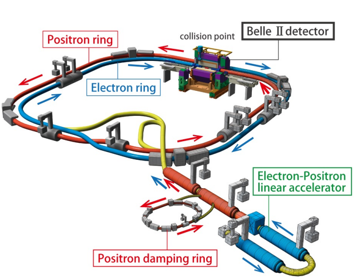
Conclusion

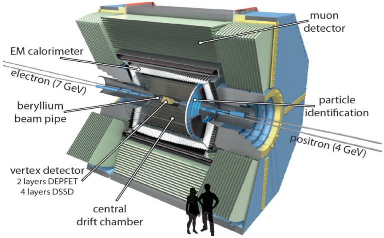
- Semileptonic B decays are important for precision determinations of $|V_{ub}|$ and $|V_{cb}|$.
- Here, the first Belle II exclusive tagged determinations of $|V_{ub}|$ and $|V_{cb}|$ were reported.
- In addition, a novel Belle II measurement of the q^2 moments of $B \to X_c \ell \nu$ decays was performed. These moments are important inputs for global fits for inclusive $|V_{cb}|$.

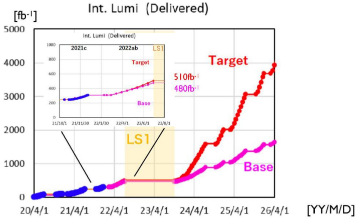


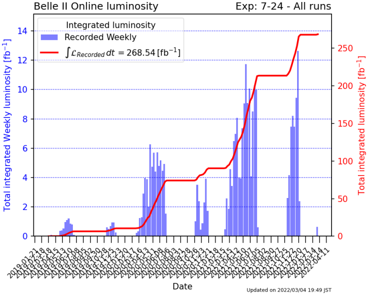


The Belle II Experiment



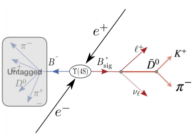


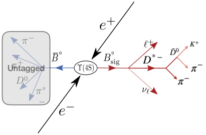




Untagged determinations of $\mathcal{B}(B \to D^{(*)}\ell\nu)$

- Untagged determinations benefit from higher efficiency
- $\epsilon_{D^0\ell^+\nu} \sim 30\%$, $\epsilon_{D^{*-}\ell^+\nu} \sim 22\%$
- But purity is lower.





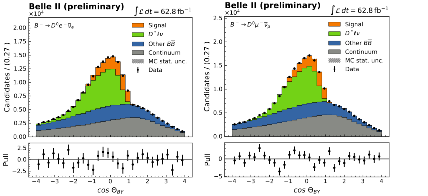
Starting from

$$0 = p_\nu^2 = (p_B^* - p_Y^*)^2$$

one can derive:

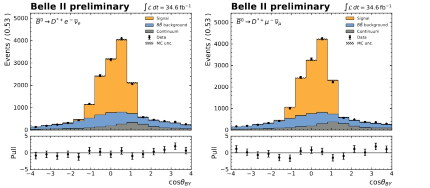
$$\cos\theta_{BY} = \frac{2E_B^* E_Y^* - m_B^2 - m_Y^2}{2|\vec{p}_B^*||\vec{p}_Y^*|}$$

$\mathcal{B}(B^- \to D^0\ell\bar{\nu}_\ell) = (2.29 \pm 0.05(\text{stat}) \pm 0.08(\text{sys})) \times 10^{-2}$



$\mathcal{B}(B^0 \to D^{*+}\ell\bar{\nu}_\ell) =$

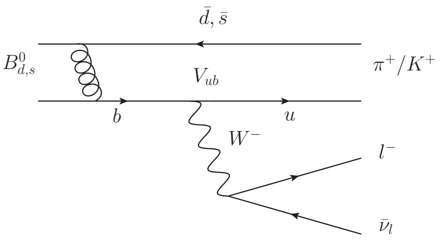
$(4.60 \pm 0.05(\text{stat}) \pm 0.17(\text{sys}) \pm 0.45(\pi_s)) \times 10^{-2}$



[arxiv2110.02648, arxiv2008.07198]

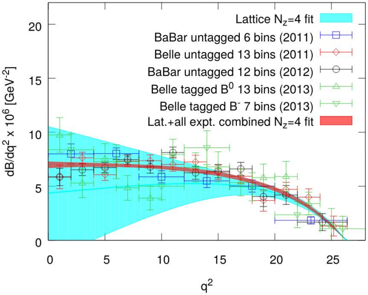
- More competitive untagged determinations of $|V_{cb}|$ and $|V_{ub}|$ in the next year.

Measuring $|V_{ub}|$ $B \to \pi \ell \nu$ decays

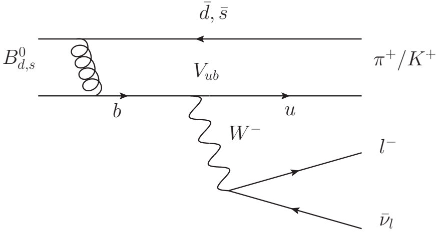


- Matrix element factorises $\mathcal{M} = -i \frac{G_F}{\sqrt{2}} V_{ub} H^\mu L_\mu$
- $H^\mu(f_i(q^2)) = \langle X | \bar{q}\gamma^\mu (1 - \gamma_5) b | B \rangle$
- q momentum transfer to the W

- Form factors $f_i(q^2)$ computed with Light Cone Sum Rules or LQCD
- For light leptons $\frac{d\mathcal{B}}{dq^2}(B \to \pi \ell \nu) \propto |\mathcal{M}|^2 \propto |V_{ub}|^2 f_+^2(q^2)$
- Simultaneously fit $\frac{d\mathcal{B}}{dq^2}$ measurements and LQCD theory inputs to determine $|V_{ub}|$ and the form factor.

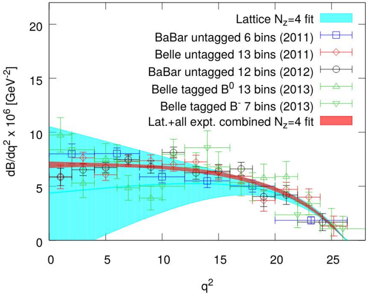


Measuring $|V_{ub}|$ $B \to \pi \ell \nu$ decays



- Matrix element factorises $\mathcal{M} = -i\frac{G_F}{\sqrt{2}} V_{ub} H^\mu L_\mu$
- $H^\mu(f_i(q^2)) = \langle X | \bar{q}\gamma^\mu(1-\gamma_5)b | B \rangle$
- q momentum transfer to the W

- Form factors $f_i(q^2)$ computed with Light Cone Sum Rules or LQCD
- For light leptons $\frac{d\mathcal{B}}{dq^2}(B \to \pi \ell \nu) \propto |\mathcal{M}|^2 \propto |V_{ub}|^2 f_+^2(q^2)$
- Simultaneously fit $\frac{d\mathcal{B}}{dq^2}$ measurements and LQCD theory inputs to determine $|V_{ub}|$ and the form factor.



CLN parameterisation for $B \to D^*\ell\nu$ decays

$$\frac{d\Gamma}{dw} = \frac{\eta_{\mathrm{EW}}^2 G_F^2}{48\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 g(w) F^2(w) |V_{cb}|^2$$

$$w = \frac{P_B \cdot P_{D^*}}{m_B m_{D^*}} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

$$g(w)F^2(w) = h_{A_1}^2(w)\sqrt{w^2-1}(w+1)^2 \left\{ 2\left[\frac{1-2wr+r^2}{(1-r)^2}\right] \times \left[1 + R_1^2(w)\frac{w-1}{w+1}\right] + \left[1 + (1-R_2(w))\frac{w-1}{1-r}\right]^2 \right\}$$

$$r = \frac{m_{D^*}}{m_B}$$

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$

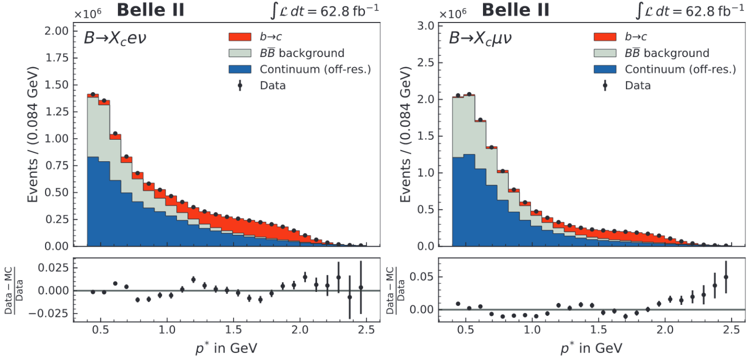
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

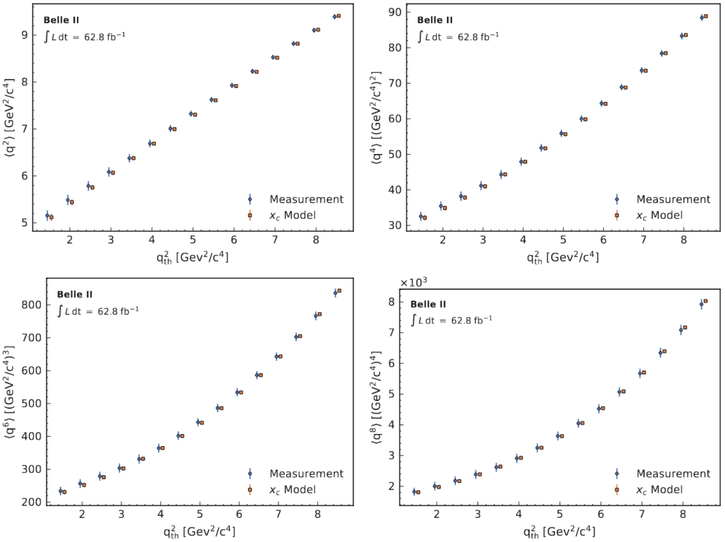
Branching fraction of $B \to X_c \ell \nu$ decays

- Independent analysis to measure $\mathcal{B}(B \to X_c \ell \nu)$ by fitting the p_ℓ^* distribution.
- $\mathcal{B}(B \to X_c \ell \nu) = (9.75 \pm 0.03(stat) \pm 0.47(syst))\%$

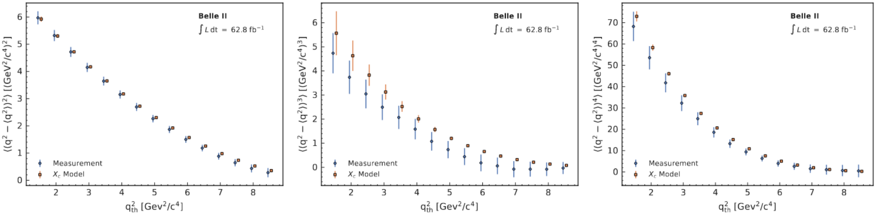


arXiv:2111.09405

q^2 moments n=1,2,3,4



Central q^2 moments n=2,3,4



Uncertainty break down for the first q^2 moment

TABLE II. Central values and uncertainties for the measurement of $\langle q^2 \rangle$. All uncertainties are given as relative uncertainties in %.

	q^2_{th} [Gev$^2/c^4$]	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
	$\langle q^2 \rangle$ [Gev$^2/c^4$]	5.16	5.49	5.79	6.09	6.38	6.69	7.01	7.32	7.62	7.93	8.23	8.53	8.82	9.10	9.39
Calibration (MC Statistics)	Calib. Curve (Stat. Unc.)	0.63	0.56	0.49	0.43	0.38	0.33	0.29	0.26	0.25	0.26	0.28	0.30	0.33	0.37	0.40
	Bias Corr. (Stat. Unc.)	0.10	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06
Calibration (X_c Model)	$\mathcal{B}(B \to D\ell\nu)$	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.04	0.03	0.02	0.02	0.01	0.01	0.00	0.00
	$\mathcal{B}(B \to D^*\ell\nu)$	0.33	0.29	0.24	0.21	0.17	0.14	0.11	0.09	0.07	0.05	0.04	0.03	0.02	0.01	0.00
	$\mathcal{B}(B \to D^{**}\ell\nu)$	0.71	0.63	0.55	0.48	0.40	0.34	0.28	0.23	0.18	0.13	0.10	0.07	0.05	0.03	0.02
	Non-Res. X_c Dropped	0.31	0.63	0.75	0.76	0.69	0.60	0.48	0.39	0.32	0.25	0.18	0.14	0.11	0.08	0.06
	Non-Res. X_c Repl. w/ D_1', D_0^*	0.34	0.49	0.51	0.45	0.37	0.29	0.18	0.10	0.04	0.02	0.00	0.03	0.03	0.03	0.01
	$B \to D\ell\nu$ Form Factor	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$B \to D^*\ell\nu$ Form Factor	0.08	0.07	0.07	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03
Calibration (Reconstruction)	PID Uncertainty	0.14	0.12	0.11	0.09	0.08	0.07	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.01	0.01
	N_γ Reweighted	0.30	0.27	0.24	0.22	0.20	0.18	0.16	0.14	0.14	0.13	0.13	0.12	0.11	0.10	0.10
	N_{tracks} Reweighted	1.09	1.00	0.92	0.85	0.78	0.72	0.65	0.60	0.55	0.51	0.47	0.44	0.41	0.38	0.35
	$E_{\text{miss}} - p_{\text{miss}}$ Reweighted	0.26	0.22	0.21	0.19	0.18	0.17	0.15	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.09
	Tracking Efficiency	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04
Background Subtraction	Spline Smooth. Factor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Bkg. Yield & Shape	1.39	1.15	0.90	0.77	0.63	0.47	0.33	0.23	0.16	0.10	0.06	0.03	0.02	0.05	0.06
Other	Non-Closure Bias	0.18	0.21	0.16	0.11	0.06	0.05	0.02	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.02
	Stat. Uncertainty	0.27	0.24	0.21	0.20	0.18	0.16	0.16	0.15	0.14	0.14	0.13	0.13	0.13	0.13	0.13
	Syst. Uncertainty	2.14	1.99	1.80	1.64	1.44	1.23	1.02	0.88	0.77	0.69	0.62	0.59	0.57	0.56	0.57
	Total Uncertainty	2.16	2.00	1.81	1.65	1.45	1.24	1.03	0.89	0.78	0.70	0.64	0.61	0.59	0.58	0.58

Uncertainty break down for the second q^2 moment

TABLE III. Central values and uncertainties for the measurement of $\langle q^4 \rangle$. All uncertainties are given as relative uncertainties in %.

	q^2_{th} [Gev$^2/c^4$]	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
	$\langle q^4 \rangle$ [(Gev$^2/c^4)^2$]	32.55	35.44	38.21	41.18	44.31	47.92	51.82	55.90	60.00	64.35	68.90	73.62	78.40	83.33	88.47
Calibration (MC Statistics)	Calib. Curve (Stat. Unc.)	0.96	0.85	0.75	0.67	0.58	0.50	0.44	0.41	0.40	0.42	0.45	0.49	0.54	0.59	0.64
	Bias Corr. (Stat. Unc.)	0.20	0.19	0.18	0.18	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.14	0.14	0.14
Calibration (X_c Model)	$\mathcal{B}(B \to D \ell \nu)$	0.18	0.16	0.15	0.13	0.12	0.10	0.08	0.07	0.06	0.04	0.03	0.02	0.01	0.01	0.01
	$\mathcal{B}(B \to D^* \ell \nu)$	0.60	0.52	0.45	0.39	0.33	0.27	0.22	0.18	0.14	0.10	0.07	0.05	0.03	0.02	0.01
	$\mathcal{B}(B \to D^{**} \ell \nu)$	1.30	1.17	1.04	0.91	0.79	0.67	0.56	0.45	0.36	0.27	0.20	0.14	0.09	0.06	0.05
	Non-Res. X_c Dropped	0.91	1.31	1.47	1.47	1.35	1.18	0.96	0.79	0.64	0.52	0.38	0.30	0.23	0.16	0.13
	Non-Res. X_c Repl. w/ D_1', D_0^*	0.69	0.87	0.89	0.79	0.66	0.51	0.31	0.17	0.07	0.03	0.02	0.06	0.07	0.06	0.03
	$B \to D \ell \nu$ Form Factor	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	$B \to D^* \ell \nu$ Form Factor	0.17	0.16	0.15	0.15	0.14	0.13	0.12	0.12	0.11	0.10	0.10	0.09	0.09	0.08	0.08
Calibration (Reconstruction)	PID Uncertainty	0.25	0.23	0.20	0.17	0.15	0.13	0.10	0.08	0.06	0.05	0.04	0.03	0.02	0.02	0.02
	N_γ Reweighted	0.61	0.57	0.52	0.49	0.45	0.40	0.36	0.33	0.32	0.30	0.28	0.26	0.25	0.23	0.22
	N_{tracks} Reweighted	2.27	2.11	1.98	1.85	1.72	1.58	1.46	1.34	1.24	1.14	1.05	0.97	0.90	0.83	0.76
	$E_{\text{miss}} - p_{\text{miss}}$ Reweighted	0.53	0.48	0.45	0.42	0.39	0.37	0.34	0.32	0.31	0.30	0.28	0.26	0.24	0.21	0.18
	Tracking Efficiency	0.28	0.26	0.24	0.22	0.20	0.19	0.17	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09
Background Subtraction	Spline Smooth. Factor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01
	Bkg. Yield & Shape	2.12	1.83	1.49	1.31	1.10	0.83	0.57	0.40	0.27	0.16	0.08	0.05	0.08	0.13	0.16
Other	Non-Closure Bias	0.32	0.37	0.30	0.23	0.13	0.11	0.06	0.05	0.04	0.05	0.04	0.04	0.04	0.03	0.05
	Stat. Uncertainty	0.49	0.46	0.43	0.40	0.37	0.35	0.34	0.33	0.31	0.31	0.30	0.29	0.29	0.29	0.29
	Syst. Uncertainty	3.86	3.68	3.42	3.16	2.82	2.46	2.09	1.82	1.61	1.44	1.30	1.21	1.15	1.10	1.07
	Total Uncertainty	3.89	3.71	3.45	3.18	2.85	2.48	2.12	1.85	1.64	1.47	1.34	1.25	1.19	1.14	1.11

Branching fraction of $B^0 \to D^{*-}\ell^+\nu$ with tagging

The branching fraction of $B^0 \to D^{*+}\ell^-\bar{\nu}$ decays is determined with

$$\mathcal{B}\left(B^0 \to D^{*-}l^+\nu_\ell\right) = \frac{\left(N^{\text{rec}} - N^{\text{bg}}\right) \times \epsilon^{-1}}{4 \times N_{B\bar{B}} \times \left(1 + f_{+0}\right)^{-1} \times \mathcal{B}\left(D^{*-} \to \pi^- \bar{D}^0\right) \times \mathcal{B}\left(\bar{D}^0 \to K^+\pi^-\right)} \quad (2)$$

The inputs to the equation are:

TABLE I. List of the input variables for the measurement of branching ratio and Γ.

Variables	Values
N^{rec}	545 (data)
N^{rec}	535 ± 38 (MC)
N^{bg}	29.4 ± 11.2
ϵ	$9.55 \pm 0.67 \times 10^{-4}$
$N_{B\bar{B}}$	$197.17 \times 10^6 \pm 2.9\%$
f_{+0}	1.058 ± 0.024
$\mathcal{B}\left(D^{*-} \to \pi^- \bar{D}^0\right)$	0.677 ± 0.005
$\mathcal{B}\left(\bar{D}^0 \to K^+\pi^-\right)$	0.03950 ± 0.00031

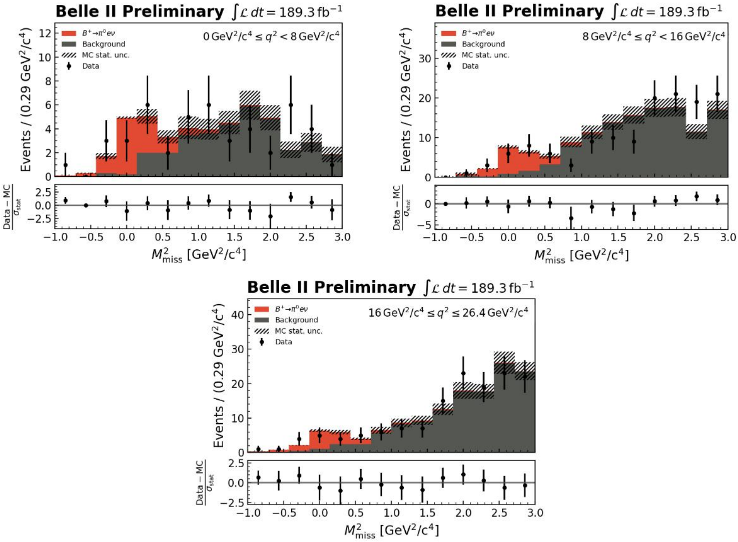
The corresponding systematics for the branching fraction are:

TABLE III. Summary of the relative systematic uncertainty on the measured branching ratio.

Systematic sources	Relative uncertainty (%)
FEI efficiency	3.9
Low momentum π efficiency	4.1
Tracking efficiency	0.9
Lepton particle identification	2.0
Background	1.2
$N_{B\bar{B}}$	2.9
f_{+0}	1.2
$\mathcal{B}\left(D^{*-} \to \pi^- \bar{D}^0\right)$	0.7
$\mathcal{B}\left(\bar{D}^0 \to K^+\pi^-\right)$	0.8
ECL energy	1.0
Form factor	0.1
MC statistics	1.8
Total	7.3

$$\mathcal{B}(B^0 \to D^{*+}\ell\nu) = 0.0527 \pm 0.0022(stat) \pm 0.0038(sys)$$

$B^+ \to \pi^0 e^+ \nu$ m^2_{miss} fits



$\frac{d\mathcal{B}}{dq^2}$ for $B \to \pi e^+ \nu$

q^2 bin	Signal efficiency	Unfolded signal yield	$\Delta\mathcal{B}$
		$B^0 \to \pi^- e^+ \nu_e$	
$0\ \text{GeV}^2 \leq q^2 < 8\ \text{GeV}^2$	$(0.189 \pm 0.002)\%$	15.5 ± 4.6	$(0.61 \pm 0.18(\text{stat}) \pm 0.03(\text{syst}))\ \times 10^{-4}$
$8\ \text{GeV}^2 \leq q^2 < 16\ \text{GeV}^2$	$(0.239 \pm 0.003)\%$	15.3 ± 4.8	$(0.48 \pm 0.15(\text{stat}) \pm 0.02(\text{syst}))\ \times 10^{-4}$
$16\ \text{GeV}^2 \leq q^2 \leq 26.4\ \text{GeV}^2$	$(0.229 \pm 0.003)\%$	10.3 ± 4.2	$(0.34 \pm 0.14(\text{stat}) \pm 0.02(\text{syst}))\ \times 10^{-4}$
Sum	–	41.1 ± 7.8	$(1.43 \pm 0.27(\text{stat}) \pm 0.07(\text{syst}))\ \times 10^{-4}$
Fit over full q^2 range	$(0.217 \pm 0.002)\%$	42.0 ± 7.9	$(1.45 \pm 0.27(\text{stat}) \pm 0.07(\text{syst}))\ \times 10^{-4}$
World average [9]	–	–	$(1.50 \pm 0.06)\ \times 10^{-4}$

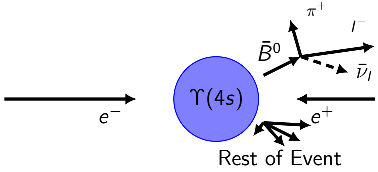
q^2 bin	Signal efficiency	Unfolded signal yield	$\Delta\mathcal{B}$
		$B^+ \to \pi^0 e^+ \nu_e$	
$0\ \text{GeV}^2 \leq q^2 < 8\ \text{GeV}^2$	$(0.329 \pm 0.004)\%$	12.9 ± 4.7	$(2.90 \pm 1.12(\text{stat}) \pm 0.19(\text{syst}))\ \times 10^{-5}$
$8\ \text{GeV}^2 \leq q^2 < 16\ \text{GeV}^2$	$(0.439 \pm 0.005)\%$	18.1 ± 5.1	$(3.05 \pm 0.91(\text{stat}) \pm 0.20(\text{syst}))\ \times 10^{-5}$
$16\ \text{GeV}^2 \leq q^2 \leq 26.4\ \text{GeV}^2$	$(0.451 \pm 0.006)\%$	14.5 ± 4.9	$(2.38 \pm 0.85(\text{stat}) \pm 0.16(\text{syst}))\ \times 10^{-5}$
Sum	–	45.5 ± 8.5	$(8.33 \pm 1.67(\text{stat}) \pm 0.55(\text{syst}))\ \times 10^{-5}$
Fit over full q^2 range	$(0.402 \pm 0.003)\%$	43.9 ± 8.3	$(8.06 \pm 1.62(\text{stat}) \pm 0.53(\text{syst}))\ \times 10^{-5}$
World average [9]	–	–	$(7.80 \pm 0.27)\ \times 10^{-5}$

Systematics fo $\frac{d\mathcal{B}}{dq^2}(B \to \pi e^+ \nu)$

Source	% of $\mathcal{B}(B^0 \to \pi^- e^+ \nu_e)$			% of $\mathcal{B}(B^+ \to \pi^0 e^+ \nu_e)$		
q^2 bin index	1	2	3	1	2	3
$N_{B\bar{B}}$	2.9					
f_{+0}	1.2					
FEI calibration	3.2			3.1		
Tracking	0.6			0.3		
π^0 efficiency	–			4.8		
Signal efficiency ϵ	1.3	1.2	1.4	1.3	1.2	1.3
Electron ID	1.0	0.4	0.4	1.0	0.5	0.5
Pion ID	0.4	0.4	0.4	–		
Total	4.8	4.7	4.8	6.7	6.7	6.7

Untagged $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ at the B factories

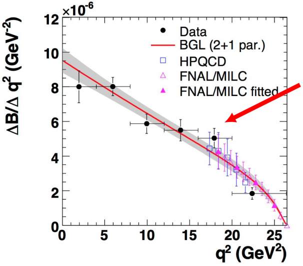
- Select good π and l candidates.

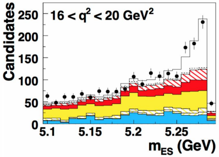


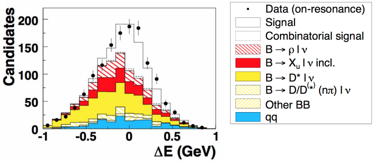
$$p_\nu = (E_{\text{miss}}, \mathbf{p}_{\text{miss}}) = p_{e^+e^-} - p_\pi - p_l - \sum p_{tracks} - \sum p_{clusters}$$

$$p_B = p_\pi + p_l + (P_{\text{miss}}, \mathbf{p}_{\text{miss}})$$

- Fit $M_{bc} = \sqrt{E^{*2}_{\text{beam}} - P^{*2}_B}$ and $\Delta E = E^*_B - E^*_{\text{beam}}$, ($* \implies$ CoM).



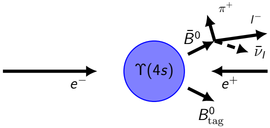


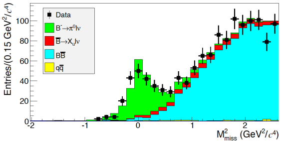


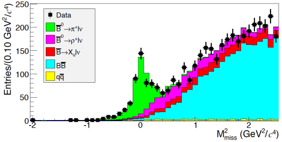
Measurement	Reference
BABAR (6 q^2 bins)	Phys. Rev. D83, 032007 (2011)
BABAR (12 q^2 bins)	Phys. Rev. D86, 092004 (2012)
Belle (13 q^2 bins)	Phys. Rev. D83, 071101 (2011)

Tagged $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ at the B factories

- $p_\nu = p_{e^+e^-} - p_\pi - p_l - p_{B_{\text{tag}}}$
- Fit Missing Mass Squared (p_ν^2)







Measurement	Reference
Belle (13 q^2 bins)	Phys. Rev. D88, 032005 (2013)