Belle II prospects for the measurements of $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$

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On behalf of the Belle II collaboration

Outline
Overview of measurements, studies and future prospects
- Belle II detector and data taking status
- Charm/τ potential and performance at Belle II
- Belle II prospects on $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$
- Summary
Prospects for measurements of $V_{us}$, $V_{cd}$ and $V_{cs}$ at Belle II

CKM 2021
25 Nov 2021

Belle II overview

**Electromagnetic Calorimeter**
CsI(Ti), waveform Sampling (barrel)
Pure CsI for end caps

**Central Drift Chamber**
Smaller cells, long lever arm, fast electronics

**Vertex Detector**
2 Layers PXD DEPFET and 4 Layers DSSD

**Particle Identification**
Time of Propagation in barrel region and ARICH in forward region

**KL and muon Detector**
Resistive Plate Chamber
Scintillator + WLSF + MPPC

Ref: Belle2 TDR: arXiv: 1011.0352
Belle II data status

- Continued data-taking through Covid-19 pandemic
- Integrated luminosity $L_{\text{int}} \sim 223 \text{ fb}^{-1}$ (Nov 18, 2021)
- Highest instantaneous luminosity $\sim 3.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
  - New world record archived in June 2021
  - SuperKEKB design luminosity: $6.5 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$

- Belle highest in June'09: $2.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$

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Projections of Integrated Luminosity Delivered by SuperKEKB to Belle II

- Target scenario: extrapolation from early 2021 run including expected improvements
- Base scenario: conservative extrapolation of SuperKEKB parameters from early 2021 run

- Date
  - Integrated Luminosity (delivered)
    - $800 \text{ fb}^{-1}$
    - $900 \text{ fb}^{-1}$
    - $1300 \text{ fb}^{-1}$

- Belle II charm/$\tau$ studies focused on detector/reconstruction performance, resolutions, and systematic effects.

Ref: https://confluence.desy.de/display/BI/Belle+II+Luminosity
charm/τ opportunities at Belle II

**Powerful SuperKEKB**

- 50 ab⁻¹ = ~50 x Belle
- e⁺e⁻ collisions (asymmetric beam energies)
  - offer stringent kinematic constraints for reconstruction of final states with neutrinos
  - acceptance and trigger criteria that introduce much less bias on flight length and kinematic properties
  - ..more in Physics Book

**Impact**

**Charm Physics**

- B-factory ▶︎ “charm-factory” (60×10⁹ pairs of c with 50 ab⁻¹)
  - excellent Dalitz plot analysis (uniform efficiency and non-biasing trigger)
  - better reconstruction of neutrinos

**τ Physics**

- B-factory ▶︎ “τ-factory” (~50×10⁹ events with 50 ab⁻¹)
  - measure wide range of observables (CP asymmetries, invariant mass spectra, lepton universality etc.)
  - precision measurements or indirect search of BSM (beyond SM) physics
  - direct search of forbidden decays

Ref: Belle2 Physics Book arxiv1808.10567
charmed/τ opportunities at Belle II

highlights of **Belle II**

- New silicon vertex detector provides better vertex resolution
- Good PID even with higher beam background environment
- More tracking volume ⇒ higher $K_s$ efficiency (w.r.t. LHCb)
- .. more in TDR and Physics Book

**Impact**

- Charm/τ Physics
  - Facilitate measurement of mixing parameters, and CP violations with neutrals in the final state
  - Belle II performance is expected to improve w.r.t. to Belle;
    - improved IP resolution (e.g. x2 better $D^0$ proper time resolution)
    - reduced statistical uncertainties
    - ..and if systematic uncertainties are reduced

*from Central Drift Chamber (CDC) and Silicon Vertex Detector (SVD)*
Prospects for measurements of $V_{us}$, $V_{cd}$ and $V_{cs}$ at Belle II

**World’s Best**

**Belle II performance**

**charm results**

- **Belle II 2019**
  - Luminosity: $34.6 \text{ fb}^{-1}$
  - Observed yield for $M$: $1230 \pm 15$ (stat.)

- **Belle II 2020**
  - Luminosity: $34.6 \text{ fb}^{-1}$
  - Observed yield for $D^0 \rightarrow K^+\pi^-$: $2500 \pm 100$ (stat.)

- **Belle II**
  - **D$^0$ lifetime**
  - **τ mass precision vs. $L$**
  - $m_\tau = 1777.28 \pm 0.75 \pm 0.33$ (sys)

**Good reconstruction performance**

- Belle II 2019:
  - $\int L \, dt = 9.6 \text{ fb}^{-1}$
  - Yield per $\text{fb}^{-1} = 1230 \pm 15$ (stat.)

- Belle II 2020:
  - $\int L \, dt = 34.6 \text{ fb}^{-1}$
  - $D^0 \rightarrow K^+\pi^-$

**Future improvements**

- World’s Best: x2 better w.r.t. Belle
**1 | $V_{us}$ | status and Belle II prospects**

- $s \leftrightarrow u$ transition
- Experimental measurements
  - kaon decays
    - traditional (also for form factor) and most accurate among all
    - but precision is limited from theory (LCQD) uncertainties (on form factor $f_+(0)$ & $f_k/f_\pi$)
  - hyperon decays
  - $\tau$ decays with strangeness in the final state (**today’s focus**)
$|V_{us}|$ from $\tau$ decays (methods)

**Exclusive method**

1. Compare BR ratio: $\tau^{-} \rightarrow K^{-}\nu_\tau$ and $\tau^{-} \rightarrow \pi^{-}\nu_\tau$

\[
\frac{\Gamma_{\tau^{-} \rightarrow K^{-}\nu_\tau}}{\Gamma_{\tau^{-} \rightarrow \pi^{-}\nu_\tau}} = \frac{|V_{us}|^2 f_K^2}{|V_{ud}|^2 f_\pi^2} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\pi^2/m_\tau^2)^2} (1 - \delta_{LD})
\]

- $|V_{us}| = 0.2234 \pm 0.0015$ (HFLAV 2021 preliminary)
  - -2.1σ from CKM unitarity
  - but large uncertainties as compared to kaons

  $|V_{us}| (K \rightarrow l3): 0.2231 \pm 0.0006$

**Inclusive method**

1. via spectral moments: $\tau \rightarrow s$ decays

\[
|V_{us}| = \sqrt{\frac{R_s}{\frac{R_{us}}{|V_{ud}|^2} - \delta_\tau}}
\]

- $|V_{us}| = 0.2192 \pm 0.0019$ (HFLAV 2021 preliminary)
  - -3.6σ lower from CKM unitarity

alternate methods [1], [2]: consistent with K and CKM unitarity

2. via branching fraction: $\tau^{-} \rightarrow K^{-}\nu_\tau$

\[
B(\tau^{-} \rightarrow K^{-}\nu_\tau) = \frac{G_F f_K^2 |V_{us}|^2 m_\tau^2}{16\pi^2} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}
\]

- $|V_{us}| = 0.2226 \pm 0.0015$ (HFLAV 2021 preliminary)
  - -2.6σ from CKM unitarity
  - but large uncertainties as compared to kaons

  $|V_{us}| (K \rightarrow l3): 0.2231 \pm 0.0006$
\[ |V_{us}| \] from \( \tau \) decays (status)

**Current status**

- **Kaon decays** (HFLAV 2021 preliminary)
  - \( |V_{us}| \): see latest numbers on plot for \( \to l3 \) & \( l2 \)
  - \( |V_{ud}| - |V_{us}| \) \( K \) anomaly \( \sim 3\sigma \)
  - \( \sim 5\sigma \) without increased \( |V_{ud}| \) systematics

- **\( \tau \) decays** (average HFLAV 2021 preliminary)
  - \( |V_{us}| = 0.2217 \pm 0.0013 \)
  - \( \tau \to s \): -3.6\( \sigma \) lower from CKM unitarity

**Belle II prospects**

- will perform LFU like analysis (use 3x1 and 1x1 topologies)
- statistical uncertainties will be improved with larger data-set
- also improved systematics from
  - PID\(^1\), trigger efficiency from detector upgrades
  - MC inputs (background estimation, modeling of decays\(^2\))

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1. PID (scale factor uncertainty will scale inverse to the statistics of the data sets)
2. Modeling of decays in the generator (KKMC, Tauola)

Belle II PID performance (efficiency/fake rates)

\[ D^+ \to D^0[K^-\pi^+]\pi^+ \]

By Alberto Lusiani in Tau 2021

NEW
2 | $V_{cs}$ | status and Belle II prospects

- Cabibbo-favoured ($c \rightarrow s$ transition)
- Experimental measurements
  - with $D$ and $D_s$ meson decays (*today’s focus*)
    - Leptonic ($D_s \rightarrow \ell \nu$) decay ~ simplest and theoretically cleanest processes
      Decay constants $f_D$ is required from Lattice QCD
    - Semi-leptonic decay ($D \rightarrow K\ell \nu$)
      Form factor $f(q^2)$ is required from Lattice QCD
  - charm baryon and $W^\pm$ decays
Overview

- Decay modes: $\rightarrow \mu \nu$, $\rightarrow e \nu$ & $\rightarrow \tau \nu$
- Decay suppressed by helicity conservation hence decay rates $\propto m_i^2$
  - $\rightarrow e \nu$ branching fraction is very small $\sim 10^{-7}$
  - $\rightarrow \tau \nu$ is favored over $\mu \nu$

Analysis method (Belle)

$$e^+e^- \rightarrow c\bar{c} \rightarrow D_{\text{tag}} X_{\text{frag}} K_{\text{frag}} D_s^{-}( \rightarrow D_s^- \gamma)$$

$$D^+, D^0, \Lambda_c^+ \text{ & } D_s^+, D_s^0 \pi, K(\text{even}), p$$

Tag: Tagged decays
Frag: Fragmented particles

Step 1: reconstruct tag side $D_{\text{tag}}$, build $X_{\text{frag}}$ and then extract $D_s^-$ via missing mass analysis
  ~ missing mass peak at $\sim D_s^-$ mass

Step 2: used signal from step 1 and search/extract $D_s^-$ yield for $\rightarrow \mu \nu$, $\rightarrow e \nu$ & $\rightarrow \tau \nu$
  ~ measure branching fraction
  $$B(D_s^+ \rightarrow f) = \frac{N(D_s^+ \rightarrow f)}{N^{\text{inc}}_{D_s} \cdot f_{\text{bias}} \cdot \varepsilon(D_s^+ \rightarrow f | \text{incl. } D_s^+)}.$$  

Step 3: calculate $f_{D_s} V_{us}$ from step 2, then two approach

1. take $f_{D_s}$ from Lattice QCD
  ~ extract $V_{us}$ and compare with CKM unitarity

2. take $V_{us}$ from CKM unitarity
  ~ extract $f_{D_s}$ and compare with Lattice QCD
\[ |V_{cs}| \via \text{leptonic decay: } D_s \rightarrow \ell \nu \]

**Current status \( \blackright D_s \rightarrow \mu \nu \)**

- several results in past years by BaBar, Belle, BESIII[1] [2][latest] and CLEO-c
- the most precise result from BESIII (2021) with 6.2 fb\(^{-1}\)
- Belle performed analysis with 913 fb\(^{-1}\)

**HFLAV 2021**

- \( |V_{cs}| = 0.9839 \pm 0.0115(\exp.) \pm 0.0020(\text{LQCD}) \)
  - average with \( D_s \rightarrow \tau \nu \)
  - \( f_{D_s} = 249.9 \pm 0.5 \text{ MeV (LQCD)} = 0.2\% \) precision
- \( f_{D_s} = 252.6 \pm 3.0 \text{ MeV} \)
  - global fit: \( |V_{cs}| = 0.973394^{+0.000074}_{-0.000096} \)

**Belle II prospects 50 ab\(^{-1}\)**

- \( \sim \) improved stats. uncertainty
  - \( \delta(|V_{cs}|) = \pm 0.004 \) (stat.)
  - \( \delta(|f_{D_s}|) = \pm 0.9 \) (stat.) MeV
- \( \sim \) systematics uncertainty (possible improvements)
  - with precision measurement of peaking backgrounds
  - in normalization (err. scaled with luminosity and are reducible with clean \( X_{tag} \))

\[
\Gamma(D_{s(\ell)}^+ \rightarrow \ell^+\nu) = \frac{G_F^2}{8\pi} f_{D_{s(\ell)}}^2 |V_{cd(s)}|^2 M_{D_{s(\ell)}}^2 \ell^{+} \left(1 - \frac{M_{\ell^+}^2}{M_{D_{s(\ell)}}^2}\right)^2
\]

**Prospects for measurements of \( V_{us}, V_{cd} \) and \( V_{cs} \) at Belle II**

**D\(_s\) \( \rightarrow \mu \nu \) Yield**

<table>
<thead>
<tr>
<th>Source</th>
<th>( \mu \nu ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>( \pm 5.32 )</td>
</tr>
<tr>
<td>Normalization</td>
<td>( \pm 1.95 )</td>
</tr>
<tr>
<td>Tag bias</td>
<td>( \pm 1.37 )</td>
</tr>
<tr>
<td>Tracking</td>
<td>( \pm 0.35 )</td>
</tr>
<tr>
<td>Efficiency</td>
<td>( \pm 1.78 )</td>
</tr>
<tr>
<td>PID</td>
<td>( \pm 1.96 )</td>
</tr>
<tr>
<td>( D_s ) background</td>
<td>( \pm 0.82 )</td>
</tr>
<tr>
<td>Comb. bkg. PDF</td>
<td>( \pm 0.02 )</td>
</tr>
<tr>
<td>Signal PDF</td>
<td>–</td>
</tr>
<tr>
<td>( \tau ) cross-feed</td>
<td>–</td>
</tr>
<tr>
<td>( B(\tau \rightarrow X) )</td>
<td>–</td>
</tr>
<tr>
<td>PDF stat.</td>
<td>–</td>
</tr>
<tr>
<td>Total syst.</td>
<td>( \pm 3.67 )</td>
</tr>
<tr>
<td>Stat. + Syst.</td>
<td>( \pm 6.46 )</td>
</tr>
</tbody>
</table>
\textbf{| } \textit{V}_{cs} \textit{ | via leptonic decay: } D_s \rightarrow \ell \nu \\

\textbf{Current status}  \quad \rightarrow D_s \rightarrow \tau \nu  \\
\quad \circ \text{several results in past years by BaBar, Belle, BESIII[1] (new: [2][3]) and CLEO-c} \\
\quad \circ \text{Belle modes; } e^+\nu\nu \rightarrow \mu^+\nu\nu \rightarrow \pi^+\nu \\
\quad \circ \text{signal } D_s \text{ extraction via fit to excess } E_{ECL} \\

\textbf{HFLAV 2021}  \\
\quad \bullet |V_{cs}| = 0.9839 \pm 0.0115 (\text{exp.}) \pm 0.0020 (\text{LQCD})  \\
\quad \circ \text{average with } D_s \rightarrow \mu\nu \\
\quad \circ f_{D_s} = 249.9 \pm 0.5 \text{ MeV (LQCD)} \\
\quad \bullet f_{D_s} = 252.6 \pm 3.0 \text{ MeV}  \\
\quad \circ \text{global fit: } |V_{cs}| = 0.973394^{+0.00074}_{-0.00096} \\

\textbf{Belle II prospects 50 ab}^{-1}  \\
\quad \circ \sim \text{improved stats. uncertainty} \\
\quad \circ \delta(|V_{cs}|) = \pm 0.003 (\text{stat.}) \sim \text{comparable to theory err.} \\
\quad \circ \delta(|f_{D_s}|) = \pm 0.6 (\text{stat.}) \text{ MeV} \sim \text{comparable to theory err.} \\
\quad \circ \sim \text{systematics uncertainty (possible improvements)}  \\
\quad \circ \text{with precision measurement of peaking backgrounds} \\
\quad \circ \text{in normalization (err. scaled with luminosity and are reducible with clean } X_{tag}) \\

\Gamma(D_{(s)}^+ \rightarrow \ell^+\nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}}^2 |V_{cs}|^2 M_{D_{(s)}}^2 M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}}^2}\right)^2
$|V_{cs}|$ via semi-leptonic decay: $D \rightarrow K \ell \nu$

**Current status**

$D \rightarrow K \ell \nu$

- several results in past years by BaBar, Belle, BESIII and CLEO-c

**HFLAV 2021**

- Form factors $f_D^{DK}(0) = 0.765 \pm 0.0031$ (ETM 17D, 18)
- $|V_{cs}| = 0.9447 \pm 0.0043$ (exp.) $\pm 0.0137$ (LQCD)

**Belle II prospects**

$e^+e^- \rightarrow c\bar{c} \rightarrow D_{\text{tag}}^{(*)} X_{\text{frag}} D_{\text{sig}}^{(*-)} \rightarrow \bar{D}_{\text{sig}}^0 \pi^-$

- MC studies with 1 ab$^{-1}$
  - based on $M_{\text{miss}}^2 = P_{\text{miss}}^2$ & $U_{\text{miss}} = E_{\text{miss}} - P_{\text{miss}}$
  - missing mass resolution is comparable with Belle
  - small continuum background contribution
  - with 50 ab$^{-1}$ data
    - larger data ($\sim 4.55 \times 10^5 D \rightarrow K \ell \nu$)
    - $\sim$ reduced stat. uncertainties

- scenario with charm factory experiments (e.g. BESIII)
  - challenging to compete with BESIII (with 20 fb$^{-1}$ data plans)
  - but Belle II will add important confirmation/constraints

\[ P_{\text{miss}} = P_{c^+} + P_{c^-} - P_{D_{\text{tag}}} - P_{X_{\text{frag}}} - P_h - P_l \]
3 | $V_{cd}$ | status and prospects

- Cabibbo-suppressed ($c \rightarrow d$ transition)
- Experimental measurements
  - Early study via neutrino production of charm ($\nu N$)
  - More precise results using $D$ meson decays (today’s focus)
    - Leptonic ($D^+ \rightarrow \ell^+ \nu$) decay
      - Decay constant $f_D$ is required from (e.g. Lattice QCD)
    - Semi-leptonic decay ($D \rightarrow \pi \ell \nu$)
      - Form factor $f(q^2)$ is required from theory (e.g. Lattice QCD)
\[ |V_{cd}| \text{ via leptonic decay: } D^+ \to \ell^+\nu \]

### Overview

- decay modes: \( \mu\nu, e\nu \& \tau\nu \)
- Belle II analysis method will be similar to \( D_s \to \ell\nu \) analysis

### Current status

- \( f_{D^+} |V_{cd}| \) : so far from charm factories only
  - \( \mu^+\nu \) : CLEO-c(2008) and BESIII (2014)
  - \( \tau^+\nu \) : CLEO-c(2008) for upper limit on BR and BESIII (2019)
  - \( e^+\nu \) : CLEO-c(2008) for upper limit on BR

  - world average \( f_{D^+} |V_{cd}| = 46.1 \pm 1.0 \pm 0.3 \pm 0.2 \) (from \( \mu^+\nu \))
  - ratio of \( BR(\mu^+\nu)/BR(\tau^+\nu) \) is compatible with SM prediction

- decay constants \( f_{D^+} \) from LQCD
  - \( f_{D^+} = 212.7 \pm 0.7 \) MeV

  - average from FNAL/MILC 17 and ETM 14E

- \( |V_{cd}|_{D \to \ell\nu} \) HFLAV (June 2021)

  - \( V_{cd} = 0.2181 \pm 0.0049(\text{exp.}) \pm 0.0007(\text{LQCD}) \)

  - also consistent with semi-leptonic measurement \( D \to \pi\ell\nu \) decays (in slide #18)
Belle II prospects

\( \Gamma(D^{+} \rightarrow \ell^{+}\nu) = \frac{G_{F}^{2}}{8\pi} f_{D_{(s)}}^{2} |V_{cd(s)}|^{2} M_{D^{+}} M_{\ell^{+}}^{2} \left(1 - \frac{M_{\ell^{+}}^{2}}{M_{D_{(s)}}^{2}}\right)^{2} \)

**Prospects for Measurements of Vus, Vcd and Vcs at Belle II**

\( |V_{cd}| \) via leptonic decay \( D^{+} \rightarrow \ell^{+}\nu \)

**Belle II Prospects**

\( D^{+} \rightarrow \mu^{+}\nu \)

- **MC Studies**
  - Belle II MC: 5.5 ab\(^{-1}\)

- **Method**
  - \( e^{+}e^{-} \rightarrow c\bar{c} \rightarrow \bar{D}X_{frag}D^{*+} (\rightarrow D^{+}\pi_{0}^{\text{slow}}) \mu^{+}\nu_{\mu} \)
  
  - Fit to missing mass \( \bar{D}X_{frag}\mu\pi_{0}^{\text{slow}} \)
  
  - Require 1 charged track from IP and with \( \mu \)-ID requirement

**MC Simulation [5.5 ab\(^{-1}\)]**

**Belle II (50 ab\(^{-1}\))**

- \( D^{+} \rightarrow \mu^{+}\nu_{\mu} \): inclusive (exclusive) decays \( \sim 3.5 \times 10^{6} \) (1250)

- Statistical error on \( \delta(f_{D^{+}} | V_{cd} |) = 0.65 \) MeV (which currently dominates in WA)

  ~ improved by factor of 2 w.r.t. to current measurement from CLEOc (1.9) and BESIII (1.2)

  ~ also competitive to BESIII plans with 20 fb-1 (~current x7) planned over next two years

**Prospects for Measurements of Vus, Vcd and Vcs at Belle II**

CKM 2021 25 Nov 2021
| $V_{cd}$ | via semi-leptonic decay: $D^0 \to \pi^- \ell^+ \nu$

decay modes: $\pi \nu \& \pi \mu \\

**Current status**

- Several results in past years by BaBar, Belle, BESIII and CLEO-c
  - Form factors $f_{\pi K}^+(0) = 0.612 \pm 0.035$ (ETM 17D, 18)
  - $|V_{cd}| = 0.2249 \pm 0.0028$(exp.) $\pm 0.0055$(LQCD)
    less precision $>2%$

**Belle II prospects**

- Belle II MC studies with 1 ab$^{-1}$ (method discussed at slide: #11)
  — missing mass resolution is comparable with Belle
- with 50 ab$^{-1}$ data-set
  — larger sample $\sim 7 \times 10^5$ (projected w/ BaBar analysis) of $D_s \to \pi \ell \nu$
    $\sim$ reduced stat. error

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Propects for measurements of Vus, Vcd and Vcs at Belle II

CKM 2021

25 Nov 2021

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SuperKEKB and Belle II provide an excellent platform for charm/τ measurements

– a good start ..

_world’s best: D^0 decay time resolution (x2 better than that of Belle/BaBar)
– more exciting results to come soon with larger luminosity in coming years.

**CKM parameters with full 50 ab^{-1}**

- \(|V_{us}| \) (from \(\tau\))
  - Belle II will provide an important insight to the current discrepancy of \(|V_{us}|\) from kaon decays and \(\tau\) decays (also inclusive vs exclusive)
  - also will add important input to the current 3σ \(|V_{ud}| - |V_{us}|_K\) anomaly

- \(|V_{cs}| \) and \(|V_{cd}| \) (from charm)
  - Statistically improved results from leptonic and semi-leptonic \(D/D_s\) decays
  - Belle II will also measure \(|V_{cd}|\) from \(D^+ \rightarrow \mu^+\nu\) decays (first attempt in B-factory)
Thank you
The CKM Matrix

- In SM: the coupling of the quarks via the charged weak current is described by CKM matrix

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- **3x3 unitarity complex matrix**
  - Unitarity constraints + freedom to redefine the complex phase (∼ 4 parameters == 3 mixing angle and 1 phase ⇒ CPV)

- **with Wolfenstein parameterization**
  - $\lambda = \sin(\theta_C) = 0.22$
  - $V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$

- **unitarity triangles**
  - $V_{qq'}V_{qq'}^\dagger = V_{qq'}^\dagger V_{qq'} = 1$
  - $q \neq q'$: 6 triangle relations ($\sum 3$ complex number = 0)
  - $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$

Prospects for measurements of Vus, Vcd and Vcs at Belle II