Review on latest results on $R(D^{(*)})$ & Outlook

Minisymposium on Precision Measurements with Leptons

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\[ R = \frac{b \rightarrow q \tau \bar{\nu}_\tau}{b \rightarrow q \ell \bar{\nu}_\ell} \]

\[ R(D^{(*)}, \pi, J/\psi) \]

1. How do we measure?

2. Latest measurements from Belle & LHCb

3. Outlook for Belle II & LHCb

\[ \Delta \chi^2 = 1.0 \text{ contours} \]

~ 3.1σ Tension

Measurements
Measurement Strategies

1. Leptonic or Hadronic $\tau$ decays?

Some properties (e.g. $\tau$ polarisation) readily accessible in hadronic decays.

2. Albeit not necessarily a rare decay of $O(\%)$ in BF, TRICKY to separate from normalisation and backgrounds

**LHCb:** Isolation criteria, displacement of $\tau$, kinematics

**B-Factories:** Full reconstruction of event (Tagging), matching topology, kinematics
3. **Semileptonic decays at B-Factories**

- $e^+/e^-$ collision produces $Y(4S) \rightarrow B\overline{B}$

- Fully reconstruct one of the two B-mesons (‘tag’) \(\rightarrow\) **possible to assign all particles** to either signal or tag B

- **Missing four-momentum** (neutrinos) can be reconstructed with high precision

\[
p_{\text{miss}} = (p_{\text{beam}} - p_{B\text{tag}} - p_{D^{(*)}} - p_\ell)
\]

✓ **Small efficiency** (~0.2-0.4%) **compensated by large integrated luminosity**
4. **Semileptonic decays at LHCb**

- No constraint from beam energy at a hadron machine, **but..**

- **Large Lorentz boost** with decay lengths in the range of **mm**
  
  ☑ Well-separated decay vertices
  
  ☑ Momentum direction of decaying particle is well known

- With known masses and other decay products can even **reconstruct four-momentum transfer squared** $q^2$ up to a two-fold ambiguity

\[
q^2 = (p_{X_b} - p_{X_q})^2
\]

Even bit more complicated for leptonic tau decays
Latest $R(D^{(*)})$ from Belle

- Reconstruct one of the two $B$-mesons (‘tag’) in semileptonic modes → possible to assign all particles in detector to tag- & signal-side

- Demand Matching topology + unassigned energy in the calorimeter $E_{\text{ECL}}$ to discriminate background from signal

$$E_{\text{extra}} = E_{\text{ECL}} = \sum_i E_{\gamma i}$$

![Graph showing signal and normalization processes](image_url)

Separation of signal & normalization

- Use kinematic properties to separate $B \rightarrow D^{(*)}\tau \nu$ signal from $B \rightarrow D^{(*)}\ell \nu$ normalization

- Construct BDT with 3 variables: $\cos \theta_{B-D^{(*)}\ell}$, $E_{\text{vis}}$, $m_{\text{miss}}^2 = p_{\text{miss}}^2$
Separation of signal & normalization

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![Graph showing $D^{*} + \ell$ separation with BDT](image)

**Table I.** Systematic uncertainties contributing to the total PDF uncertainty due to calibration factors. To validate the calibration factors, we build candidates using MC samples, and perform a fit projections and data points with statistical uncertainties in the reference in 2D comparisons, together with the most recent Belle results on the branching fraction of $D^{(*)}$ decays. The uncertainties on the branching fraction of $D^{(*)}$ decays significantly to the total PDF uncertainty due to which are not well known and hence contribute significantly.

$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$

$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$

*Most precise measurement to date*
LHCb Measurement of $R(D^*)$  

- Tau reconstructed via $\tau \rightarrow \pi^+ \pi^+ \pi^- (\pi^0) \nu$, only two neutrinos missing

Although a semileptonic decay is studied, nearly no background from $B \rightarrow D^* X \mu \nu$

- Main background: prompt $X_b \rightarrow D^* \pi \pi \pi + \text{neutrals}$

BF $\sim$ 100 times larger than signal, all pions are promptly produced

- Suppressed by requiring minimum distance between $X_b$ & $\tau$ vertices ($>4 \sigma_{\Delta z}$)

$\sigma_{\Delta z}$: resolution of vertices separation

- Reduces this background by three orders of magnitude
**LHCb Measurement of** $R(D^*)$

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- **Remaining double charm bkgs:**

  - $X_b \to D^* D_s^+ X \sim 10 \times \text{Signal}$
  - $X_b \to D^* D^+ X \sim 1 \times \text{Signal}$
  - $X_b \to D^* D_{s0}^+ X \sim 0.2 \times \text{Signal}$

  **Reduces this background by three orders of magnitude**
Remaining backgrounds reduced via isolation & MVA

Require signal candidates to be **well isolated**

i.e. reject events with extra charged particles pointing to the B and/or $\tau$

Events with additional neutral energy are suppressed with a MVA

More information about that in backup
LHCb Measurement of $R(D^*)$

- Extraction in 3D fit to MVA: $q^2 : \tau$ decay time
  - Invariant masses of $3\pi$ system
  - Invariant mass of $D^*3\pi$ system
  - Neutral isolation variables
  - Both reconstructed with some tricks (more in backup)
  - 4 Bins 8 Bins 8 Bins

- Components:
  1. Signal component for $\tau \to \pi^+\pi^+\pi^- (\pi^0)\nu$
  11. Background components

- ~ 1296 ± 86 Signal events
- Using normalisation mode and light lepton BFs:
  - More information about normalization in backup

$R(D^*) = 0.286 \pm 0.019$ (stat) $\pm 0.025$ (syst) $\pm 0.021$ (norm)
and the systematic uncertainty associated with measurement of (R) is straightforward. However, it becomes unclear such correlations. A correlation model will not be able to properly quantify validated or constrained by control regions. Thus, a simple

\[
\frac{R(D)}{R(D)_{SM}} = 0.299 \pm 0.003 \\
\frac{R(D^*)}{R(D^*)_{SM}} = 0.258 \pm 0.005
\]

HFLAV arithmetic average of SM Calculations

More Recent SM Calculations:
BaBar B \rightarrow D^*  
https://arxiv.org/abs/1004.0002  
- R(D^*)=0.253\pm0.005

Gambino, Jung, Schacht using Belle 2019 data  
- R(D^*)=0.254\pm0.007\pm0.006

Bordone, Jung, van Dyk using Belle 2019 data  
https://arxiv.org/abs/1908.00388  
- RD=297\pm0.003, RD^*=0.250\pm0.003

See also: https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html

Note that there is a difference in stat. coverage for the 2D (39.3%) versus 1D measurements (68.3%)
The hadronic form factors are, in some measurements, valuated as

\[
\mathcal{R}(D^{(*)}) = \frac{\mathcal{R}(D^{(*)})_{SM}}{\mathcal{R}(D^{(*)})_{SM}} = 0.299 \pm 0.003
\]

\[
\mathcal{R}(D^{(*)})_{SM} = 0.258 \pm 0.005
\]

The uncertainties in the predictions of these background contributions vary considerably. As discussed in (left), we show the dependence of the world average assuming such correlation and obtain \(\hat{\rho}_{D^{(*)}}\) of 3.2 standard deviations taking into account the small uncertainties of the theoretical predictions.

**VI.B Combination and Interpretation of the Results**

In the individual measurements, \(\mathcal{R}(D^{(*)})\) is treated as a free, but constrained, parameter of the average (see main text for more details).

The measurements of \(\mathcal{R}(D^{(*)})\) are estimated with different assumptions for the unknown correlation and obtain \(\hat{\rho}_{D^{(*)}}\) ranging from 0.258 to 0.32, resulting in compatibilities with the SM of 3.6 standard deviations (close to the value quoted by (left))

\[
\mathcal{R}(J/\psi) = 0.71 \pm 0.25
\]

\[
\mathcal{R}(J/\psi)_{SM} = 0.2582 \pm 0.0038
\]

Outlook

Much larger LHCb & Belle II data sets are coming soon

Will push precision of measured ratios considerably

LHCb

Belle II

several ab⁻¹ &
several 10 x fb⁻¹
Outlook

Novel ideas are emerging how to make best use of the available data

Model-independent interpretations via angular analyses or direct determinations of NP couplings

LHCb

Belle II

More Slides
Meet the “Measurement Matrix”

<table>
<thead>
<tr>
<th></th>
<th>Hadronic or inclusive tagging</th>
<th>SL tagging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptonic</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Hadronic</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>

\[ q^2 = (p_B - p_{D^*})^2 \]

\[ P_{D^*} \quad P_{\ell} \]

\[ D \quad D^* \quad \pi \]

Polarisation

LHCb:
- Phys.Rev.Lett.120,171802 (2018) (D*, Hadronic \(\tau\))

Belle:
- Phys. Rev. D 97, 012004 (2018) (D* had tag)
- Phys.Rev.D 92, 072014 (2015) (D/D* had tag, \(q^2\))
- Phys.Rev. D94,072007 (2016) (D*, SL tag, \(p_{D^*}\), \(p_\ell\))
- Phys.Rev.Lett.120,171802 (2018)

BaBar:
- Phys.Rev.D 88, 072012 (2013) (D/D* had tag, \(q^2\))


Belle:
- Phys.Rev. D 93, 032007 (2016)
- Phys.Rev. D 94,072007 (2016) (\(\pi\) had tag)

& older work, e.g.
Key elements of the update (details still under study):

Belle II Germany Meeting, Sep. 14th, 2020:

- Keep essential investments for upgrade of Linac, Belle II
- Partial upgrade of RF power (2 stations)
- Modify QCS
- Aim at an ecological operation by limiting running cost
- Store beam currents of LER 2.8A and HER 2.0A
- Enlarge radius of QCS beam pipes
- Luminosity priority on integrated luminosity, rather than peak

Fatal quenches
- Protect QCS against off-orbit particles
- Mitigate beam-beam effect in high bunch-current regime

L_{\text{peak}}: 8 \times 10^{35} \text{cm}^{-2}\text{s}^{-1} (requires redesign of RVC)

Updated Luminosity Projection

Run 1 2011 2012
Run 2 2013 2014
Run 3 2015 2016
Run 4 2017 2018
Run 5 2019 2020
Run 6 2021 2022

Updated plan for SuperKEKB submitted to the MEXT Roadmap Committee

Goal: prepare LoI's by end of 2020

- Improve performance
- Increase resilience against background

Opportunity for detector upgrade in 2026

- Increase resilience against background
- Improve performance

Polarization and/or luminosity upgrades?
Limiting Systematics

<table>
<thead>
<tr>
<th>Result</th>
<th>Experiment</th>
<th>$\tau$ decay</th>
<th>Tag</th>
<th>MC stats</th>
<th>Systematic uncertainty [%]</th>
<th>Total uncert. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{R}(D)$</td>
<td>$\text{BABAR}^a$</td>
<td>$\ell\nu\nu$</td>
<td>Had.</td>
<td>5.7</td>
<td>2.5</td>
<td>5.8</td>
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<tr>
<td></td>
<td>$\text{Belle}^b$</td>
<td>$\ell\nu\nu$</td>
<td>Semil.</td>
<td>4.4</td>
<td>0.7</td>
<td>0.8</td>
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<tr>
<td></td>
<td>$\text{Belle}^c$</td>
<td>$\ell\nu\nu$</td>
<td>Had.</td>
<td>4.4</td>
<td>3.3</td>
<td>4.4</td>
</tr>
<tr>
<td>$\mathcal{R}(D^*)$</td>
<td>$\text{BABAR}^a$</td>
<td>$\ell\nu\nu$</td>
<td>Had.</td>
<td>2.8</td>
<td>1.0</td>
<td>3.7</td>
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<td></td>
<td>$\text{Belle}^b$</td>
<td>$\ell\nu\nu$</td>
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<td>2.3</td>
<td>0.3</td>
<td>1.4</td>
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<tr>
<td></td>
<td>$\text{Belle}^c$</td>
<td>$\ell\nu\nu$</td>
<td>Had.</td>
<td>3.6</td>
<td>1.3</td>
<td>3.4</td>
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<tr>
<td></td>
<td>$\text{Belle}^d$</td>
<td>$\pi\nu, \rho\nu$</td>
<td>Had.</td>
<td>3.5</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>$\text{LHCb}^e$</td>
<td>$\pi\pi(\pi^0)\nu$</td>
<td>—</td>
<td>4.9</td>
<td>4.0</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>$\text{LHCb}^f$</td>
<td>$\mu\nu\nu$</td>
<td>—</td>
<td>6.3</td>
<td>2.2</td>
<td>2.1</td>
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</tbody>
</table>
## Latest $R(D^{(*)})$ from Belle: Systematics

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$B \to D^{**} \bar{\nu}_\ell$</td>
<td>PDF modeling</td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{R}(D)$</td>
<td>$\epsilon_{\text{sig}}/\epsilon_{\text{norm}}$</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total systematic</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total statistical</td>
<td>12.1</td>
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<td></td>
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<tr>
<td>$\mathcal{R}(D^*)$</td>
<td>$\epsilon_{\text{sig}}/\epsilon_{\text{norm}}$</td>
<td>4.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>Total</td>
<td>8.1</td>
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</table>
## LHCb Measurement of $R(D^*)$: Systematics

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Uncertainty [%]</th>
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</thead>
<tbody>
<tr>
<td>Double-charm bkg.</td>
<td>5.4</td>
</tr>
<tr>
<td>Simulated sample size</td>
<td>4.9</td>
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<tr>
<td>Corrections to simulation</td>
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<tr>
<td>$B \rightarrow D^{**}l\nu$ bkg.</td>
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<tr>
<td>Normalization yield</td>
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<tr>
<td>Trigger</td>
<td>1.6</td>
</tr>
<tr>
<td>PID</td>
<td>1.3</td>
</tr>
<tr>
<td>Signal FFs</td>
<td>1.2</td>
</tr>
<tr>
<td>Combinatorial bkg.</td>
<td>0.7</td>
</tr>
<tr>
<td>Modeling of $\tau$ decay</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Total systematic</strong></td>
<td><strong>9.1</strong></td>
</tr>
<tr>
<td>$\mathcal{B}(B \rightarrow D^*\pi\pi\pi)$</td>
<td>3.9</td>
</tr>
<tr>
<td>$\mathcal{B}(B \rightarrow D^*\ell\nu)$</td>
<td>2.3</td>
</tr>
<tr>
<td>$\mathcal{B}(\tau^+ \rightarrow 3\pi\nu)/\mathcal{B}(\tau^+ \rightarrow 3\pi\pi^0\nu)$</td>
<td>0.7</td>
</tr>
<tr>
<td><strong>Total external</strong></td>
<td><strong>4.6</strong></td>
</tr>
<tr>
<td><strong>Total statistical</strong></td>
<td><strong>6.5</strong></td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>12.0</strong></td>
</tr>
</tbody>
</table>
LHCb Measurement of $R(D^*)$

- Actually measure BF relative to $B^0 \rightarrow D^{*\pi^+\pi^+\pi^-}$

$$K_{had}(D^*) = \frac{BR(B^0 \rightarrow D^{*\tau^+\nu_\tau})}{BR(B^0 \rightarrow D^{*\pi^+\pi^-\pi^+})} = \frac{N(B^0 \rightarrow D^{*\tau^+\nu_\tau})}{N(B^0 \rightarrow D^{*\pi^+\pi^-\pi^+})} \times \frac{1}{BR(\tau^+ \rightarrow \pi^+\pi^+\pi^+\nu_\tau)} \times \frac{\epsilon(B^0 \rightarrow D^{*\pi^-\pi^+\pi^-})}{\epsilon(B^0 \rightarrow D^{*\tau^+\nu_\tau})}$$

- Measured to about 4% precision

Most precise measurement from BaBar: Phys. Rev. D94 (2016) 091101

- Dedicated control samples for remaining backgrounds

$X_b \rightarrow D^{*-}D_s^{+}X$  ---  Use $D_s^{+} \rightarrow 3\pi$ and fit $m(D^{*}D_s)$ to constrain individual contributions

$X_b \rightarrow D^{*-}D^{+}X$  ---  Use $D^{+} \rightarrow K3\pi$ to correct $q^2$, but float in fit

- Extraction in 3D maximum likelihood fit

  to MVA : $q^2 : \tau$ decay time

  Invariant masses of 3$\pi$ system

  Invariant mass of $D^{*}3\pi$ system

  Neutral isolation variables
LHCb Measurement of \( R(D^*) \): \( q^2 \) & \( \tau \) decay time

4-fold ambiguity:

\[
|\vec{p}_\tau| = \frac{(m_{3\pi}^2 + m_\tau^2)|\vec{p}_{3\pi}| \cos \theta \pm E_{3\pi} \sqrt{(m_\tau^2 - m_{3\pi}^2)^2 - 4m_\tau^2|\vec{p}_{3\pi}|^2 \sin^2 \theta}}{2(E_{3\pi}^2 - |\vec{p}_{3\pi}|^2 \cos^2 \theta)}
\]

\[
|\vec{p}_{B^0}| = \frac{(m_{D^{*\tau}}^2 + m_{B^0}^2)|\vec{p}_{D^{*\tau}}| \cos \theta' \pm E_{D^{*\tau}} \sqrt{(m_{B^0}^2 - m_{D^{*\tau}}^2)^2 - 4m_{B^0}^2|\vec{p}_{D^{*\tau}}|^2 \sin^2 \theta'}}{2(E_{D^{*\tau}}^2 - |\vec{p}_{D^{*\tau}}|^2 \cos^2 \theta')}
\]

Can be approximated by doing:

\[
\theta_{max} = \arcsin \left( \frac{m_{\tau}^2 - m_{3\pi}^2}{2m_\tau|\vec{p}_{3\pi}|} \right) \quad \theta'_{max} = \arcsin \left( \frac{m_{B^0}^2 - m_{D^{*\tau}}^2}{2m_{B^0}|\vec{p}_{D^{*\tau}}|} \right)
\]

Possible to reconstruct rest frame variables such as tau decay time and \( q^2 \).
These variables have negligible biases, and sufficient resolution to preserve good discrimination between signal and background.
LHCb Measurement of $R(D^*)$: Control samples

Use exclusive $D_s \rightarrow 3\pi$ decays to select a $X_b \rightarrow D^- D_s^+ X$ control sample.

Determine the different $X_b \rightarrow D^*^- D_s^+ X$ contributions from a fit to $m(D^* Ds)$:

- $B^0 \rightarrow D^* D_s$, $B^0 \rightarrow D^* D_s^*$, $B^0 \rightarrow D^* D_{s0}^*$, $B^0 \rightarrow D^* D_{s1}'$, $B_s \rightarrow D^* D_s X$, $B \rightarrow D** D_s X$

only 20% of $D_s$ originates directly from $B$, 40% originates from $D_s^*$, 40% from $D_{s**}$.

- Uncertainties in the fit parameters propagated to final analysis.

LHCb-PAPER-2017-017
**LHCb Measurement of** $R(D^*)$: **Control samples**

$X_b \to D^* - D^0 X$ decays can be isolated by selecting exclusive $D^0 \to K - 3\pi$ decays (kaon recovered using isolation tools).

A correction to the $q^2$ distributions is applied to the Monte Carlo to match data.

In contrast to the $D_{s}^+$ case, most $3\pi$ final states in $D^+$ and $D^0$ decays originate from $D^{+,0} \to K^{0,+} 3\pi$.

For the $D^0$, the inclusive 4 prongs BR constrains strongly the rate of $3\pi$ events.

Unfortunately, this constraint does not exist for the $D^+$ mesons, $K3\pi\pi^0$ is poorly known, the inclusive BR is not measured.

We let the $D^+$ component float in the fit.