$B^+ \rightarrow \ell^+ \nu_\ell \gamma$ at Belle and prospects of $B^+ \rightarrow \ell^+ \nu_\ell(\gamma)$

6th KEK Flavor Factory Workshop
Felix Metzner | 15th February 2019
Introduction

- B meson pairs are produced at the $\Upsilon(4S)$ resonance with no additional particles
- Measurement of missing energy modes possible
- New *tagging algorithm* for Belle II developed\(^a\)
- Opposite B meson can now be reconstructed with higher efficiency compared to the Belle approach
- New method applied to (converted) Belle MC/data and later Belle II
- Update of the Belle hadronically tagged $B^+ \to \ell^+ \nu_\ell \gamma$ analysis\(^b\)
- Determination of the first inverse moment $\lambda_B$ of the light-cone distribution amplitude of the B meson

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This Analysis: M. Gelb, F. U. Bernlochner, P. Goldenzweig, F. Metzner *et al.*

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The Decay $B^+ \rightarrow \ell^+ \nu_\ell \gamma$

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{em} G_F^2 m_B^4 |V_{ub}|^2}{48\pi^2} x_\gamma^3 (1 - x_\gamma) [F_A^2 + F_V^2]$$

with $x_\gamma = 2E_\gamma/m_B$

Previous Belle result (2015):
$$\Delta B(B^+ \rightarrow \ell^+ \nu_\ell \gamma) < 3.5 \cdot 10^{-6}$$

Form Factors (valid for large photon energies)

$$F_V(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B} R(E_\gamma, \mu) + \left[ \xi(E_\gamma) + \frac{Q_b m_B f_B}{2E_\gamma m_b} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} \right]$$

$$F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B} R(E_\gamma, \mu) + \left[ \xi(E_\gamma) - \frac{Q_b m_B f_B}{2E_\gamma m_b} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} + \frac{Q_\ell f_B}{E_\gamma} \right]$$

Upper limit Belle 2015

$E_\gamma > 1.7$ GeV

$E_\gamma > 1$ GeV

Method | $\lambda_B$ (GeV)
--- | ---
QCD factorization | $\approx 0.2$
QCD sum rules | $0.46 \pm 0.11$
BaBar (2009)$^a$ | $> 0.115$
Belle (2015)$^b$ | $> 0.238$


Analysis Strategy

**Reconstruction & Selection**

\[ B^+ \rightarrow \ell^+ \nu_\ell \gamma \]

+ Full Event Interpretation

**Background Suppression**

- Multivariate Methods
  - \[ B^+ \rightarrow \pi^0 \ell^+ \nu_\ell \]
  - \[ B^+ \rightarrow \eta \ell^+ \nu_\ell \]
  - \[ e^+ e^- \rightarrow q\bar{q} \]

**Control Region**

\[ B^+ \rightarrow \pi^0 \ell^+ \nu_\ell \]

**Signal Extraction**

Likelihood Fit

\[ \lambda_B \text{ Determination} \]

\[ B^+ \rightarrow \ell^+ \nu_\ell \gamma \] at Belle and prospects of \[ B^+ \rightarrow \ell^+ \nu_\ell (\gamma) \] - Felix Metzner
Analysis of missing energy mode relies on
- reconstruction of second $B_{\text{tag}}$ meson
- in hadronic decay channels

Inclusive Tag
$\epsilon = O(100)\%$
Consistency of $B_{\text{tag}}$

Semileptonic Tag
$\epsilon = O(1)\%$
Knowledge of $B_{\text{tag}}$

Hadronic Tag
$\epsilon = O(0.1)\%$
Exact knowledge of $B_{\text{tag}}$
The Tagging Algorithm: Full Event Interpretation

- Hierarchical reconstruction of $B_{\text{tag}}$ with a network of classifiers
- Successor of the Belle Full Reconstruction (FR)
- Training and application
- Hadronic and semi-leptonic tag modes
- **Generic FEI:**
  1) FEI trained and applied on full event
  2) Signal selection
- **Signal-specific FEI (new):**
  1) Signal selection
  2) FEI trained and applied on **rest-of-event** → trained on specific event topology
- Each $B_{\text{tag}}$ candidate has an assigned probability $P_{\text{FEI}}$

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**B–Tagging efficiency $\epsilon$ on MC**

<table>
<thead>
<tr>
<th>Tag</th>
<th>FR$^a$</th>
<th>gen. FEI Belle</th>
<th>gen. FEI Belle II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hadronic $B^+$</td>
<td>0.28%</td>
<td>0.76%</td>
<td>0.66%</td>
</tr>
<tr>
<td>SL $B^+$</td>
<td>0.67%</td>
<td>1.80%</td>
<td>1.45%</td>
</tr>
<tr>
<td>Hadronic $B^0$</td>
<td>0.18%</td>
<td>0.46%</td>
<td>0.38%</td>
</tr>
<tr>
<td>SL $B^0$</td>
<td>0.63%</td>
<td>2.04%</td>
<td>1.94%</td>
</tr>
</tbody>
</table>

$a$: Belle Full Reconstruction algorithm.

$B^+ \rightarrow \ell^+ \nu_\ell \gamma$ at Belle and prospects of $B^+ \rightarrow \ell^+ \nu_\ell (\gamma)$ - Felix Metzner
Calibration of the Tagging Algorithm

Why calibration?
Difference in tagging efficiency on data and MC:
- Hadronic branching ratios
- Dynamics of hadronic decays
- Detector simulation
- ...

Procedure
1) Reconstruct $B_{\text{sig}}$ in well-known channel
2) Apply tagging algorithm
3) Extract the number of events on MC and data via a fit of the $M^2_{\text{miss}}$ distribution
4) Calculate the correction factor for calibration channel:

$$\epsilon = \frac{N_{\text{Data}}}{N_{\text{MC}}}$$

$$\epsilon = 0.825 \pm 0.014 \pm 0.049$$

$\epsilon$ incorporates all corrections on the tag-side $B_{\text{tag}}$.
Missing Mass — MC Expectation

Signal simulated with $\Delta B(B^+ \rightarrow \ell^+\nu\gamma)|_{E_\gamma > 1.0\,\text{GeV}} = 5 \times 10^{-6}$

Increased signal reconstruction efficiency by a factor of 3 compared to previous Belle analysis without increasing the background.
Improved Measurement Strategy

Improved measurement strategy

To constrain the peaking background from $B^+ \to \pi^0 \ell^+ \nu_\ell$ decays in the analysis we fit an additional sample of reconstructed $B^+ \to \pi^0 \ell^+ \nu_\ell$ decays.

We have two samples:

- $B^+ \to \ell^+ \nu_\ell \gamma$ selection (nominal analysis)
- $B^+ \to \pi^0 \ell^+ \nu_\ell$ selection (control region)

In addition we can use the extracted $\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu_\ell)$.

Two parameters

Measure two quantities:

$$\Delta \mathcal{B}(B^+ \to \ell^+ \nu_\ell \gamma)_{E_\gamma > 1.0 \text{ GeV}} \quad \text{and} \quad R_\pi = \frac{\Delta \mathcal{B}(B^+ \to \ell^+ \nu_\ell \gamma)_{E_\gamma > 1.0 \text{ GeV}}}{\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu_\ell)} \quad (1)$$

This allows to extract $\lambda_B$ independent of $|V_{ub}|$. In addition, some systematics cancel in the ration $R$. 

Fit on Data

**Fit on Data**

\[ B^+ \rightarrow e^+\nu_e \gamma \]

\[ B^+ \rightarrow \mu^+\nu_\mu \gamma \]

\[ B^+ \rightarrow \pi^0\ell^+\nu_\ell \]

\[ B^+ \rightarrow \pi^0\ell^+\nu_\ell \gamma \]

\[ \ell \rightarrow B(B^+ \rightarrow \pi^0\ell^+\nu_\ell)(10^{-5}) \sigma \Delta B(B^+ \rightarrow \ell^+\nu_\ell\gamma)(10^{-6}) \sigma \]

<table>
<thead>
<tr>
<th>\ell</th>
<th>( B(B^+ \rightarrow \pi^0\ell^+\nu_\ell)(10^{-5}) )</th>
<th>( \sigma )</th>
<th>( \Delta B(B^+ \rightarrow \ell^+\nu_\ell\gamma)(10^{-6}) )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>( 8.3^{+0.9}_{-0.8} \pm 0.9 )</td>
<td>8.0</td>
<td>( 1.7^{+1.6}_{-1.4} \pm 0.7 )</td>
<td>1.1</td>
</tr>
<tr>
<td>\mu</td>
<td>( 7.5 \pm 0.8 \pm 0.6 )</td>
<td>9.6</td>
<td>( 1.0^{+1.4}_{-1.0} \pm 0.4 )</td>
<td>0.8</td>
</tr>
<tr>
<td>e, \mu</td>
<td>( 7.9 \pm 0.6 \pm 0.6 )</td>
<td>12.6</td>
<td>( 1.4 \pm 1.0 \pm 0.4 )</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Previous results for** \( B^+ \rightarrow \pi^0\ell^+\nu_\ell \)

<table>
<thead>
<tr>
<th>( B(B^+ \rightarrow \pi^0\ell^+\nu_\ell)(10^{-5}) )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle excl.(^a)</td>
<td>( 8.0 \pm 0.8 \pm 0.4 )</td>
</tr>
<tr>
<td>PDG</td>
<td>( 7.80 \pm 0.27 )</td>
</tr>
</tbody>
</table>

Limit Calculation

Bayesian Limit

\[ 0.9 = \frac{\int_0^{\Delta B_{\text{limit}}} L_{\text{PDF}}(\Delta B) \, d\Delta B}{\int_0^\infty L_{\text{PDF}}(\Delta B) \, d\Delta B} \]

<table>
<thead>
<tr>
<th>(\ell)</th>
<th>(\Delta B(B^+ \to \ell^+ \nu_\ell \gamma) \times 10^{-6}) limit (%90) C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar (2009)(^a)</td>
<td>Belle (2015)(^b)</td>
</tr>
<tr>
<td>e</td>
<td>-</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-</td>
</tr>
<tr>
<td>e, (\mu)</td>
<td>&lt; 14</td>
</tr>
</tbody>
</table>

Limits are estimated with total systematic error.

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\(B^+ \to \ell^+ \nu_\ell \gamma\) at Belle and prospects of \(B^+ \to \ell^+ \nu_\ell (\gamma)\) - Felix Metzner

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## Systematics

<table>
<thead>
<tr>
<th>Source</th>
<th>Multiplicative</th>
<th>Additive</th>
<th>$\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu_{\ell})$ in $10^{-5}$</th>
<th>$\Delta \mathcal{B}(B^+ \to \ell^+ \nu_{\ell} \gamma)$ in $10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\bar{B}B}$</td>
<td>Common</td>
<td>Reconstructed Tag Channel</td>
<td>±0.11</td>
<td>±0.02</td>
</tr>
<tr>
<td>LID Efficiency</td>
<td>Common</td>
<td>Peaking Background BDT</td>
<td>±0.16</td>
<td>±0.02</td>
</tr>
<tr>
<td>Tracking Efficiency</td>
<td>Common</td>
<td>PDF Templates</td>
<td>±0.03</td>
<td>±0.00</td>
</tr>
<tr>
<td>Calibration</td>
<td>Specific</td>
<td>B $\to X_u \ell^+ \nu_{\ell}$</td>
<td>±0.49</td>
<td>±0.09</td>
</tr>
<tr>
<td>Reconstruction Efficiency</td>
<td>Specific</td>
<td>Signal Model</td>
<td>±0.20</td>
<td>±0.01</td>
</tr>
</tbody>
</table>

**Combined**

<table>
<thead>
<tr>
<th>Source</th>
<th>$\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu_{\ell})$ in $10^{-5}$</th>
<th>$\Delta \mathcal{B}(B^+ \to \ell^+ \nu_{\ell} \gamma)$ in $10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>±0.62</td>
<td>±0.36</td>
</tr>
</tbody>
</table>

Systematic uncertainties are directly incorporated into the likelihood.
Extraction of $\lambda_B$

$$R_\pi = \frac{\Delta B(B^+ \rightarrow \ell^+ \nu_\ell \gamma)}{B(B^+ \rightarrow \pi^0 \ell^+ \nu_\ell)} = \frac{\Delta \Gamma(\lambda_B)}{\Gamma(B^+ \rightarrow \pi^0 \ell^+ \nu_\ell)}$$

$$R_\pi^{\text{measured}} = (1.7 \pm 1.4) \times 10^{-2}$$

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_B$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>$0.36^{+0.25+0.03}_{-0.08-0.03}$</td>
</tr>
<tr>
<td>Model II</td>
<td>$0.38^{+0.25+0.05}_{-0.06-0.08}$</td>
</tr>
<tr>
<td>Model III</td>
<td>$0.32^{+0.24+0.05}_{-0.07-0.08}$</td>
</tr>
</tbody>
</table>

based on theoretical input from:
Beneke et al., JHEP 07:154 (2018)

Result of Belle (2015) was $\lambda_B > 0.238$ GeV

$\lambda_B > 0.24$ GeV @ 90% C.L.
Prospects of $B^+ \to \ell^+ \nu \ell \gamma$ at Belle II

Analysis is **statistically limited**.

⇒ **Extrapolation for Belle II:**
  - scale statistical uncertainty with luminosity: $\sqrt{711 \text{ fb}^{-1}} / \mathcal{L}$
  - unchanged central value
  - unchanged systematic uncertainty
Prospects of $B^+ \rightarrow \ell^+ \nu_\ell \gamma$ at Belle II

$\Rightarrow$ Estimate improved statistical uncertainties for the full analysis

$\Rightarrow$ Propagate results to $V_{ub}$ and $\lambda_B$
$B^+ \rightarrow \mu^+ \nu_\mu$ at Belle

and

Prospects for Belle II
$B^+ \to \mu^+ \nu_\mu$ at Belle

**Rare decay** with SM expectation of

$$\mathcal{B}(B^+ \to \mu^+ \nu_\mu) \approx \mathcal{O}(10^{-7}).$$

Latest result (Belle 2018):

$$\mathcal{B}(B^+ \to \mu^+ \nu_\mu) = (6.46 \pm 2.22 \pm 1.6) \times 10^{-7}$$


⇒ Requires high reconstruction efficiency!

Clean signature of **two-body decay** of $B_{\text{sig}}$

⇒ $p^B_\mu = m_B/2 \approx 2.64$ GeV

in the $B_{\text{sig}}$ rest frame

⇒ Experimental access to CKM matrix element $V_{ub}$

⇒ Sensitive to New Physics (e.g. 2HDM, Sterile Neutrinos)
$B^+ \rightarrow \mu^+ \nu_\mu$ at Belle

To provide a sufficiently high reconstruction efficiency a **inclusive B-tagging** algorithm is applied.

Knowledge of the $B_{\text{tag}}$ meson’s momentum allows to boost into the $B_{\text{sig}}$ rest frame.

- **Inclusive Tag**
  - Efficiency $\epsilon = \mathcal{O}(100\%)$
  - Consistency of $B_{\text{tag}}$

- **Semileptonic Tag**
  - Efficiency $\epsilon = \mathcal{O}(1\%)$
  - Knowledge of $B_{\text{tag}}$

- **Hadronic Tag**
  - Efficiency $\epsilon = \mathcal{O}(0.1\%)$
  - Exact knowledge of $B_{\text{tag}}$
Prospects of $B^+ \rightarrow \mu^+ \nu_\mu$ at Belle II

Based on these new results we make predictions for the relative uncertainties of $B(B^+ \rightarrow \mu^+ \nu_\mu)$ and $V_{ub}$ for this statistically limited decay mode, assuming

- unchanged central values and
- 3% irreducible systematic uncertainty

by scaling reducible $\sigma$ with $\sqrt{711\ \text{fb}^{-1}/L}$ and propagating the effect to $V_{ub}$. 

| $B^+ \rightarrow \ell^+ \nu_\ell \gamma$ at Belle and prospects of $B^+ \rightarrow \ell^+ \nu_\ell (\gamma)$ | Felix Metzner | 15th February 2019 | 19/21 |
Summary

- First application of (signal-specific) FEI.
- Improved upper 90% C.L. limit for $B^+ \rightarrow \ell^+ \nu_\ell \gamma$
- Improved method for $\lambda_B$ extraction!

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$\Delta \mathcal{B}(B^+ \rightarrow \ell^+ \nu_\ell \gamma)$ limit ($10^{-6}$) @90% C.L.</th>
<th>BaBar (2009)</th>
<th>Belle (2015)</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>-</td>
<td>&lt; 6.1</td>
<td>&lt; 4.3</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>-</td>
<td>&lt; 3.4</td>
<td>&lt; 3.4</td>
<td></td>
</tr>
<tr>
<td>e, $\mu$</td>
<td>&lt; 14</td>
<td>&lt; 3.5</td>
<td>&lt; 3.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda_B$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD factorization</td>
</tr>
<tr>
<td>QCD sum rules</td>
</tr>
<tr>
<td>BaBar</td>
</tr>
<tr>
<td>Belle (2015)</td>
</tr>
<tr>
<td>This work</td>
</tr>
</tbody>
</table>
By utilizing the new Belle II software and the B2BII conversion package, we can
- still squeeze out new results from the Belle data set and
- get the analysis software warmed up for Belle II data.

Thank You for Your attention!
Backup — Fit on Data for $B^+ \rightarrow \pi^0 \ell^+ \nu_\ell$

$B^+ \rightarrow \pi^0 e^+ \nu_e$

$B^+ \rightarrow \pi^0 \mu^+ \nu_\mu$

B$^+ \rightarrow \ell^+ \nu_\ell \gamma$ at Belle and prospects of $B^+ \rightarrow \ell^+ \nu_\ell (\gamma)$.  

Felix Metzner 15th February 2019