Precision measurements of the CKM parameters  
(mainly $\gamma/\phi_3$ measurements)

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Abstract. The CKM angle $\gamma/\phi_3$ is the only one that is accessible with  
tree level decays in a theoretically clean way such that it provides a  
precision test of $CP$ violation in the standard model. The Belle II experiment  
is a substantial upgrade of the Belle detector and will operate at the  
SuperKEKB asymmetric-energy $e^+e^-$ collider. The accelerator has already successfully completed the first phase of commissioning in 2016  
and first $e^+e^-$ collisions in Belle II happened during April 2018. The  
design luminosity of SuperKEKB is $8 \times 10^{35}$ cm$^{-2}$s$^{-1}$ and the Belle II experiment aims to record 50 ab$^{-1}$ of data, a factor of 50 more than its predecessor (Belle). The key method to measure $\phi_3$ is through interference between the $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$ decays which occurs if the final state of the charm-meson decay is accessible to both the $D^0$ and $\bar{D}^0$ mesons. To achieve the best sensitivity, a large variety of $D$ and $B$ decay modes are required, which is possible at the Belle II experiment as almost any final state can be reconstructed including those with photons. With the ultimate Belle II data sample of 50 ab$^{-1}$, a determination of $\phi_3$ with a precision of 1$^\circ$ or better is foreseen. We explain herein the details of the planned measurements at Belle II.

Keywords: CKM matrix, $\gamma$, $\phi_3$, Belle II

1 Introduction

At present, the current uncertainty on $\gamma/\phi_3$ is approximately 5$^\circ$, still worse by a factor 10 with respect to $\phi_1 = (21.9 \pm 0.7)^\circ$ [1]. From CKMfitter [1], we find that the uncertainties on the CKM parameters measured from tree-level processes are larger than those from measurements of loop level diagrams. Thus, it is important to reduce the error on $\phi_3$ to test the validity of the standard model (SM). One of the reasons for this is the relative small branching fraction of the decays involved in the measurement owing to non-diagonal CKM matrix elements, since

$$\phi_3 \equiv \arg \left( \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right),$$

(1)
where $V_{ij}$ is the weak vertex factor for a quark transition $i \to j$. Thus, with more data, we can improve the precision of the measurement, with the uncertainty being dominated by the available statistics.

The angle $\phi_3$ can be extracted via interference between the color-favored $B^- \to D^0 K^-$ and color-suppressed $B^- \to D^0 K^-$ decays that are shown in Fig. 1. These are pure tree-level processes, hence theoretically clean. The correction to these processes is only $O(10^{-7})$ [2]. If the amplitude for the color-favored decay is $A$, then for the color-suppressed one, it is $A r_B e^{i(\delta_B - \phi_3)}$, where $\delta_B$ is the strong phase difference between the decay processes, and

$$r_B \equiv \left| \frac{A_{\text{sup}}}{A_{\text{fav}}} \right|.$$  \hspace{1cm} (2)

For $B \to DK$ decays, $r_B \sim 0.1$, whereas for $B \to D\pi$, $r_B \sim 0.05$. Though $B \to D\pi$ decays are not very sensitive to $r_B$ and $\phi_3$, we can use them as the control sample for $B \to DK$ to eliminate most of the systematic uncertainties.

![Fig. 1. Color-favored (left) and -suppressed (right) $B^- \to DK^-$ processes.](image)

The remainder of this document is structured as follows: Sec. 2 describes the methods and constraints to extract $\phi_3$, Sec. 3 describes the potential $\phi_3$ sensitivity from the Belle II experiment, as well as preliminary results from the Belle II phase 2 data. Section 4 gives the summary.

## 2 Methods for $\phi_3$ extraction

We classify the methods used to extract $\phi_3$ according to the $D$ meson final state:

(i) **GLW** [3] method: $CP$ eigenstates such as $K^+ K^-$, $\pi^+ \pi^-$, $K_S^0 \pi^0$.

(ii) **ADS** [4] method: doubly-Cabibbo-suppressed states such as $K^+ X^-$, where $X^-$ can be $\pi^-, \pi^0, \pi^- \pi^+, \pi^- \pi^0$.

(iii) **GGSZ** [5] method: self-conjugate multibody states such as $K_S^0 \pi^+ \pi^-$, $K^+_S K^+ K^-$, $K_S^0 \pi^+ \pi^- \pi^0$.

In both GLW and ADS methods, $\phi_3$-sensitive parameters can be extracted by taking a ratio between the suppressed and favored decay rates and a measurement of asymmetries between them. We obtain four GLW parameters $R_{CP}^{\pm} = 1 + r_B^2 \pm 2r_B \delta_B \cos \phi_3$ and $A_{CP}^{\pm} = \pm 2r_B \sin \delta_B \sin \phi_3 / R_{CP}^{\pm}$, and two ADS parameters $R_{\text{ADS}} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3$ and $A_{\text{ADS}} = 2r_B r_D \sin(\delta_B \delta_D) \sin \phi_3 / R_{\text{ADS}}$.
for the $\phi_3$ extraction. Here, $r_D$ and $\delta_D$ are the ratio of the amplitudes of the suppressed and favored $D$ decays, and the $D$ strong phase, respectively. These are external inputs from charm measurements.

An inclusive approach leads to almost zero sensitivity for the GGSZ modes. Thus, we bin the Dalitz space into regions with differing strong phases, which allows $\phi_3$ to be determined from a single channel in a model-independent manner. This eliminates the model-dependent systematic uncertainty in the measurement. Fraction of $D^0$ and $\bar{D}^0$ events in bin $i$, called $K_i$ and $\bar{K}_i$, can be obtained from $D^{\pm\pm} \rightarrow D\pi^\pm$ decays at $B$-factories, which reduces the statistical uncertainty thanks to their large data samples. But, we need the information of these strong phases, $c_i$ and $s_i$, as external inputs from the charm factory experiments CLEO-c or BESIII, where the quantum-entangled $D^0\bar{D}^0$ pairs are produced via $e^+e^-\rightarrow \psi(3770)\rightarrow D^0\bar{D}^0$. Here, $c_i$ and $s_i$ correspond to the amplitude weighted average cosine and sine of the strong phase difference between $D^0$ and $\bar{D}^0$ decay in the $i^{th}$ bin. An optimal binning scheme is needed to obtain the maximal sensitivity. Figure 2 shows the Dalitz plot and the $c_i$ and $s_i$ values for the golden mode $B^{\pm} \rightarrow D(K^{0}_{S}\pi^{+}\pi^{-})K^{\pm}$ [6].

![Fig. 2. Binned Dalitz plot and corresponding $c_i$, $s_i$ values for $D \rightarrow K^{0}_{S}\pi^{+}\pi^{-}$ decays.](image)

The Belle combined measurement is $\phi_3 = (73.4^{+13.0}_{-15.2})^\circ$ [1], which is dominated by the GGSZ method. Similarly, the BaBar Collaboration combined all their measurements to give a value $\phi_3 = (69.2^{+17.0}_{-16.0})^\circ$ [7]. The LHCb Collaboration finds $\phi_3 = (74.0^{+5.0}_{-5.8})^\circ$ [8] by combining all their measurements. While $B$-factories used their full data sets, LHCb results are based on their Run I data. Combining these three results, we obtain $\phi_3 = (73.5^{+4.2}_{-5.1})^\circ$ [1], which is currently dominated by the results from LHCb.

### 3 Sensitivity from the Belle II experiment

The Belle II experiment will start collecting data from early 2019 with all its sub-detectors. It will accumulate 50 ab$^{-1}$ data, about 50 times that of its predecessor,
with an instantaneous luminosity of $8 \times 10^{35}$ cm$^{-2}$s$^{-1}$. Belle II will also have better $K/\pi$ separation capability with the central drift chamber, imaging time-of-propagation and ring imaging Cherenkov counters, which work in different $K/\pi$ momentum ranges. An improved $K^0_S$ reconstruction efficiency is expected, mainly due to the larger acceptance of the silicon vertex detector. All this will result in a substantially improved precision measurements. More details can be found in Ref. [9].

Currently, the $\phi_3$ sensitivity is dominated by the statistical uncertainty from the number of reconstructed $B$ decays. Thus, by going from a Belle integrated luminosity of 711 fb$^{-1}$ to 50 ab$^{-1}$, the sensitivity is expected to get a significant boost. The major background is continuum events, coming from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) due to their large cross-section. As signal hides behind this large background, we need to eliminate the latter as much as we can to get a better sensitivity. The aim is to reach a precision of 1° with the full data sample, combining improvements obtained from $K/\pi$ separation, $K^0_S$ reconstruction, and continuum suppression. This would allow us to probe for possible new physics contributions, that can potentially cause a shift in the value of $\phi_3$ by $\pm 4^\circ$ [10]. We perform a toy study with the golden mode $B^\pm \rightarrow D (K^0_S \pi^+ \pi^-) K^\pm$, which results in the expected sensitivity as a function of integrated luminosity in Fig. 3.

By adding more modes, such as $B \rightarrow D^* K$, $B^\pm \rightarrow D (K^0_S K^+ K^-) K^\pm$, we can further improve the sensitivity and approach the 1° precision.

![Fig. 3. $\phi_3$ sensitivity with the amount of data collected at Belle II. The red arrow indicates the expected sensitivity from the 50 ab$^{-1}$ sample.](image_url)

Recently, Belle II successfully recorded data with all but for the vertex detector during April-May 2018. It has accumulated a data sample corresponding to an integrated luminosity of 472 pb$^{-1}$. Figure 4 shows a comparison between data and Monte Carlo (MC) simulations for the reconstruction of $K^0_S$ candidates.
in Belle II. The invariant mass resolution is already showing a good agreement between data and MC events.

![Fig. 4. Comparison of $K_S^0$ reconstruction in Belle II data and MC events.](image1)

We perform the reconstruction of $D^{\pm} \rightarrow D\pi^{\pm}, D \rightarrow K_S^0\pi^+\pi^-$ and the CP mode $D^0 \rightarrow K_S^0\pi^0$, which are shown in Fig. 5. The plotted variables are $M_i$ ($i = K_S^0\pi^+\pi^-$ or $K_S^0\pi^0$), where $M_i$ is the invariant mass of the final state $i$, and $\Delta M$, the reconstructed mass difference between $M(i)\pi^\pm$ and $M_i$. The signal-to-background ratio is already good in both cases, and the reconstruction of $D \rightarrow K_S^0\pi^0$ shows the capability of Belle II for neutral particle reconstruction.

![Fig. 5. $M_i$ (left) and $\Delta M$ (right) distributions for the $D^*$ tagged modes $D \rightarrow K_S^0\pi^+\pi^-$ (top) and $D \rightarrow K_S^0\pi^0$ (bottom).](image2)
We perform the “rediscovery” of the $B$ meson from these data. We have accumulated about 100 $B$ candidates in which the majority are from the $B^\pm \to D\pi^\mp$ mode, which is the control channel for the $\phi_3$ extraction; corresponding distributions of fit variables are shown in Fig. 6. The variables are the energy difference $\Delta E = E_B^* - E_{beam}^*$ and the beam-energy constrained mass $M_{bc} = \sqrt{(E_{beam}^*/c^2)^2 - (p_B^*/c)^2}$, where $E_B^*$ and $p_B^*$ are the energy and momentum of the $B$ meson candidate, and $E_{beam}^*$ is the beam energy, all calculated in the center-of-mass frame.

![Fig. 6. $\Delta E$ (left) and $M_{bc}$ (right) distributions for various $B$ modes.](image)

4 Summary

At Belle and Belle II, the GGSZ method will have the largest impact on the $\phi_3$ sensitivity. Simulation studies show that a precision of 3° is achievable in Belle II with the $D \to K^0_S\pi^+\pi^-$ mode alone, even without the full benefit of particle identification, $K^0_S$ finding and continuum suppression. By combining it with all other modes and improved reconstruction algorithm, we can go closer to the goal of 1° precision. Preliminary results from the recently concluded run of Belle II without the vertex detector show promising results.

References

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