

$B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ at the
Belle Experiment: Branching Fraction
Measurements and Forward-Backward
Asymmetry in Helicity Angle

by

Kim Smith

ORCID: [0000-0003-0446-9474](https://orcid.org/0000-0003-0446-9474)

*Submitted in total fulfilment of the requirements
of the degree of Doctor of Philosophy*

September 2024

School of Physics

THE UNIVERSITY OF MELBOURNE

Abstract

This thesis presents the measurement of the forward-backward asymmetry in the helicity angle of the hadronic B decay $B^0 \rightarrow D^- \pi^+ \pi^0$ and the branching fractions of $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$. The analysis is performed on a dataset of 570 fb^{-1} collected at the $\Upsilon(4S)$ resonance by the KEKB asymmetric e^+e^- collider and collected by the Belle detector.

The analysis methodology was developed and validated using Monte Carlo simulated samples, in which the signal reconstruction and background suppression was optimised in a fully blind way. To account for the non-uniform relationship between the reconstruction efficiency and the 3-body final state kinematics, an efficiency matrix in bins of the cosine of the helicity angle and the invariant mass of the $\pi^+ \pi^0$ was employed. A series of extended maximum likelihood fits in these bins was performed to extract the results from the data.

This analysis was first performed on a partial dataset of 74 fb^{-1} before proceeding with the full 570 fb^{-1} dataset. The measured values in partial data are:

$$\begin{aligned}\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0) &= (9.95 \pm 0.43 \pm 0.56) \times 10^{-3} \\ \mathcal{B}(B^0 \rightarrow D^- \rho^+) &= (7.87 \pm 0.37 \pm 0.44) \times 10^{-3} \\ A_{FB}^{\pi\pi}(B^0 \rightarrow D^- \pi^+ \pi^0) &= -0.028 \pm 0.005 \pm 0.051\end{aligned}$$

where the first uncertainties are statistical and the second are systematic. The results in full data are:

$$\begin{aligned}\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0) &= (8.92 \pm 0.23 \pm 0.57) \times 10^{-3} \\ \mathcal{B}(B^0 \rightarrow D^- \rho^+) &= (7.91 \pm 0.21 \pm 0.47) \times 10^{-3} \\ A_{FB}^{\pi\pi}(B^0 \rightarrow D^- \pi^+ \pi^0) &= -0.048 \pm 0.002 \pm 0.047\end{aligned}$$

This constitutes the world's first measurement of the branching fraction and helicity asymmetry of the $B^0 \rightarrow D^- \pi^+ \pi^0$, and the most precise measurement of the $B^0 \rightarrow D^- \rho^+$ branching fraction.

Declaration of Authorship

I, Kim Smith, declare that this thesis titled, ‘ $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ at the Belle Experiment: Branching Fraction Measurements and Forward-Backward Asymmetry in Helicity Angle’ and the work presented in it are my own. I confirm that:

- The thesis comprises only my original work towards the Doctor of Philosophy except where indicated in the preface;
- Due acknowledgement has been made in the text to all other material used; and
- The thesis is fewer than the maximum word limit in length (100,000 words), exclusive of tables, maps, bibliographies and appendices as approved by the Research Higher Degrees Committee.

Kim Smith

Preface

Due to a range of factors including the cost of resources, scale of data acquisition, and interdisciplinary knowledge required, experimental particle physics in the modern day cannot be done in isolation. Large scientific bodies, such as the Belle collaboration, provide analysts the access to resources and specialised expertise they need to conduct their research.

- **Chapter 1** is an original introduction to the Standard Model of particle physics, as well as a review of the theory behind the forward-backward asymmetry in the helicity angle formulated by Tobias Huber, Javier Virto and K. Keri Vos.
- **Chapter 2** is an original summary of the Belle detector technical specifications.
- **Chapters 3-6** constitute an original analysis of the $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$ decays under the supervision of Prof. Martin Sevier.

Unless otherwise stated, all work presented here is the original work of the author.

Acknowledgements

First and foremost, I would like to give my deepest thanks to my project supervisor Martin Sevier, who I could always rely on to have my back and steer me in the right direction when the going got tough. I don't think anyone could've predicted the challenges I would face when starting this Ph.D, but Martin's patience, insights, and encouragement gave me the energy to see it through to the end.

To my friends and colleagues, I give my thanks for keeping me grounded in reality when my head was swimming in simulations. Particular thanks goes to Chia-Ling for always being there to provide assistance on esoteric errors, and to Tristan whose experience with decade old code bases proved invaluable.

Finally, to my dearest parents, none of this would have been possible without your unwavering support (and commiserations!) over all these years.

Contents

1	Introduction	1
1.1	The Standard Model	1
1.2	B Physics	4
1.3	Charge-Parity Violation and the Unitarity Triangle	4
1.4	$B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$	6
1.5	Theoretical Motivation	8
1.5.1	SCET & QCD Factorisation	8
1.5.2	Differential Decay Rate	9
1.5.3	Helicity Angle	11
1.6	Statement of Intention	12
2	The Belle Experiment	13
2.1	The KEKB Accelerator	13
2.2	The Belle Detector	14
2.2.1	Beam Pipe	14
2.2.2	Silicon Vertex Detector - SVD	16
2.2.3	Central Drift Chamber - CDC	17
2.2.4	Electromagnetic Calorimeter - ECL	19
2.2.5	Aerogel Cherenkov Counter - ACC	21
2.2.6	Time-of-Flight Detector - TOF	22
2.2.7	Extreme Forward Calorimeter - EFC	23
2.2.8	K_L^0 Muon Detector - KLM	24
3	$B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ Analysis Strategy	27
3.1	Event Selection	27
3.1.1	MC Matching and Truth Information	28
3.1.2	Dataset Skimming	28
3.1.3	Data Samples	29
3.1.4	Event Reconstruction	30
3.1.5	π^0 Momentum Correction	32
3.2	Background Studies	34
3.2.1	Self-crossfeed Background	34

3.2.2	Continuum Background	36
3.2.3	Generic $B\bar{B}$ Background	39
3.2.4	$D\pi^0$ Resonances	41
3.3	Best Candidate Selection	43
3.4	Efficiencies & Expected Yields	44
3.5	Signal Extraction	46
3.5.1	Fitting Strategy	46
3.5.2	sPlot Considerations	48
3.5.3	Bin Event Migration	49
3.5.4	Reconstruction Efficiency Matrix	52
3.5.5	PDF Modelling	55
3.6	Fitter Tests	57
3.6.1	ΔE Fits Validation	57
3.6.2	$A_{FB}^{\pi\pi}$ Extraction Validation	58
3.6.3	$B^0 \rightarrow D^- \pi^+ \pi^0$ Yield Extraction Validation	60
3.6.4	$B^0 \rightarrow D^- \rho^+$ Yield Extraction Validation	63
4	$B^+ \rightarrow \bar{D}^0 \pi^+$ Control Sample Studies	65
4.1	Event Selection	65
4.1.1	Data Samples	65
4.1.2	Event Reconstruction	66
4.1.3	π^0 Momentum Correction	67
4.2	Background Studies	68
4.2.1	Self-crossfeed Background	68
4.2.2	Continuum Background	69
4.2.3	Generic $B\bar{B}$ Background	69
4.3	Best Candidate Selection	71
4.4	Reconstruction Efficiencies	71
4.5	Fitting Strategy	73
4.6	Comparison with Data	77
5	Systematic Uncertainties	83
5.1	Charged Track Efficiency	84
5.2	π^0 Reconstruction Efficiency	85
5.3	Number of $B\bar{B}$ Pairs	85
5.4	Value of R_{BB}	86
5.5	Particle Identification Efficiency	86
5.6	Continuum Suppression	87
5.7	ΔE PDF Modelling	88

5.8	ΔE Fitter Bias	91
5.9	Rare Generic $B\bar{B}$	92
5.10	Bin Event Migration	92
5.11	Efficiency Matrix	93
5.12	$M_{\pi\pi}$ Fitter Bias	94
5.13	Summary of Systematic Uncertainties	94
6	Results and Conclusions	96
6.1	Box Open Result - $B^0 \rightarrow D^- \pi^+ \pi^0$ Branching Fraction	96
6.2	Box Open Result - $A_{FB}^{\pi\pi}$ Helicity Asymmetry	97
6.3	Box Open Result - $B^0 \rightarrow D^- \rho^+$ Branching Fraction	98
A	Binned ΔE PDF Modelling for $B^0 \rightarrow D^- \pi^+ \pi^0$	100
B	Binned ΔE PDF Modelling for $B^+ \rightarrow \bar{D}^0 \pi^+$	103
C	Reconstruction Efficiency Update Formula	106
D	$A_{FB}^{\pi\pi}$ Statistical Uncertainty	108
	Bibliography	110

List of Figures

1.1	Latest plot of the Unitarity Triangle from the CKMfitter group showing the constraints in the complex plane. The red hashed region (around ϕ_2) of the global combination corresponds to 68% confidence limit.	7
1.2	Dominant Feynman diagrams for the (left) $B^0 \rightarrow D^- \rho^+$ decay, and (right) $B^0 \rightarrow D^- \pi^+ \pi^0$ decay.	8
1.3	Diagram showing the $B^0 \rightarrow D^- \pi^+ \pi^0$ decay in the (left) lab frame and the (right) $\pi^+ \pi^0$ reference frame.	12
2.1	A top-down diagram of the KEKB collider displaying the position of the LER and HER along with the placement of the interaction point [1]. . . .	15
2.2	Side view of the Belle detector.	16
2.3	Configuration of the SVD2 layers.	17
2.4	Top and side schematics of the CDC.	19
2.5	Side view of the ECL layout and end-caps.	20
2.6	Top and side schematics of the CsI(Tl) counter crystals.	21
2.7	Side-on diagram of the ACC barrel and end-cap regions with readout using fine-mesh photomultiplier tubes (FM-PMT).	22
2.8	Schematic of a barrel aerogel model (left) and end-cap module (right). . . .	22
2.9	Dimensions of the time-of-flight (TOF) counter modules and trigger scintillation counter (TSC).	23
2.10	Isometric diagram of the EFC crystal placement (left) and a side-on view of the mounting dimensions (right).	24
2.11	Breakdown of the layers of a single RPC stack in the KLM.	26
3.1	(left) Momentum dependence of kaon efficiency and pion fake rate. Circles correspond to kaon efficiency and the triangles to pion fake rate. (right) Likelihood ratio PID(K), versus momenta for daughter tracks from $D^0 \rightarrow K^- \pi^+$ decays, tagged by the charge of the slow π^+ s. The open circles correspond to kaons and the cross-points to pions.	31
3.2	Invariant mass of the D^- in $B^0 \rightarrow D^- \pi^+ \pi^0$ decays.	33
3.3	Comparison of ΔE and M_{bc} distributions for signal and continuum background.	33

3.4	A comparison of the $\Delta E - M_{bc}$ relation for uncorrected (left) and corrected (right) M_{bc} performed on signal MC. Correlation factors are 4.44% and 1.15% respectively.	34
3.5	Distribution of the (blue) ΔE signal distribution, the (red) SCF distribution comparing the impact of selection criteria on the SCF background.	35
3.6	Diagram of a (left) continuum event and a (right) $B\bar{B}$ event highlighting the strongly collimated jets for continuum and uniformly spherical $B\bar{B}$	36
3.7	FBDT value for signal and continuum background in both the training set and test set.	40
3.8	(left) The figure of merit (FoM) for the test set. (right) ROC curves for both the training and test sets. The area under the curve (AUC) for the training set is 0.930 and for the test set 0.926.	40
3.9	Distributions of (left) M_{bc} and (right) ΔE for $D^- \pi^+ \pi^0$ signal and generic $B\bar{B}$ background demonstrating the impact of cuts on generic $B\bar{B}$ and signal MC after having passed the initial skim given in Section 3.1.2. Cut values shown in dashed lines.	41
3.10	Distribution of the ΔM variable for a sample of generic $B\bar{B}$ which includes the signal channels. A veto is placed between 0.13 and 0.15 GeV/ c^2 to exclude the D^*	43
3.11	Dalitz plots of generic $B\bar{B}$. (left) $M_{D^- \pi^+}$ vs $M_{\pi^+ \pi^0}$ and (right) $M_{D^- \pi^0}$ vs $M_{\pi^+ \pi^0}$	43
3.12	Number of signal candidates per event prior to BCS in $B^0 \rightarrow D^- \pi^+ \pi^0$ MC with an average candidate multiplicity of 1.21.	45
3.13	Normalised ΔE distributions of (left) generic $B\bar{B}$ and (right) continuum backgrounds in MC with (red) and without (blue) BCS applied.	45
3.14	Normalised distributions of (left) $\cos \theta_h$ and (right) $M_{\pi\pi}$ for (blue) $B^0 \rightarrow D^- \pi^+ \pi^0$ and (red) $B^0 \rightarrow D^- \rho^+$ channels.	48
3.15	ΔE distributions in SCF MC, showing the large variation across phase-space.	49
3.16	Example toy fit to MC in $M_{\pi\pi}$ with (red) $D^- \pi^+ \pi^0$ and (blue) $D^- \rho^+$ signal channels.	50
3.17	Fraction of reconstructed signal candidates that are due to self-crossfeed background in each bin of $\cos \theta_h$ and $M_{\pi\pi}$	51
3.18	ΔE distributions for (blue) signal and (red) SCF MC for (left) $\cos \theta_h < -0.8$ and (right) $\cos \theta_h > 0.8$	51
3.19	Fraction of events that migrated out (left) and in (right) of each bin after reconstruction for pure signal MC sample.	52

3.20	$D^- \rho^+$ yield map prior to reconstruction (top left) and after reconstruction with the event migration correction applied (top right). Log projections of each variable are also given where an artificial scale factor has been applied to provide visual separation between the pre-reconstruction (red) and post-reconstruction + event migration correction (blue) distributions.	53
3.21	Efficiency matrix for pure signal derived from Belle generic MC which includes $D^- \pi^+ \pi^0$, $D^- \rho^+$ and all other signal channels known to Belle. . .	54
3.22	$\cos \theta_h$ distributions before applying efficiency matrix (left), after around the rho mass peak (middle), and after below the mass peak (right).	54
3.23	Full kinematic range ΔE PDF fits to each component. The PDF shapes are listed in Table IX. The χ^2 values given are per degree of freedom. . .	56
3.24	PDF binned fits to $M_{\pi\pi}$ for (left) $D^- \rho^+$ and (right) all other $D^- \pi^+ \pi^0$ final states. The χ^2 values given are per degree of freedom.	58
3.25	Average distributions of 20 toy ensemble fits comparing the fitted distributions (red points) to the true distributions in MC (blue).	59
3.26	Distributions of extracted $\cos \theta_h$ for (top left) $A_{FB}^{\pi\pi} = -0.115$, (top middle) $A_{FB}^{\pi\pi} = -0.057$, (top right) $A_{FB}^{\pi\pi} = 0.002$ MC samples, (bottom left) $A_{FB}^{\pi\pi} = 0.057$ and (bottom right) $A_{FB}^{\pi\pi} = 0.118$	60
3.27	Linearity test for the $A_{FB}^{\pi\pi}$ parameter with backgrounds included. The generated $A_{FB}^{\pi\pi}$ is the asymmetry present at generation time. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors.	61
3.28	Linearity test for the $B^0 \rightarrow D^- \pi^+ \pi^0$ yield performed on MC samples containing expected backgrounds. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors but may not be visible in this plot due to relative size.	62
3.29	Linearity test for the $B^0 \rightarrow D^- \rho^+$ yield extraction. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors.	64
4.1	Invariant mass distribution for D^0 in $B^+ \rightarrow \bar{D}^0 \pi^+$ prior to applying analysis selection cuts. Peak mean is at $1.862 \text{ GeV}/c^2$ in agreement with the PDG world average.	67
4.2	M_{bc} projections in bins of ΔE are shown to make the effect of the π^0 momentum correction apparent. The corrected and uncorrected distributions are overlaid in red and blue, respectively.	68
4.3	Distribution of the (blue) ΔE signal distribution and the (red) SCF distribution comparing the impact of selection criteria on the SCF background for $B^+ \rightarrow \bar{D}^0 \pi^+$	69

4.4	Comparison of the BDT output for both MC and continuum data. The continuum data was selected from the M_{bc} and ΔE ranges which has negligible contributions for signal and $B\bar{B}$	70
4.5	Distribution of the (blue) ΔE signal distribution and the (red) continuum distribution comparing the impact of the BDT cut before (left) and after (right) it is applied on the continuum background for $B^+ \rightarrow \bar{D}^0 \pi^+$	70
4.6	Distributions of (left) M_{bc} and (right) ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ signal and generic $B\bar{B}$ background. Cut values shown in dashed lines.	71
4.7	Number of signal candidates per event prior to BCS in $B^+ \rightarrow \bar{D}^0 \pi^+$ MC with an average candidate multiplicity of 1.187.	72
4.8	ΔE distributions for each component of the fit to $B^+ \rightarrow \bar{D}^0 \pi^+$. Continuum and generic $B\bar{B}$ components were modelled on 6 streams of Belle MC. Signal and SCF components were modelled on dedicated signal MC. The PDF shapes are listed in Table XVI. The χ^2 values given are per degree of freedom.	74
4.9	Fraction of reconstructed signal candidates that are due to self-crossfeed background in each bin of $\cos \theta_h$ and $M_{\pi\pi}$ as found in signal MC.	75
4.10	Fraction of events that migrated out (left) and in (right) of each bin after reconstruction for pure signal MC sample.	76
4.11	Comparison of the efficiency matrix for $B^+ \rightarrow \bar{D}^0 \pi^+$ modelled on signal MC with no modifications (left) and after applying the event migration correction (right). Weighted average efficiencies are 0.111 in both cases.	76
4.12	Linearity test for the $B^+ \rightarrow \bar{D}^0 \pi^+$ yield performed on MC samples containing expected backgrounds. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors but may not be visible in this plot due to relative size.	78
4.13	Total integrated unbinned fit to ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data.	80
4.14	Fits to ΔE in bins of $M_{\pi\pi}$ and $\cos \theta_h$ for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data. Bins shown here are in the $\cos \theta_h$ range 0.8 to 1.0 and range in $M_{\pi\pi}$ from 0.25 GeV/ c^2 to 1.40 GeV/ c^2 . The χ^2 values given are per degree of freedom.	81
4.15	Sum of fitted PDFs in bins of $\cos \theta_h$ and $M_{\pi\pi}$ to ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data.	82
5.1	Transverse momentum distribution for the π^+ daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle. The dashed line at 200 MeV/ c indicates the threshold for high momentum tracks [2].	84

5.2	Total momentum distribution for the π^0 daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle. Binned according to the divisions given in S. Ryu et al. [3].	86
5.3	Total momentum distribution for the π^+ daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle.	88
5.4	Data-MC discrepancy as a function of the signal rejection fraction.	89
5.5	Total systematic uncertainty of $B^0 \rightarrow D^- \pi^+ \pi^0$ in bins of $M_{\pi\pi}$ and $\cos \theta_h$. Quadratic sum of all uncertainties across all bins is 4.2%. For the $B^0 \rightarrow D^- \rho^+$ channel, the sum is done in bins below $1 \text{ GeV}/c^2$ in $M_{\pi\pi}$ which results in 3.6% uncertainty. Finally, the $A_{FB}^{\pi\pi}$ result is further restricted to the region $ \cos \theta_h < 0.8$, leading to a total of 2.8%.	90
5.6	Total systematic uncertainty of $B^+ \rightarrow \bar{D}^0 \pi^+$ in bins of $M_{\pi\pi}$ and $\cos \theta_h$. Quadratic sum of all uncertainties across all bins is 1.35%.	91
6.1	$\cos \theta_h$ distributions for (left) partial Belle data and (right) full Belle data.	99
6.2	ΔE distributions for (left) partial Belle data and (right) full Belle data. Data points and PDFs are the sum across all bin fits in $\cos \theta_h$ and $M_{\pi\pi}$	99
6.3	$M_{\pi\pi}$ distributions for (left) partial Belle data and (right) full Belle data. The χ^2 values given are per degree of freedom.	99
A.1	Sample of 20 ΔE PDFs for generic $B\bar{B}$ MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	100
A.2	Sample of 20 ΔE PDFs for continuum MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	101
A.3	Sample of 20 ΔE PDFs for self-cross feed MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	101
A.4	Sample of 20 ΔE PDFs for $B^0 \rightarrow D^- \pi^+ \pi^0$ signal MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	102
B.1	Sample of 20 ΔE PDFs for generic $B\bar{B}$ MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	103
B.2	Sample of 20 ΔE PDFs for continuum MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	104
B.3	Sample of 20 ΔE PDFs for self-cross feed MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	104
B.4	Sample of 20 ΔE PDFs for $B^0 \rightarrow D^- \pi^+ \pi^0$ signal MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.	105

List of Tables

I	Summary of the elementary particles that make up the Standard Model. Anti-matter partners of each particle have been omitted.	3
II	Comparison of branching fractions for each signal decay channel between Belle MC values (circa 2010) and current PDG world averages.	30
III	Composition of the SCF background from different sources. The proportion is the fraction of the total SCF background that contains each source. Decay in flight indicates that the B was reconstructed using a particle that was created in the detector during its flight. Missing neutrino, photon, massive FSPs and K_L s all indicate that these particle types were missed in the reconstruction of the B candidate. Misidentified FSP indicates that a FSP was assigned the wrong identity (e.g. confusion of $\pi^\pm$ s and $K^\pm$ s). Wrong daughter particle refers to instances where the B was incorrectly reconstructed using a daughter particle from the other B [4].	35
IV	Hyperparameter values used during training of the BDT in continuum suppression.	39
V	Kinematic cuts applied to reduce generic $B\bar{B}$ background.	41
VI	Top ten most abundant decay modes found within generic $B\bar{B}$ background for each of the two kinematic regions listed. This corresponds to the two peaking regions found in the Dalitz plots in Figure 3.11. . . .	42
VII	Performance of best candidate selection methods. Proportion of times the correct B candidate is chosen when presented with multiple candidates on each MC sample. Pure signal is a sample of MC in which the other B is forced to decay to a $\nu\bar{\nu}$ state, thus eliminating all potential cross-contamination from the other B	44
VIII	Expected yields for SVD2 Belle data. $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ yields are taken from generic Belle MC streams.	46
IX	ΔE PDF shapes for all components used in both fits to full kinematic range and bin-by-bin fits.	55

X	Results of the $A_{FB}^{\pi\pi}$ validation performed on MC samples containing expected backgrounds. Uncertainties in the third column are the standard error from 10 ensemble toy fits.	60
XI	Results of the $B^0 \rightarrow D^- \pi^+ \pi^0$ yield validation. The input yields are obtained by applying the event migration correction and efficiency matrix directly without performing any ΔE fits. The listed uncertainties are the standard error of the 10 toy fits.	62
XII	Results of the $B^0 \rightarrow D^- \rho^+$ yield validation. The input yields are obtained by taking the values from Table XI and scaling them by the relative branching fraction of the $B^0 \rightarrow D^- \rho^+$ decay found in generic MC. The listed uncertainties are the standard error of the 10 toy fits. . .	63
XIII	Comparison of branching fractions for each signal decay channel between Belle MC values (circa 2010) and current PDG world averages.	66
XIV	Kinematic cuts applied to $B^+ \rightarrow \bar{D}^0 \pi^+$ to reduce generic $B\bar{B}$ background.	71
XV	Expected yield for SVD2 Belle data. $B^+ \rightarrow \bar{D}^0 \pi^+$ yield calculated in an analogous way to Equation 3.13 using the most recent PDG world average result.	72
XVI	Fitted PDF shapes for control mode components.	73
XVII	Results of the $B^+ \rightarrow \bar{D}^0 \pi^+$ yield validation. The input yields are obtained by applying the event migration correction and efficiency matrix directly without performing any ΔE fits. The listed uncertainties are the standard error of the 10 toy fits.	77
XVIII	Expected yields from MC compared to extracted yields found in data for the two fit strategies.	77
XIX	PID correction factors for $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$	87
XX	Systematic uncertainties for the $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction measurements and $A_{FB}^{\pi\pi}$ observable in partial 74 fb^{-1} Belle data.	94
XXI	Systematic uncertainties for the $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction measurements and $A_{FB}^{\pi\pi}$ observable in 570 fb^{-1} SVD2 Belle data.	95
XXII	Systematic uncertainties for the $B^+ \rightarrow \bar{D}^0 \pi^+$ branching fraction measurement in 570 fb^{-1} SVD2 Belle data.	95

Chapter 1

Introduction

The Standard Model (SM) of particle physics is an established scientific theory that describes the behaviour of the fundamental particles and their interactions. Joseph Thomson's groundbreaking discovery of the electron in 1897 marked the beginning of the journey into subatomic physics, from the simple classical models of a few ball-like particles to the current quantum interpretations involving more than a dozen elementary particles. The construction of powerful detectors has allowed particle physicists to probe the structure of matter at the most extreme ends of the scales, both in terms of size and energy. These multi-billion-dollar detector experiments have provided crucial evidence in support of the SM as the most accurate model of particle physics, and as such has granted us a deeper understanding into the nature of the universe and its structure.

1.1 The Standard Model

The combined theories describing three of the four fundamental forces of nature and their interactions with elementary particles are collectively called the Standard Model and is among history's most rigorously tested scientific theories.

The fundamental particles that comprise the SM can be categorised at a high level by their spin quantum numbers that define the fermions and bosons. Bosons are force-carrying particles that mediate the interaction between particles and have either $+1$ spin (vector bosons) or 0 spin (scalar bosons). The fermions are characterised by their

half-integer spin and can be further broken down into the leptons and quarks, of which only the quarks interact with the strong nuclear force.

There are six flavours of quarks of increasing mass, with the lightest being the up (u) and down (d) types on the order of a few MeV, up to the heaviest top (t) quark at 173 GeV. These quarks bind together into composite particles known as hadrons, the most common of which are the mesons (quark-antiquark pairs) and baryons (three quarks). While mesons are all short-lived particles that have lifetimes no greater than 10^{-8} sec, there exist baryons that are, for all intents and purposes, completely stable. This includes the proton (uud) and the neutron (udd) which combine to give rise to all stable atoms and subsequent molecules. For largely historic reasons, the charges of the quarks are chosen so that the composite hadrons can be assigned integer values. Hence, the up, strange, and bottom quarks have $+2/3$ charge, and the down, charm, and top quarks have $-1/3$ charge. Furthermore, the phenomenon of colour confinement means that they can never be found in isolation. Instead, as the quarks are separated, the stored energy in the intermediary binding gluons eventually increases to a point where a quark-antiquark pair is produced resulting in two new hadrons, leading to large cascading showers of particles in high energy collisions.

The six quarks are accompanied by three charged and three neutral leptons of increasing mass. The charged leptons are the electron, muon and tauon, each of which has a corresponding neutral neutrino. Neutrinos have been experimentally found to undergo oscillations, in which they change their species, implying that they must possess some small non-zero mass. This observation is in conflict with the SM, which predicts no neutrino mass, and therefore constitutes an area of great interest for both experimentalists and theoreticians.

We now return to the bosons to discuss their properties. Within the SM, there is no description of gravity, and so only the electromagnetic force and the strong and weak nuclear forces are assigned corresponding gauge bosons. Electromagnetism is mediated by the photon, which is both massless and electrically neutral. Analogous to the electric charge is a property known as colour that is only possessed by gluons, the mediators of the strong nuclear force. In total there are eight differently coloured gluons all of which are massless. Lastly, the weak nuclear force has three massive bosons: W^+ , W^- and Z^0 , whose electric charges are indicated by their superscripts. In 2012 the Higgs boson (H^0)

TABLE I: Summary of the elementary particles that make up the Standard Model. Anti-matter partners of each particle have been omitted.

Fermions							
	Gen	Name	Symbol	Spin	Charge (e)	Mass (MeV)	Interactions Experienced
Leptons	1st	Electron	e^-	1/2	-1	0.511	EM, W
		Electron Neutrino	ν_e	1/2	0	$< 2.2 \times 10^{-6}$	W
	2nd	Muon	μ^-	1/2	-1	105.7	EM, W
		Muon Neutrino	ν_μ	1/2	0	< 0.19	W
	3rd	Tau	τ^-	1/2	-1	1.78×10^3	EM, W
		Tau Neutrino	ν_τ	1/2	0	< 18.2	W
Quarks	1st	Up	u	1/2	+2/3	2.16	EM, W, S
		Down	d	1/2	-1/3	4.67	EM, W, S
	2nd	Strange	s	1/2	+2/3	93	EM, W, S
		Charm	c	1/2	-1/3	1.27×10^3	EM, W, S
	3rd	Bottom	b	1/2	+2/3	4.18×10^3	EM, W, S
		Top	t	1/2	-1/3	172.9×10^3	EM, W, S
Bosons							
Vector		Photon	γ	1	0	0	EM
		Gluon	g	1	0	0	S
		W boson	$W^\pm$	1	± 1	80.4×10^3	EM, W
		Z boson	Z	1	0	91.2×10^3	W
Scalar		Higgs boson	H	0	0	125.25×10^3	Higgs Field

was discovered [5, 6], a new neutral boson which had been previously predicted in 1964 by three independent papers [7–9] as a mechanism to give mass to the gauge bosons. Due to the fact that the Higgs is not itself a gauge boson but is instead a scalar boson, it is therefore not a force carrier and hence does not mediate any force. For the prediction of the particle, Peter Higgs and Francois Englert were awarded the Nobel prize in 2013. Although there is no quantum (i.e. subatomic) theory of gravity contained within the SM, at large scales, General Relativity describes it accurately. A complete summary of the 17 elementary particles (not including their antimatter counterparts) is given in Table I.

In broad terms, this is the Standard Model as it currently stands. Despite its success in the subatomic realm [10–12], it has nothing to say on the existence of dark matter, dark energy, or baryonic matter-antimatter asymmetry throughout the observable universe and, as previously discussed, does not account for the neutrino mass.

1.2 B Physics

In the world of particle physics, the enigmatic B -meson plays a crucial role in unravelling the secrets of the universe. These mesons are defined by their possession of a valence b (bottom) quark and are exemplified by the $\bar{B}^0(\bar{d}b)$, $B^-(\bar{u}b)$, $\bar{B}_s^0(\bar{s}b)$, and $B_c^-(\bar{c}b)$.

When the B decays, the b quark transitions to a lower mass quark via the emission of a W boson. Among these decays, those leading to fully hadronic states are of particular interest, constituting approximately 75% of all B decays. This vast array of decays offers a plethora of opportunities to investigate various particle physics phenomena.

Detecting and studying B -mesons presents several challenges. Due to their large total kinematic phase space, the branching fractions of many B decays are quite small. Typical branching fractions are on the order of 10^{-3} , while rare decays can be as low as 10^{-6} . This necessitates the design of specialised physics detectors capable of handling high luminosity and achieving exceptional precision. To meet these demands, collaborations of Physicists and Engineers designed two $e^\pm$ and e^+e^- collider facilities and established the Belle and BaBar [13] experiments at KEK and SLAC, respectively. These collected data over the dates 1999 - 2010 and 1999 - 2008 respectively. Analyses, such as the one described in this thesis, from both experiments are ongoing. New experiments LHCb [14] and Belle II [15] employing even larger data samples continue to investigate B -meson decays.

1.3 Charge-Parity Violation and the Unitarity Triangle

The historical interest in B -mesons emerged in the early 1980s when theorists, Carter and Sanda [16] proposed that investigations into these particles had the potential to act as probes for the existence of charge-parity violation (CPV). This is a violation of the principle that particle interactions should proceed at the same rate when all internal quantum numbers are replaced with their charge conjugates (C transformation) as well as reversing the handedness/parity of all particles (P transformation). The presence of CPV is one of three conditions laid out by A. D. Sakharov [17] to explain the matter-antimatter asymmetry observed throughout our universe [18]. The conditions are as follows:

1. There must have been a presence of baryon number violating processes.
2. There must have been a presence of charge and charge-parity violating processes.
3. There must have been interactions occurring out of thermal equilibrium.

These three conditions are necessary, but may not be sufficient, to explain the current day baryon asymmetry. For example, if CP symmetry was not violated, one would expect the rate at which a B^0 decays to a $\pi^+\pi^-$ final state would be equal to the corresponding charge conjugated, parity flipped process, i.e.

$$\Gamma(B^0 \rightarrow \pi^+\pi^-) = \Gamma(\bar{B}^0 \rightarrow \pi^-\pi^+)$$

However, in 1964 James Cronin and Val Fitch observed that in neutral kaon decays, CP was being indirectly violated [19]. They observed that the long lived neutral kaon, K_L , which was believed to be a CP odd state underwent the decay

$$K_L \rightarrow \pi^+\pi^-$$

with a branching ratio of $\sim 2 \times 10^{-3}$, which is a pure CP even eigenstate, demonstrating the violation of CP symmetry in weak decays. Since then, CPV has been observed in the B system as well.

Within the framework of the SM, CPV can manifest in three places: the quark sector, the lepton sector and the strong sector. By studying the physics of B decays with B factories like the Belle detector, physicists have been able to probe the quark sector in order to precisely quantify the amount of CPV present. This is done by using a matrix to parametrize the strength of quark decays as they change flavour:

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \quad (1.1)$$

This is the Cabibbo-Kobayashi-Maskawa (CKM) matrix and its elements describe transitions of quarks of one flavour to another where the probability of such a transition is proportional to $|V_{ij}|^2$. It was proposed by Makoto Kobayashi and Toshihide Maskawa [20] as an extension to ideas developed by Nicola Cabibbo [21]. This requires three

generations, and predicts the existence of the charm quark as the partner to the strange quark, as well as the top and bottom quarks required to explain CPV. During quark transitions, the number of quarks is conserved which results in a unitarity condition that the CKM matrix obeys. This states that the matrix is equal to its own conjugate transpose, i.e. $V_{CKM}^* V_{CKM} = V_{CKM} V_{CKM}^* = I$ where I is the identity matrix. This gives rise to a set of six independent equations that must all be satisfied, where each one defines a triangle in the complex plane. By convention, the triangle defined by the equation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (1.2)$$

has been dubbed “The Unitarity Triangle” by particle physicists. Its three angles are labelled ϕ_1 , ϕ_2 and ϕ_3 using the Belle convention (β , α and γ respectively at BaBar) and are defined as

$$\phi_1 \equiv \text{Arg} \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \quad \phi_2 \equiv \text{Arg} \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \quad \phi_3 \equiv \text{Arg} \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right). \quad (1.3)$$

Each of the sides and angles of this triangle can be experimentally measured and so a natural test of the SM is to check that it closes in the complex plane. The latest results are compiled by the CKMfitter group [22] and a plot of the Unitarity Triangle with its current constraints is shown in Figure 1.1.

1.4 $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$

Decays involving b to c quark transitions (charm decays), dominate over transitions to other quarks and thus provide numerous opportunities for extensive physics analyses. Firstly, such studies play a crucial role in extracting CPV parameters, which are vital for understanding the violation of fundamental symmetries in particle interactions. Secondly, the determination of the CKM matrix, responsible for describing the mixing of quark flavours, relies on data obtained from these decays. Finally, B to charm decays are backgrounds when exploring rare B decays, such as $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ \pi^-$. Understanding and modelling the charm decay processes are thus essential for accurate analysis in the search for new physics beyond the standard model.

Despite the presence of the common $b \rightarrow c$ transition, the decay $B^0 \rightarrow D^- \pi^+ \pi^0$ has had no previously published branching fraction measurement. The most recent related

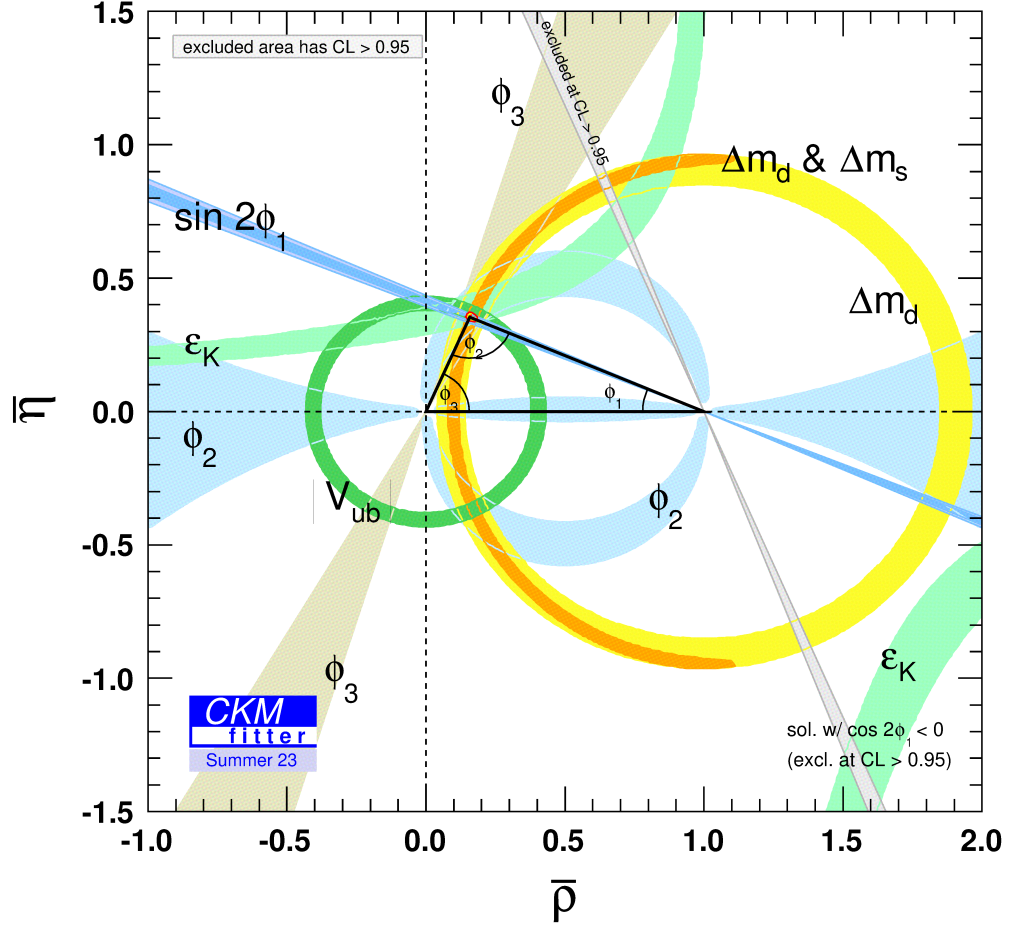


Figure 1.1: Latest plot of the Unitarity Triangle from the CKMfitter group showing the constraints in the complex plane. The red hashed region (around ϕ_2) of the global combination corresponds to 68% confidence limit.

measurement was performed by the CLEO collaboration in 1994 who studied a sample of 79 background subtracted events and found a branching ratio for the intermediate resonance state $B^0 \rightarrow D^- \rho^+$ [23] which the Particle Data Group (PDG) reports with a branching fraction of $(7.6 \pm 1.2) \times 10^{-3}$ [24]. It has theoretical interest for reasons beyond its unobserved status, including its relevance as a control mode in rare B to charmless hadronic decays such as $B^+ \rightarrow \rho^+ \rho^0$ [25]. The dominant Feynman diagrams for the decay of the B^0 to $D^- \pi^+ \pi^0$ and $D^- \rho^+$ are given in Figure 1.2. On the theory side, due to systematics in the three-body system, direct calculation of the branching fractions of these decays is either not possible or computationally intractable [26].

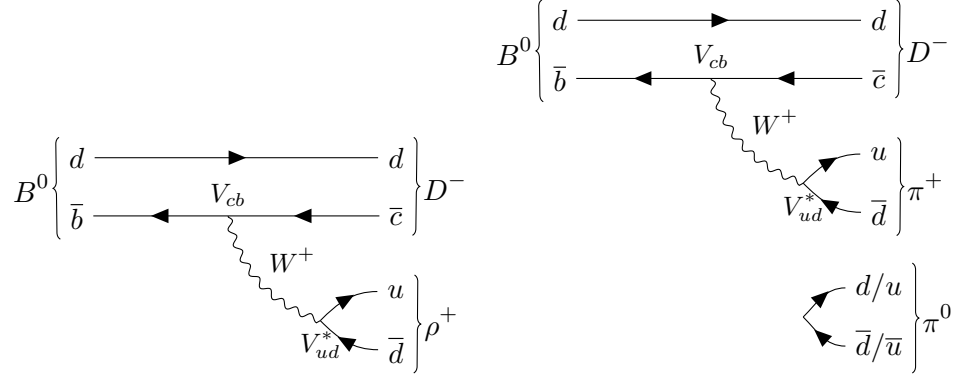


Figure 1.2: Dominant Feynman diagrams for the (left) $B^0 \rightarrow D^- \rho^+$ decay, and (right) $B^0 \rightarrow D^- \pi^+ \pi^0$ decay.

1.5 Theoretical Motivation

A study performed by T. Huber et. al [26] explored the applicability of the SCET/QCD factorisation method to heavy-to-heavy, three-body, non-leptonic B decays and presented a derivation of their decay amplitude factorisations up to next-to-next-to-leading order (NNLO). Additionally, they provide a discussion of certain observables which are dependent on perturbative higher-order corrections. These observables are constructed from ratios of Dalitz plot bins and can probe potential sources of isospin violating corrections. A full derivation of all the results of the paper are beyond the scope of this analysis, but a summary of its main results will be given as they will serve as key motivation.

1.5.1 SCET & QCD Factorisation

The combination of quantum mechanics, special relativity and classical field theories like electromagnetism gives rise to quantum field theory (QFT). Within the framework of QFT, particles are treated as excitations of an underlying quantum field and their interactions are governed by the Lagrangian which dictates their time evolution. Quantum chromodynamics (QCD) is a specific type of QFT that describes the strong interaction and explains how quarks interact through the exchange of gluons.

SCET (Soft-Collinear Effective Theory) and QCD factorisation are two theoretical frameworks used for making quantitative predictions of certain observables involving particles over largely separated energy scales. The two approaches are similar in that they both

employ QCD calculations performed with perturbation theory, a mathematical technique for finding approximate solutions to problems that do not permit exact methods. QCD factorisation, or QCDF, was developed for the computation of hadronic matrix elements which represent the transition amplitudes of a given initial state to a particular final state. These amplitudes encode information about the strong interaction between the quarks and gluons within the hadrons involved in the process, and when squared gives the probability per unit time for the initial state to decay to the final state.

By making use of the QCDF/SCET method, one can factorise the amplitudes in terms of local form factors, light-cone distribution amplitudes (LCDAs) and short distance coefficients which have been calculated up to next-to-next-to-leading order (NNLO) in QCD. In the calculations, the strength of the strong interaction is characterised by the dimensionless coupling constant α_s whose strength drops off at high energy, in a phenomenon known as asymptotic freedom. At low orders of α_s , one can factorise the matrix elements into a form factor multiplied by a decay constant. However, at higher orders this simple factorisation breaks down and more sophisticated techniques are required to find the corrections which may involve re-summing large logarithms, a process that the SCET formalism was, in part, designed to handle.

This approach has historically been confined to two-body decays but in recent times the extension to the three-body case has also been explored. This was motivated by data collected from experiments starting with CLEO, Belle and BaBar and more recently at Belle II and LHCb. T. Huber et. al [26] considers the theoretically simplest three-body case for heavy-to-heavy transitions using $B^0 \rightarrow D^- \pi^+ \pi^0$ as an example. Due to the similar energy scales of the B and D -mesons, the system can be treated as a two-body decay into a D and a resonant pair of light mesons and thus allows for the use of SCET/QCDF techniques.

1.5.2 Differential Decay Rate

We begin by defining two kinematic variables, k^2 the total momentum of the dimeson system, and θ_h the angle between the 3-momentum of the neutral pion and the B -meson in the dimeson rest frame. By writing the kinematic Källén functions λ_{ij} [27] in terms of these two variables, it is possible to parameterise the phase-space of the three-body

system, and therefore the decay amplitudes, entirely in (k^2, θ_h) . The differential decay rate can now be expressed in terms of these two variables.

$$\frac{d\Gamma(\bar{B} \rightarrow D^+ M^- \pi^0)}{dk^2 d\cos\theta_h} = \frac{\sqrt{\lambda_{BD}(k^2)\lambda_{M\pi}(k^2)}}{64(2\pi)^3 k^2 m_B^3} |\mathcal{A}(\bar{B} \rightarrow D^+ M^- \pi^0)|^2 \quad (1.4)$$

where the Källén functions are defined as $\lambda_{ij}(k^2) = (m_i^2 - m_j^2)^2 + k^4 - 2(m_i^2 + m_j^2)k^2$. The amplitude can then be written in terms of the Fermi constant G_F , well established $B \rightarrow D$ form factors and CKM matrix elements,

$$\mathcal{A}(\bar{B} \rightarrow D^+ M^- \pi^0) = -G_F V_{ux}^* V_{cb} (m_B^2 - m_D^2) F_0^{B \rightarrow D}(k^2) a_1(D^+ M^- \pi^0) \quad (1.5)$$

The details of this derivation are non-trivial and will not be discussed here but can be found in Ref. [26]. We now consider the narrow-width limit in which the total width of the intermediate resonance V (not to be confused with the CKM matrix) goes to zero. Resonances are extremely short-lived particles with lifetimes on the order of 10^{-23} s that leave no detectable track in the detector, and so must be inferred on the basis of energy and momentum conservation laws. In the narrow-width limit, the QCDF amplitude a_1 becomes

$$|a_1(D^+ V^- (\rightarrow M^- \pi^0))|^2 \xrightarrow{\Gamma_V^{tot} \rightarrow 0} \frac{24\pi^2 f_V^2 m_V^2 \cos^2 \theta_h}{\sqrt{\lambda_{M\pi}(m_V^2)}} |a_1(D^+ V^-)|^2 \mathcal{B}(V \rightarrow M\pi) \delta(k^2 - m_V^2) \quad (1.6)$$

where f_V is the decay constant and δ is the Dirac delta function. Substituting this expression back into Equation 1.4 and making a change of variables from k^2 to s , the invariant squared mass of the dimeson, we get the narrow width limit for the decay rate:

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow V^- (\rightarrow M^- \pi^0))}{ds d\cos\theta_h} &\xrightarrow{\Gamma_V^{tot} \rightarrow 0} G_F^2 |V_{ux}^* V_{cb}|^2 (m_B^2 - m_D^2)^2 |F_0^{B \rightarrow D}|^2 |a_1(D^+ V^-)|^2 \\ &\times \cos^2 \theta_h \frac{3f_V^2 \sqrt{\lambda_{BD}(m_V^2)}}{64\pi m_B^3} \mathcal{B}(V \rightarrow M\pi) \delta(s - m_V^2) \end{aligned} \quad (1.7)$$

This result will be needed in the following section.

1.5.3 Helicity Angle

In the decay of the B^0 to the $D^-\pi^+\pi^0$ final state, one can consider the angle θ_h between the 3-momentum of the B and the π^0 mesons. This is known as the helicity angle and can be used to define regions of interest in the decay topology which can be seen in Figure 1.3 for both the lab frame and the $\pi^+\pi^0$ reference frame. We define $z \equiv \cos \theta_h$ and call the ‘forward’ region to be where $0 < z < 1$ and similarly the ‘backward’ region corresponds to $-1 < z < 0$. We can now define the following ratio of integrated differential decay rates,

$$\mathcal{R}_{MM'} [z_1, z_2; z'_1, z'_2] (k^2) \equiv \frac{\int_{z_1}^{z_2} \frac{d\Gamma(\bar{B} \rightarrow D^+ M^- \pi^0)}{dk^2 dz} dz}{\int_{z'_1}^{z'_2} \frac{d\Gamma(\bar{B} \rightarrow D^+ M'^- \pi^0)}{dk^2 dz} dz}, \quad (1.8)$$

where the expression for the decay rate was derived in the previous section. In the case where $M = M'$, we see that all prefactors cancel leaving only the following:

$$\mathcal{R}_{MM} [z_1, z_2; z'_1, z'_2] (k^2) = \frac{\int_{z_1}^{z_2} |a_1(D^+ M^- \pi^0)|^2 dz}{\int_{z'_1}^{z'_2} |a_1(D^+ M^- \pi^0)|^2 dz} \quad (1.9)$$

One can express the amplitude as a factorisation into partial waves.

$$\mathcal{A}(k^2, \theta_h) = \sum_{\ell=0}^{\infty} \mathcal{A}^{(\ell)}(k^2) P_{\ell}(\cos \theta_h), \quad (1.10)$$

where $P_{\ell}(x)$ are Legendre polynomials. However, in the case $M\pi = \pi^+\pi^0$ we have an isospin-one state and thus the summation runs over odd ℓ only. The Legendre polynomials are defined such that the odd terms are all anti-symmetric functions and thus, when squaring the amplitude where there is no interference between odd and even partial waves, we find only symmetric terms. Therefore, one can say that the ratio previously defined should obey the following relation:

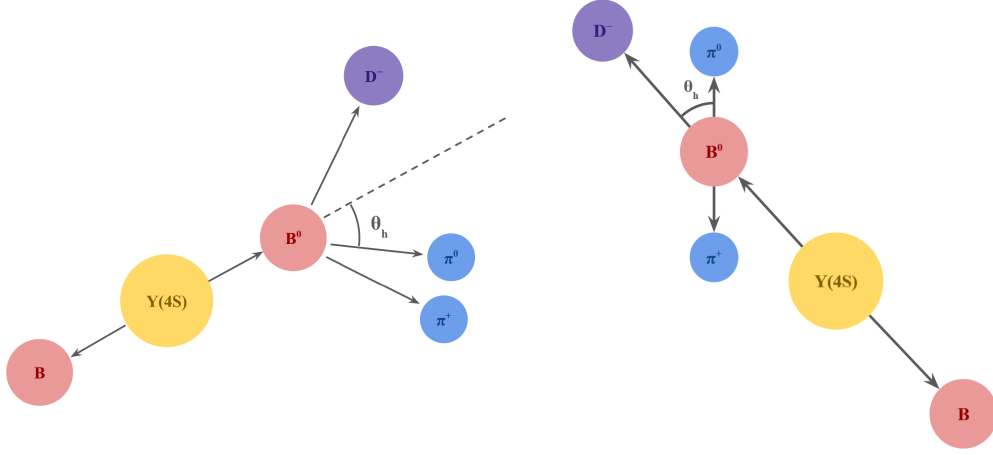


Figure 1.3: Diagram showing the $B^0 \rightarrow D^- \pi^+ \pi^0$ decay in the (left) lab frame and the (right) $\pi^+ \pi^0$ reference frame.

$$\mathcal{R}_{\pi\pi}[0, z; -z, z](k^2) = \mathcal{R}_{\pi\pi}[-z, 0; -z, z](k^2) = \frac{1}{2}. \quad (1.11)$$

Consequently, one should expect that the asymmetry between the forward and backward helicity regions should vanish under the Standard Model,

$$A_{FB}^{\pi\pi}(k^2) = \mathcal{R}_{\pi\pi}[0, 1; -1, 1](k^2) - \mathcal{R}_{\pi\pi}[-1, 0; -1, 1](k^2) = 0. \quad (1.12)$$

This asymmetry is an observable quantity that can be experimentally measured and constitutes a test for the existence of any potential isospin-violating corrections.

1.6 Statement of Intention

This analysis has three key goals:

1. To make the first measurement of the total branching fraction of $B^0 \rightarrow D^- \pi^+ \pi^0$.
2. To measure the branching fraction of $B^0 \rightarrow D^- \rho^+$ with much higher precision.
3. To make the first measurement of $A_{FB}^{\pi\pi}$ in the helicity angle which is predicted to be zero.

Chapter 2

The Belle Experiment

The Belle experiment is one of several scientific projects undertaken by the High Energy Accelerator Research Organization (KEK) located in Tsukuba, Japan. The main design goal of the experiment was to facilitate the study of a wide range of physics targets including rare B -meson decays, and in particular CP-violation, by producing them in large quantities in an asymmetric collider. Such a machine is known as a B-factory and Belle was able to achieve this by colliding $8.0\text{ GeV } e^-$ with $3.5\text{ GeV } e^+$ at the $\Upsilon(4S)$ resonance energy which subsequently decays into a pair of B -mesons 96% of the time. The experiment consists of two main facilities: the KEKB accelerator [28] and the Belle detector. Although the experiment actively gathered data from 1999 to 2010, its results are still studied to this day, which has resulted in over 600 published studies to the field of high energy particle physics.

2.1 The KEKB Accelerator

The KEKB accelerator is an asymmetric e^+e^- collider located in Ibaraki Prefecture, Tsukuba, Japan with a total circumference of 3km and operates at the $\Upsilon(4S)$ resonance. The two beams are aligned side by side as shown in Figure 2.1 with the High and Low Energy Rings (HER/LER) carrying 8.0 GeV electrons and 3.5 GeV positrons respectively. The beams collide with a centre of mass (CoM) energy of $\sqrt{s}=10.58\text{ GeV}$ which is just above the $\Upsilon(4S)$ resonance. The resulting production of B -mesons is close to an even split between charged and neutral species with a corresponding ratio

of 1.058 ± 0.024 , charged to neutral [24]. The B -mesons are Lorentz boosted due to the asymmetric nature of the colliding beam energies, with the CoM travelling at a velocity of $0.425c$ in the laboratory reference frame. The two beams consist of a stream of separated particle bunches that contain on average 10^{11} particles per bunch, where they eventually collide at the interaction point (IP) at a beam crossing angle of 22 mrad . In order to maximise the overlapping cross sectional area of the beam bunches, a series of magnets are positioned before the IP that work to rotate and align them for maximal luminosity. The Belle experiment was designed for a peak luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, but by the end of its run life had attained a peak of $2.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, twice the original goal. Over the course of the experiment, 711 fb^{-1} of 772×10^6 B pairs were collected.

2.2 The Belle Detector

2.2.1 Beam Pipe

The beam pipe design was driven by the need for accurate decay vertex readings. With the inverse relationship between vertex resolution and radial distance, the vertex detectors were positioned as close as possible to the interaction point (IP). Additionally, the z-vertex resolution was improved by minimising the impact of Coulomb scattering through lowering the beam wall thickness. The beam pipe consists of a hollow beryllium metal cylinder with an inner and outer wall, and a coolant that flows between the two. The inner wall has a radius of 20 mm and a thickness of 0.5 mm while the outer wall has a radius of 23 mm and the same thickness. The beryllium walls of the $-4.6 \text{ cm} \leq z \leq 10.1 \text{ cm}$ central region were selected for their low atomic number, resulting in less coulomb scattering.

The walls of the beam pipe surrounding the IP are at risk of beam-induced heating, which could result in harm to nearby vertex detectors. A helium gas cooling system is used to mitigate this, which is capable of maintaining a maximum temperature change between the inner and outer walls of the beam pipe of 25°C , assuming a uniformly distributed 100 W heat load and 2 g/s helium flow. The IP is surrounded by two $20 \mu\text{m}$ thick gold masks that are used to reduce the intensity of synchrotron radiation photons with energies below 2 keV . The background caused by particle interactions with residual gas particles and the tails of the beam with the wall of the pipe, was found to be 130 kHz

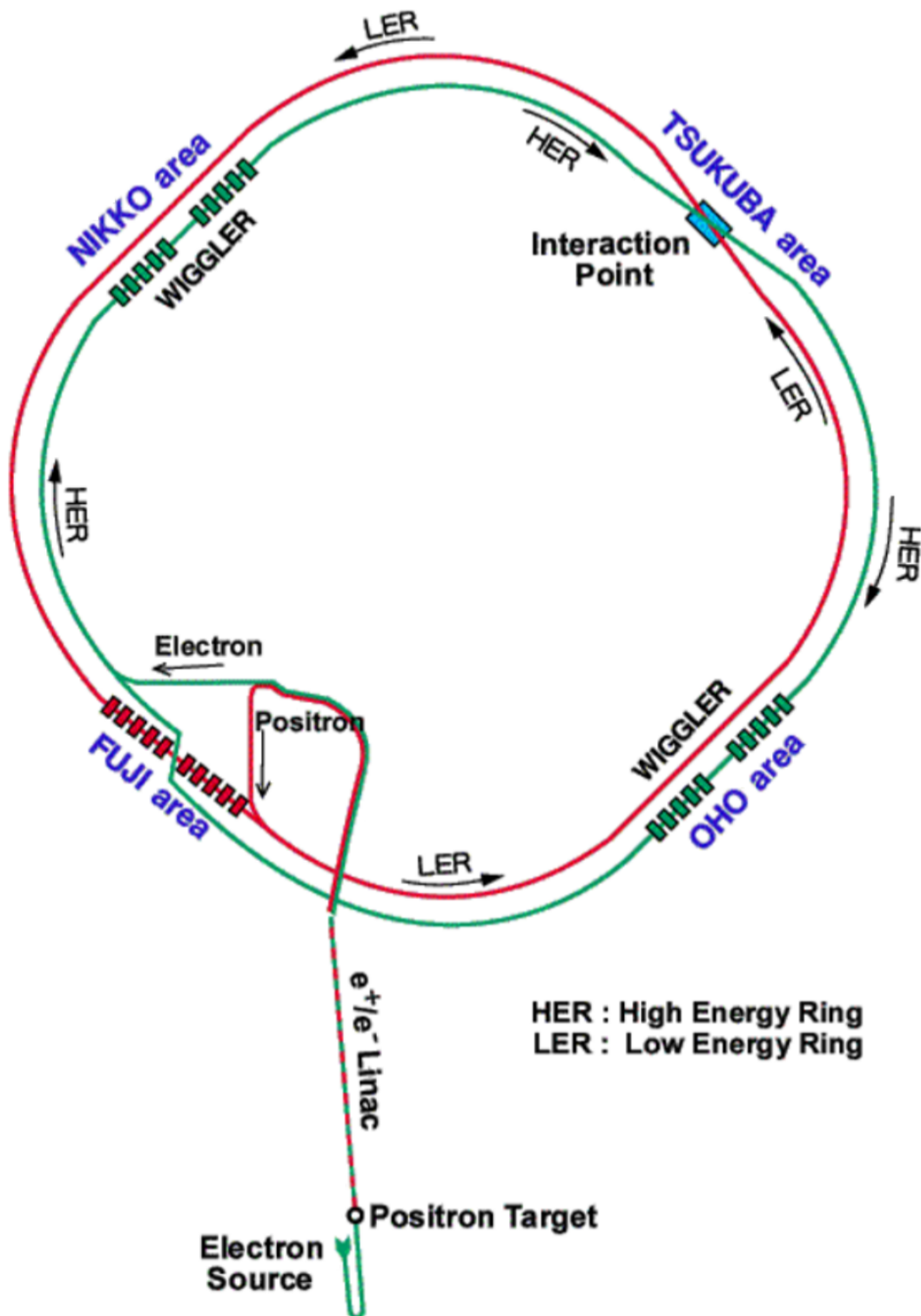


Figure 2.1: A top-down diagram of the KEKB collider displaying the position of the LER and HER along with the placement of the interaction point [1].

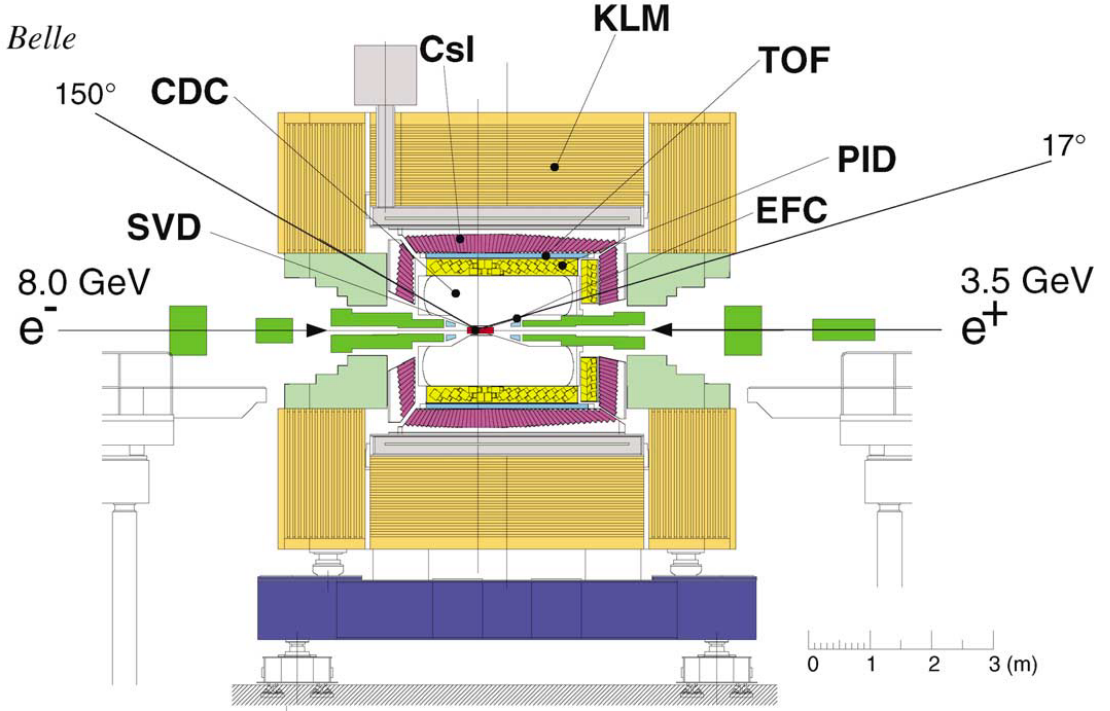


Figure 2.2: Side view of the Belle detector.

by maintaining a vacuum of 10^{-9} Torr. There are additional masks designed to be movable so as to reduce radiation to the detector during beam injection. During an upgrade in 2003, a new beam pipe was installed, which had a reduced inner radius of 15 mm and outer radius of 18.5 mm. Furthermore, the gold mask was replaced with a thinner 10 μm one and the active helium coolant was replaced with liquid paraffin.

2.2.2 Silicon Vertex Detector - SVD

One of the main design goals of the Belle experiment is to make careful observations of time-dependent CP asymmetries in B -meson decays. Achieving this requires precise measurements of the decay vertices of the two B -mesons and the difference in their z -vertex positions. The SVD can do this to a resolution of $\sim 100 \mu\text{m}$ which is sufficient for this purpose after taking into account the Lorentz boost produced by the asymmetric beam energies. This boosts the z -vertex difference from 2 μm to 200 μm . It is also a vital component of the charged particle tracking system.

The layout of the SVD is that of a series of double-sided silicon strip detectors (DSSD) in the form of concentric stacked layers surrounding the beam pipe shown in Figure 2.3. The DSSDs function like a p-n junction in a semiconductor operating under a 75 V reverse

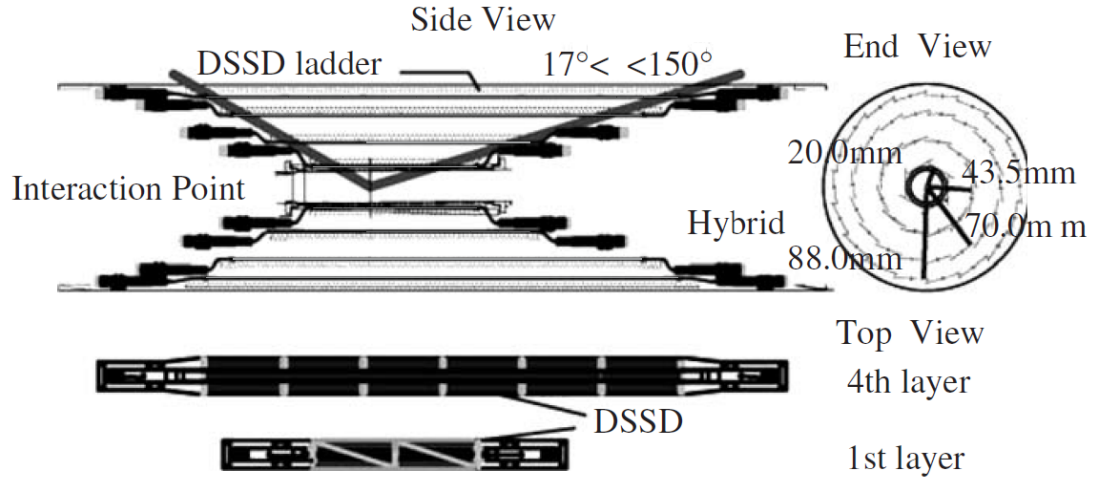


Figure 2.3: Configuration of the SVD2 layers.

bias which creates a depletion zone between the two oppositely doped sides of the strips. Within this zone, any charged particles passing through will liberate electrons and holes which then are attracted to the oppositely charged strips. These induce pulses that localise the interaction point of the primary charged particle. These hits are detected across multiple layers of DSSDs from which one can reconstruct the particle's trajectory and point of origin.

The SVD construction was split into two phases; prior to the 2003 upgrade it was known as SVD1 and after the upgrade, SVD2. As this study exclusively uses SVD2 Belle data, the following is a description of the upgraded SVD2 architecture. SVD2 contained four layers of DSSD modules located at 20 mm, 43.5 mm, 70 mm and 88 mm from the IP. These layers contained 6, 12, 18 and 18 modules respectively and occupied an angular acceptance range of $17^\circ < \theta < 150^\circ$ which coincides with the tracking volume of the CDC. The DSSDs are each constructed with 1280 strip sensors with 640 readout pads on each side. The two sides of the strips are called the z-side and the ϕ -side, with the z-side having an active region of $53.5 \times 32.0 \text{ mm}^2$ and the ϕ -side, $54.5 \times 32.0 \text{ mm}^2$. The full size of each DSSD is $57.5 \times 33.5 \text{ mm}^2$ with 102 in total.

2.2.3 Central Drift Chamber - CDC

Almost all measurements made by the Belle experiment require a precise determination of the flight trajectory and momenta of charged particles. The CDC was designed to meet this need and does so for particles with a transverse momentum greater than

100 MeV/ c and a resolution of

$$\frac{\sigma_{p_T}}{p_T} \sim 0.005 \sqrt{1 + p_T^2}, \quad (2.1)$$

where p_T is the transverse momentum of the particle and σ_{p_T} is its resolution. The transverse momentum can be determined from the curvature of charged tracks in the 1.5 T magnetic field, which in addition to the track's helicity, can also determine the z -component of the momentum.

The CDC consists of a series of drift cells arranged in a cylindrical layout. In total, there are 50 layers and 8400 cells each of which contain a number of cathode field wires and anode sense wires. This covers an angular range of $17^\circ \leq \theta \leq 150^\circ$ and is pictured in Figure 2.4. The wires are arranged in both axial and stereo directions to allow for the trajectory to be measured in the z direction. The cells are filled with a gas mixture of equal parts helium and ethane, chosen for its low atomic number and long radiation length (640 m), the former of which helps to minimise multiple Coulomb scattering that hinders momentum resolution. The ethane absorbs UV light from the electron avalanche near the anode wires which prevents delocalised ionisation away from the primary particle tracks.

When charged particles pass through the CDC drift cells, they liberate electrons and ionise the gas. The electrons initially drift towards the anode sense wire. Near the wire, the electric field increases as $1/r$ and provides sufficient energy for the electron to initiate further ionisations. These in turn create more ionisations and result in an avalanche of charge, multiplying the initial signal by a factor of 10^4 . The timing of the pulse on the wire is recorded and employed with the known calibration of the electron drift-velocity to determine the distance of the primary charged particle to the anode wire with a resolution of approximately 100 μm . This cascade of electrons is detected as a pulse, the size of which is proportional to the rate of energy loss in the gas per distance travelled, $(\frac{dE}{dx})$ which in turn is related to the particle's velocity. The exact relationship is given by the Bethe formula [29],

$$-\frac{dE}{dx} = \frac{4\pi N_A z^2 e^5}{m_e v^2} \frac{Z}{A} \ln \left(\frac{2m_e v^2}{I(1 - \beta^2)} - \beta^2 \right), \quad (2.2)$$

which relates the energy loss over distance, dE/dx , to the charge of the particle, z ,

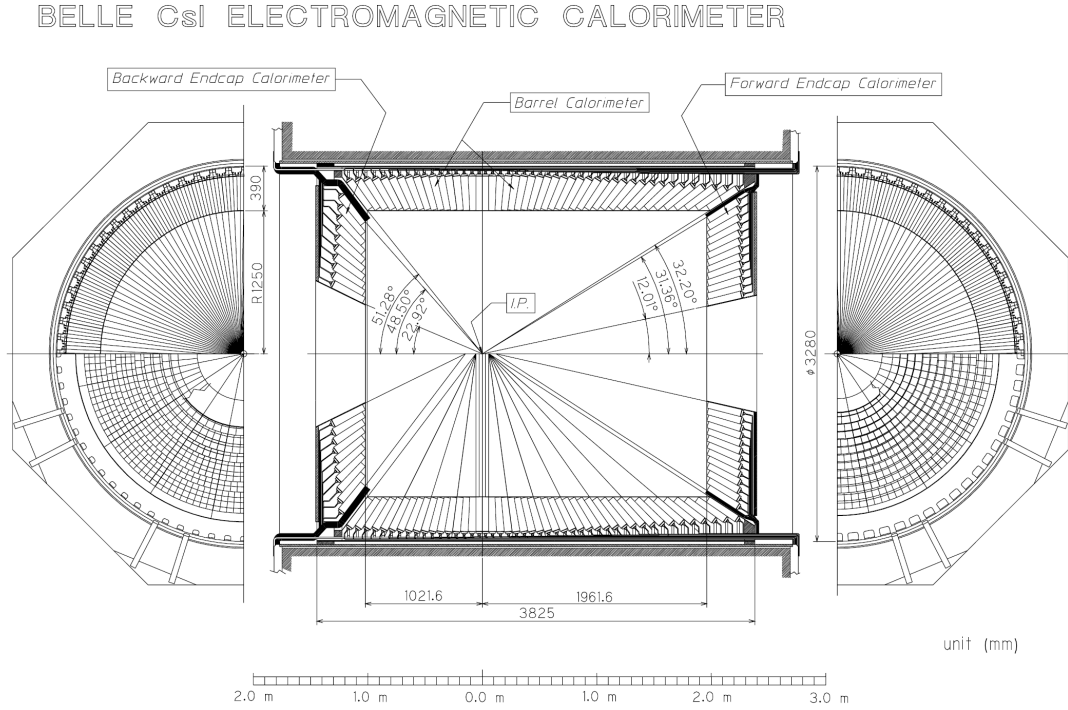


Figure 2.5: Side view of the ECL layout and end-caps.

these events. There are 8736 CsI(Tl) counters in total, all pointed towards the IP. The crystals are distributed between three segments: the barrel, and two end-caps. The barrel section is 3 m long, has an inner radius of 1.25 m, covers an angular range of $32.2^\circ < \theta < 128.7^\circ$ and contains 6624 crystal counters. The forward and backward end-caps, placed at +2 m and -1 m, extend the coverage to $12.4^\circ < \theta < 155.1^\circ$ and contain 1152 and 960 crystal counters, respectively.

Photons interacting with the crystals undergo pair production into electron and positron pairs. These subsequently initiate bremsstrahlung processes, creating more photons, which in turn undergo further pair production. The result is a cascade of electrons and positrons. The length-scale of these interactions is given by the radiation length of the crystal. As the charged particles propagate they lose energy, some of which is converted into scintillation light. This in turn is detected by two photodiodes mounted on the back of the crystals. The size of the scintillation light pulse is proportional to the energy of the incident photon or electron/positron. To improve scintillation light collection and reduce leakage, each crystal is wrapped in a layer of 200 μm thick Goretex Teflon. Two additional layers of 25 μm thick aluminium and Mylar are applied for light and electrical shielding.

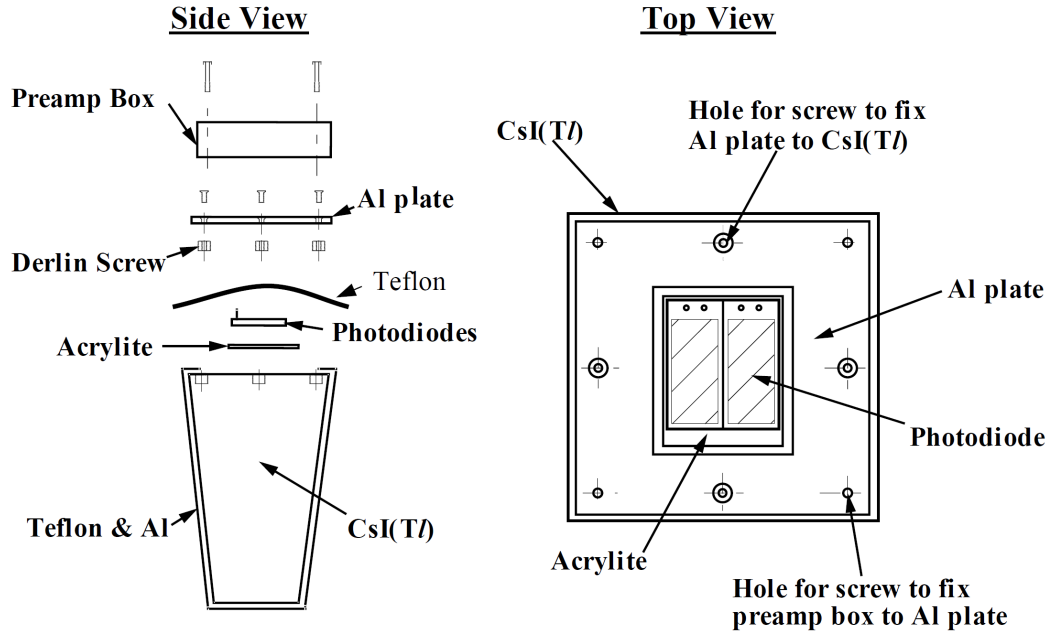


Figure 2.6: Top and side schematics of the CsI(Tl) counter crystals.

2.2.5 Aerogel Cherenkov Counter - ACC

The key responsibility of the ACC is to distinguish between $\pi^\pm$ and $K^\pm$ with momenta $> 1.2 \text{ GeV}/c$ that are outside the range of the CDC or the capabilities of the time-of-flight detector. Cherenkov radiation emission occurs when a charged particle passes through a transparent dielectric medium at velocities greater than that of the speed of light in that medium. This can only happen if

$$n > \frac{1}{\beta} = \sqrt{1 + \left(\frac{m}{p}\right)^2}, \quad (2.3)$$

where n is the refractive index of the medium and m and p are the mass and momentum of the particle, respectively. By carefully tuning the refractive indices of the aerogel media to be between 1.01 and 1.03, only pions with momenta $> 1.2 \text{ GeV}/c$ will produce Cherenkov radiation, while more massive kaons will not. The exact arrangement of the aerogel counters and their n values is shown in Figure 2.7.

The general construction of the ACC includes a central barrel section and an end-cap section. The barrel covers an angular range of $33.3^\circ < \theta < 127.9^\circ$ and contains 960 aerogel counter modules. The end-cap covers the range from $13.6^\circ < \theta < 33.4^\circ$ and

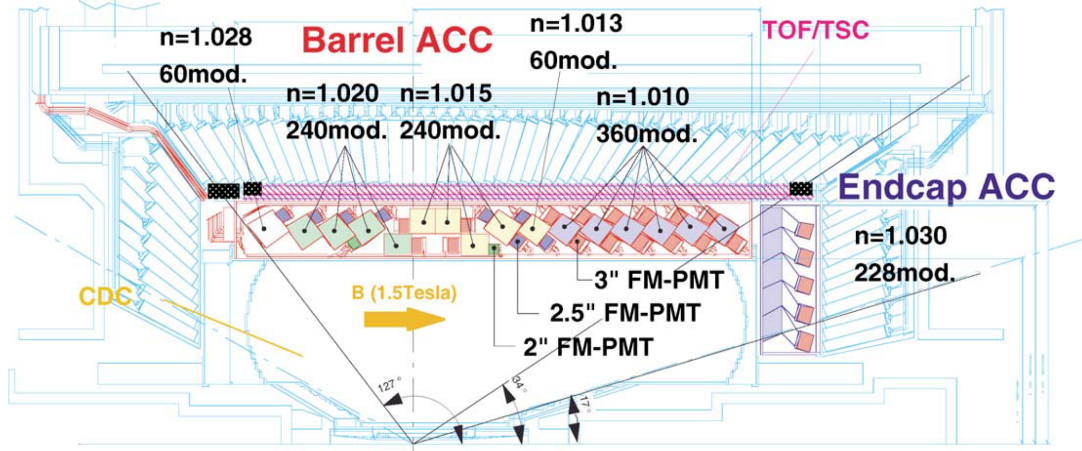


Figure 2.7: Side-on diagram of the ACC barrel and end-cap regions with readout using fine-mesh photomultiplier tubes (FM-PMT).

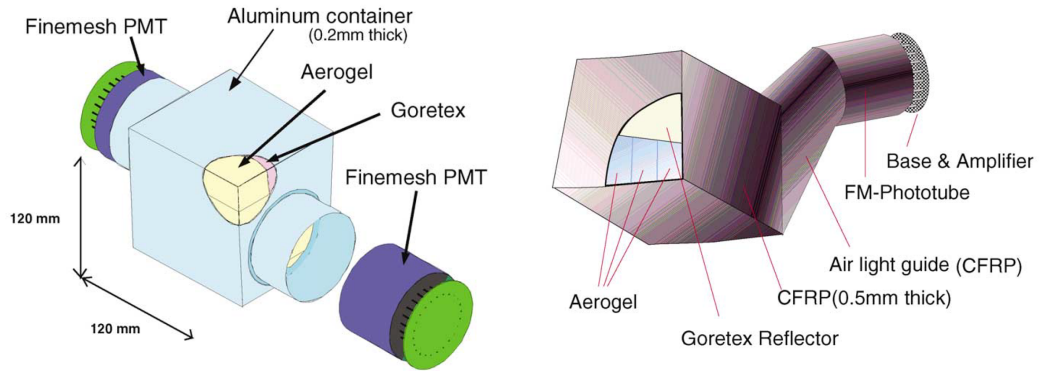


Figure 2.8: Schematic of a barrel aerogel model (left) and end-cap module (right).

contains 288 modules stacked in 5 concentric layers. Each barrel aerogel module is contained within a 0.2 mm thick aluminium box with two R6680 fine-mesh photomultiplier tubes (FM-PMTs) produced by Hamamatsu Photonics for collection of the Cherenkov radiation in the 1.5 T magnetic field which can be seen in Figure 2.8.

2.2.6 Time-of-Flight Detector - TOF

The time-of-flight (TOF) system records the time taken for a particle to travel from the IP to the TOF counters. This provides useful information for particle identification for approximately 90% of all particles resulting from $\Upsilon(4S)$ decays. From special relativistic considerations we know,

$$T = \frac{L}{c} \sqrt{1 + \frac{mc^2}{p}}. \quad (2.4)$$

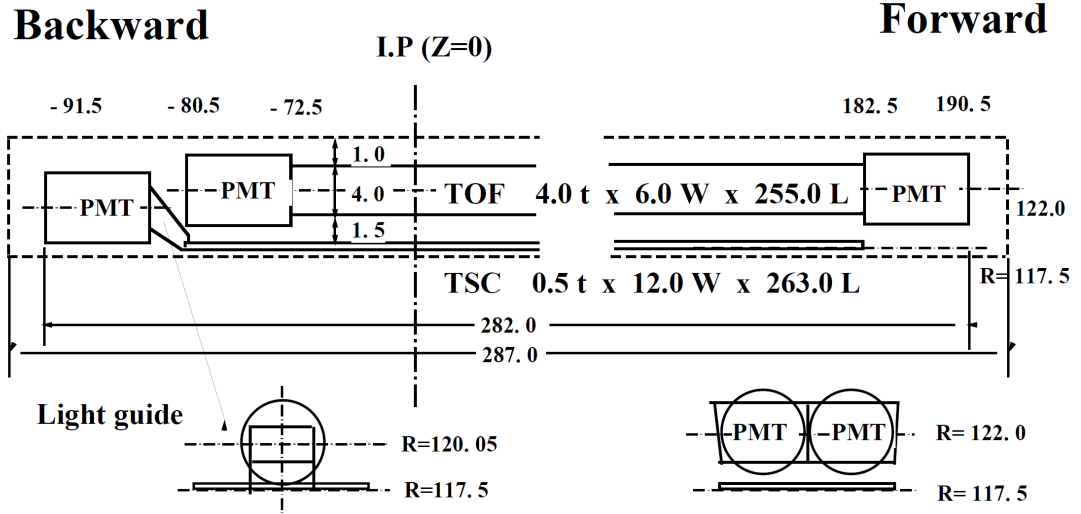


Figure 2.9: Dimensions of the time-of-flight (TOF) counter modules and trigger scintillation counter (TSC).

By knowing the time-of-flight T , the flight path length L and the particle's momentum p , its mass is determined. For satisfactory performance, a temporal resolution of 100 ps is desired. To achieve this, a series of 128 TOF modules line the inner wall of the ECL barrel, with each module containing two plastic scintillation counters and one trigger scintillation counter (TSC). The modules are equipped with Hamamatsu type R6680 FM-PMTs for readout. The exact dimensions are given in Figure 2.9. This configuration covers the polar angle range $34^\circ < \theta < 120^\circ$. Particles that can reach the TOF counters have a minimum transverse momentum of about $0.28 \text{ GeV}/c$ and any particle below this threshold can only be picked up by the CDC.

2.2.7 Extreme Forward Calorimeter - EFC

In order to remain experimentally sensitive to certain decays that require a measurement of the final state missing energy such as $B \rightarrow \tau \nu$, it is necessary to minimise the loss of energy due to particles missing the detector. For this reason the extreme forward calorimeter (EFC) is designed to extend beyond the polar angle range of the ECL. The EFC occupies the angular range $6.4^\circ < \theta < 11.5^\circ$ in the forward direction and $163.3^\circ < \theta < 171.2^\circ$ in the backward direction. In this region the radiation from the IP is too high for the CsI(Tl) scintillation crystals to survive, and so a different calorimeter design is required. It consists of 80 scintillation crystals, in both the forward and backward regions (see Figure 2.10), constructed from pure bismuth germanate ($\text{Bi}_4\text{Ge}_3\text{O}_{12}$), chosen

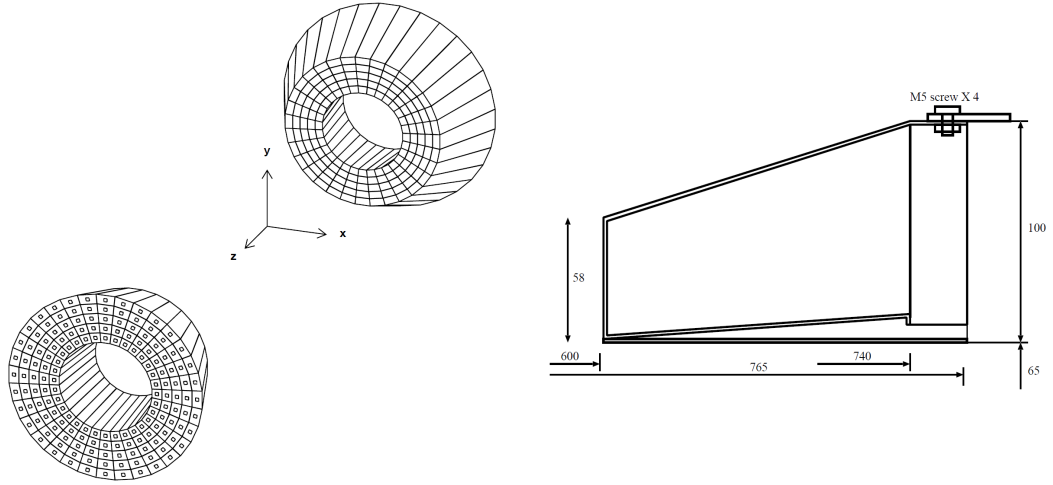


Figure 2.10: Isometric diagram of the EFC crystal placement (left) and a side-on view of the mounting dimensions (right).

for its radiation hardness (up to the kGy level), high energy resolution and low cost relative to other materials considered. To achieve this level of radiation hardness, some accuracy must be traded off compared to the CsI(Tl) crystals used in the ECL. In addition to aiding in the detection of $B \rightarrow \tau \nu$ processes the EFC also acts as a mask, shielding the CDC from excessive backgrounds. Small-angle electron-positron scattering is common enough of an occurrence that it can act as a proxy for luminance. By detecting these events the EFC also serves as a beam and luminosity monitoring system for the Belle experiment as a whole.

2.2.8 K_L^0 Muon Detector - KLM

The KLM is responsible for the detection of K_L mesons and muons with momenta greater than 600 MeV/ c . Due to their high penetration and long life-times, these particles will pass through the inner detector and ECL without being fully absorbed and are detected in the thick KLM formed by instrumenting the iron magnetic flux return interleaved with plates of glass-electrode-resistive plate counters (RPCs).

Within the barrel region ($45^\circ < \theta < 125^\circ$) there are 15 RPC layers and 14 interlaced layers of 4.7 cm thick iron plates (shown in Figure 2.11). The forward and backward end-caps (extending to $20^\circ < \theta < 155^\circ$) contain 14 layers of both RPCs and iron plates. These plates constitute 3.9 strong interaction lengths (normally incident) in addition to the 0.8 lengths provided by the ECL and also serve as the magnetic flux return path

for the superconducting solenoid. The RPC layers are constructed from two parallel electrode glass plates with a 2 mm gap filled with a non-combustible gas mixture of 62% HFC-134a (CH_2FCF_3), 30% argon and 8% butane-silver (70% n-butane, 30% iso-butane).

When charged particles enter the gas-filled gap they proceed to ionise it and cause a discharge event. This is recorded as the 2-dimensional location with the plate and time, but due to the large variation in the size of ionising showers, no useful energy reading is made. K_L s are identified as discharge events that lack a corresponding track from the CDC and have relatively little penetration through the RPC layers. Muons can be distinguished from other similar charged light hadrons by their inability to interact via the strong nuclear force. Thus, they leave both a track in the CDC and a corresponding long trail of discharges through the KLM layers.

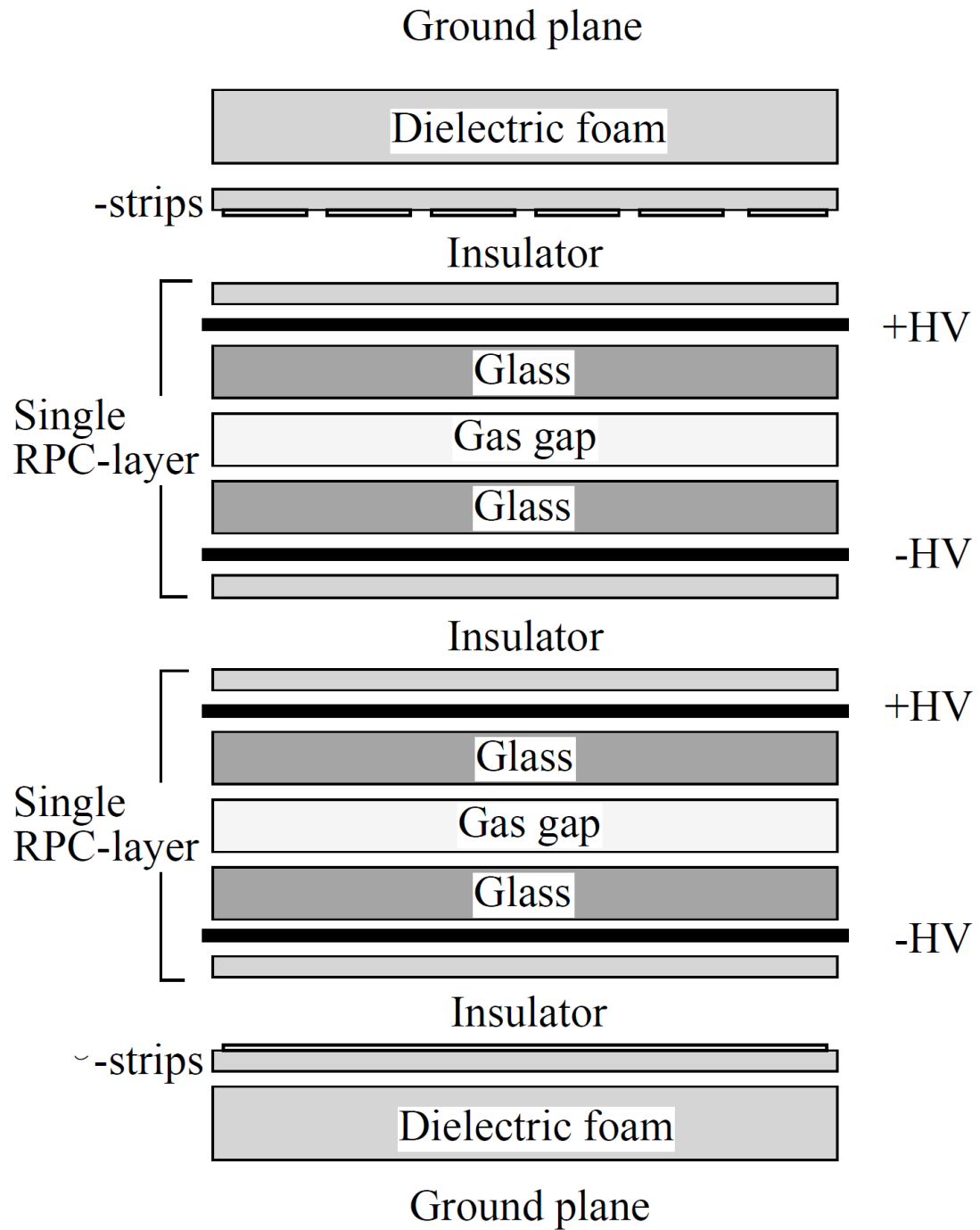


Figure 2.11: Breakdown of the layers of a single RPC stack in the KLM.

Chapter 3

$B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ Analysis Strategy

Presented here is an in-depth description of the complete $B^0 \rightarrow D^- \pi^+ \pi^0$ analysis procedure.

3.1 Event Selection

To safeguard against the possibility of researcher bias, the real Belle data is left unseen until the analysis is validated with Monte Carlo (MC) data and a “Control Mode” which is a well known B decay mode similar to the $B^0 \rightarrow D^- \pi^+ \pi^0$ under study. MC data refers to a detailed simulation of the physics experiment. Belle employs the GEANT3 software library for this purpose [30]. This simulation includes a detailed geometry of the experiment, along with the experimental running conditions of the KEKB accelerator at the time the data was collected. The simulation proceeds in two phases. The underlying physics of the process was generated with the EvtGen software [31] which is followed by a detailed simulation of the interactions of the generated particles with the Belle detector via GEANT3. MC data can also be employed to assess the accuracy, precision, and efficiency of the analysis techniques, as well as to identify any potential biases or systematic errors that may be present. Additionally, it can be used to explore the sensitivity of the results to different analysis criteria.

In this analysis, MC data was generated and skimmed with the Belle standard data analysis software framework, BASF [32] first before being converted into Belle II mini data summary tables (mDST). This is done in order to make use of the improved Belle II software, which in particular includes cleaner continuum suppression and more accessible MC truth information for better background validation.

3.1.1 MC Matching and Truth Information

Within the Belle II analysis software framework (BASF2) [33], MC matching is the process of associating reconstructed particle candidates back to their original simulated particles created at the generator level [4]. This matching procedure varies when comparing single tracks to more complex, composite particles. In the case of tracks, the method involves confirming that the hits used in a reconstructed track correspond to a single generated particle, thereby ensuring the track purity [34]. For composite particles, the process involves a preliminary step where an algorithm is used to find the MC match by establishing the relation between the reconstructed composite particle and the corresponding MC particle. Following this, an evaluation step is employed to categorise incorrectly reconstructed candidates. During this process, the algorithm performs recursive checks for any daughters that are themselves composite particles, ensuring all daughters of the initial particle are correctly matched. The output of this final evaluation step is a series of error flags that indicate potential issues in the matching process, such as internal errors, misidentifications, or decays in flight. Collectively, this is known as MC-truth information and is a crucial part of validating particle physics analyses. In this analysis, MC-truth variables were primarily used to isolate individual backgrounds for dedicated studies, as well as provide insights into the composition of those backgrounds.

3.1.2 Dataset Skimming

A skim is a set of loose selection criteria applied to data and MC for the purposes of greatly reducing its file size before proceeding with a more refined analysis. This saves on both disk space and computational resources, while also making the data more manageable for analysts to work with. In this analysis the skim selection criteria employed was mostly the same as those found in Section 3.2.3, except with looser $\Delta E > -0.3 \text{ GeV}$

and $M_{bc} > 5.27 \text{ GeV}/c^2$ cuts on the B -meson (defined in Section 3.1.4). This resulted in a reduction of 91% in the file sizes of background MC samples.

3.1.3 Data Samples

The analysis was performed on an integrated luminosity of 569.617 fb^{-1} of SVD2 Belle data consisting of 619.62×10^6 $B\bar{B}$ pairs. Belle MC samples were first skimmed in BASF [32] and then processed via the B2BII software package [35] in order to convert the Belle mDST file format into a format readable by the Belle II software framework BASF2 [33]. Signal MC samples were generated as 1 million events in which one B decays via the signal mode, $B^0 \rightarrow D^- \pi^+ \pi^0$ including all intermediate resonance channels comprising the best estimates of the processes expected by the Belle collaboration. In this, 78% were $B^0 \rightarrow D^- \rho^+$ events which were singled out for dedicated analysis using MC-truth information provided by BASF2. In our MC simulations, only the most common decay mode, $D^- \rightarrow K^+ \pi^- \pi^-$, was employed since this was the only mode employed in the $B^0 \rightarrow D^- (K^+ \pi^- \pi^-) \pi^+ \pi^0$ reconstruction. This constitutes $\sim 10\%$ of all D^- decays. The other B was left to decay according to previously measured branching fractions (i.e. generically). Because this analysis is not statistically limited, the systematic uncertainties were reduced by only including a single decay mode for the D -meson. All other D^- channels include additional π^0 s and K_S^0 s which would further increase the overall systematic uncertainties if included. A summary of the branching fraction values used by the Belle collaboration for MC simulation (circa 2010) for each signal channel is given in Table II. As a point of comparison, the current world average PDG values are also given where known. Due to the large uncertainty in the PDG signal channel branching fraction ($\sim 20\%$), only the Belle values were utilised when analysing signal MC. This was further motivated by comparing Belle generic MC (see Section 3.2.3) to a 10% sample of real Belle data and finding good agreement in their kinematic phase-space.

The underlying physics of the MC simulation was performed by the EvtGen software package [31] using appropriate physics models as described in the EvtGen documentation. The models were chosen in accordance with those used in generic MC generated by the Belle collaboration. For most signal channels this included the effects of final state electromagnetic interactions by utilising the PHOTOS physics model [36].

TABLE II: Comparison of branching fractions for each signal decay channel between Belle MC values (circa 2010) and current PDG world averages.

B^0 Channel	Belle	PDG	PDG Error
$D^- \rho^+$	7.5×10^{-3}	7.6×10^{-3}	1.2×10^{-3}
$D^{*-} \pi^+$	2.76×10^{-3}	2.74×10^{-3}	0.13×10^{-3}
$D^- \pi^+ \pi^0$	1×10^{-3}	-	-
$D_2^*(2460)^- \pi^+$	8.5×10^{-4}	-	-
$D^- D^+$	2.3×10^{-4}	2.11×10^{-4}	0.18×10^{-4}
$D_0^*(2300)^0 \pi^0$	5×10^{-5}	-	-
$D_2^*(2460)^0 \pi^0$	3×10^{-5}	-	-
Total	1.242×10^{-2}	-	-
D^- Channel			
$K^+ \pi^- \pi^-$	5.31×10^{-2}	-	-
$K_0^*(1430)^0 \pi^-$	3.78×10^{-2}	1.25×10^{-2}	0.06×10^{-2}
$K^*(892)^0 \pi^-$	1.95×10^{-2}	1.04×10^{-2}	0.12×10^{-2}
$K^{*'}(1720)^0 \pi^-$	1.47×10^{-2}	-	-
Total	12.51×10^{-2}	9.38×10^{-2}	0.16×10^{-2}

In addition to the signal decay, there are multiple sources of backgrounds that must be distinguished from the signal. The B -meson features many other decay channels, which include daughter particles such as pions and kaons, whose reconstruction could lead to misidentification. The events from alternate decays of B -mesons is called the generic background. The continuum background is the result of the hadronisation of the quark/anti-quark pairs created by the original collision. The background studies were performed on MC officially produced by the Belle collaboration for the purpose of doing physics analyses. Belle created sets of MC known as ‘streams’ where each stream contains as many events as the complete full Belle experiment run. For this analysis 6 streams of continuum events and 6 streams of generic B decays were used. B decays to hadronic states involving only $b \rightarrow u, d, s$ transitions are much rarer ($\text{BF} \approx 10^{-6}$) and as such require much larger samples of MC to analyse. For this, an additional 50 streams of these rare $B\bar{B}$ Belle MC was analysed.

3.1.4 Event Reconstruction

The reconstruction of the $B^0 \rightarrow D^- \pi^+ \pi^0$ signal was performed via an iterative process starting with the final state particles (FSPs). Lists of charged FSPs ($\pi^\pm$ and $K^\pm$) were

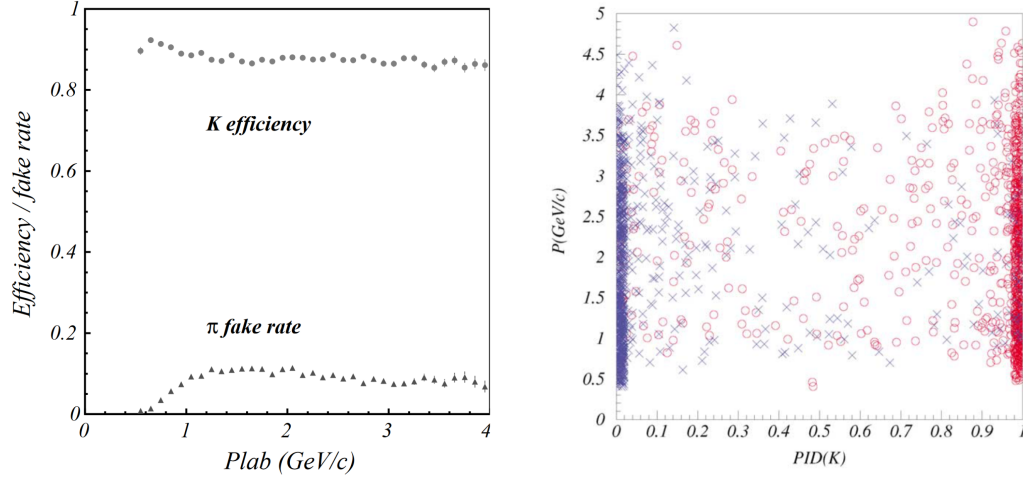


Figure 3.1: (left) Momentum dependence of kaon efficiency and pion fake rate. Circles correspond to kaon efficiency and the triangles to pion fake rate. (right) Likelihood ratio $PID(K)$, versus momenta for daughter tracks from $D^0 \rightarrow K^- \pi^+$ decays, tagged by the charge of the slow π^+ s. The open circles correspond to kaons and the cross-points to pions.

filled based on track information given by various Belle subdetectors. The charged pions and kaons were chosen on the basis of a likelihood ratio given by the hadron identification system defined as

$$R_{K/\pi} = \frac{L_K}{(L_K + L_\pi)}, \quad (3.1)$$

where L_K and L_π are the likelihoods of each track type given by the ACC, TOF and CDC subdetectors. Tracks for which $R_{K/\pi} > 0.6$ were identified as $K^\pm$ s and $R_{K/\pi} < 0.4$ were considered as $\pi^\pm$ s. Momentum dependence in the lab frame of kaon efficiency and pion fake rate are given in Figure 3.1 [37] along with the PID likelihood ratio for daughter tracks from a study of $D^0 \rightarrow K^- \pi^+$ decays found within the ACC subdetector [1].

π^0 s were reconstructed from two photons with the requirement that their energy is greater than 50 MeV if detected in the ECL barrel region and greater than 100 MeV if detected in the forward or backward endcap regions. Additionally, the π^0 invariant mass was required to be within 2 standard deviations of the world average value of 135 MeV/c² [24].

A candidate $D^\pm$ was identified if any combination of $K^\mp$ and two $\pi^\pm$'s had an invariant mass in the range 1.86 – 1.88 GeV/c². The distribution of this is shown in Figure 3.2. Finally, the B^0 candidate was constructed in a similar way from the $D^\mp$, $\pi^\pm$ and π^0 .

The analysis software ensured that the charged pions used in the reconstruction of the D and the B are not the same.

It is important to note that the reconstruction of the $B^0 \rightarrow D^- \rho^+$ channel was chosen to mimic the reconstruction of the $B^0 \rightarrow D^- \pi^+ \pi^0$ signal as closely as possible. Therefore, all particle reconstructions and selection criteria were identical between the two channels and was disentangled during the fitting stage of the analysis.

The primary selection cuts placed on the B candidates were made on kinematic variables that have good discriminating power between signal and background events. Additionally, we require that they be uncorrelated to reduce systematic bias. The beam constrained mass, M_{bc} and the beam energy difference, ΔE satisfy these requirements and are defined as

$$M_{bc} \equiv \sqrt{E_{beam}^2 - p_B^2} \quad (3.2)$$

and

$$\Delta E \equiv E_B - E_{beam} \quad (3.3)$$

with their distribution shapes for both the signal and continuum background demonstrated in Figure 3.3. These variables are available to be exploited in Belle analyses due to the clean e^+e^- collisions that provide a well known initial state with no missing energies, and thus a well defined E_{beam} value. The reconstructed B^0 candidate was required to fall within the ΔE range $-0.2 - 0.2$ GeV and the M_{bc} range $5.272 - 5.287$ GeV/ c^2 .

3.1.5 π^0 Momentum Correction

Photons are detected in the ECL by the identification of clusters of energy deposited in the ECL crystals. Some of the energy from the original photon can be lost via leaks from the cluster, causing the energy of the photons to be underestimated. This in turn causes the momentum of the π^0 s to be underestimated. A correction to the pion momentum can be included which addresses this as well as having the effect of decorrelating the ΔE and M_{bc} variables. The expression for the corrected M_{bc} is

$$M_{bc}^{corr} = \sqrt{E_{beam}^2 - \left| \vec{p}_{B_{corrected}}^0 \right|^2} \quad (3.4)$$

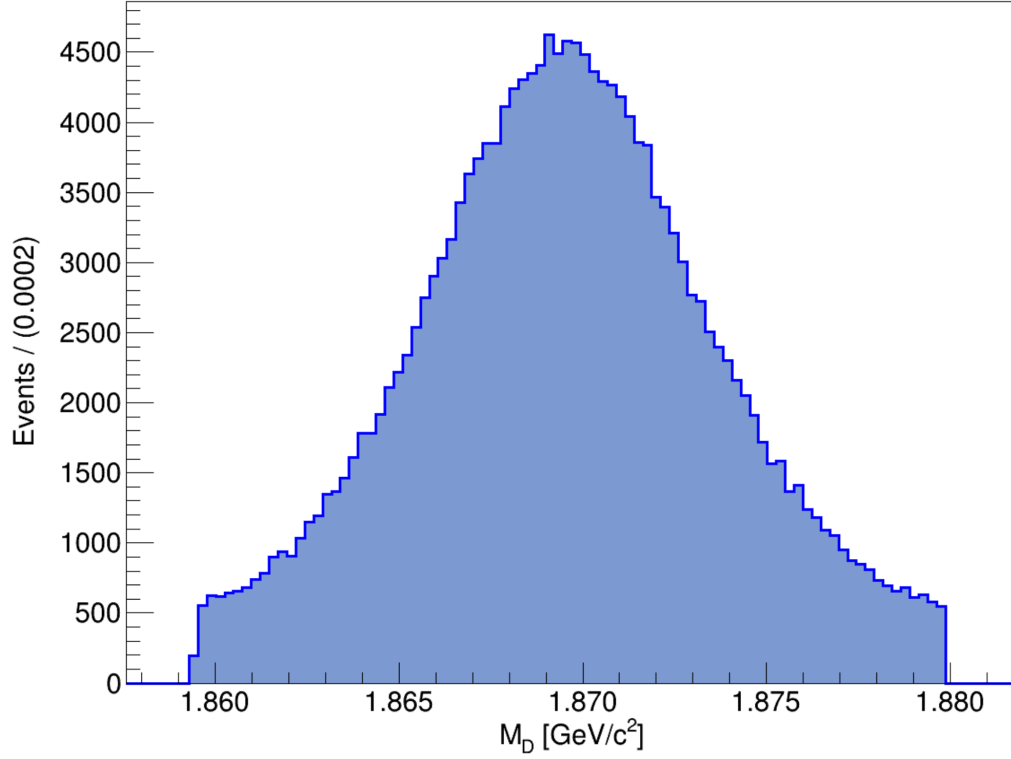


Figure 3.2: Invariant mass of the D^- in $B^0 \rightarrow D^- \pi^+ \pi^0$ decays.

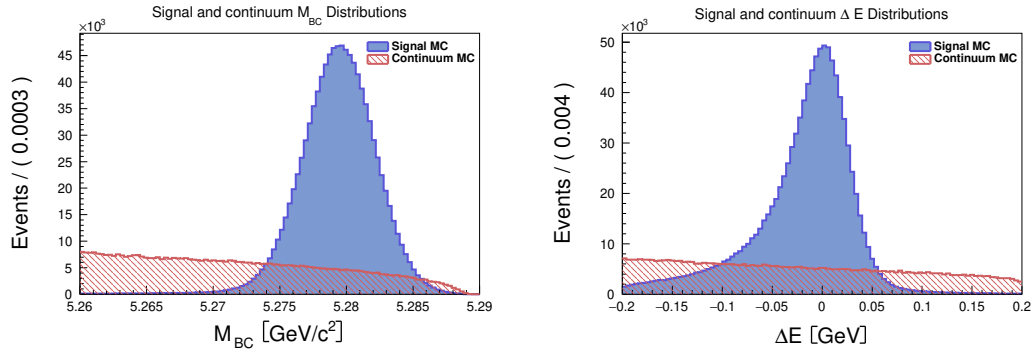


Figure 3.3: Comparison of ΔE and M_{bc} distributions for signal and continuum background.

where the $\vec{p}_{B_{corrected}^0}$ term is given by

$$\vec{p}_{B_{corrected}^0} = \vec{p}_{D^-} + \vec{p}_{\pi^+} + \frac{\vec{p}_{\pi^0}}{|\vec{p}_{\pi^0}|} \sqrt{(E_{beam} - E_{D^-} - E_{\pi^+})^2 - m_{\pi^0}^2}. \quad (3.5)$$

Here, the π^0 energy has been recalculated as the difference in energy between the beam energy and the other FSPs. A comparison of the corrected and uncorrected M_{bc} variables is shown in Figure 3.4 demonstrating the reduction in the correlation to ΔE . Going

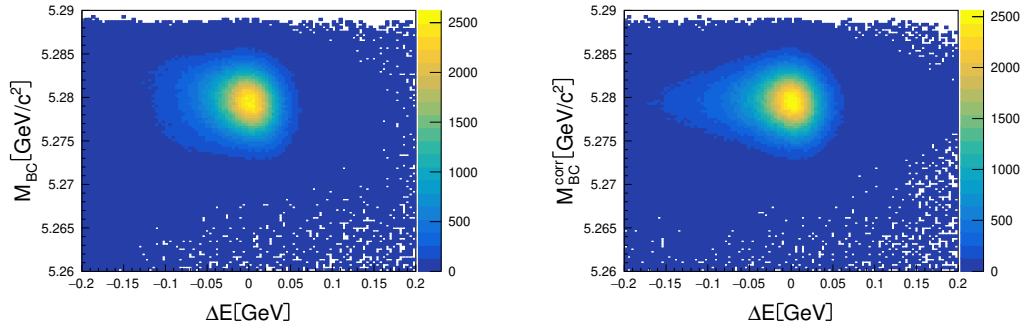


Figure 3.4: A comparison of the $\Delta E - M_{bc}$ relation for uncorrected (left) and corrected (right) M_{bc} performed on signal MC. Correlation factors are 4.44% and 1.15% respectively.

forward, the beam constrained mass, M_{bc} will be assumed to mean the momentum-corrected version.

3.2 Background Studies

This section provides a description of each type of background considered in the analysis and the techniques used to reduce them.

3.2.1 Self-crossfeed Background

In every event there are two B -mesons, but in general only one of them could decay via the signal channel while the other is far more likely to decay generically. When reconstructing the signal side B it is possible that tracks and energy depositions are taken from the non-signal B (the other “side” of the decay) instead, resulting in misreconstructed events known as self-crossfeed (SCF). There are other processes that can result in misreconstruction such as incorrect particle identification and fake track reconstruction. Misidentification of particles occurs when a particle species is assigned to a track that is different from its true identity, while fake tracks are reconstructed tracks that do not correspond to real particles in the event. These other sources are also included in the SCF background for the purposes of this analysis. All types of misreconstruction can be isolated for further study in simulation using the available MC-truth information (see Section 3.1.1) provided by BASF2.

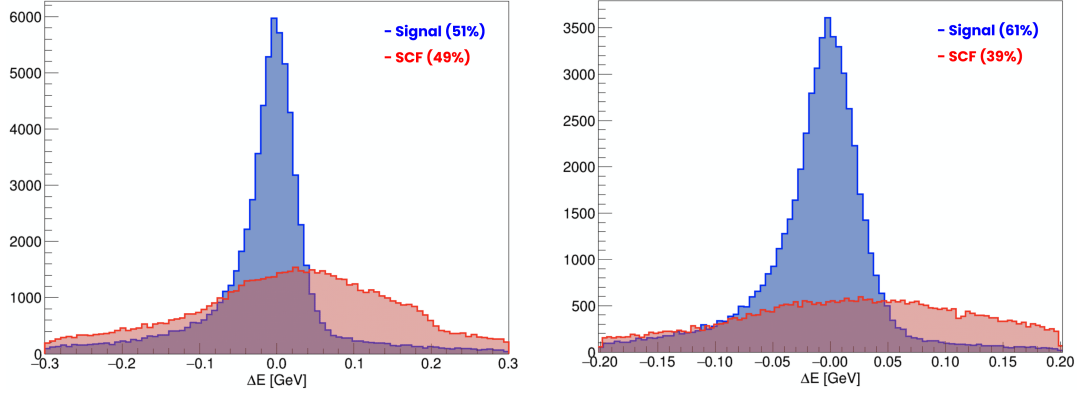


Figure 3.5: Distribution of the (blue) ΔE signal distribution, the (red) SCF distribution comparing the impact of selection criteria on the SCF background.

TABLE III: Composition of the SCF background from different sources. The proportion is the fraction of the total SCF background that contains each source. Decay in flight indicates that the B was reconstructed using a particle that was created in the detector during its flight. Missing neutrino, photon, massive FSPs and K_L s all indicate that these particle types were missed in the reconstruction of the B candidate. Mis-identified FSP indicates that a FSP was assigned the wrong identity (e.g. confusion of $\pi^\pm$ s and $K^\pm$ s). Wrong daughter particle refers to instances where the B was incorrectly reconstructed using a daughter particle from the other B [4].

Source	Proportion
Decay in flight	42.4%
Missing neutrino	26.4%
Missing photon	77.6%
Missing massive FSP	72.8%
Missing K_L	23.9%
Mis-identified FSP	23.3%
Wrong daughter particle	99.9%

It was found that after reconstruction and the application of all selection criteria, SCF constituted 39% of all signal candidates. This large contribution is characteristic of three-body signal modes with multiple charged pions and kaons in the final state which increases the likelihood of reconstruction from the wrong side of the event. A breakdown of the composition of the SCF background is given in Table III and a comparison of the signal to SCF distributions before and after the application of selection criteria are given in Figure 3.5. In general, the causes of SCF are non-exclusive and are often due to a combination of factors simultaneously.

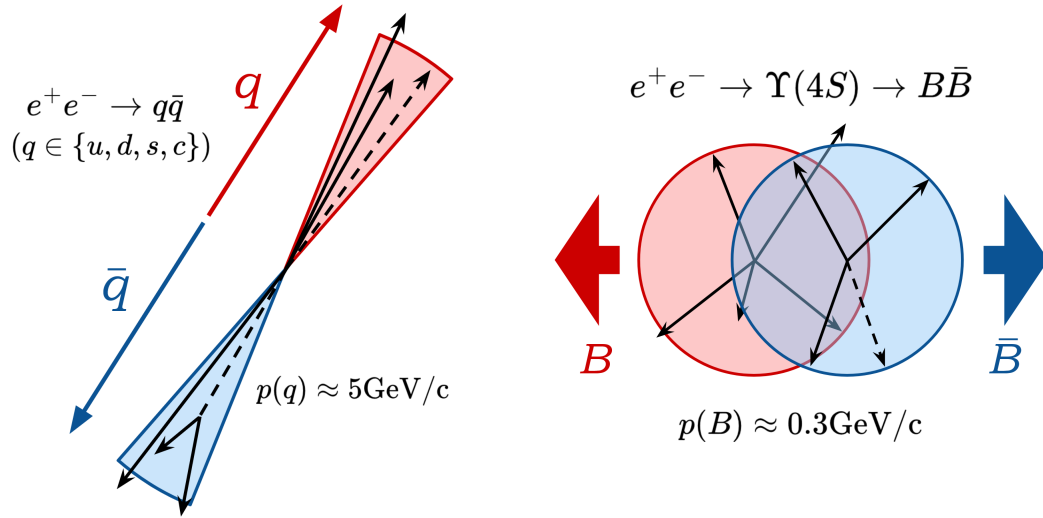


Figure 3.6: Diagram of a (left) continuum event and a (right) $B\bar{B}$ event highlighting the strongly collimated jets for continuum and uniformly spherical $B\bar{B}$.

3.2.2 Continuum Background

Continuum events are those in which the electron and positron annihilate and directly produce a quark anti-quark pair ($e^-e^+ \rightarrow q\bar{q}$, $q = u, d, s, c$) on-resonance. The quark pair then hadronise into a shower of particles with a distinctive jet-like decay topology as illustrated in Figure 3.6. These continuum events constitute the majority source of combinatorial backgrounds. The back-to-back hadronic jets describe an event shape that can be exploited using a trained classifier algorithm to distinguish it from signal events which have a more spherical shape.

A multivariate analysis was used to distinguish between continuum and signal events and in this analysis the classifier employed was a Boosted Decision Tree (BDT) [38, 39]. A BDT is a machine learning technique that works by training a series of smaller decision trees on random subsets of the data. Each of these weak trees are trained to correct the errors made by the previous tree in the sequence which ‘boosts’ the overall performance.

A new tree is constructed in each iteration of the boosting process to correct the erroneously classified events of the previous tree. An initial assumption is made that every event is of equal importance, and as such, they are each assigned an equal weight. As the training process continues, events that were previously classified incorrectly by prior models are updated with larger weights and likewise correctly classified events are assigned smaller weights. This has the effect of increasing the probability in subsequent

iterations of selecting a feature and threshold value to split upon that results in correctly separating signal from background events.

A number of detector-level variables are used in both Belle and Belle II which acted as inputs for the training of the FastBDT, Belle II's implementation of the BDT algorithm [40]. However, care must be taken to avoid training on variables that are correlated with the fitting variable ΔE as this could lead to sculpting of the background distributions into resembling signal. To this end, any variables found to have large correlations were excluded from the BDT, and what follows is a description of those that were included.

- **Thrust Variables:** For a collection of N particles with momenta $\vec{p}_i (i = 1, \dots, N)$, one can define the thrust axis $\vec{T}$ as the unit vector that maximises the sum of the projections of all particles onto the axis [41]. The thrust axis scalar T is hence given by

$$T = \frac{\sum_{i=1}^N |\vec{T} \cdot \vec{p}_i|}{\sum_{i=1}^N |\vec{p}_i|} \quad (3.6)$$

For the training of the BDT, two thrust related variables were employed: the first is the cosine of the angle between the thrust axis of the B candidate decay particles and the z -axis, $\cos\text{TBz}$, and the second is the magnitude of the thrust axis from the rest-of-event, thrustOm .

- **CLEO Cones:** The CLEO collaboration developed a set of variables that calculated the sum of the absolute values of momentum for particles located within angular regions around the thrust axis, with 10° separations, which generated 9 concentric cones [42]. The cone facing the thrust axis is merged with its counterpart in the opposite direction.
- **Modified Fox-Wolfram Moments:** These are a series of variables based on the Fox-Wolfram moments [43, 44] which are constructed from the momenta and relative angles of B daughters. They are defined for a system of N particles as

$$H_l = \sum_{i,j}^N |p_i| |p_j| P_l(\cos \theta_{i,j}) \quad (3.7)$$

where $\theta_{i,j}$ is the angle between particles with momenta p_i and p_j and P_l is the l^{th} order Legendre polynomial. The Belle collaboration created modified versions

of these variables H_{xl}^{so} and H_l^{oo} , where s denotes the B candidate daughters, o denotes the particles from the other B , termed the rest-of-event (ROE), and x is the charge of the reconstructed particle [41]. H_{xl}^{so} is defined as

$$H_{xl}^{so} = \begin{cases} \sum_i \sum_{jx} |p_{jx}| P_l(\cos \theta_{i,jx}) & (l = \text{even}) \\ \sum_i \sum_{jx} Q_i Q_{jx} |p_{jx}| P_l(\cos \theta_{i,jx}) & (l = \text{odd}) \end{cases} \quad (3.8)$$

where i denotes the B daughter, jx denotes the ROE for each x category, charged or neutral and Q is the charge of the corresponding particle. Similarly, H_l^{oo} is defined as

$$H_l^{oo} = \begin{cases} \sum_j \sum_k |p_j| |p_k| P_l(\cos \theta_{j,k}) & (l = \text{even}) \\ \sum_j \sum_k Q_j Q_k |p_j| |p_k| P_l(\cos \theta_{j,k}) & (l = \text{odd}) \end{cases} \quad (3.9)$$

where j and k run over the ROE particles.

The data was divided into training and test sets for training and validation purposes. The BDT was trained on 10 million signal MC events and streams 0 - 4 of continuum Belle MC. Test data is 1 million signal MC events and stream 5 of continuum Belle MC. A set of configuration settings are used to structure the BDT model and its learning process, collectively termed the ‘‘hyperparameters’’ of the model. The hyperparameters of the BDT are given in Table IV and were previously chosen by the Belle II collaboration in order to optimise the speed of training while maximising its performance.

To balance the rejection of background events with the retention of signal events, a Figure of Merit (FoM) is calculated for each potential value of the BDT output. It is calculated as,

$$FoM = \frac{N_{sig}}{\sqrt{N_{sig} + N_{bkg}}} \quad (3.10)$$

where N_{sig} and N_{bkg} denote the number of signal and continuum background events, respectively. Evaluation of the BDT performance and the ROC curve is shown in the FoM in Figure 3.8 which shows a consistent performance between the training set and the test set without over-fitting on the training data. The maximum FoM value occurs at 0.45 but the curve is relatively flat from 0.2 to 0.7. Since this analysis is not statistically limited, a more conservative cut of 0.27 was chosen in order to reduce potential sources

TABLE IV: Hyperparameter values used during training of the BDT in continuum suppression.

Hyperparameter	Value
Number of trees	200
Number of cut levels	8
Depth of tree	3
Shrinkage factor during boosting step	0.1
Fraction of data used in stochastic training	0.5

of data-MC discrepancy systematic errors. This results in rejecting 64% of continuum events whilst retaining 90% of signal events.

3.2.3 Generic $B\bar{B}$ Background

Generic $B\bar{B}$ background refers to any B decay that proceeds via the $b \rightarrow c$ quark transition and mimics the signal, which after continuum, constitute the majority of background. Clearly our backgrounds should be free of signal events, however, the two signal modes for this analysis, $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$, are both included in generic MC. Thus, in order to obtain a pure sample of $B\bar{B}$, it was necessary to remove all signal decays before continuing. This includes all instances of the signal produced via an intermediate resonance. BASF2 provides access to MC-truth information that allows one to identify and reject any such events from being included in the MC data set. By selecting only events that do not identify as signal according to this MC-truth information, we are able to guarantee a sample of generic $B\bar{B}$ that is free of both the signal and the associated SCF.

The $B\bar{B}$ background can be reduced by applying a series of kinematic cuts which are listed in Table V, the most significant of which are the cuts on M_{bc} and ΔE . For M_{bc} , signal candidates were required to lie between 5.272 and 5.287 GeV/ c^2 resulting in 79% generic background rejection and 87% signal retention of events that pass the skim criteria. For ΔE , a cut range of between -0.2 and 0.2 GeV resulted in 37% generic background rejection and 97% signal retention. These values are calculated after applying the data skim described in Section 3.1.2 and their impact can be seen in Figure 3.9.

There will inevitably exist other $B\bar{B}$ decays that can mimic our signal and by investigating the MC decays we can identify the most common ones. It was found that

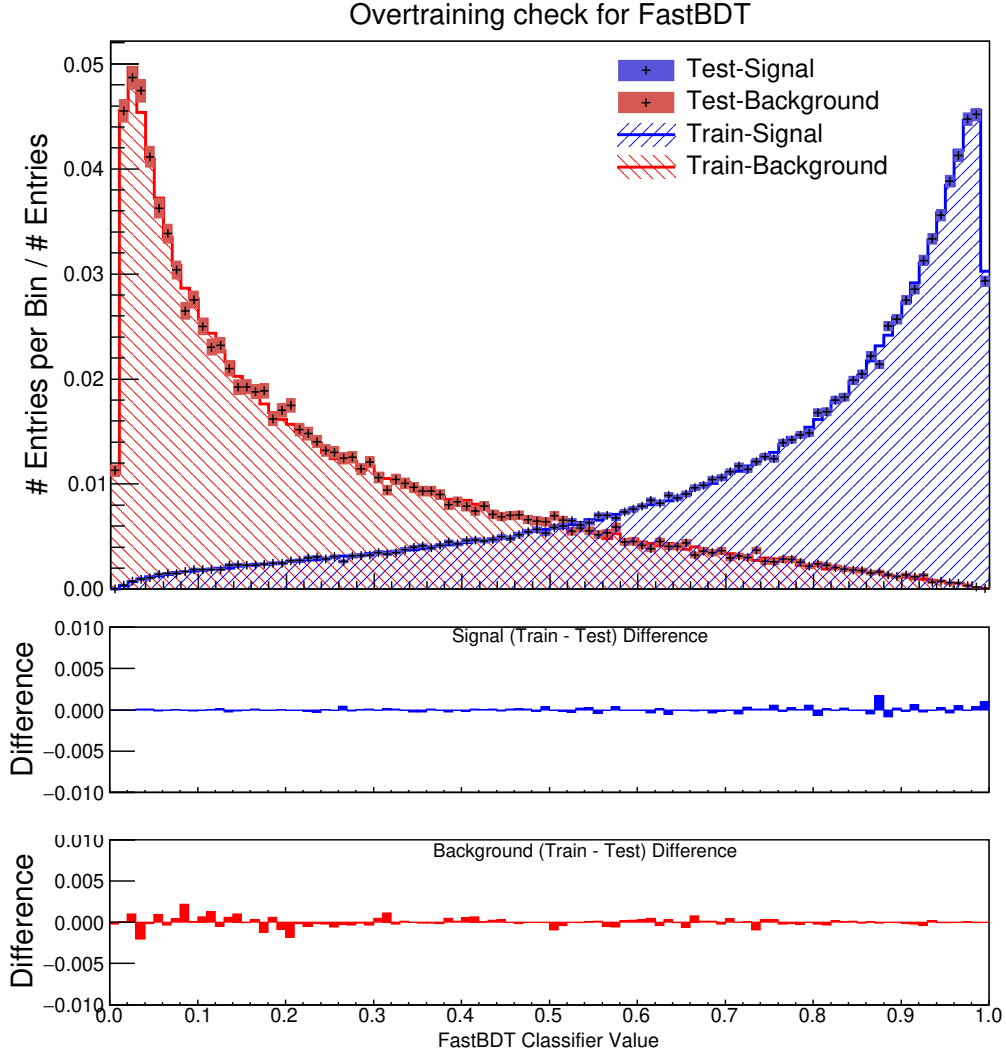


Figure 3.7: FBDT value for signal and continuum background in both the training set and test set.

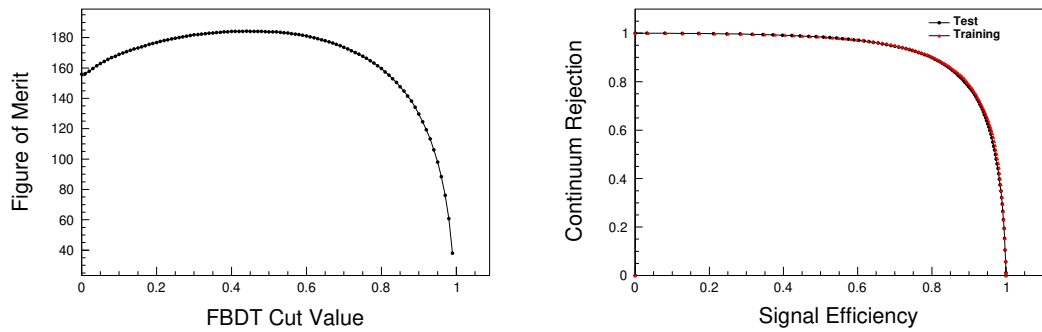
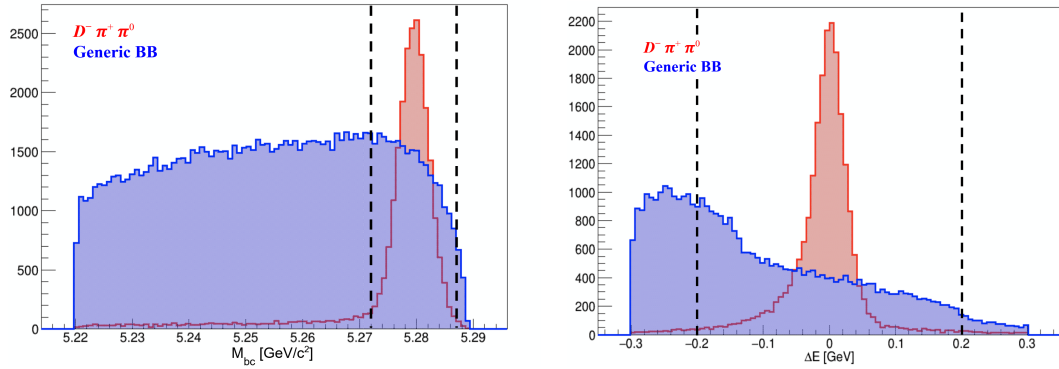


Figure 3.8: (left) The figure of merit (FoM) for the test set. (right) ROC curves for both the training and test sets. The area under the curve (AUC) for the training set is 0.930 and for the test set 0.926.

TABLE V: Kinematic cuts applied to reduce generic $B\bar{B}$ background.

Variable	Selected Range
M_{bc}	$5.272 - 5.287 \text{ GeV}/c^2$
ΔE	$-0.2 - 0.2 \text{ GeV}$
$M_{D^\pm}$	$1.86 - 1.88 \text{ GeV}/c^2$
M_{π^0}	$115 - 152 \text{ MeV}/c^2$
p_{π^0}	$> 0.2 \text{ GeV}/c$
$p_{\pi^\pm}$	$0.3 - 3.5 \text{ GeV}/c$
$p_{K^\pm}$	$0.3 - 3.5 \text{ GeV}/c$


Figure 3.9: Distributions of (left) M_{bc} and (right) ΔE for $D^- \pi^+ \pi^0$ signal and generic $B\bar{B}$ background demonstrating the impact of cuts on generic $B\bar{B}$ and signal MC after having passed the initial skim given in Section 3.1.2. Cut values shown in dashed lines.

$B^0 \rightarrow D^{*(2010)-} \rho^+$, $B^0 \rightarrow D^- a_1(1260)^+$, $B^- \rightarrow D^{*(2007)0} \rho^-$, $B^- \rightarrow D^0 \rho^-$ and $B^- \rightarrow D^{*(2010)+} \pi^- \pi^- \pi^0$ combined together constitute $\sim 20\%$ of the total $B\bar{B}$ background. The signal $D^\pm$ and π^0 was selected to be within 2 standard deviations of the nominal invariant masses. Low momentum final state $K^\pm$ and $\pi^\pm$ particles were excluded by placing cuts on the lab frame momentum. These cuts were the same as those employed in a Belle analysis of $B \rightarrow \bar{D} \pi^0$ by T. Bloomfield [45], which has a similar final state.

3.2.4 $D\pi^0$ Resonances

There exists the possibility of other intermediate, non S-wave resonances that peak in the central ΔE signal region which may interfere with the extraction of the forward-backward asymmetry parameter. If this is the case, then the partial-wave expansion outlined in Equation 1.10 does not converge [26]. It may be the case that the ρ resonance will dominate and make these effects negligible but this needs to be tested. To investigate

TABLE VI: Top ten most abundant decay modes found within generic $B\bar{B}$ background for each of the two kinematic regions listed. This corresponds to the two peaking regions found in the Dalitz plots in Figure 3.11.

$M_{D^- \pi^+} > 4.7 \text{ GeV}$ $0.9 \text{ GeV} < M_{\pi^+ \pi^0} < 1.5 \text{ GeV}$		$M_{D^- \pi^0} < 2.3 \text{ GeV}$ $1.1 \text{ GeV} < M_{\pi^+ \pi^0} < 1.6 \text{ GeV}$	
$D^- \rho^+$	20.50%	$D^*(2010)^- \pi^+$	15.67%
$D^*(2010)^- \pi^+$	12.99%	$D^- \rho^+$	14.14%
$D^- \mu^+ \nu_\mu$	6.05%	$D^- \mu^+ \nu_\mu$	4.70%
$D^- e^+ \nu_e$	5.51%	$D^*(2010)^- \rho^+$	4.60%
$D^*(2010)^- \mu^+ \nu_\mu$	5.37%	$D^*(2010)^- \mu^+ \nu_\mu$	4.33%
$D^*(2010)^- \rho^+$	5.24%	$D^*(2010)^- e^+ \nu_e$	4.30%
$D^*(2010)^- e^+ \nu_e$	4.53%	$\bar{D}_0^*(2400)^0 \pi^+$	2.63%
$\bar{D}_0^*(2400)^0 \pi^+$	3.01%	$\bar{D}^*(2007)^0 \pi^+$	2.60%
$\bar{D}^*(2007)^0 \pi^+$	2.34%	$\bar{D}_2^*(2460)^0 \pi^+$	1.66%
$\bar{D}^0 \rho^+$	2.36%	$D^- \pi^+ \pi^+$	1.63%

this effect we search for the presence of charged D^* resonances decaying to $D\pi^0$ states in a sample of generic $B\bar{B}$ MC. We define the following difference of invariant masses between D^* and its daughter D ,

$$\Delta M = M(D^*) - M(D) \quad (3.11)$$

and look for a peak in its distribution, the result of which is shown in Figure 3.10. A clear peak is seen around $0.142 \text{ GeV}/c^2$ which is consistent with the expected value for ΔM from the PDG of $0.14 \text{ GeV}/c^2$.

Additionally, two scatter plots were made plotting the invariant mass of the $D^- \pi^{+/0}$ versus the $\pi^+ \pi^0$ (i.e. Dalitz plots [46, 47]) shown in Figure 3.11. These were inspected for the presence of any other resonance structures. The two peaking regions were investigated further by looking into the MC-truth information and tabulating the top ten most abundant decays modes listed in Table VI. We see that the $D^*(2010)^- \pi^+$ is at most 10.8% of the peak. We also see the presence of some semi-leptonic decays which can be attributed to the loose ΔE kinematic cuts (see Table V). These modes were accounted for as part of the $B\bar{B}$ background PDFs when performing the final fits to ΔE , but in an effort to minimise undesired peaking structures in the $M_{\pi\pi}$ distribution, a ΔM veto was applied around the peak at $[0.13, 0.15] \text{ GeV}$. Additionally, this veto was applied to a sample of pure signal MC and was found to have a negligible effect on the reconstruction efficiency ($< 0.01\%$).

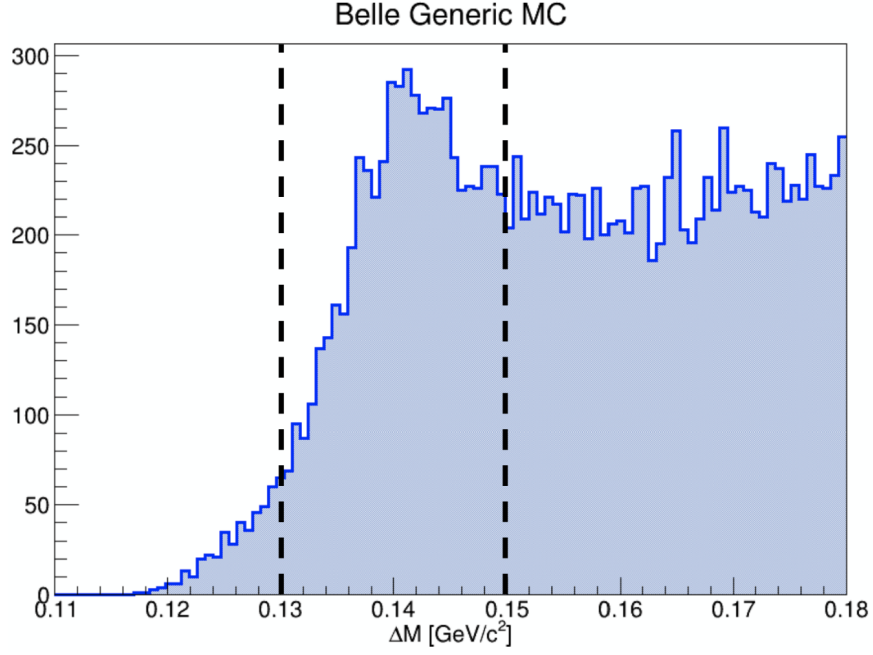


Figure 3.10: Distribution of the ΔM variable for a sample of generic $B\bar{B}$ which includes the signal channels. A veto is placed between 0.13 and 0.15 GeV/c^2 to exclude the D^* .

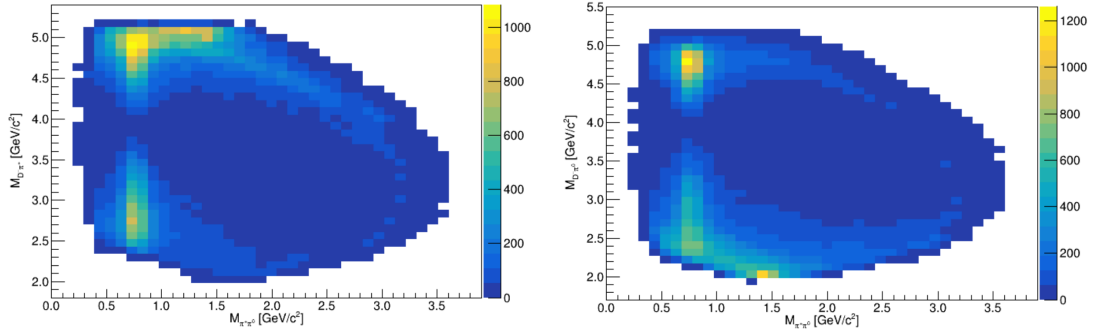


Figure 3.11: Dalitz plots of generic $B\bar{B}$. (left) $M_{D^- \pi^+}$ vs $M_{\pi^+ \pi^0}$ and (right) $M_{D^- \pi^0}$ vs $M_{\pi^+ \pi^0}$.

3.3 Best Candidate Selection

The reconstruction process is capable of creating multiple signal candidates per event, even after applying selection cuts. Best candidate selection (BCS) is the method of choosing the most likely correct signal candidate from all possibilities.

There are numerous possible choices for BCS. Two methods were tested for ranking B candidates: Best D mass and best vertex fit. The best D mass is the reconstructed D mass that is closest to the D mass stated by the PDG of $1.869 \text{ GeV}/c^2$ [24]. The vertex fit was performed using the Tree Fitter algorithm developed for BASF2 [48]. The

TABLE VII: Performance of best candidate selection methods. Proportion of times the correct B candidate is chosen when presented with multiple candidates on each MC sample. Pure signal is a sample of MC in which the other B is forced to decay to a $\nu\bar{\nu}$ state, thus eliminating all potential cross-contamination from the other B .

	Best D Mass (%)	Best Vertex Fit (%)
Pure $B^0 \rightarrow D^- \pi^+ \pi^0$	97	95
Generic $B\bar{B}$	79	70

two methods were tested on a sample of pure signal MC (with the other side going to a neutrino anti-neutrino state) and a sample of generic $B\bar{B}$. The pure signal sample was used as a baseline performance metric while the generic sample acted as a proxy for what one might expect in real data. The performance of the two methods were evaluated by looking at the proportion of times that the true signal candidate was correctly chosen when presented with an event that contained a true signal. The results are shown in Table VII along with the average candidate multiplicity before applying BCS in Figure 3.12. Additionally, the amount of self-crossfeed background (described in Section 3.2.1) found in signal MC was compared between the two methods and found that the best D mass method was slightly superior (20.1%) to the vertex fit method (20.8%). As a final validation, the potential for background sculpting was investigated. To make sure the application of the BCS method didn't modify the backgrounds into anything more signal-like, the ΔE distributions for generic $B\bar{B}$ and continuum were compared before and after applying BCS and are shown in Figure 3.13. In both cases, it shows that the application of BCS did not sculpt the backgrounds into resembling signal. Based on these results the best D mass method was chosen for BCS.

3.4 Efficiencies & Expected Yields

After applying all selection criteria and background suppression, the overall reconstruction efficiency, not including the $D^- \rightarrow K^+ \pi^- \pi^-$ decay fraction of 9.4% was found to be $(6.23 \pm 0.03)\%$ for $B^0 \rightarrow D^- \pi^+ \pi^0$ and $(6.21 \pm 0.03)\%$ for $B^0 \rightarrow D^- \rho^+$. The expected yields for $B^0 \rightarrow D^- \rho^+$ signal and associated backgrounds are listed in Table VIII. Due to the lack of any prior measurement for the overall $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction, the value given in Table VIII is the amount found in one stream of generic $B\bar{B}$ MC. The same is done for the $B^0 \rightarrow D^- \rho^+$ yield. The expected continuum and pure generic $B\bar{B}$

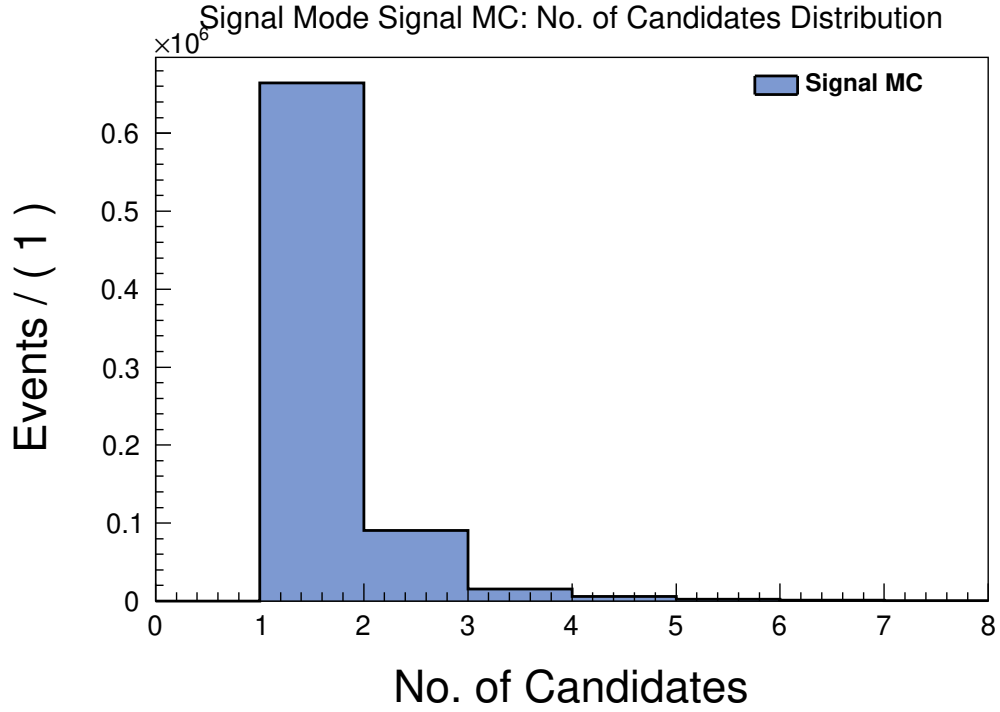


Figure 3.12: Number of signal candidates per event prior to BCS in $B^0 \rightarrow D^- \pi^+ \pi^0$ MC with an average candidate multiplicity of 1.21.

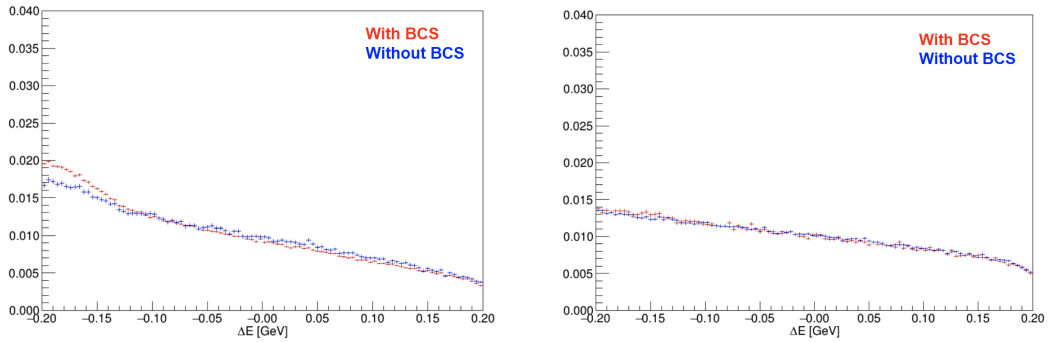


Figure 3.13: Normalised ΔE distributions of (left) generic $B\bar{B}$ and (right) continuum backgrounds in MC with (red) and without (blue) BCS applied.

background yields are the average over all six available streams of Belle SVD2 MC. Due to the non-linear nature of the reconstruction efficiency over phase-space, this cannot easily be interpreted as the expectation found in real data. Instead, these values will be used as an ansatz baseline for validations to MC datasets consisting of a cocktail of signal and backgrounds designed to mimic real data, known as ensemble toy MC.

TABLE VIII: Expected yields for SVD2 Belle data. $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ yields are taken from generic Belle MC streams.

Component	Expected yield
$B^0 \rightarrow D^- \pi^+ \pi^0$	58 299
$B^0 \rightarrow D^- \rho^+$	35 559
Self-crossfeed	40 388
Generic $B\bar{B}$	43 318
Continuum	50 193

3.5 Signal Extraction

The overall fitting strategy for extracting $A_{FB}^{\pi\pi}$ as well as the branching fractions for $B^0 \rightarrow D^- \rho^+$ and $B^0 \rightarrow D^- \pi^+ \pi^0$, inclusive of all intermediate resonances, is given in this section.

3.5.1 Fitting Strategy

The efficiency of finding $B^0 \rightarrow D^- \pi^+ \pi^0$ events changes as a function of the kinematics of the $D^- \pi^+ \pi^0$ final state. This is termed the phase-space of the event. The distribution of $D^- \pi^+ \pi^0$ events is not uniform across this phase-space but it is not known what it is a priori. The method described below enables one to determine the efficiencies needed to determine the branching ratios for $B^0 \rightarrow D^- \pi^+ \pi^0$, $B^0 \rightarrow D^- \rho^+$ and $A_{FB}^{\pi\pi}$.

To extract the signal yield, a series of unbinned maximum likelihood fits to ΔE in bins of $\cos \theta_h$ and the invariant mass of the $\pi^+ \pi^0$ system, $M_{\pi\pi}$, were performed and then a subsequent correction to account for bin event migration was applied. This was then divided by the efficiency for each bin as will be described in Section 3.5.4 in order to extract the true yield for that bin. The fit model consisted of the total $B^0 \rightarrow D^- \pi^+ \pi^0$ signal and three background components: self-crossfeed, continuum and generic $B\bar{B}$. The $\cos \theta_h$ and $M_{\pi\pi}$ distributions for samples of pure $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ are given in Figure 3.14 for comparison. The fit was done in 10 bins of $\cos \theta_h$ from -1 to $+1$ and 20 bins of $M_{\pi\pi}$ from 0.25 to $3.5 \text{ GeV}/c^2$ with 10 bins above $1 \text{ GeV}/c^2$ and 10 bins below for a total of 200 bins. This binning was chosen to have a high enough resolution to capture the ρ mass peak while avoiding too many bins in regions of phase-space with low yield. The need to bin the $B^0 \rightarrow D^- \pi^+ \pi^0$ phase-space in such a way

was further motivated by inspecting the SCF distributions and noting how they varied across phase-space as well as from the total integrated distribution. This is shown in Figure 3.15.

The total signal yield is calculated as the linear sum of fitted yields over all bins and its associated statistical uncertainty is the sum in quadrature over all bin fit errors. From the fitted signal distribution the asymmetry, $A_{FB}^{\pi\pi}$ can be calculated as

$$A_{FB}^{\pi\pi} = \frac{N_F - N_B}{N_F + N_B} \quad (3.12)$$

where N_F and N_B are the number of events in the forward or backward regions with $\cos \theta_h > 0$ or $\cos \theta_h < 0$ respectively. Only events in the low invariant mass region below $1 \text{ GeV}/c^2$ were included in the calculation of $A_{FB}^{\pi\pi}$ since this is where the ρ invariant mass dominates as specified in Ref. [26].

To obtain the final yield for $B^0 \rightarrow D^- \rho^+$, a binned extended maximum likelihood fit was performed on the efficiency corrected $M_{\pi\pi}$ distributions for $M_{\pi\pi} < 1 \text{ GeV}/c^2$. This captures the ρ resonance peak and avoids the change in binning above $1 \text{ GeV}/c^2$, an example of which is shown in Figure 3.16 on a toy MC sample. The yields for $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ are converted into branching fractions according to the following formula,

$$\mathcal{B} = \frac{N_{sig}}{2 \times R_{B^0 \bar{B}^0} \times N_{B\bar{B}} \times \mathcal{B}(D^- \rightarrow K^+ \pi^- \pi^-)} \quad (3.13)$$

where $N_{B\bar{B}}$ is the total number of $B\bar{B}$ pairs produced during Belle SVD2 and $R_{B^0 \bar{B}^0}$ is the ratio of neutral $B\bar{B}$ pairs to the total. The factor of 2 in the denominator takes into account that for every $\Upsilon(4S)$ produced there are two subsequent B -mesons, either of which could decay via the signal mode.

Taken as is, this fitting strategy treats the signal and SCF components as being independent of each other, when in reality they are highly correlated. In order to capture this correlation in the fit procedure, the SCF yield was parametrised in terms of the signal yield and the SCF fraction. This fraction was then restricted to float within 10 percentage points of the value obtained from MC, a map of which is shown in Figure 3.17. The

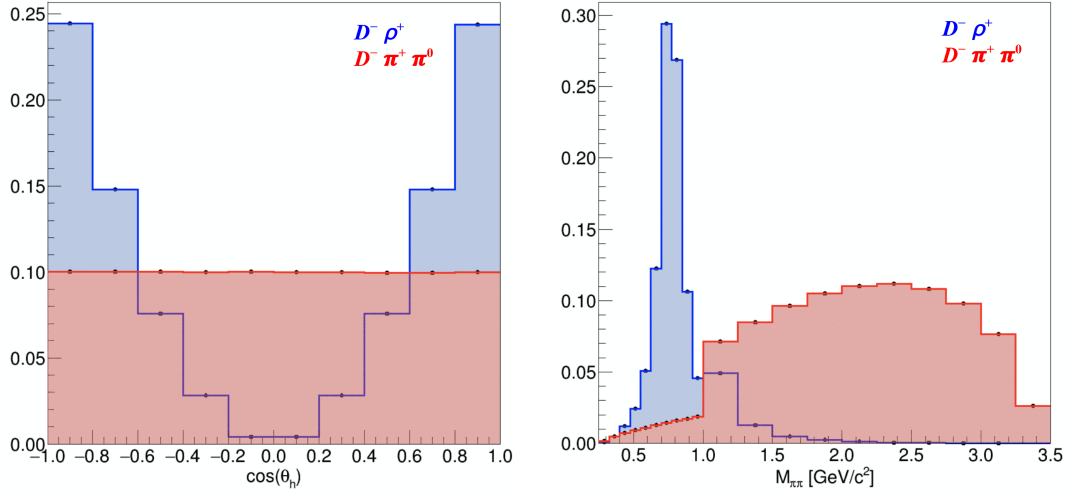


Figure 3.14: Normalised distributions of (left) $\cos \theta_h$ and (right) $M_{\pi\pi}$ for (blue) $B^0 \rightarrow D^- \pi^+ \pi^0$ and (red) $B^0 \rightarrow D^- \rho^+$ channels.

explicit form of the parametrisation is given as,

$$N_{SCF} = f_{SCF} N_{sig+SCF} \quad (3.14)$$

$$= f_{SCF} \frac{N_{sig}}{1 - f_{SCF}} \quad (3.15)$$

which allows us to extract the signal yield from a sum of the signal and SCF PDFs

$$N_{sig} PDF_{sig} + N_{SCF} PDF_{SCF} = N_{sig} \left(PDF_{sig} + \frac{f_{SCF}}{1 - f_{SCF}} PDF_{SCF} \right). \quad (3.16)$$

3.5.2 sPlot Considerations

The sPlot method is a powerful statistical technique for disentangling signal from background in a single discriminating variable [49]. It does this by using the yields obtained from a likelihood fit to calculate weights (called sWeights) for each event based on its probability of being signal or background, allowing for the separation of their contributions in the variable of interest. This technique was considered as an alternative to performing multiple fits in bins of $\cos \theta_h$ and $M_{\pi\pi}$ but was ultimately deemed unsuitable due to large correlations found between ΔE and $\cos \theta_h$. Additionally, the background shapes were found to vary substantially across phase-space as demonstrated in Figure 3.18 where the signal and SCF background is compared in the low and high $\cos \theta_h$ regions. This adds to the complications of employing the sPlot technique.

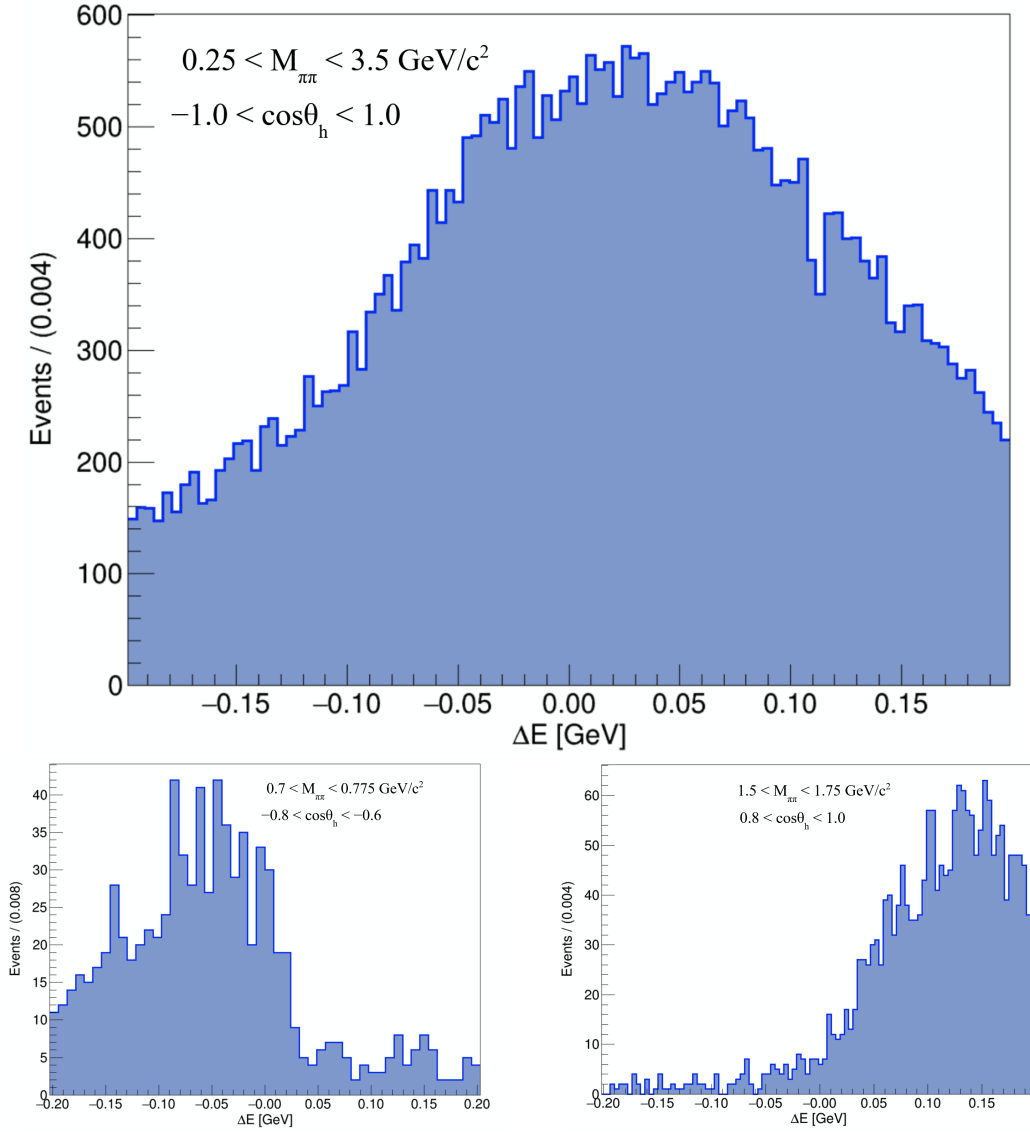


Figure 3.15: ΔE distributions in SCF MC, showing the large variation across phase-space.

3.5.3 Bin Event Migration

During the reconstruction process, it is inevitable that there will be some positional shift in kinematic phase-space. This shift could bias the extracted value of $A_{FB}^{\pi\pi}$. The correction for this effect employed MC truth information and was made as follows.

The number of events in each bin that migrated out of that bin after reconstruction was counted and then divided by the number of events that originated from that bin. A similar count was performed to determine the fraction of events that originated from outside the bin and migrated in. These two fraction maps are shown in Figure 3.19 for the dedicated signal MC data sample and can be used to modify the signal yield

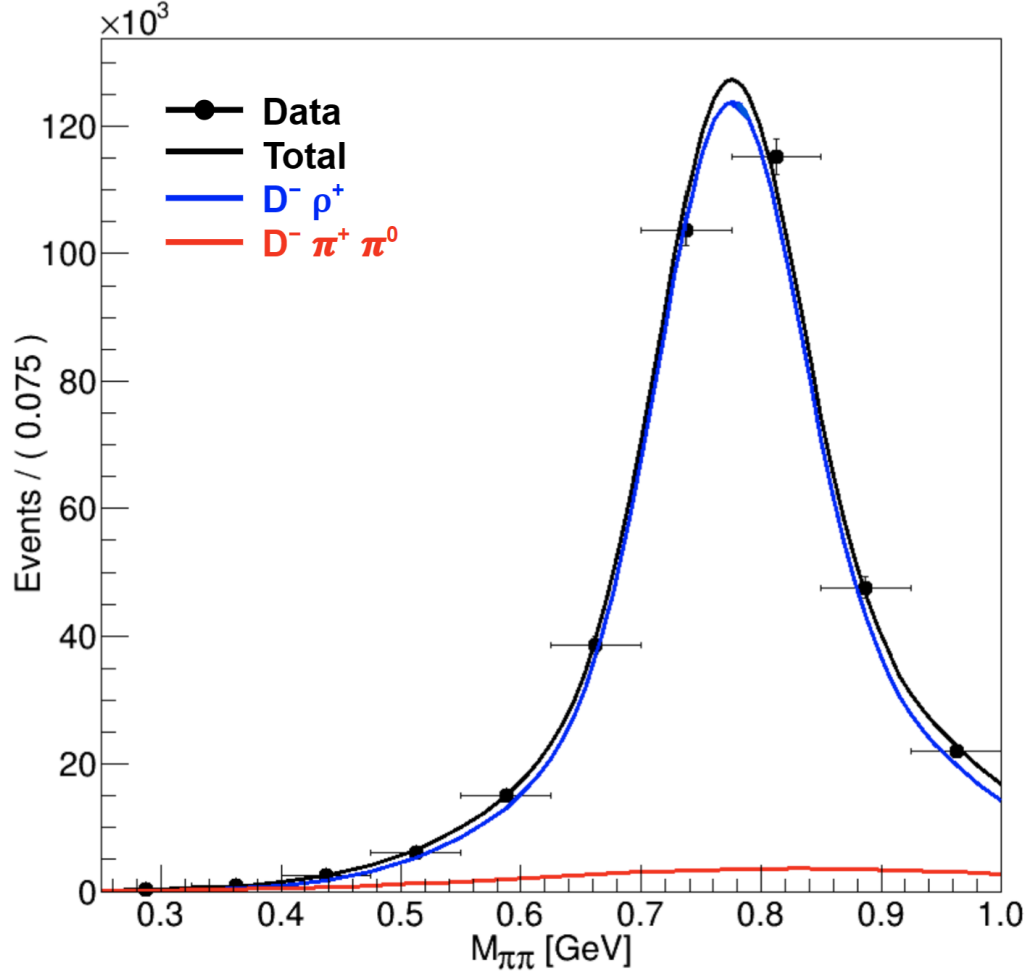


Figure 3.16: Example toy fit to MC in $M_{\pi\pi}$ with (red) $D^- \pi^+ \pi^0$ and (blue) $D^- \rho^+$ signal channels.

map after fitting in ΔE , the result of which should resemble the distribution prior to reconstruction. To derive an update formula one can express the reconstructed yield, N_{rec} , in terms of the initial yield prior to reconstruction, N_{init} , and the number of events that migrate into and out of each bin during reconstruction, N_{in} and N_{out} :

$$N_{rec} = N_{init} - N_{out} + N_{in} \quad (3.17)$$

$$N_{init} - N_{out} = N_{rec} - N_{in} \quad (3.18)$$

$$N_{init} \left(1 - \frac{N_{out}}{N_{init}} \right) = N_{rec} \left(1 - \frac{N_{in}}{N_{rec}} \right). \quad (3.19)$$

Hence, in each bin N_{rec} can be re-calculated according to the formula,

$$N_{init} = N_{rec} \left(\frac{1 - f_{in}}{1 - f_{out}} \right) \quad (3.20)$$

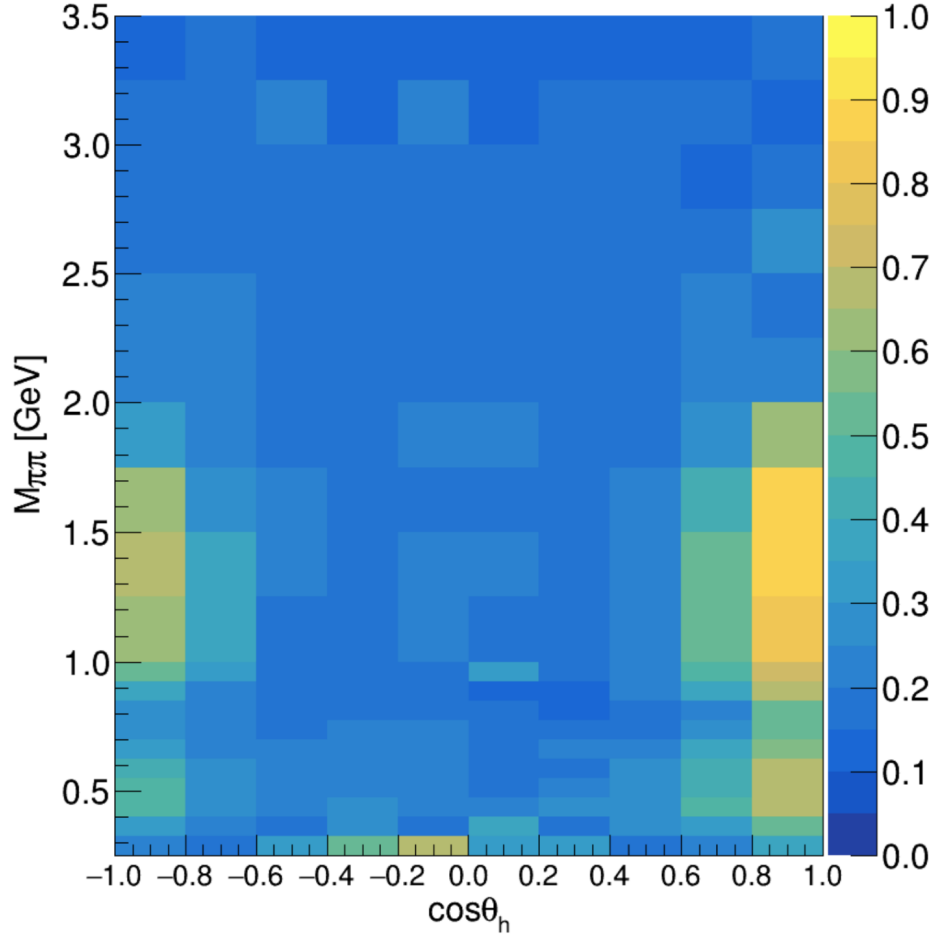


Figure 3.17: Fraction of reconstructed signal candidates that are due to self-crossfeed background in each bin of $\cos \theta_h$ and $M_{\pi\pi}$.

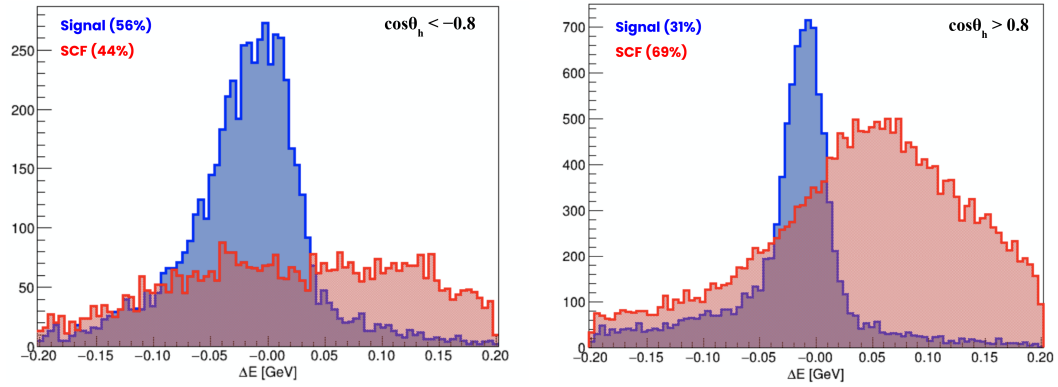


Figure 3.18: ΔE distributions for (blue) signal and (red) SCF MC for (left) $\cos \theta_h < -0.8$ and (right) $\cos \theta_h > 0.8$.

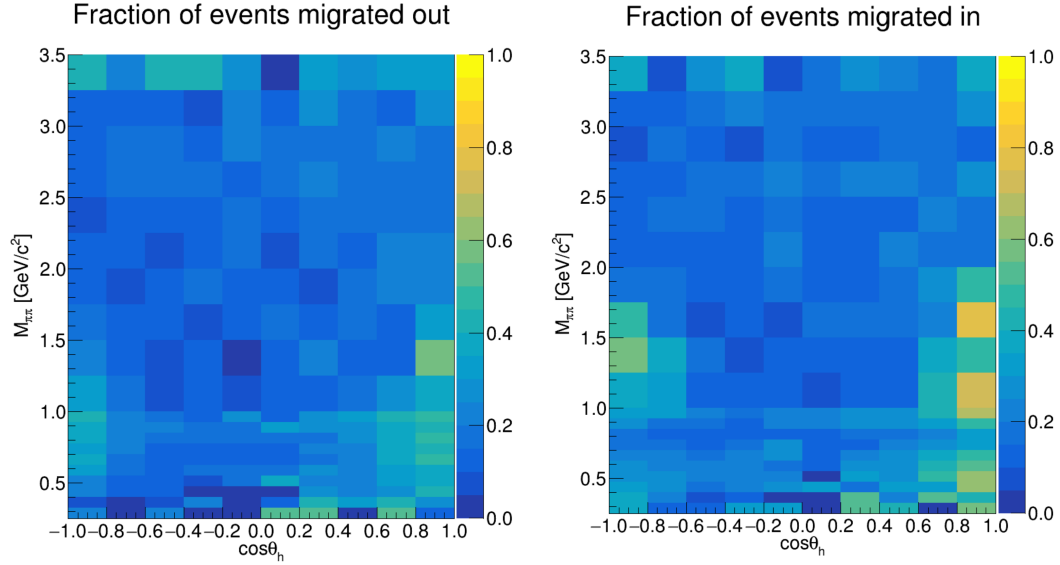


Figure 3.19: Fraction of events that migrated out (left) and in (right) of each bin after reconstruction for pure signal MC sample.

where f_{in}/f_{out} is the fraction of events that originated from inside/outside and migrated into/out of the bin after reconstruction. A comparison of the pre-reconstruction and event migration corrected yield maps are shown in Figure 3.20 which show almost perfect agreement indicating that the yield correction is able to compensate for the effects of event migration. An artificial scale factor has been applied to provide visual separation between the two distributions, without which they would be identical except for the last bin in $M_{\pi\pi}$ which makes less than a 0.1% contribution to the $D^- \pi^+ \pi^0$ yield.

3.5.4 Reconstruction Efficiency Matrix

A crucial parameter in the yield extraction is the reconstruction efficiency. Instead of considering the overall efficiency across all available phase-space, an efficiency matrix in bins of $\cos\theta_h$ and $M_{\pi\pi}$ was used. For reference, the definition of θ_h given by T. Huber et al. [26] is the angle between the three-momenta of the B -meson and the π^0 in the rest frame of the $\pi^+ \pi^0$. This was calculated with a custom function integrated with the BASF2 software framework. The binned efficiency matrix is able to account for variations in reconstruction efficiency as we pass through regions containing higher order resonance structures.

The efficiency in each bin is given by the reconstructed yield divided by the total number of MC generated events in that bin. This has been performed for the dedicated signal

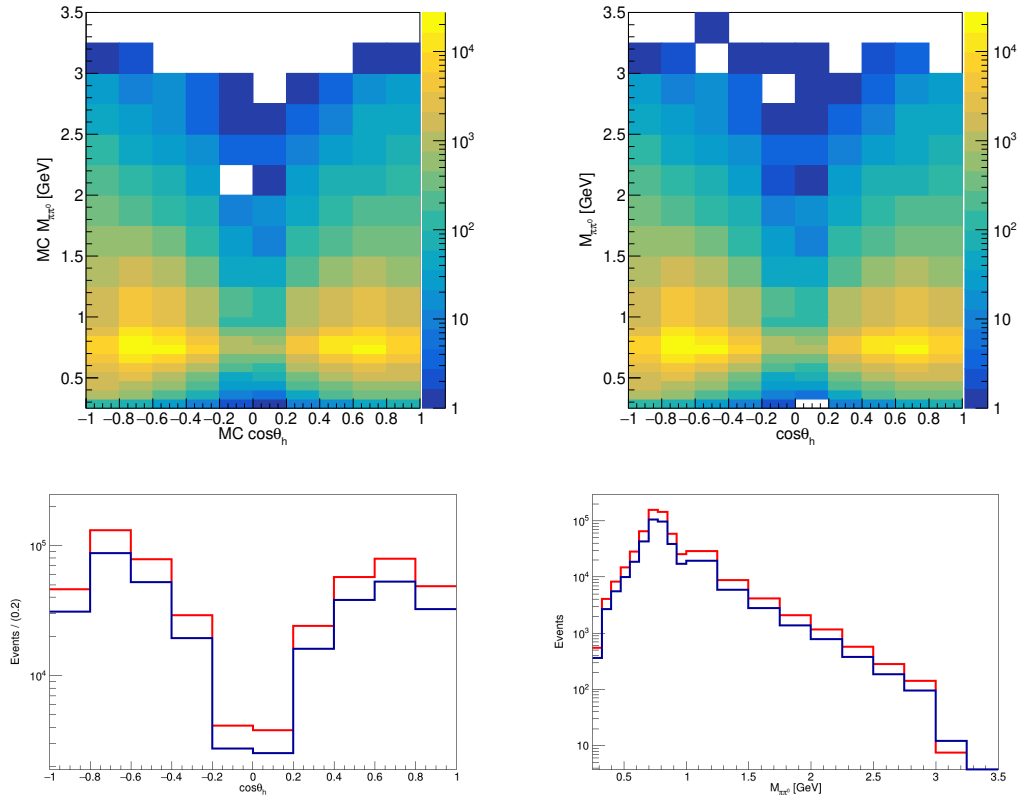


Figure 3.20: $D^- \rho^+$ yield map prior to reconstruction (top left) and after reconstruction with the event migration correction applied (top right). Log projections of each variable are also given where an artificial scale factor has been applied to provide visual separation between the pre-reconstruction (red) and post-reconstruction + event migration correction (blue) distributions.

MC sample which was generated with all signal channels found in generic MC. The resultant efficiency matrix is used in the final yield extraction for both $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ and is shown in Figure 3.21. Its effect on the reconstructed $\cos \theta_h$ distribution is shown in Figure 3.22 which demonstrates its ability to recover the generated signal distribution, both around the ρ mass peak as well as the low $M_{\pi\pi}$ region. The performance of this matrix was validated on an MC sample of pure $B^0 \rightarrow D^- \rho^+$ of a known generated yield. After performing the summation in Equation 3.13 the efficiency corrected yield extracted from the $D^- \rho^+$ component was found to be within 1% of the generated number of events, and thus provides confidence in its performance.

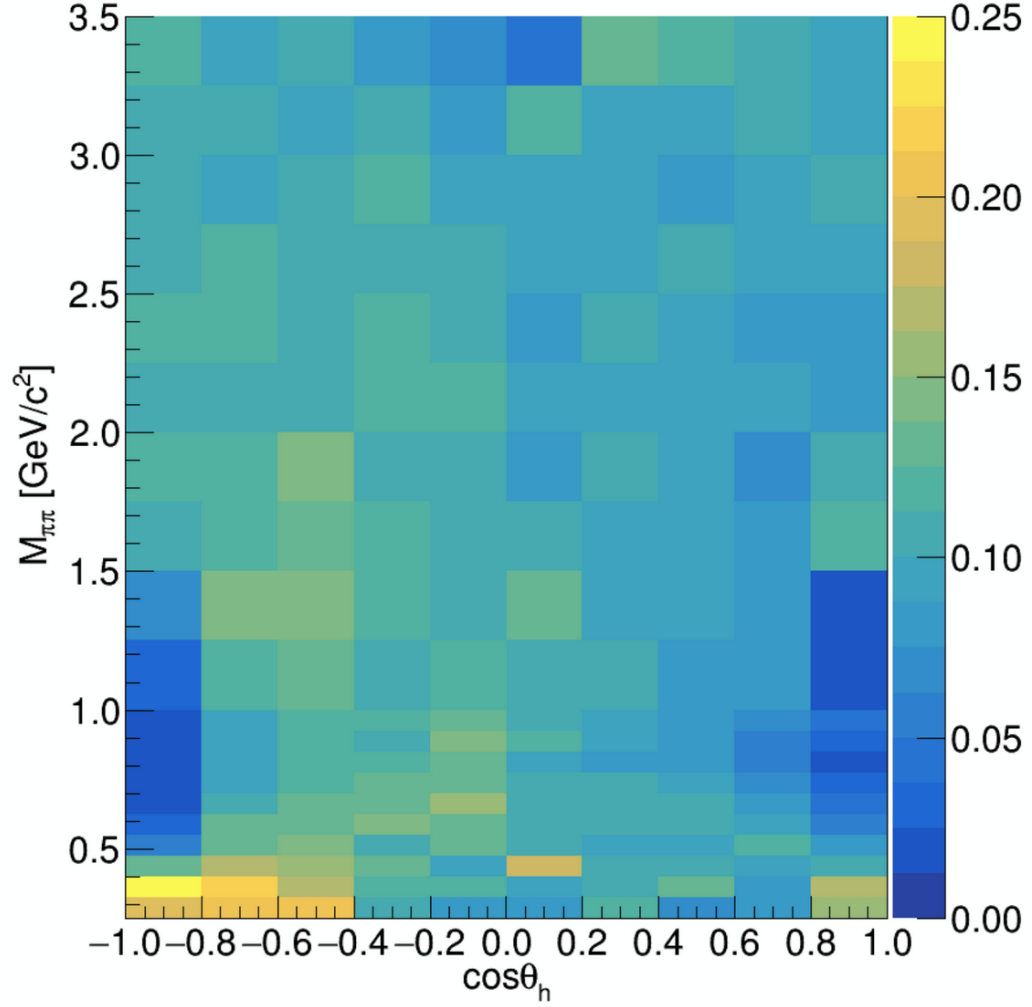


Figure 3.21: Efficiency matrix for pure signal derived from Belle generic MC which includes $D^- \pi^+ \pi^0$, $D^- \rho^+$ and all other signal channels known to Belle.

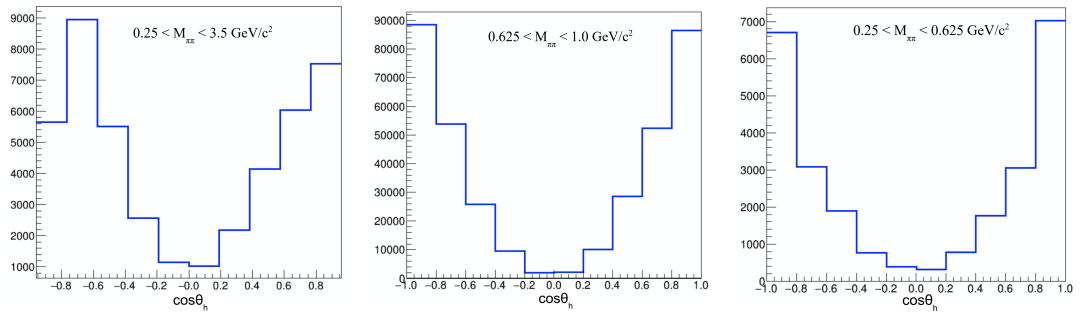


Figure 3.22: $\cos \theta_h$ distributions before applying efficiency matrix (left), after around the rho mass peak (middle), and after below the mass peak (right).

TABLE IX: ΔE PDF shapes for all components used in both fits to full kinematic range and bin-by-bin fits.

Component	PDF Shape (ΔE)
$B^0 \rightarrow D^- \pi^+ \pi^0$	Crystal Ball [50] + 2 $\times$ Gaussians
Generic $B\bar{B}$	2nd Order Chebychev + Gaussian
Continuum	3rd Order Chebychev
Self-crossfeed	3 $\times$ Gaussians

3.5.5 PDF Modelling

In this section we describe the process of modelling the component PDFs for each of the steps outlined in Section 3.5.1, namely:

1. fitting the ΔE component distributions across the full kinematic range.
2. fitting the ΔE component distributions in each bin of $\cos \theta_h$ and $M_{\pi\pi}$ with shape parameters based on those found in the previous step.
3. fitting to the final $M_{\pi\pi}$ projected distribution.

3.5.5.1 Full Component ΔE PDFs

One-dimensional unbinned maximum likelihood fits were performed to each component in ΔE across all kinematically allowed phase space. The resulting PDFs served as a baseline on which to model subsequent PDFs within each bin of $\cos \theta_h$ and $M_{\pi\pi}$. The signal and self-crossfeed (SCF) component fits were performed on a dedicated signal MC sample based on the Belle generic MC. The generic $B\bar{B}$ and continuum background fits were performed on representative MC samples taken from six streams of Belle MC for each. A summary of the PDF shapes for each component is given in Table IX and their fits are shown in Figure 3.23. After all individual component fits were performed, the best fit values for all shape parameters were fixed for the combined unbinned maximum likelihood ΔE fit, with the exception of their yields and the signal component's core Crystal Ball mean and width which were both floated, to account for potential data-MC differences.

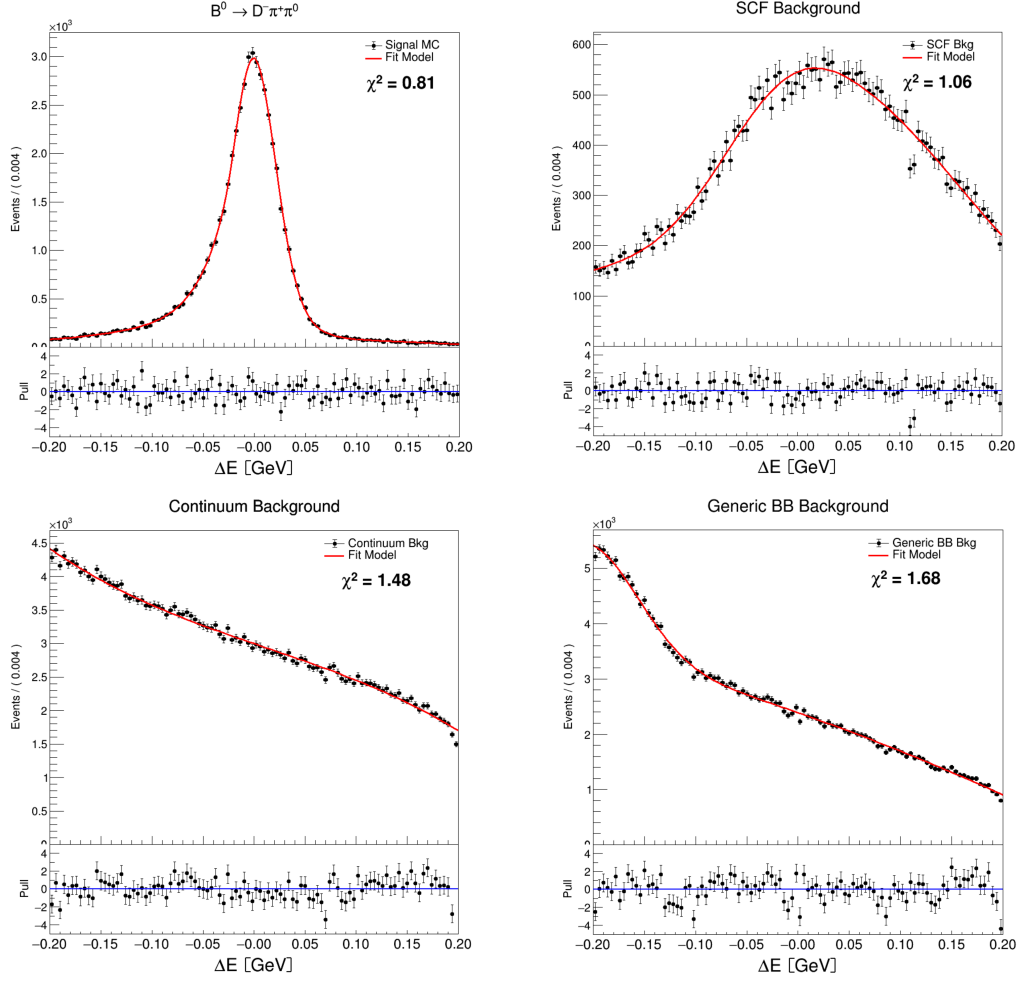


Figure 3.23: Full kinematic range ΔE PDF fits to each component. The PDF shapes are listed in Table IX. The χ^2 values given are per degree of freedom.

3.5.5.2 ΔE PDFs in Bins of $\cos \theta_h$ and $M_{\pi\pi}$

For each bin, the PDF used in the fit is the sum total of the component PDFs weighted by their respective yields,

$$P_{total} = \sum_i N_i P_i \quad (3.21)$$

where the summation runs over the four components (signal, generic $B\bar{B}$, continuum, SCF). The individual component PDFs were modelled on the same MC samples described in Section 3.1.3 and a representative selection of these 200 fits can be found for each component in Appendix A. The PDF shapes per bin were taken to be the same as those found in Table IX for the total kinematic range. However, in order to account for the variance across phase-space in their distributions, the shape parameters were allowed to be fit within a range of 10-20 times the statistical uncertainty returned by

the fits to the full kinematic range. In low statistics bins (< 50 events), convergence of the fit is not guaranteed and so a single Gaussian with a broad width was used in those bins. An attempt was made at fitting a Chebychev polynomial to these bins for the continuum component but it was found that in many cases, the fit would not converge.

3.5.5.3 $M_{\pi\pi}$ PDFs

The fits to ΔE in bins of $\cos \theta_h$ and $M_{\pi\pi}$ produces as its output a binned signal distribution in those same variables. However, ΔE alone cannot distinguish between $D^- \rho^+$ and the other signal channels, so in order to make an exclusive branching fraction measurement, it is necessary to disentangle the $D^- \rho^+$ mode from the others. This was accomplished by performing a second binned maximum likelihood fit to $M_{\pi\pi}$ which required dedicated PDFs, one for the $D^- \rho^+$ component and a second “background” component that consisted of all other $D^- \pi^+ \pi^0$ channels.

To obtain the $M_{\pi\pi}$ distributions for each component, the $M_{\pi\pi}$ vs $\cos \theta_h$ distribution was projected onto the x-axis and restricted to the region $M_{\pi\pi} < 1 \text{ GeV}/c^2$. The fit was limited to this low mass region where the $\rho^\pm$ resonance dominates while avoiding the discontinuity in the bin widths above this region. An attempt was made to fit a relativistic Breit-Wigner to the $\rho^\pm$ but it was found that the fitting package, RooFit [51], could not converge on a physically plausible width. As an alternative to the Breit Wigner, the $D^- \rho^+$ peak was fit using a double Gaussian while the $D^- \pi^+ \pi^0$ component was fit with a single Gaussian as shown in Figure 3.24. However, due to the uncertainty in how the underlying $D^- \pi^+ \pi^0$ component will manifest in real data, both the mean and width were left floated with a wide range.

3.6 Fitter Tests

3.6.1 ΔE Fits Validation

To extract the total $B^0 \rightarrow D^- \pi^+ \pi^0$ signal component, fits to ΔE were performed in the 200 bins of $\cos \theta_h$ and $M_{\pi\pi}$. For validation of this fit, a mixed MC sample was prepared with each component randomly sampled from the full MC samples described in Section 3.1.3 with expected yields found in Table VIII. A comparison of the $\cos \theta_h$ and

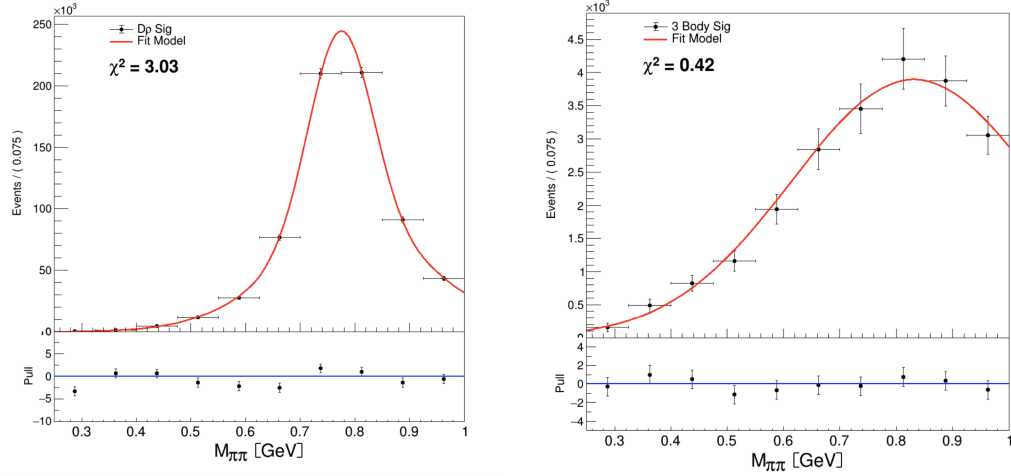


Figure 3.24: PDF binned fits to $M_{\pi\pi}$ for (left) $D^- \pi^+ \pi^0$ and (right) all other $D^- \pi^+ \pi^0$ final states. The χ^2 values given are per degree of freedom.

$M_{\pi\pi}$ distributions from before and after the fits are shown in Figure 3.25 which are the averaged results from 20 runs. For $\cos \theta_h$, additional comparisons were made for specific regions of $M_{\pi\pi}$ including where the $\rho^\pm$ resonance peaks between 0.625 and 1.0 GeV/ c^2 , and in the low mass region between 0.25 and 0.625 GeV/ c^2 . For the higher mass regions, the agreement is excellent indicating that the fit is able to extract the reconstructed signal distribution accurately. However, in the low mass region an overestimation of the yield in the outer bins was seen. Consequently, the $|\cos \theta_h| > 0.8$ bins have been excluded from the final calculation of $A_{FB}^{\pi\pi}$.

3.6.2 $A_{FB}^{\pi\pi}$ Extraction Validation

To validate that the methodology described in Section 3.5.1 is capable of extracting any potential helicity asymmetry found in real data, a study was performed on five $B^0 \rightarrow D^- \pi^+ \pi^0$ MC samples generated with different values of $A_{FB}^{\pi\pi}$. The samples were generated using EvtGen v02.00.00 [31] with 1 million signal events each and skimmed in BASF2 to introduce an artificial helicity bias. This resulted in samples with the values $A_{FB}^{\pi\pi} = -0.115, -0.057, 0.002, 0.057$ and 0.118 . These were then passed through GEANT3 for simulation of the detector interactions, followed by a BASF level reconstruction skim, and finally converted back into BASF2 file format with B2BII.

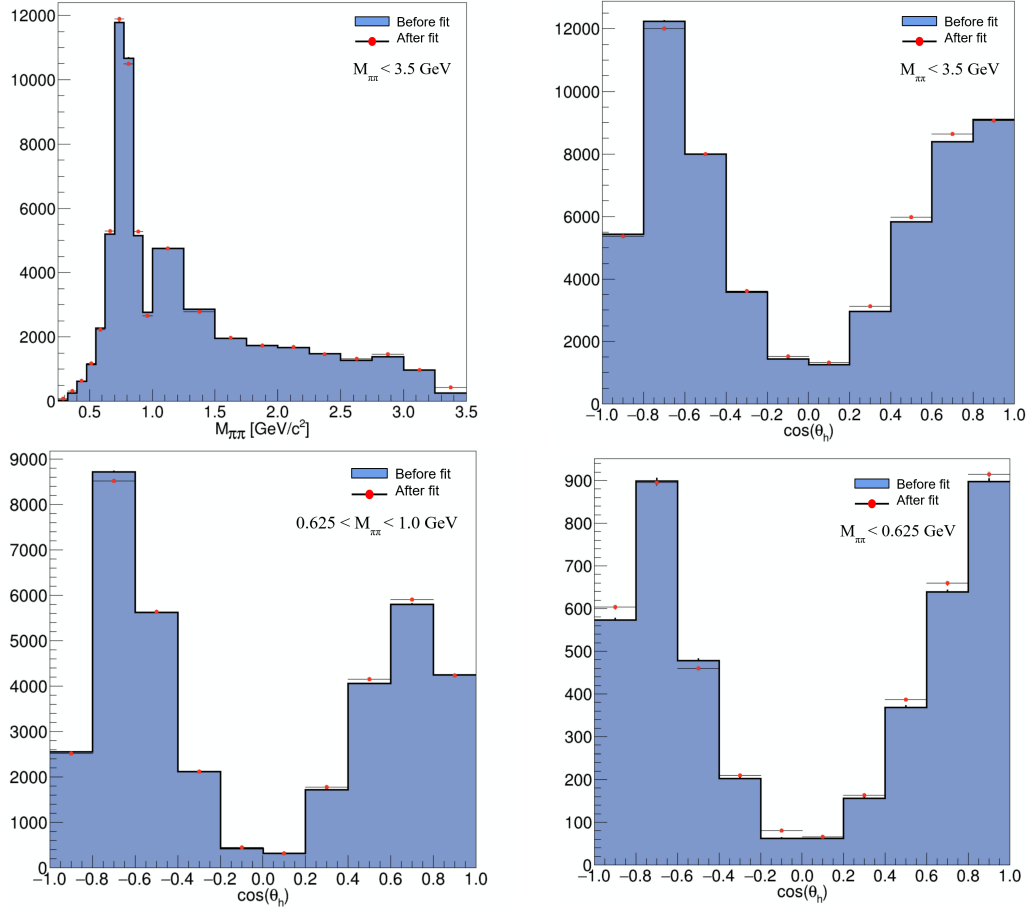


Figure 3.25: Average distributions of 20 toy ensemble fits comparing the fitted distributions (red points) to the true distributions in MC (blue).

The five converted samples were then passed through the same BASF2 reconstruction described in Section 3.1.4 including continuum suppression and best candidate selection techniques. Finally, to extract $A_{FB}^{\pi\pi}$, we apply the full fitting strategy outlined in Section 3.5.1 to each of the five biased samples with expected backgrounds present. The projected distributions for $\cos\theta_h$ are shown for each of the five samples in Figure 3.26 for comparison. When fitting to samples including backgrounds, 10 ensemble toy fits were performed and the average extracted $A_{FB}^{\pi\pi}$ was calculated along with the corresponding standard error. The asymmetry is calculated according to Equation 3.12, but note that only the region $M_{\pi\pi} < 1 \text{ GeV}/c^2$ and $|\cos\theta_h| < 0.8$ were considered for the reasons outlined in Section 3.5.1.

The numerical results of the extracted $A_{FB}^{\pi\pi}$ are given in Table X. A linearity plot of these results against the original generated values is shown in Figure 3.27 with a line of best fit given. A perfect extraction would return a fitted line with a slope of 1 and intercept

TABLE X: Results of the $A_{FB}^{\pi\pi}$ validation performed on MC samples containing expected backgrounds. Uncertainties in the third column are the standard error from 10 ensemble toy fits.

Generated $A_{FB}^{\pi\pi}$	Extracted $A_{FB}^{\pi\pi}$
-0.115	-0.121 ± 0.004
-0.057	-0.059 ± 0.003
0.002	0.023 ± 0.003
0.057	0.052 ± 0.004
0.118	0.120 ± 0.003

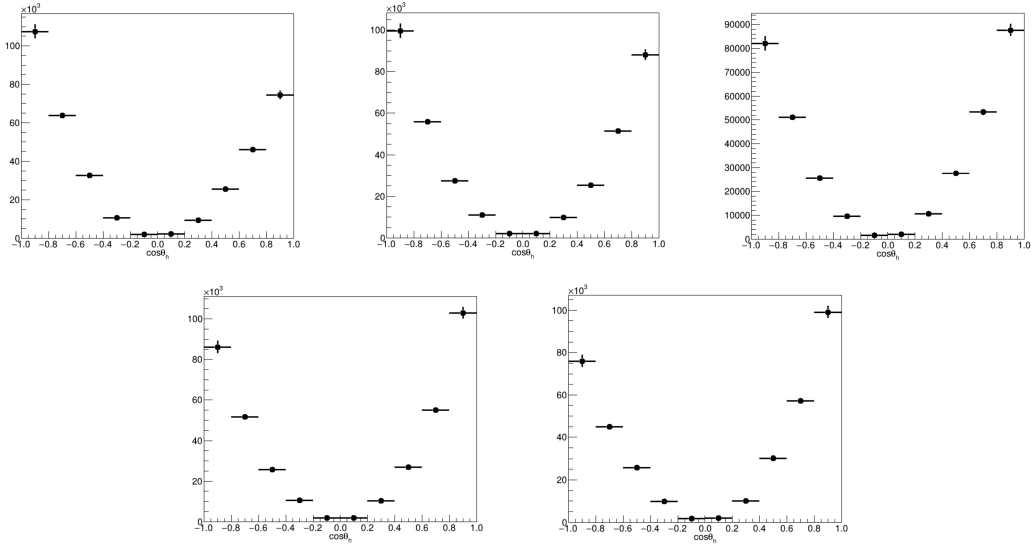


Figure 3.26: Distributions of extracted $\cos\theta_h$ for (top left) $A_{FB}^{\pi\pi} = -0.115$, (top middle) $A_{FB}^{\pi\pi} = -0.057$, (top right) $A_{FB}^{\pi\pi} = 0.002$ MC samples, (bottom left) $A_{FB}^{\pi\pi} = 0.057$ and (bottom right) $A_{FB}^{\pi\pi} = 0.118$.

of 0. The result obtained here is consistent with 1 and 0 to within statistical uncertainty and so no correction is needed. However, an outlier was found around the value expected by the standard model, and so we conservatively take the maximum difference between the generated and extracted value, 2.1%, as the systematic uncertainty for this.

3.6.3 $B^0 \rightarrow D^- \pi^+ \pi^0$ Yield Extraction Validation

There is no prior measurement of the total $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction and hence there is no expectation on the yield. It is therefore important to validate the performance of the fit strategy on a range of possible $B^0 \rightarrow D^- \pi^+ \pi^0$ yields that may be found in real data. The validation was done ranging from 60% to 140% of the signal yield obtained from Belle SVD2 generic $B\bar{B}$ streams listed in Table VIII. Background component yields

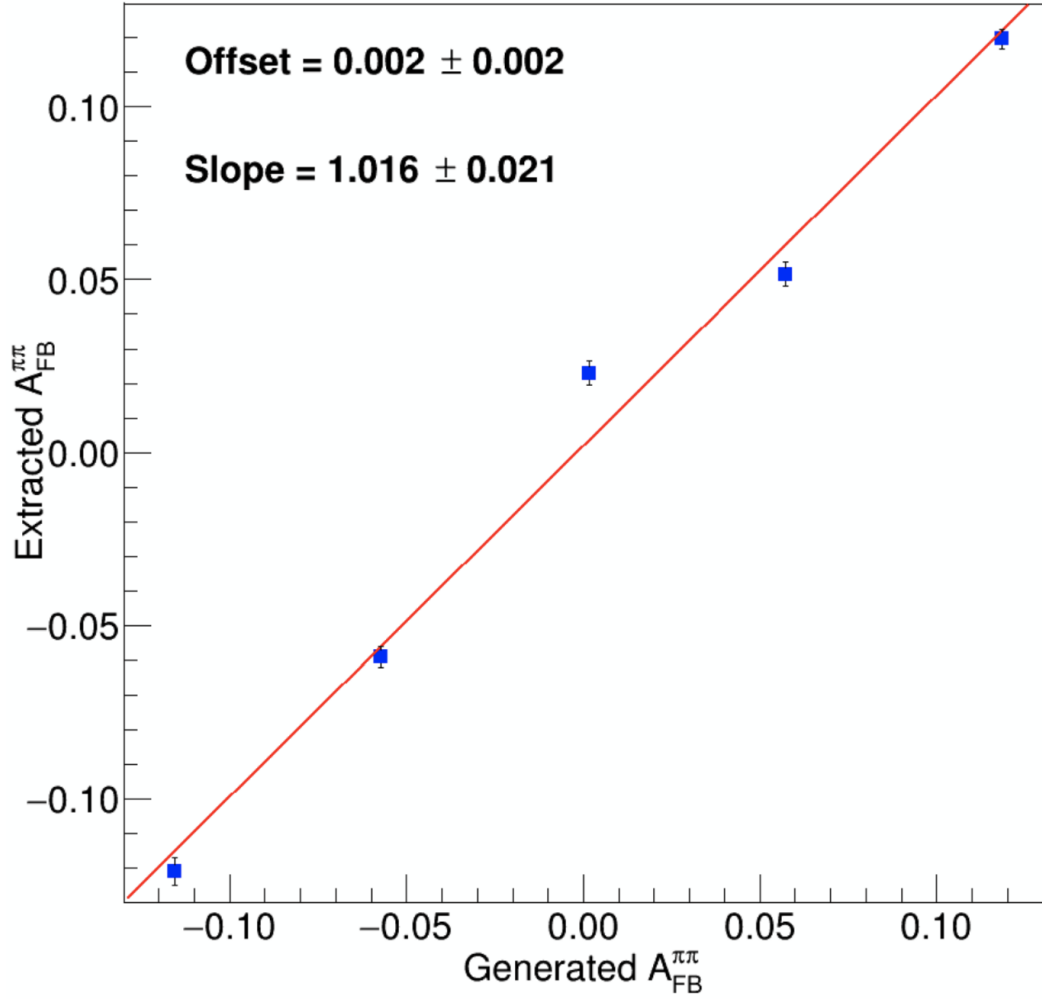


Figure 3.27: Linearity test for the $A_{FB}^{\pi\pi}$ parameter with backgrounds included. The generated $A_{FB}^{\pi\pi}$ is the asymmetry present at generation time. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors.

were held fixed at their expected yields with the exception of the SCF background which was varied in the same proportion as the signal component. For each $B^0 \rightarrow D^- \pi^+ \pi^0$ yield, the full fits in bins of ΔE was performed 10 times with events randomly sampled from the total signal MC data sample. The resulting distributions are 2D histograms in $\cos\theta_h$ and $M_{\pi\pi}$ which then have the event migration corrections applied. As a final step, a division by the efficiency matrix was done and the average across the 10 samples was recorded. The numerical results of the validation are listed in Table XI and a plot of the extracted yield against the input yield is shown in Figure 3.28 along with a line of best fit. The non-zero offset of the line indicates a bias is present which is calculated as the offset divided by the midpoint of the input values. This gives a 1.9% bias and the extracted yield will be corrected for this.

TABLE XI: Results of the $B^0 \rightarrow D^- \pi^+ \pi^0$ yield validation. The input yields are obtained by applying the event migration correction and efficiency matrix directly without performing any ΔE fits. The listed uncertainties are the standard error of the 10 toy fits.

Input $N_{D^- \pi^+ \pi^0}$	Extracted $N_{D^- \pi^+ \pi^0}$
536 246	552 328 $\pm$ 2996
625 620	651 734 $\pm$ 2696
714 994	736 468 $\pm$ 4261
804 369	834 884 $\pm$ 4241
893 743	914 734 $\pm$ 3246
983 117	998 233 $\pm$ 4508
1 072 492	1 087 417 $\pm$ 5129
1 161 866	1 191 261 $\pm$ 3777
1 251 240	1 279 049 $\pm$ 4384

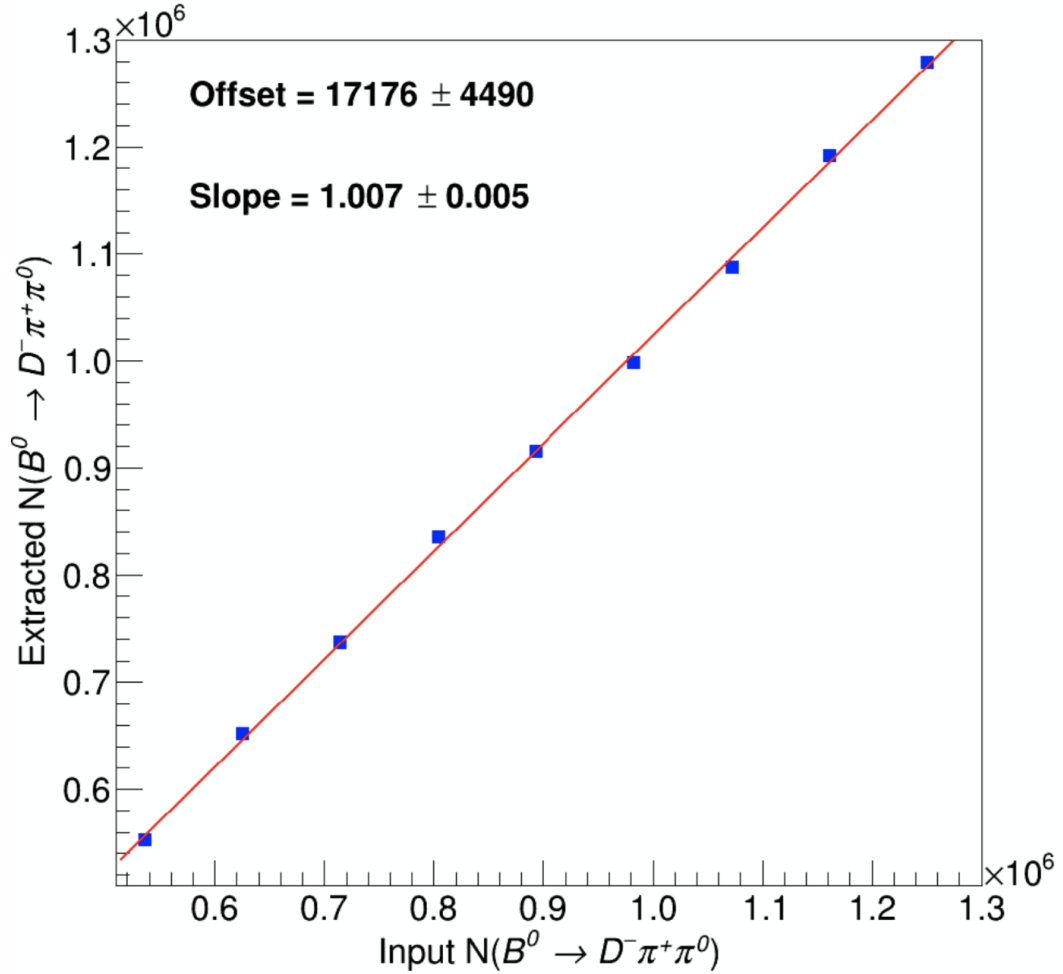


Figure 3.28: Linearity test for the $B^0 \rightarrow D^- \pi^+ \pi^0$ yield performed on MC samples containing expected backgrounds. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors but may not be visible in this plot due to relative size.

TABLE XII: Results of the $B^0 \rightarrow D^- \rho^+$ yield validation. The input yields are obtained by taking the values from Table XI and scaling them by the relative branching fraction of the $B^0 \rightarrow D^- \rho^+$ decay found in generic MC. The listed uncertainties are the standard error of the 10 toy fits.

Input $N_{D^- \rho^+}$	Extracted $N_{D^- \rho^+}$
353 245	374 010 $\pm$ 2628
415 028	431 010 $\pm$ 1577
471 025	483 644 $\pm$ 4505
534 514	541 238 $\pm$ 3896
595 040	611 462 $\pm$ 4277
650 319	670 272 $\pm$ 4258
711 620	731 430 $\pm$ 5520
777 754	794 256 $\pm$ 3322
833 731	853 174 $\pm$ 5621

3.6.4 $B^0 \rightarrow D^- \rho^+$ Yield Extraction Validation

For the validation of the extracted yield of the $B^0 \rightarrow D^- \rho^+$ we adopt the same strategy as was employed in Section 3.6.3, with the additional step of performing the binned fit to $M_{\pi\pi}$ to disentangle the $D^- \rho^+$ channel from the others. Since the fit is performed below $1 \text{ GeV}/c^2$, we must apply a scale factor to the extracted yield to take into account the events above $1 \text{ GeV}/c^2$. From MC studies we find that 82% of the fitted PDF lies below $1 \text{ GeV}/c^2$ in $M_{\pi\pi}$. When varying the input signal yield, care was taken to ensure that only the $B^0 \rightarrow D^- \rho^+$ channel was varied while all other $B^0 \rightarrow D^- \pi^+ \pi^0$ channels were held constant. An example fit can be seen in Figure 3.16.

The numerical results of the validation are listed in Table XII. A plot of the extracted yield against the input yield is shown in Figure 3.29 along with a line of best fit. The non-zero offset of the line indicates a bias is present which is calculated as the offset divided by the midpoint of the input values. This gives a 2.7% bias and the extracted yield will be corrected for this.

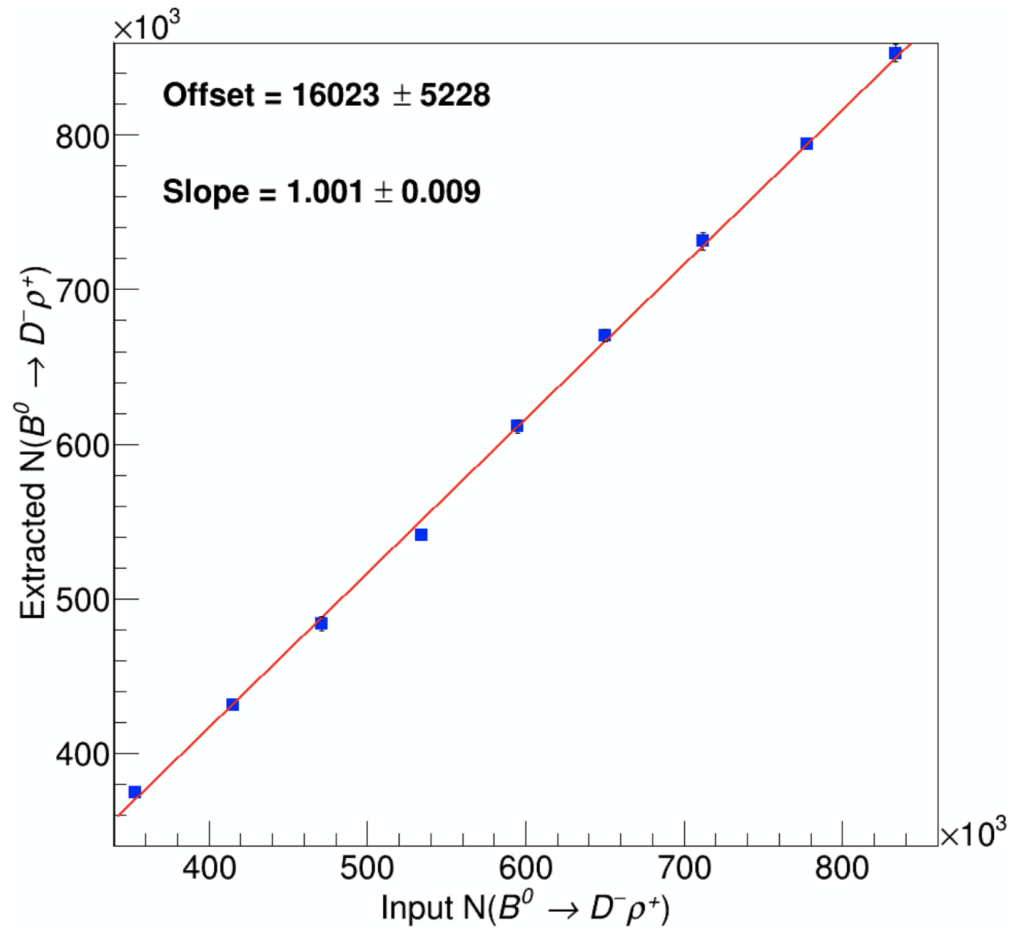


Figure 3.29: Linearity test for the $B^0 \rightarrow D^- \rho^+$ yield extraction. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors.

Chapter 4

$B^+ \rightarrow \bar{D}^0 \pi^+$ Control Sample Studies

Monte Carlo studies, while invaluable, may not accurately model detector interactions, potentially impacting the extracted branching fraction. To investigate this potential discrepancy and further validate the analysis methodology end-to-end, the same fitting strategy was applied to a control mode. An ideal control mode would exhibit similar final state kinematics while also possessing a well-measured branching fraction. This also helps in determining the systematic uncertainty stemming from the continuum suppression BDT. To this end, the mode $B^+ \rightarrow \bar{D}^0 \pi^+$ was chosen, with the D^0 being reconstructed from $K^- \pi^+ \pi^0$. This choice was strategic, as it closely mimics the B^0 signal decay in terms of the final state π^0 . Additionally, this mode benefits from precise measurements. Specifically, the current world average for the branching fraction of $B^+ \rightarrow \bar{D}^0 \pi^+$ stands at $(4.61 \pm 0.10) \times 10^{-3}$, and for $D^0 \rightarrow K^+ \pi^- \pi^0$, it is 0.144 ± 0.006 [24].

4.1 Event Selection

4.1.1 Data Samples

The control mode analysis was performed on an integrated luminosity of 569.617 fb^{-1} of SVD2 Belle data consisting of 619.62×10^6 $B\bar{B}$ pairs. Belle MC samples were again

TABLE XIII: Comparison of branching fractions for each signal decay channel between Belle MC values (circa 2010) and current PDG world averages.

B^+ Channel	Belle	PDG	PDG Error
$\bar{D}^0 \pi^+$	4.84×10^{-3}	4.61×10^{-3}	0.10×10^{-3}
D^0 Channel			
$K^- \rho^+$	8.96×10^{-2}	11.2×10^{-2}	0.7×10^{-2}
$\bar{K}^*(892)^0 \pi^0$	1.98×10^{-2}	1.95×10^{-2}	0.25×10^{-2}
$K^*(892)^- \pi^+$	5.17×10^{-2}	2.31×10^{-2}	$(+0.40, -0.20) \times 10^{-2}$
$K_0^*(1430)^- \pi^+$	5.8×10^{-3}	4.8×10^{-3}	2.2×10^{-3}
$K_2^*(1430)^- \pi^+$	4×10^{-4}	-	-
$\bar{K}_0^*(1430)^0 \pi^+$	3.4×10^{-3}	5.9×10^{-3}	$(+5.0, -1.6) \times 10^{-3}$
$K^- \pi^+ \pi^0$	6.6×10^{-3}	11.5×10^{-3}	$(+6.0, -2.0) \times 10^{-3}$
Total	17.73×10^{-2}	14.4×10^{-2}	0.6×10^{-2}

skimmed in BASF [32] before being processed via the B2BII software package [35]. Signal MC samples were generated as 1 million events in which one B decays via the signal mode, $B^+ \rightarrow \bar{D}^0 \pi^+$ including all intermediate resonance channels comprising the best estimate of the processes expected by the Belle collaboration. Only the decay mode $D^0 \rightarrow K^- \pi^+ \pi^0$ was employed since this closely mimics the $B^0 \rightarrow D^- \pi^+ \pi^0$ signal decay. The other B was left to decay generically according to the branching fractions specified by the PDG. A summary of the branching fraction values used by the Belle collaboration for MC simulation (circa 2010) for each signal channel is given in Table XIII. As a point of comparison, the current world average PDG values are also given where known. In order to stay consistent with the analysis methodology of the $B^0 \rightarrow D^- \pi^+ \pi^0$ decay (see Section 3.1.3), only the Belle values were utilised when analysing signal MC. For modelling the physics in MC simulations, the EvtGen software package was employed in the same way as described in Section 3.1.3. For this analysis, background studies were performed on 6 streams of continuum events and 6 streams of generic B decays along with an additional 50 streams of rare $B\bar{B}$ decays.

4.1.2 Event Reconstruction

The reconstruction of the $B^+ \rightarrow \bar{D}^0 \pi^+$ signal was performed in much the same way as the signal analysis discussed in Section 3.1.4. Charged tracks for which $R_{K/\pi} > 0.6$ were identified as $K^\pm$ s and $R_{K/\pi} < 0.4$ were considered as $\pi^\pm$ s. π^0 s were reconstructed from

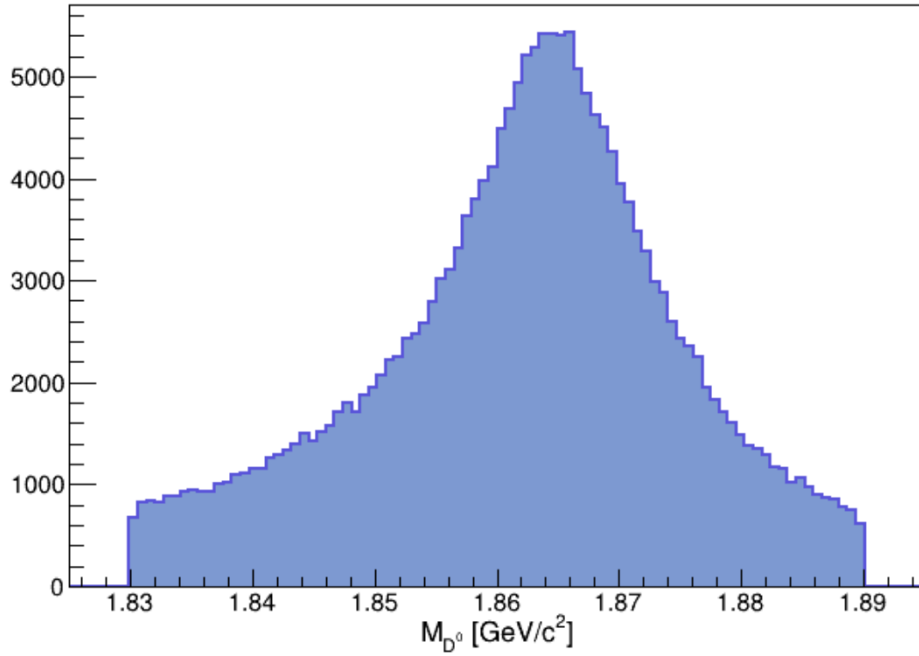


Figure 4.1: Invariant mass distribution for D^0 in $B^+ \rightarrow \bar{D}^0 \pi^+$ prior to applying analysis selection cuts. Peak mean is at $1.862 \text{ GeV}/c^2$ in agreement with the PDG world average.

two photons with the requirement that their energy is greater than 50 MeV if detected in the ECL barrel region and greater than 100 MeV if detected in the forward or backward endcap regions. Additionally, the π^0 invariant mass was required to be within 2 standard deviations of the world average value of 135 MeV[24]. A candidate D^0 was identified if any combination of $K^\mp$, $\pi^\pm$ and π^0 had an invariant mass in the range $1.835 - 1.89 \text{ GeV}/c^2$. The distribution of this is shown in Figure 4.1. Finally, the $B^\pm$ candidate was reconstructed in a similar way from the D^0 and $\pi^\pm$. The reconstructed $B^\pm$ candidate was required to fall within the ΔE range $-0.2 - 0.2 \text{ GeV}$ and the M_{bc} range $5.272 - 5.287 \text{ GeV}/c^2$.

4.1.3 π^0 Momentum Correction

This signal mode contains a π^0 in the final state, so in principal a similar momentum correction for M_{bc} is required, the details of which can be found in Section 3.1.5. A comparison of the M_{bc} in bins of ΔE before and after the π^0 momentum correction is shown in Figure 4.2 to demonstrate the effect. For this control mode the effect was found

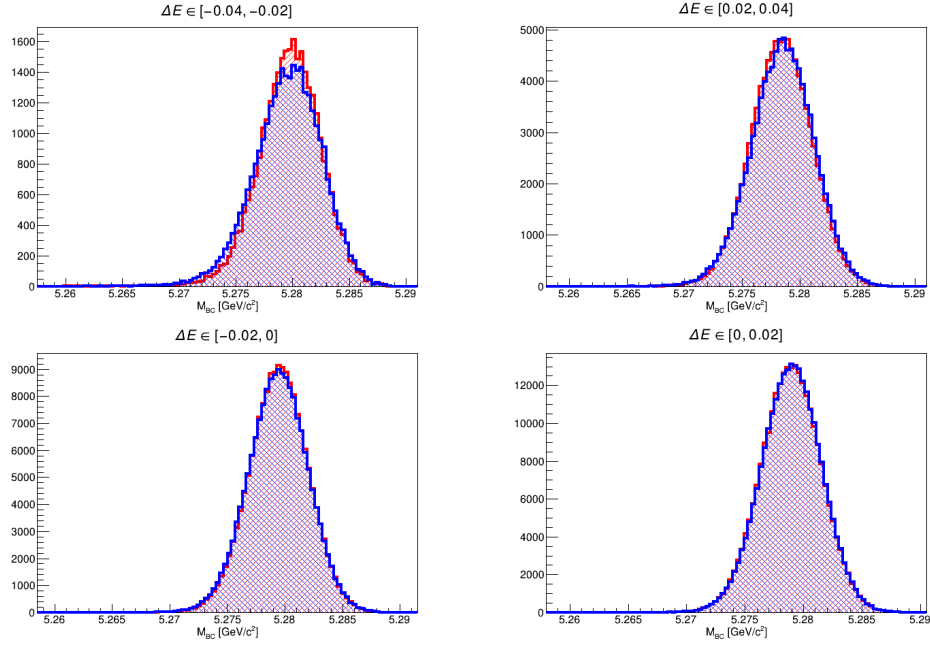


Figure 4.2: M_{bc} projections in bins of ΔE are shown to make the effect of the π^0 momentum correction apparent. The corrected and uncorrected distributions are overlaid in red and blue, respectively.

to be minimal but in order to remain consistent with the reconstruction methodology of the $B^0 \rightarrow D^- \pi^+ \pi^0$ analysis, it was included regardless.

4.2 Background Studies

As with the $B^0 \rightarrow D^- \pi^+ \pi^0$ analysis, the control mode must also contend with extracting signal in the presence of backgrounds. This section describes how each background source is handled.

4.2.1 Self-crossfeed Background

For the control mode, the self-crossfeed background once again includes all kinds of misreconstructions such as fake tracks, misidentified final state particles, and signal candidates reconstructed from daughter particles from the other side of the event. It was found that after reconstruction and the application of all selection criteria, SCF backgrounds constituted 17% of all signal candidates. A comparison of the signal to SCF distributions before and after the application of selection criteria is given in Figure 4.3.

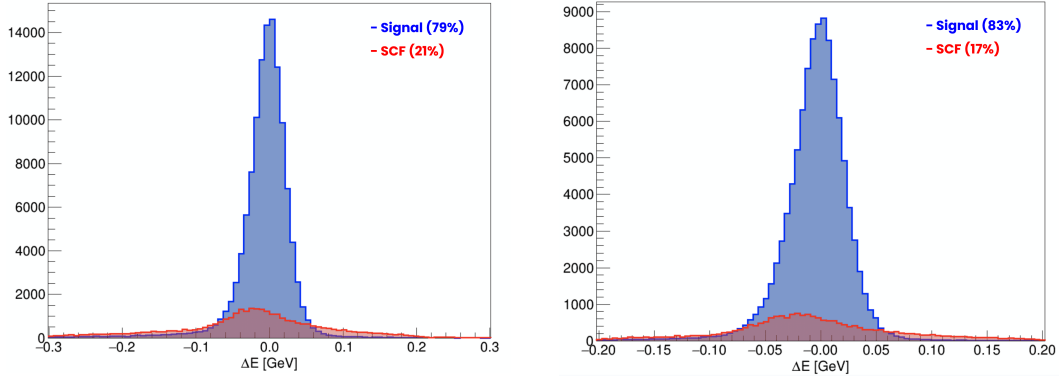


Figure 4.3: Distribution of the (blue) ΔE signal distribution and the (red) SCF distribution comparing the impact of selection criteria on the SCF background for $B^+ \rightarrow \bar{D}^0 \pi^+$.

4.2.2 Continuum Background

The continuum suppression is performed using the same FastBDT trained in Section 3.2.2 using the same weights as for $B^0 \rightarrow D^- \pi^+ \pi^0$. A comparison of the BDT output distribution is shown for a sample of MC with expected yields for each component and real SVD2 Belle data in Figure 4.4. The same BDT cut value of 0.27 is chosen to remain consistent with Section 3.2.2, which in this case results in 90% signal retention and 71% continuum rejection. The continuum background distribution shape in ΔE , before and after applying the cut are shown in Figure 4.5.

4.2.3 Generic $B\bar{B}$ Background

Removal of $B^+ \rightarrow \bar{D}^0 \pi^+$ events from the generic $B\bar{B}$ background sample was performed in the same way as in Section 3.2.3 along with similar kinematic cuts applied which are given in Table XIV. These cuts have previously been optimised in a Belle analysis of $B \rightarrow \bar{D} \pi^0$ by T. Bloomfield [45] which has a similar final state. A slightly broader selection is applied for the D^0 mass due to the wider mass peak seen in Figure 4.1. The ΔE and M_{bc} cuts and their impacts on generic background are shown in Figure 4.6. The ΔE cut has been kept intentionally loose to allow for the generic peak on the left which is needed to distinguish it from continuum background. This is important for the fitting procedure which is described later in Section 3.5.1. The cut on ΔE retains 99% of signal and rejects 47% of $B\bar{B}$ and 26% of continuum backgrounds, while the cut on M_{bc} retains 97% of signal and rejects 35% of $B\bar{B}$ and 54% of continuum backgrounds.

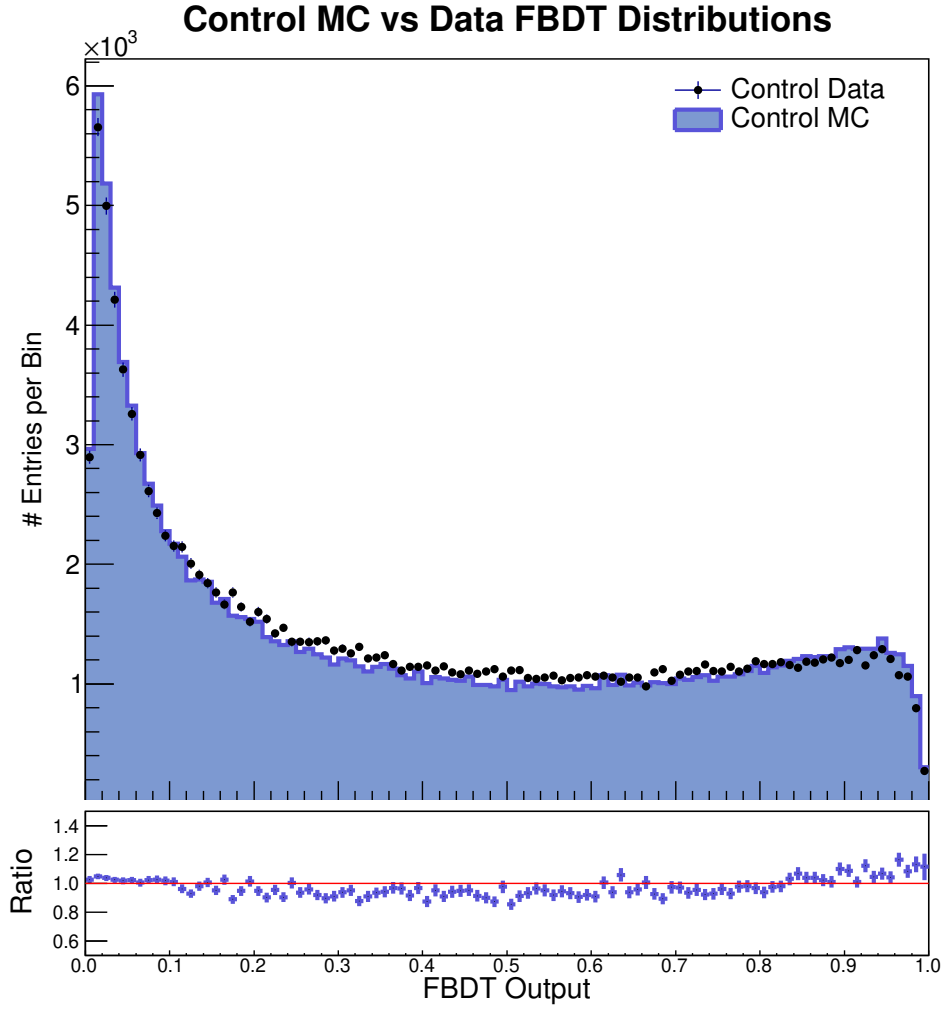


Figure 4.4: Comparison of the BDT output for both MC and continuum data. The continuum data was selected from the M_{bc} and ΔE ranges which has negligible contributions for signal and $B\bar{B}$.

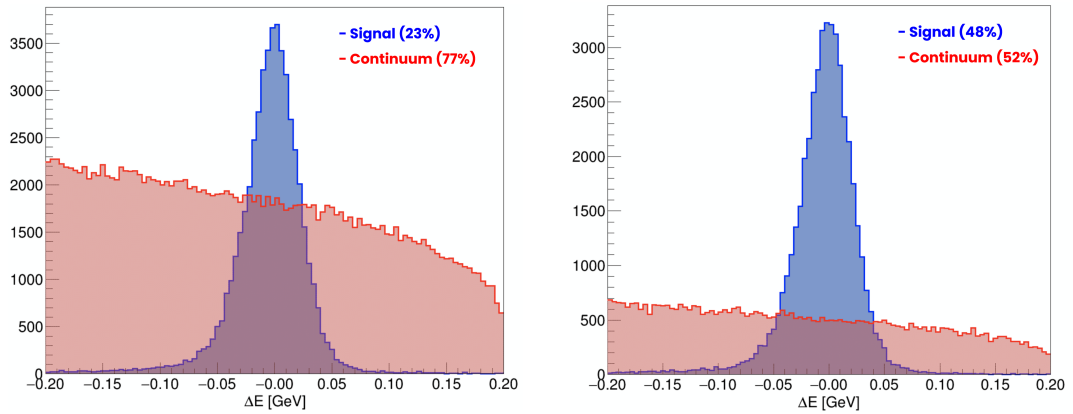


Figure 4.5: Distribution of the (blue) ΔE signal distribution and the (red) continuum distribution comparing the impact of the BDT cut before (left) and after (right) it is applied on the continuum background for $B^+ \rightarrow \bar{D}^0 \pi^+$.

TABLE XIV: Kinematic cuts applied to $B^+ \rightarrow \bar{D}^0 \pi^+$ to reduce generic $B\bar{B}$ background.

Variable	Selected Range
M_{bc}	$5.272 - 5.287 \text{ GeV}/c^2$
ΔE	$-0.2 - 0.2 \text{ GeV}$
M_{D^0}	$1.835 - 1.89 \text{ GeV}/c^2$
M_{π^0}	$115 - 152 \text{ MeV}/c^2$
p_{π^0}	$> 0.2 \text{ GeV}/c$
$p_{\pi^\pm}$	$0.3 - 3.5 \text{ GeV}/c$
$p_{K^\pm}$	$0.3 - 3.5 \text{ GeV}/c$

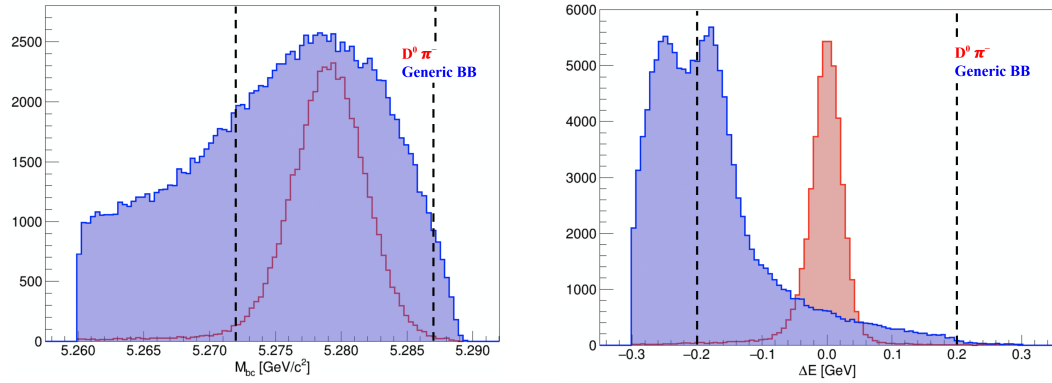


Figure 4.6: Distributions of (left) M_{bc} and (right) ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ signal and generic $B\bar{B}$ background. Cut values shown in dashed lines.

4.3 Best Candidate Selection

BCS is performed using the same method as used in Section 3.3, the closest D mass to the PDG D mass. For this control mode the nominal D^0 mass is $1.865 \text{ GeV}/c^2$. The candidate multiplicity before applying the BCS is shown in Figure 4.7. Its performance was evaluated on one stream of generic $B\bar{B}$ MC, where it was able to identify the correct signal candidate in 86.4% of cases.

4.4 Reconstruction Efficiencies

After applying selection cuts, best candidate selection and continuum suppression, but neglecting the $D^0 \rightarrow K^- \pi^+ \pi^0$ branching ratio, the $B^+ \rightarrow \bar{D}^0 \pi^+$ signal efficiency is $(10.56 \pm 0.05)\%$ which leads to an expected signal yield of 44 695 events. This is calculated according to the following formula assuming the PDG branching fraction of 4.61×10^{-3}

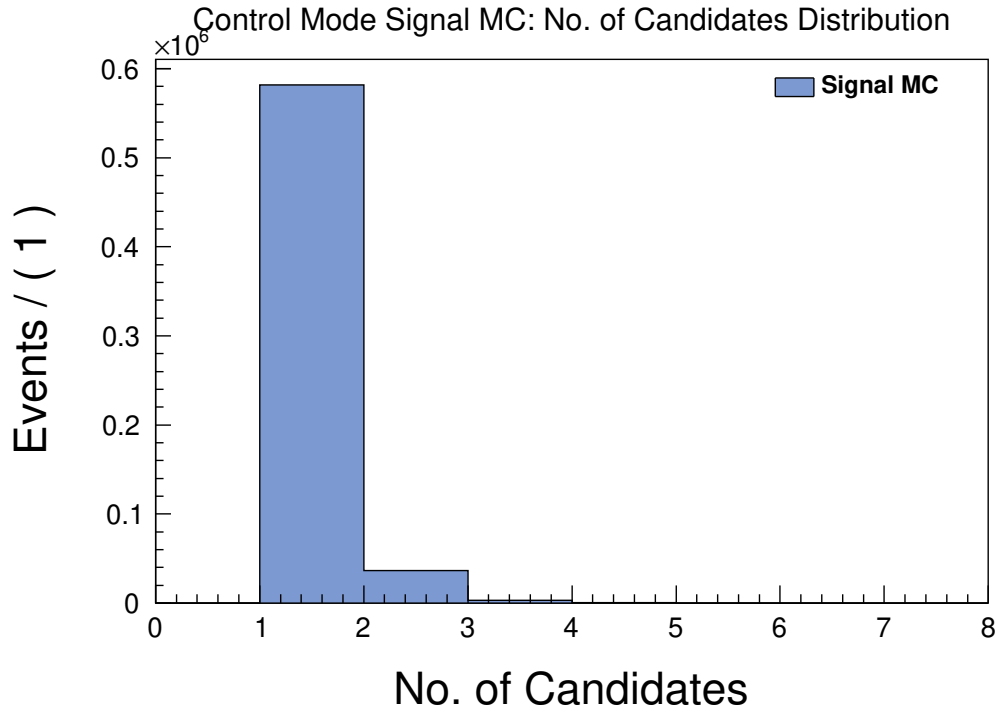


Figure 4.7: Number of signal candidates per event prior to BCS in $B^+ \rightarrow \bar{D}^0 \pi^+$ MC with an average candidate multiplicity of 1.187.

TABLE XV: Expected yield for SVD2 Belle data. $B^+ \rightarrow \bar{D}^0 \pi^+$ yield calculated in an analogous way to Equation 3.13 using the most recent PDG world average result.

Component	Expected yield	Proportion
$B^+ \rightarrow \bar{D}^0 \pi^+$	44 695	38.1%
Continuum	21 978	18.7%
Generic $B\bar{B}$	41 467	35.3%
Self-crossfeed	9284	7.9%

for the $B^+ \rightarrow \bar{D}^0 \pi^+$ [24],

$$N_{expected} = 2 \times R_{B^+ B^-} \times N_{B\bar{B}} \times \mathcal{B}(B^+ \rightarrow \bar{D}^0 \pi^+) \times \mathcal{B}(\bar{D}^0 \rightarrow K^+ \pi^- \pi^0) \times \epsilon_{recon}. \quad (4.1)$$

The full breakdown of expected yields for each component is given in Table XV. The expected continuum and pure generic $B\bar{B}$ background yields are the average over all 6 available streams of Belle SVD2 MC.

TABLE XVI: Fitted PDF shapes for control mode components.

Component	PDF Shape (ΔE)
$B^+ \rightarrow \bar{D}^0 \pi^+$	Crystal Ball [50] + 2× Gaussians
Generic $B\bar{B}$	2nd Order Chebychev + 2× Gaussians
Continuum	3rd Order Chebychev
Self-cross Feed	3× Gaussians

4.5 Fitting Strategy

Two separate fit strategies are performed to Belle SVD2 data as both a point of comparison and as an additional method to validate the analysis. First, a full unbinned maximum likelihood fit to ΔE was performed in the same way as done most recently by T. Bloomfield [45]. In the second fit strategy, fits to ΔE are again performed but are now done in bins of $M_{\pi\pi}$ and $\cos\theta_h$ in a completely analogous way to the main $B^0 \rightarrow D^- \pi^+ \pi^0$ signal analysis described in Section 3.5.1, which acts as validation of the extraction of the total branching fraction. For this control mode, $M_{\pi\pi}$ is defined as the invariant mass of the $\pi^+ \pi^0$ in the $D^0 \rightarrow K^- \pi^+ \pi^0$ decay.

For the total integrated fit to ΔE , the signal and SCF components are modelled on MC samples generated with Evtgen [31] and simulated with GEANT3 [30] as employed in BASF. The continuum and generic $B\bar{B}$ backgrounds were modelled on 6 streams of each as provided by the Belle collaboration. The ΔE PDFs for each component are shown in Figure 4.8 and the shapes used in each case are given in Table XVI. In accordance with the analysis by T. Bloomfield, the SCF background is treated as part of the signal component during the fit and as such, the reconstruction efficiency is determined with this included and is now found to be $(12.78 \pm 0.06)\%$. During the fit to data, all shape parameters were held fixed to the values found during PDF modelling with the exception of the component yields and the mean and width of the signal crystal ball function.

The fits in bins of $M_{\pi\pi}$ and $\cos\theta_h$ were performed in much the same way as described in Section 3.5.1. The full kinematic phase-space was segmented into 10 bins of $\cos\theta_h$ between -1.0 and 1.0 , and 16 bins of $M_{\pi\pi}$ between $0.25 \text{ GeV}/c^2$ and $1.40 \text{ GeV}/c^2$ for a total of 160 bins. Within each bin, the fit components were modelled with the same PDF shapes as described in the total integrated fit which are summarised in Table XVI. While modelling the PDFs, the shape parameters were left to float in a range of 10-20

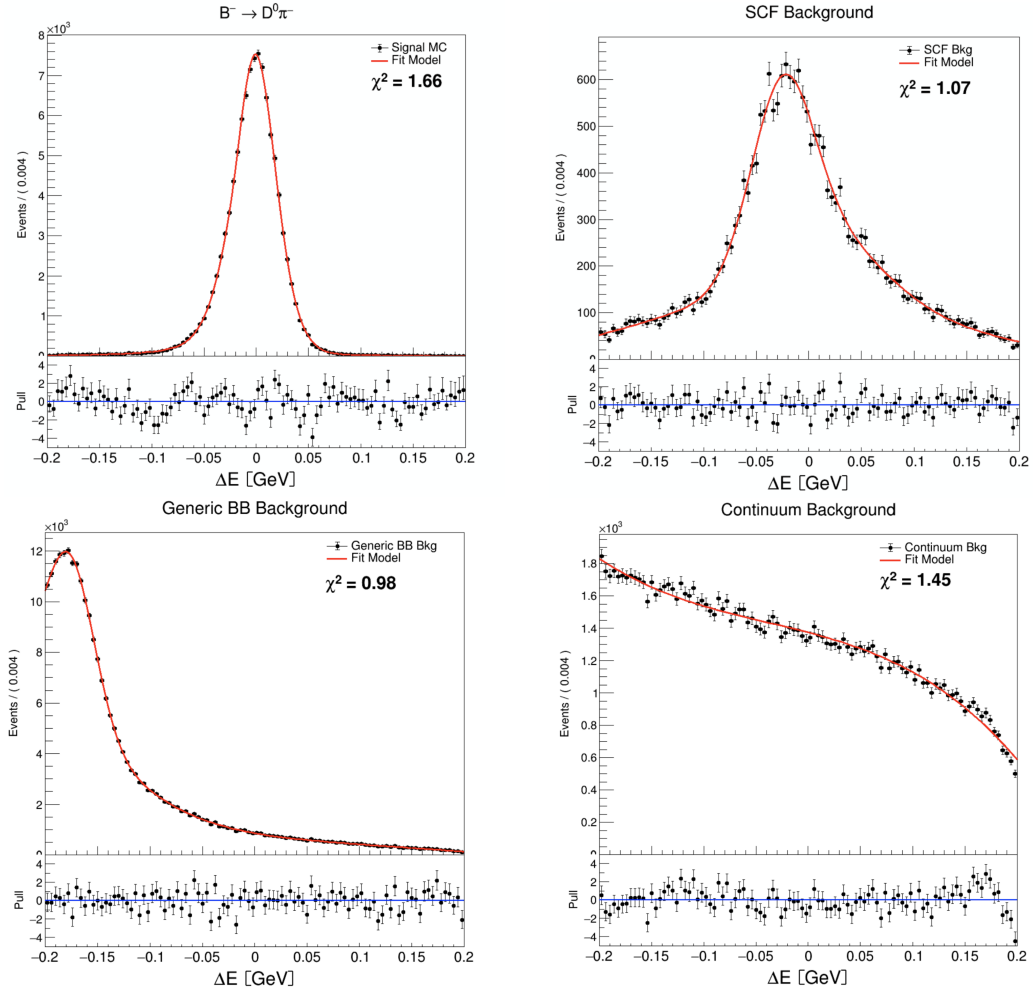


Figure 4.8: ΔE distributions for each component of the fit to $B^+ \rightarrow \bar{D}^0 \pi^+$. Continuum and generic $B\bar{B}$ components were modelled on 6 streams of Belle MC. Signal and SCF components were modelled on dedicated signal MC. The PDF shapes are listed in Table XVI. The χ^2 values given are per degree of freedom.

standard deviations around the central value found in the unbinned fit as this was found to give acceptably low χ^2 values while maintaining stability of the fits. A representative selection of these 160 fits can be found for each component in Appendix B. The SCF yield was parametrised by the signal yield and SCF fraction as described by Equation 3.15 for the same reasons given there. A plot of this fraction across all bins is shown in Figure 4.9.

After performing binned signal yield fits, an event migration correction was applied as described in Section 3.5.3. The fraction of events migrating in and out of every bin is shown in Figure 4.10. Neglecting minor boundary effects, the total yield across all bins before and after the migration correction has been applied should remain the same. It was found that only 0.03% of events were lost as a result of the migration correction when

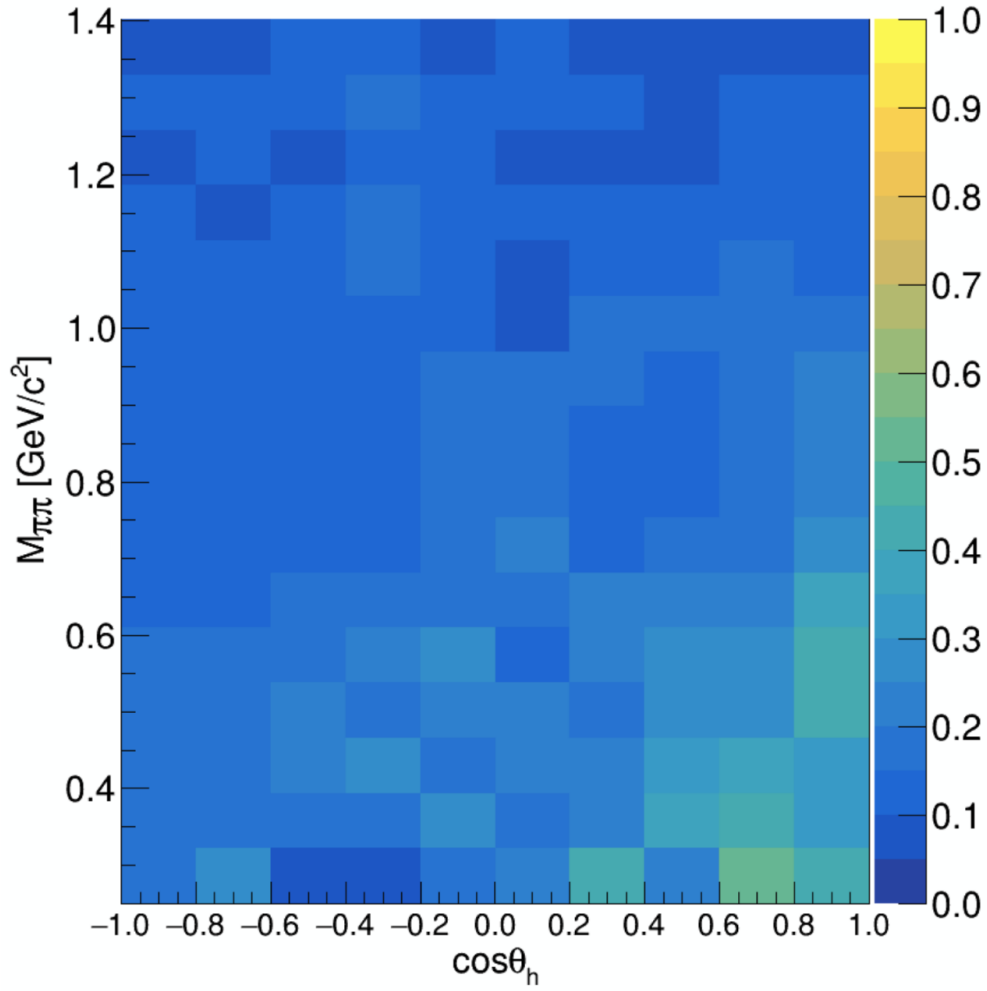


Figure 4.9: Fraction of reconstructed signal candidates that are due to self-crossfeed background in each bin of $\cos \theta_h$ and $M_{\pi\pi}$ as found in signal MC.

applied to a sample of signal MC, indicating good performance. This corrected yield was then used in the generation of the reconstruction efficiency matrix. A comparison of the effects of the migration correction is shown in Figure 4.11. For each plot, the average efficiency weighted by the number of events in each bin is found to be 0.111 for both the unmodified matrix and for the migration corrected matrix.

It is important to validate the performance of the fit strategy on a range of possible $B^+ \rightarrow \bar{D}^0 \pi^+$ yields that may be found in real data. However, due to the well measured branching fraction in this case, the validation was performed in a smaller range, 80% to 120% of the expected signal yield (see Table XV), when compared to the $B^0 \rightarrow D^- \pi^+ \pi^0$ validation. Background component yields were held fixed at their expected values with the exception of the SCF background which was varied in the same proportion as the

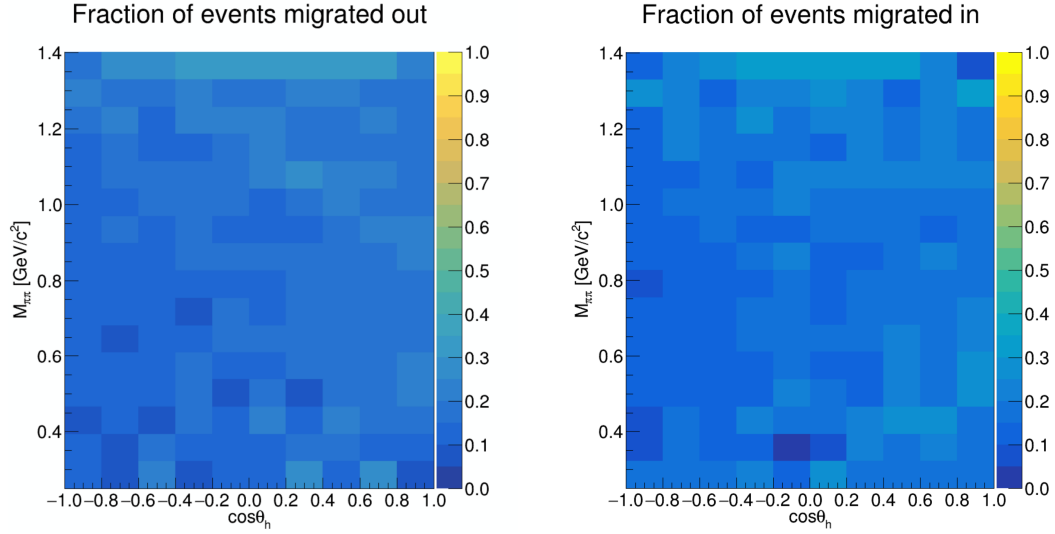


Figure 4.10: Fraction of events that migrated out (left) and in (right) of each bin after reconstruction for pure signal MC sample.

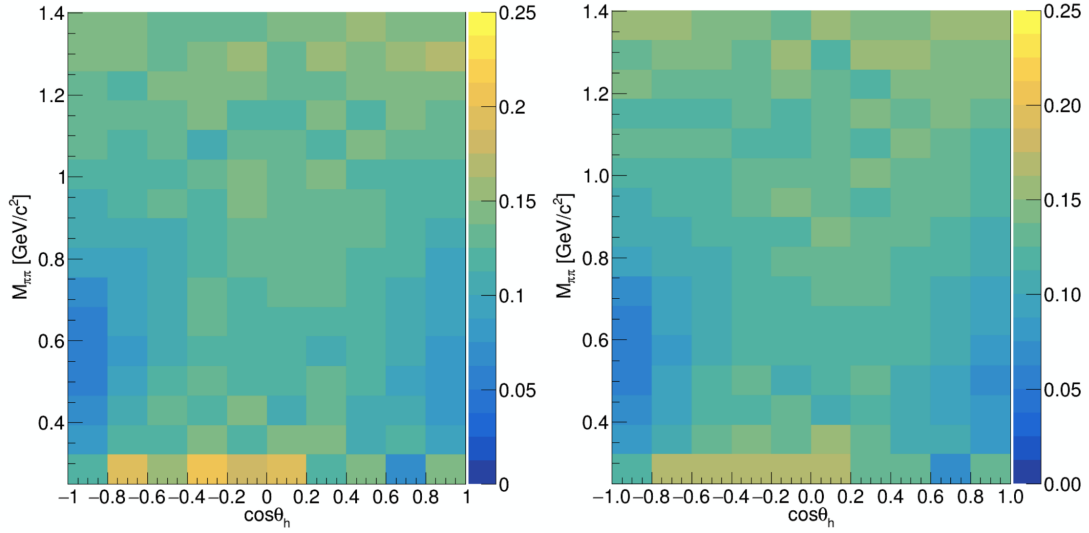


Figure 4.11: Comparison of the efficiency matrix for $B^+ \rightarrow \bar{D}^0 \pi^+$ modelled on signal MC with no modifications (left) and after applying the event migration correction (right). Weighted average efficiencies are 0.111 in both cases.

signal component. For each $B^+ \rightarrow \bar{D}^0 \pi^+$ yield, the full fits in bins of ΔE were performed 10 times with events randomly sampled from the total signal MC data sample. The resulting distributions are 2D histograms in $\cos \theta_h$ and $M_{\pi\pi}$ which then have the event migration corrections applied. As a final step, a division by the efficiency matrix was performed and the average across the 10 samples was recorded. The numerical results of the validation are listed in Table XVII and a plot of the extracted yield against the input yield is shown in Figure 4.12 along with a line of best fit. To determine the bias present, the equation of this line will be used in conjunction with the extracted $B^+ \rightarrow \bar{D}^0 \pi^+$ yield

TABLE XVII: Results of the $B^+ \rightarrow \bar{D}^0 \pi^+$ yield validation. The input yields are obtained by applying the event migration correction and efficiency matrix directly without performing any ΔE fits. The listed uncertainties are the standard error of the 10 toy fits.

Input $N_{D^0 \pi^-}$	Extracted $N_{D^0 \pi^-}$
334 822	$333\,724 \pm 1143$
376 675	$374\,511 \pm 979$
418 528	$414\,004 \pm 484$
460 381	$456\,082 \pm 1184$
502 234	$496\,651 \pm 1075$

TABLE XVIII: Expected yields from MC compared to extracted yields found in data for the two fit strategies.

Component	Expected Yield	Fitted Yield (un-binned)	Fitted Yield (binned)
$B^+ \rightarrow \bar{D}^0 \pi^+$	39605	—	40809 ± 371
SCF	8227	—	9136 ± 96
$B^+ \rightarrow \bar{D}^0 \pi^+ + \text{SCF}$	47833	46121 ± 308	49945 ± 223
Continuum	21978	29849 ± 408	24797 ± 632
Generic $B\bar{B}$	41467	39361 ± 318	40594 ± 355

in data. The bias will be calculated as the relative change between the yield extracted from the ΔE fits (the y-axis of Figure 4.12) and the corresponding input yield (the x-axis of Figure 4.12). This bias will be compensated for with a correction term in the branching fraction.

4.6 Comparison with Data

The full Belle SVD2 data is fit to in ΔE , first as an unbinned fully integrated maximum likelihood fit, and then again but in multiple bins of $M_{\pi\pi}$ and $\cos\theta_h$. The unbinned fit to ΔE is shown in Figure 4.13 while a representative selection of binned fits from the range $0.8 < \cos\theta_h < 1.0$ in high statistics bins are shown Figure 4.14. The total sum of fitted PDFs from all bins is shown in Figure 4.15 which shows good agreement, indicating the fit strategy performs well globally. The extracted yields for both methods are compared against the expected values obtained from MC studies in Table XVIII.

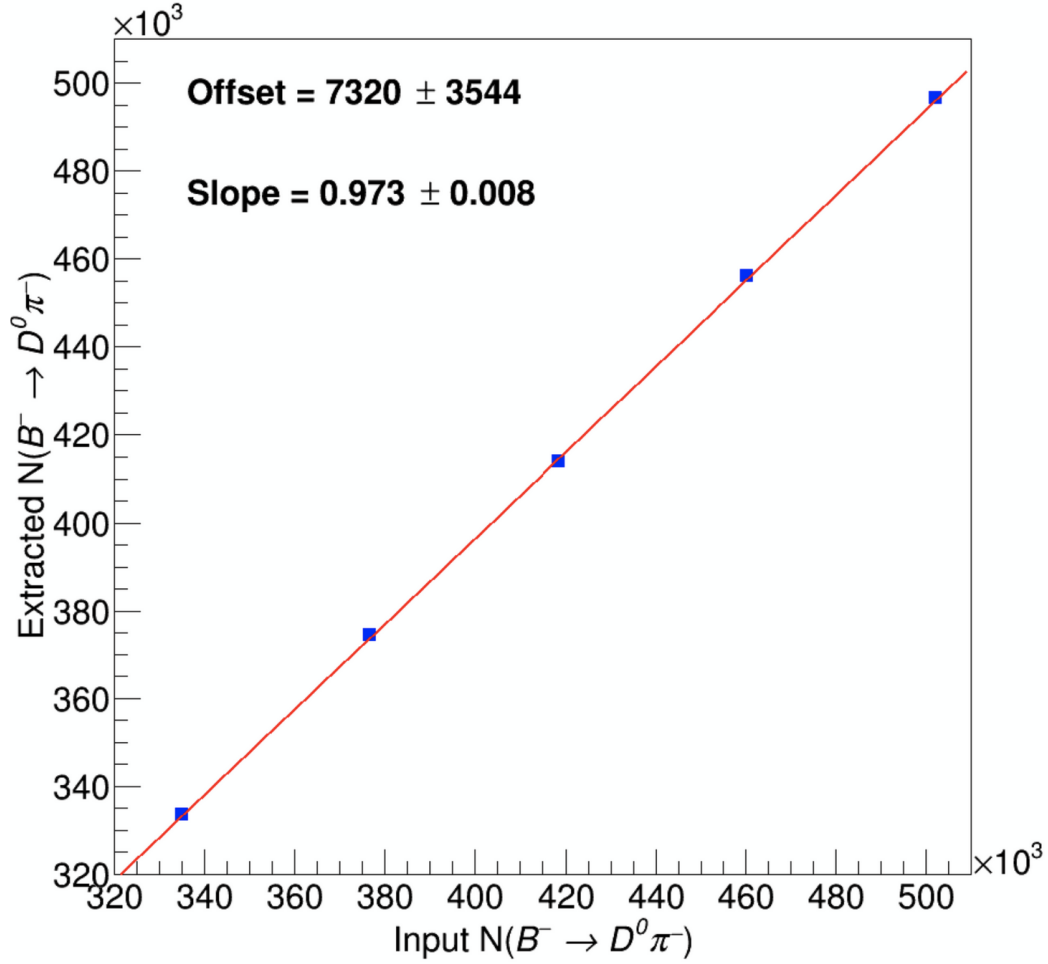


Figure 4.12: Linearity test for the $B^+ \rightarrow \bar{D}^0 \pi^+$ yield performed on MC samples containing expected backgrounds. Each data point is the mean of 10 toy ensemble runs. Error bars are the associated standard errors but may not be visible in this plot due to relative size.

For the unbinned result, the branching fraction is calculated according to the following formula:

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0 \pi^+) = \frac{N_{fitted}}{2 \times R_{B^0 \bar{B}^0} \times N_{B \bar{B}} \times \mathcal{B}(D^0 \rightarrow K^- \pi^+ \pi^0) \times \epsilon \times R_{eff}^{\pi^0} \times R_{eff}^{PID}} \quad (4.2)$$

where ϵ is the total reconstruction efficiency inclusive of SCF, $R_{eff}^{\pi^0}$ is a correction factor for the π^0 reconstruction efficiency and R_{eff}^{PID} is a correction due to differences in the PID tracking efficiency between MC and data. This results in a measured branching fraction of

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0 \pi^+) = (4.44 \pm 0.03) \times 10^{-3} \quad (4.3)$$

where the uncertainty is statistical. This value is in agreement with the previously measured value by T. Bloomfield [45] that found a value of

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0(K^+\pi^-\pi^0)\pi^+) = (4.46 \pm 0.02(stat.) \pm 0.23(syst.)) \times 10^{-3} \quad (4.4)$$

which employed the same unbinned fit strategy and assumed a constant average efficiency over all kinematic regions of $D^0 \rightarrow K^-\pi^+\pi^0$ phase-space.

For the result obtained from the fit in bins of $M_{\pi\pi}$ and $\cos\theta_h$, the event migration correction is first applied to the fitted yield across all bins before finally dividing by the efficiency matrix to extract the full signal yield in all bins. We find:

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0\pi^+) = (4.75 \pm 0.04 \pm 0.17) \times 10^{-3} \quad (4.5)$$

where the first uncertainty is statistical and the second is systematic. A bias of 0.8% was found using the line of best fit in Figure 4.12, Section 4.5, and has been corrected for. This binned approach takes into account the non-uniform acceptance of the Belle detector across phase-space. This enables correct modelling of the $D^0 \rightarrow K^-\pi^+\pi^0$ decay which has a strong $D^0 \rightarrow K^*\pi$ resonance and hence a very non-uniform distribution in kinematic phase-space. For a detailed discussion of the systematic uncertainties that contributed, see Chapter 5. This value is in agreement with the world average result given by the PDG [24] that reports a value of

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0\pi^+) = (4.61 \pm 0.10) \times 10^{-3} \quad (4.6)$$

which has included the most recent result from T. Bloomfield.

The results from each fit strategy produce slightly different central values. This is to be expected, as the unbinned approach does not consider two important factors: the non-uniform acceptance of the detector and the uneven distribution of yield across the phase-space. Nevertheless, this result is included to make a direct comparison to the earlier Belle study by T. Bloomfield while also providing confidence in the overall $B^0 \rightarrow D^-\pi^+\pi^0$ analysis methodology.

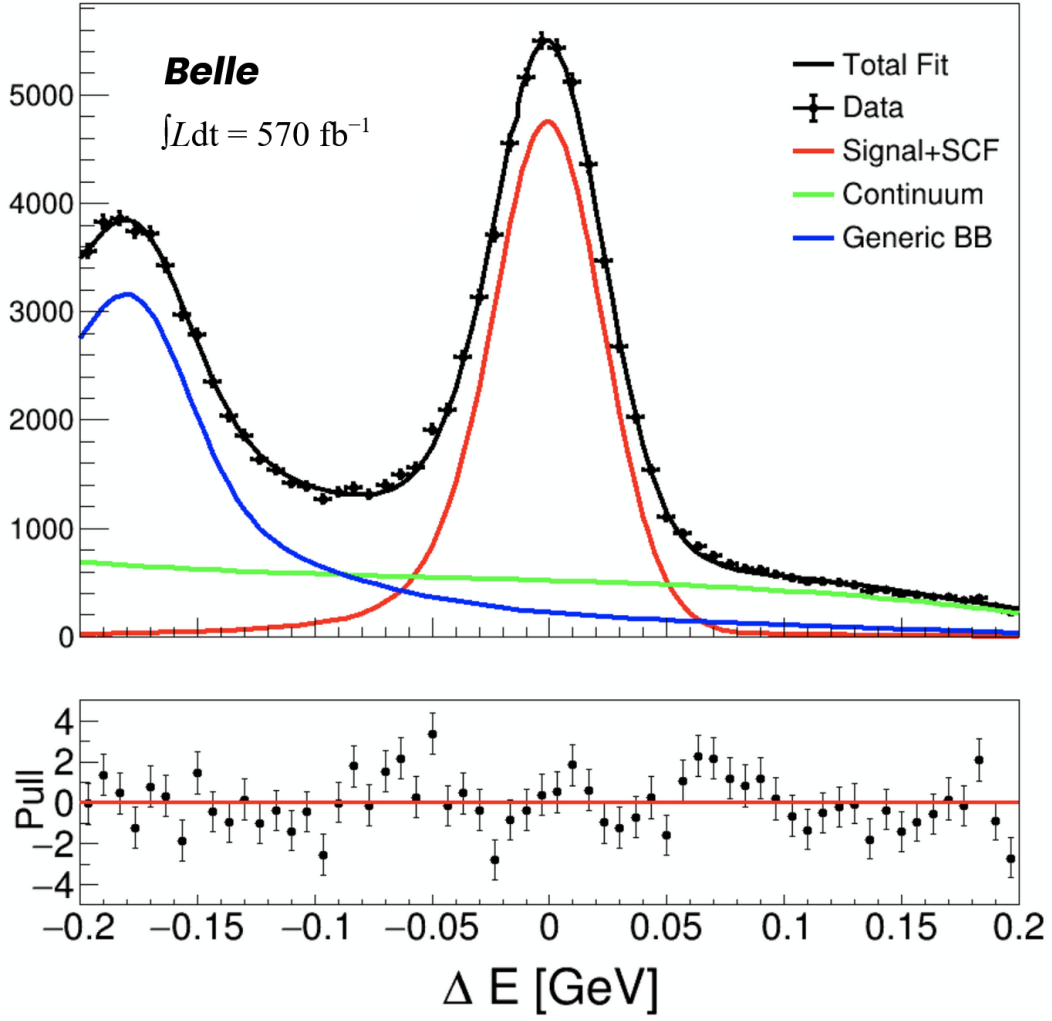


Figure 4.13: Total integrated unbinned fit to ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data.

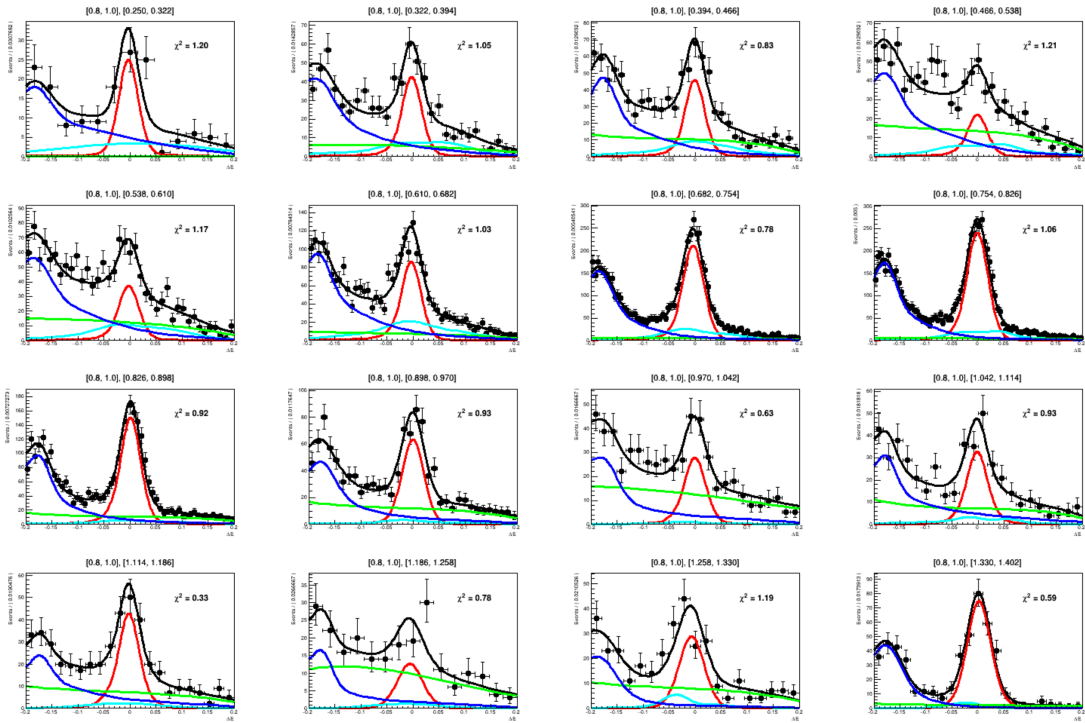


Figure 4.14: Fits to ΔE in bins of $M_{\pi\pi}$ and $\cos\theta_h$ for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data. Bins shown here are in the $\cos\theta_h$ range 0.8 to 1.0 and range in $M_{\pi\pi}$ from 0.25 GeV/c² to 1.40 GeV/c². The χ^2 values given are per degree of freedom.

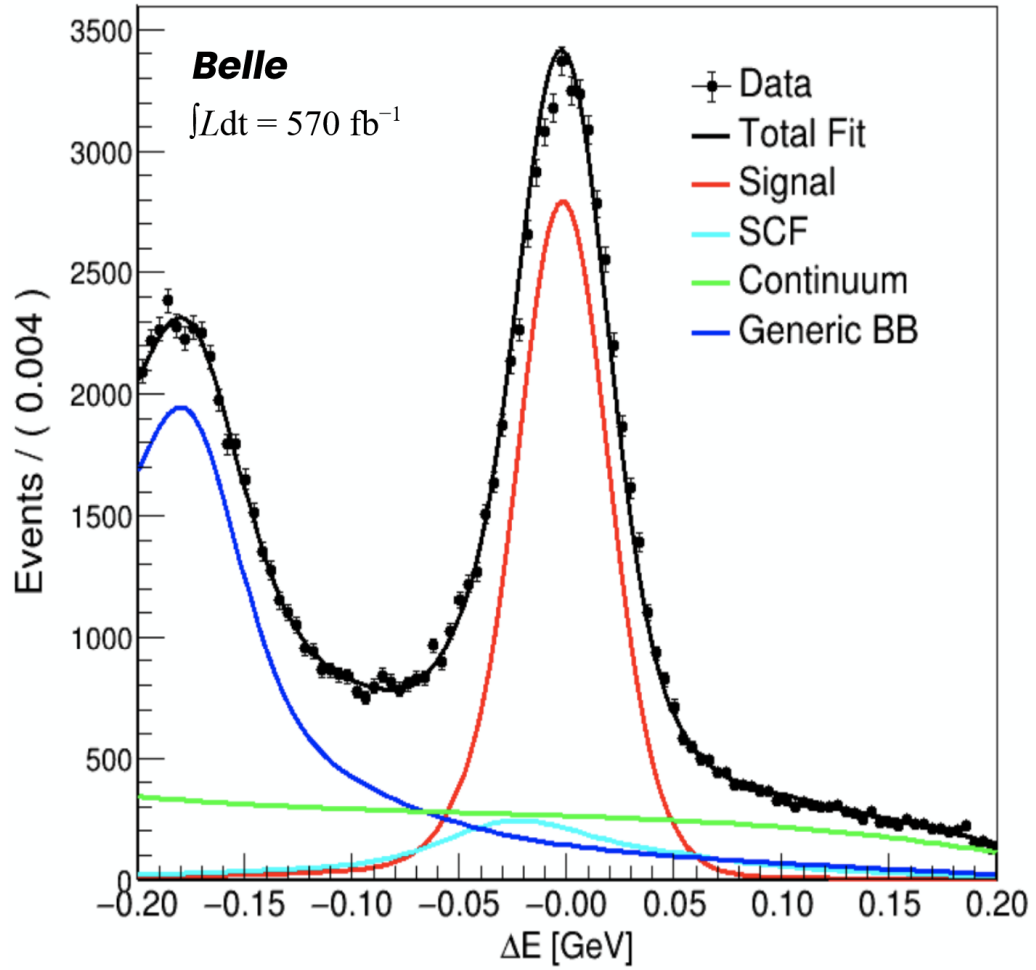


Figure 4.15: Sum of fitted PDFs in bins of $\cos \theta_h$ and $M_{\pi\pi}$ to ΔE for $B^+ \rightarrow \bar{D}^0 \pi^+$ in Belle SVD2 data.

Chapter 5

Systematic Uncertainties

Besides the inherent statistical variations stemming from the finite nature of the dataset, changes resulting from the analytical approach contribute to the overall uncertainty as well. These variations, referred to as systematic uncertainties, can arise from various sources. They are particularly important whenever there is a need to make an assumption or choose among different analytical methodologies. Unlike statistical variation, these systematic effects will cause a consistent bias in the results which must be accounted for on a case-by-case basis. This analysis is not statistically limited, and so a careful consideration of the systematic uncertainties, (systematics), is necessary. This chapter lists the considered contributions accompanied by a discussion of how each was derived. For the $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ branching fractions, and the $A_{FB}^{\pi\pi}$ asymmetry, the dominant systematic was found to be due to the fixed parameters in the ΔE PDF parameters. For the $B^+ \rightarrow \bar{D}^0 \pi^+$ control mode branching fraction, this was instead dominated by the uncertainty in the continuum suppression cut.

A number of systematic effects do not provide a net bias to the extracted yield in the forward or backward helicity regions, and so do not contribute to the $A_{FB}^{\pi\pi}$ systematic uncertainty. Where this is true, it has been noted.

Full summaries of all systematic sources for full 570 fb^{-1} SVD2 Belle data are given in Table [XXI](#) and Table [XXII](#). Additionally, the systematics on partial 74 fb^{-1} data is given in Table [XX](#).

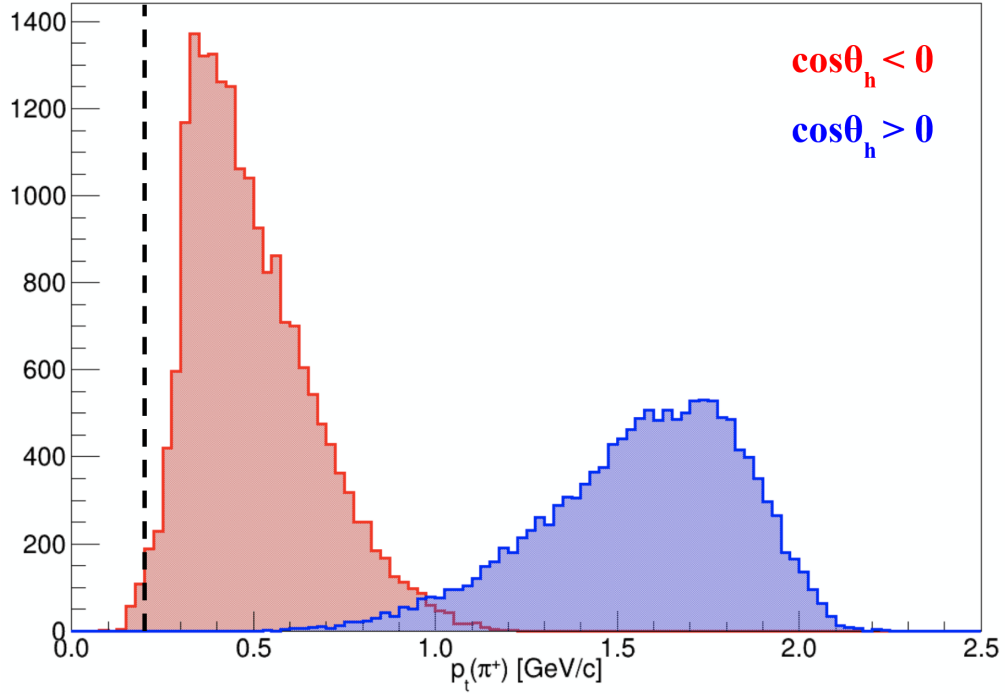


Figure 5.1: Transverse momentum distribution for the π^+ daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle. The dashed line at 200 MeV/c indicates the threshold for high momentum tracks [2].

5.1 Charged Track Efficiency

The reconstruction efficiency of charged tracks is well modelled by MC simulation, but MC-data differences do exist and are weakly dependent on a particle's transverse momentum, p_t . This introduces an uncertainty which has been documented in a previous study based on the reconstruction rate of D^* decays in MC and data. This study found an uncertainty of 0.35% per high momentum track [2]. Between each track, the uncertainties are completely correlated and so they are added linearly instead of quadratically. For $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$, this amounts to a 1.40% total systematic and for $B^+ \rightarrow \bar{D}^0 \pi^+$, 1.05%.

To evaluate the contribution to $A_{FB}^{\pi\pi}$, the transverse momentum distributions for the forward and backward helicity regions are compared in the lab reference frame as shown in Figure 5.1. This shows that over 99% of π^+ s coming from the B^0 have $p_t > 200$ MeV/c in both helicity regions. This is the threshold for high momentum tracks [2] and so an uncertainty of 0.35% is applied per charged track for a total of 1.40%, similar to the case for $B^0 \rightarrow D^- \pi^+ \pi^0$.

5.2 π^0 Reconstruction Efficiency

A previous study that measured the π^0 reconstruction efficiency in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays found the data to MC ratio in bins of momentum in the lab reference frame [3]. For this analysis, the ratio will be taken as a corrective factor to the final extracted yield. To find the appropriate correction factor, a weighted sum of all ratios across all momentum bins was performed where each weight was calculated as the proportion of signal π^0 s that fall in that bin. For $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$, this comes to 0.96 ± 0.014 and 0.95 ± 0.014 , respectively, and provides a 1.49% systematic uncertainty per final state π^0 .

To evaluate potential contributions to $A_{FB}^{\pi\pi}$, the total π^0 momentum distributions for the forward and backward helicity regions are compared in the lab reference frame as shown in Figure 5.2 with binning according to the divisions given in S. Ryu et al. [3]. The correction factors were calculated in the same way for each region and resulted in 0.95 ± 0.014 in the forward region and 0.96 ± 0.014 in the backward region. With this level of precision it is not possible to make a clear separation between the two values. As such, the correction factors are not applied in the calculation of $A_{FB}^{\pi\pi}$ and makes no contribution to its uncertainty.

5.3 Number of $B\bar{B}$ Pairs

There exists an uncertainty associated with the number of $B\bar{B}$ pairs produced. This arises due to uncertainties in physical parameters of the detector, branching fractions and trigger efficiencies. For Belle SVD2 the total number is $(619.620 \pm 9.441) \times 10^6$ providing a 1.52% uncertainty [2]. This makes no contribution to the $A_{FB}^{\pi\pi}$ uncertainty.

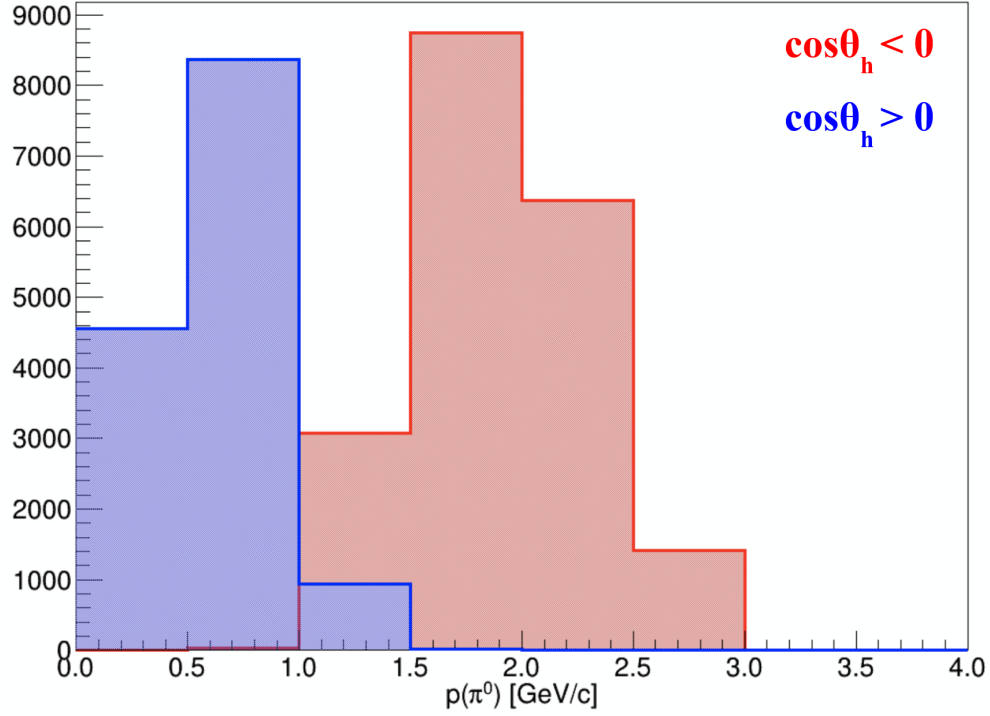


Figure 5.2: Total momentum distribution for the π^0 daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle. Binned according to the divisions given in S. Ryu et al. [3].

5.4 Value of R_{BB}

The fraction of $\Upsilon(4S)$ decays that result in either a $B^0\bar{B}^0$ or B^+B^- pair is given by the PDG [24] as

$$\mathcal{B}(\Upsilon(4S) \rightarrow B^0\bar{B}^0) = 0.486 \pm 0.006 \quad (5.1)$$

$$\mathcal{B}(\Upsilon(4S) \rightarrow B^+B^-) = 0.514 \pm 0.006 \quad (5.2)$$

Proportionally, this provides a 1.23% systematic uncertainty for the $B^0 \rightarrow D^-\pi^+\pi^0$ result and 1.17% for the $B^+ \rightarrow \bar{D}^0\pi^+$ result. This makes no contribution to the $A_{FB}^{\pi\pi}$ uncertainty.

5.5 Particle Identification Efficiency

The Particle Identification Efficiency, (PID), is slightly different in data compared to its predicted value in MC. To address this, the PID group has provided a software package which can be used to calculate the correction in the reconstruction efficiency of

TABLE XIX: PID correction factors for $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$.

Particle	$B^0 \rightarrow D^- (K^+ \pi^- \pi^-) \pi^+ \pi^0$	$B^+ \rightarrow \bar{D}^0 (K^+ \pi^- \pi^0) \pi^+$
$K^\pm (D \text{ daughter})$	1.008 ± 0.010	1.006 ± 0.009
$\pi^\pm (B \text{ daughter})$	0.956 ± 0.009	0.969 ± 0.009
$\pi^\pm (D \text{ daughter})$	0.984 ± 0.010	0.954 ± 0.008
$\pi^\pm (D \text{ daughter})$	0.954 ± 0.008	—
Total	0.905 ± 0.019	0.929 ± 0.020

charged tracks in the final state [52]. This correction factor was found for each track in bins of particle momentum and helicity angle for each specified decay and is shown in Table XIX. Additionally, the uncertainty in the MC-data ratio is provided for each and a total of 1.56% and 1.30% was found for $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$, respectively.

To evaluate potential contributions to $A_{FB}^{\pi\pi}$, the total π^+ momentum distributions for the forward and backward helicity regions are compared in the lab reference frame as shown in Figure 5.3. The correction factors were calculated in the same way for each region and resulted in 0.892 ± 0.014 in the forward region and 0.925 ± 0.015 in the backward region. With this level of precision it is possible to make a clear separation between the two values. As such, the correction factors will be applied in the calculation of $A_{FB}^{\pi\pi}$. The systematic uncertainties in the two regions are not obviously independent of each other and so the average of the two values, 1.43%, is taken as the systematic contribution.

5.6 Continuum Suppression

When placing a cut on the continuum suppression variable there is a risk of amplifying data-MC discrepancies. To estimate the effect of this, the control mode analysis was performed with a BDT cut value of 0.27 (10% signal rejection) and again with a more aggressive value of 0.6 (40% signal rejection). Between the two, we found a discrepancy of 6% in the extracted signal yield. This is taken to be a conservative approximation of the level of data-MC discrepancy associated with the 0.6 BDT cut for the signal modes. Under the assumptions that applying no cut on the BDT output would introduce no additional discrepancy, and that below the 0.6 cut level the discrepancy is linearly proportional to the signal rejection fraction, a conservative estimate of the discrepancy

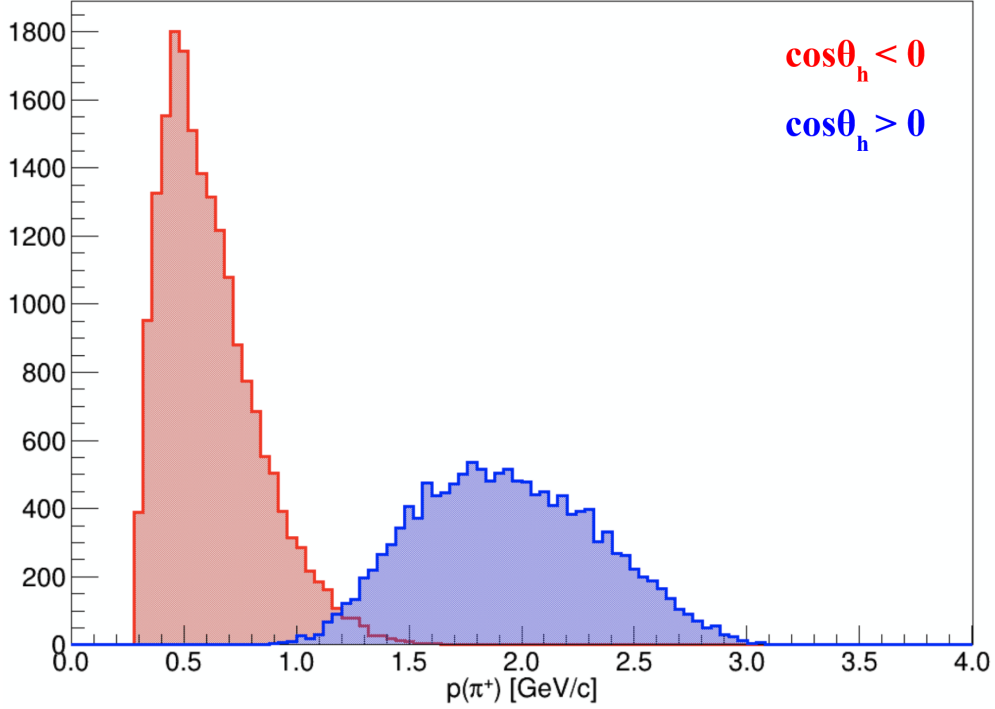


Figure 5.3: Total momentum distribution for the π^+ daughter of the B^0 given in the reference frame of the lab for the (blue) forward and (red) backward regions in the helicity angle.

at 0.27 can be extrapolated to be 1.5%. This discrepancy is taken to be the systematic uncertainty for both $B^+ \rightarrow \bar{D}^0 \pi^+$ and $B^0 \rightarrow D^- \pi^+ \pi^0$ results and is graphically demonstrated in Figure 5.4. This makes no contribution to the $A_{FB}^{\pi\pi}$ uncertainty.

5.7 ΔE PDF Modelling

Many of the parameters employed to define the PDF distributions were fixed in the fits to data. Inaccuracies in the PDF shapes derived from MC simulation (see Table IX) can introduce a systematic uncertainty. To account for this we perform a study on the effects of varying the bin-by-bin ΔE PDF shape parameters on the final extracted signal yield. When determining these parameters via the fit to MC data, the correlations between the parameters were recorded in a matrix σ . Consider the set of shape parameter values in each bin of $M_{\pi\pi}$ and $\cos\theta_h$, $X = \{x_1, x_2, \dots, x_i\}$. The extracted yield when performing a fit to X is denoted the nominal yield,

$$fit(X) = N_{nom} \quad (5.3)$$

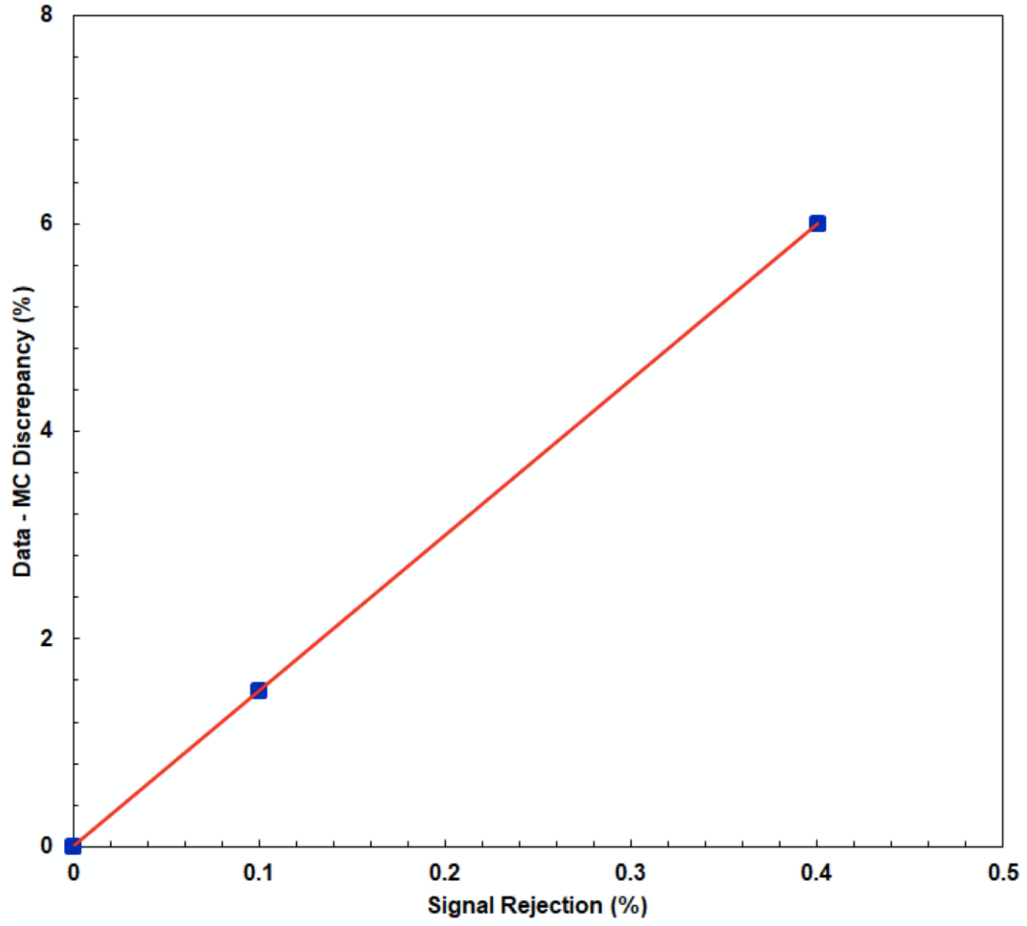


Figure 5.4: Data-MC discrepancy as a function of the signal rejection fraction.

Now consider shifting all parameters in X by some amount S such that

$$fit(X + S_i) = N_i^+ \quad (5.4)$$

where the label i denotes the parameter relative to which S is calculated. The expression for finding the shift S in parameter i relative to parameter j is given by

$$S_{ij} = \frac{\delta x_i}{x_i} \sigma_{ij} x_j \quad (5.5)$$

where δx is the statistical uncertainty in x and σ_{ij} is the correlation coefficient between parameter i and j . The systematic uncertainty due to parameter i is calculated as the average deviation from N_{nom} after applying the shift in the positive and negative directions,

$$\delta_i = \frac{1}{2} (|N_i^+ - N_{nom}| + |N_i^- - N_{nom}|) \quad (5.6)$$

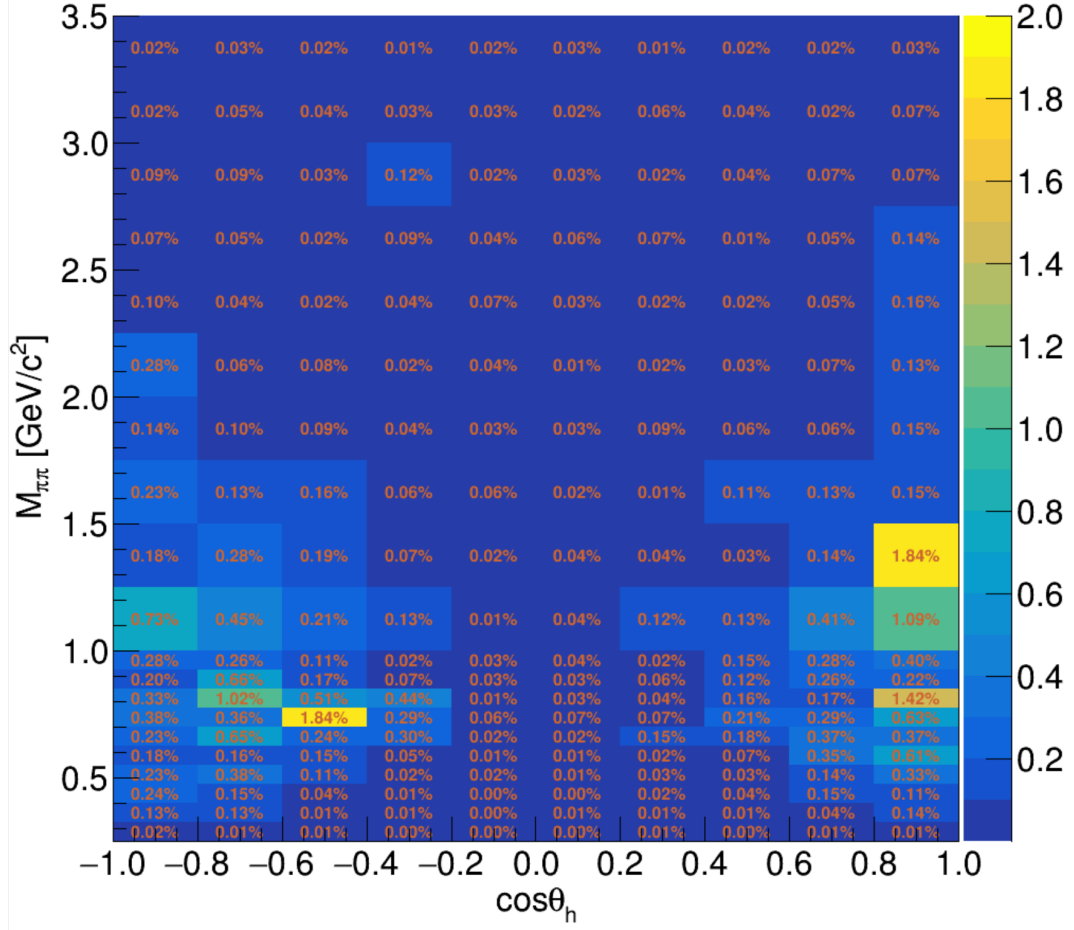


Figure 5.5: Total systematic uncertainty of $B^0 \rightarrow D^- \pi^+ \pi^0$ in bins of $M_{\pi\pi}$ and $\cos \theta_h$. Quadratic sum of all uncertainties across all bins is 4.2%. For the $B^0 \rightarrow D^- \rho^+$ channel, the sum is done in bins below $1 \text{ GeV}/c^2$ in $M_{\pi\pi}$ which results in 3.6% uncertainty. Finally, the $A_{FB}^{\pi\pi}$ result is further restricted to the region $|\cos \theta_h| < 0.8$, leading to a total of 2.8%.

The uncertainty over all parameters is given by the quadratic sum of their individual uncertainties $\sqrt{\sum_i \delta_i^2}$ as shown in Figure 5.5 and 5.6 for every bin of $M_{\pi\pi}$ and $\cos \theta_h$ for both $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^+ \rightarrow \bar{D}^0 \pi^+$. The total uncertainty across all bins is calculated in the same way and was found to be 1.35% for the control mode $B^+ \rightarrow \bar{D}^0 \pi^+$ and 4.2% for the signal mode $B^0 \rightarrow D^- \pi^+ \pi^0$. For the $B^0 \rightarrow D^- \rho^+$ yield extraction, only the bins below $1 \text{ GeV}/c^2$ in $M_{\pi\pi}$ contribute, resulting in a 3.6% systematic uncertainty. Finally, the $A_{FB}^{\pi\pi}$ is further restricted to the region $|\cos \theta_h| < 0.8$, leading to a total of 2.8%.

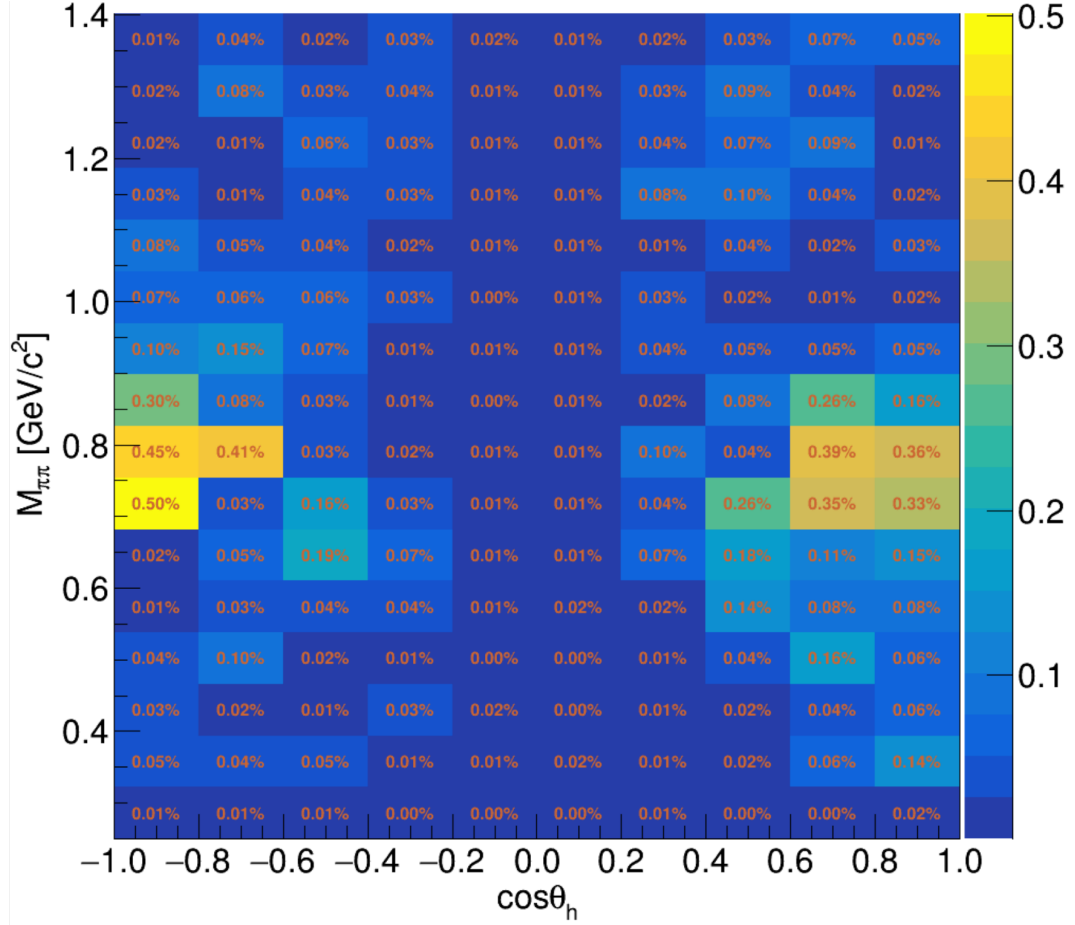


Figure 5.6: Total systematic uncertainty of $B^+ \rightarrow \bar{D}^0 \pi^+$ in bins of $M_{\pi\pi}$ and $\cos\theta_h$. Quadratic sum of all uncertainties across all bins is 1.35%.

5.8 ΔE Fitter Bias

For the fits to ΔE , a series of linearity tests were performed for each of the results on samples of toy MC, the details of which are given in Section 3.6.1. The systematic uncertainty is conservatively taken to be half the size of the bias found in each case. This amounts to 0.95% for the $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ branching fractions and 0.4% for the $B^+ \rightarrow \bar{D}^0 \pi^+$ control mode. For $A_{FB}^{\pi\pi}$ the full bias of 2.1% is taken as the uncertainty. The $B^0 \rightarrow D^- \rho^+$ result has an additional systematic contribution due to the subsequent fit to $M_{\pi\pi}$ which is discussed in Section 5.12.

5.9 Rare Generic $B\bar{B}$

After reconstruction and selection criteria are applied, it was found that rare generic B decays contribute less than 0.1% of the total expected yield and was therefore omitted in the full fit. This omission, in principal, contributes a small systematic uncertainty to all reported results, but within the context of this analysis, is completely dominated by all other systematics.

5.10 Bin Event Migration

To determine the systematic effect of the bin event migration, the extraction of the $B^0 \rightarrow D^- \pi^+ \pi^0$ and $A_{FB}^{\pi\pi}$ results were performed both with and without applying the migration correction to the fitted bin yields. The systematic uncertainty for each result was taken to be half the difference in the final extracted value, as a proportional fraction. Because this systematic is determined by noting the effect of the migration correction on each data set separately, the value obtained will differ between partial and full Belle data. Additionally, the derived uncertainty is a complicated function of the exact extracted yield distribution across all bins and thus small statistical fluctuations can have unpredictable effects on its precise value. For $B^0 \rightarrow D^- \pi^+ \pi^0$ and $A_{FB}^{\pi\pi}$, the systematic uncertainty came out to 0.26% and 1.05% in partial 74 fb^{-1} Belle data, respectively, and 2.8% and 1.5% in full 570 fb^{-1} data. In the case of the control mode $B^+ \rightarrow \bar{D}^0 \pi^+$, it was found to have a negligibly small effect of $< 0.1\%$ in both partial and full data.

In the case of the $B^0 \rightarrow D^- \rho^+$ branching fraction, due to the level of migration across the $1 \text{ GeV}/c^2 \ M_{\pi\pi}$ boundary, the application of the migration correction is essential to obtaining a sensible result, and so a different comparison was needed. The migration correction factor is defined in terms of the fraction of events that migrate into and out of any given bin during event reconstruction. This was modelled on a signal MC sample that contained a mixture of all signal channels including the $B^0 \rightarrow D^- \pi^+ \pi^0$ direct decay as well as the $B^0 \rightarrow D^- \rho^+$ decay. However, to determine the systematic for the $B^0 \rightarrow D^- \rho^+$, a different signal sample was employed which contained only $B^0 \rightarrow D^- \rho^+$ signal decays with no contributions from any other channels. Comparing the final extracted yield when using this migration correction found a change of 0.86%, corresponding to a systematic of 0.43% in partial data and 0.28% in full data.

5.11 Efficiency Matrix

The performance of both the efficiency matrix and the SCF fraction may differ between what is found in data and MC, and this contributes an uncertainty. To estimate the size of this effect, an update to the efficiency matrix was made based on the fitted SCF fraction from data and was subsequently compared to the extracted result that was obtained if the update wasn't applied. This update is motivated by noting that the efficiency in each bin of $\cos \theta_h$ and $M_{\pi\pi}$ implicitly assumes the SCF fraction in MC via the relation,

$$\epsilon_{Sig}^{MC} = \frac{N_{Sig}^{MC}}{N_{gen}} = \frac{(1 - f_{SCF}^{MC})N_{tot}^{MC}}{N_{gen}} \quad (5.7)$$

where N_{Sig}^{MC} is the pure signal yield in MC, N_{tot}^{MC} is the total signal and SCF yield in MC, and N_{gen} is the generated signal yield. However, during the fit to data, the SCF fraction is floated in a range centred around the value found in MC. Hence, we can write a similar relation for data,

$$\epsilon_{Sig}^{data} = \frac{(1 - f_{SCF}^{data})N_{tot}^{MC}}{N_{gen}}. \quad (5.8)$$

This provides an equation to update the efficiency according to the SCF fraction in data

$$\epsilon_{Sig}^{data} = \frac{(1 - f_{SCF}^{data})\epsilon_{Sig}^{MC}}{(1 - f_{SCF}^{MC})}. \quad (5.9)$$

The derivation of this formula is also given in Appendix C. This update was applied as a correction to the full results where half the difference in the results was taken as the systematic uncertainty. Because this systematic is determined by noting the effect of the SCF fraction update on each data set separately, the value obtained will differ between partial and full Belle data. In partial 74 fb^{-1} data this comes to 0.43%, 1.7% and 0.8% for the $B^0 \rightarrow D^- \pi^+ \pi^0$, $B^0 \rightarrow D^- \rho^+$ and $B^+ \rightarrow \bar{D}^0 \pi^+$ branching fractions, respectively, and 3.2% for $A_{FB}^{\pi\pi}$. In full 570 fb^{-1} data this comes to 1.3%, 2.6% and 0.8% for the $B^0 \rightarrow D^- \pi^+ \pi^0$, $B^0 \rightarrow D^- \rho^+$ and $B^+ \rightarrow \bar{D}^0 \pi^+$ branching fractions, respectively and 1.8% for $A_{FB}^{\pi\pi}$.

TABLE XX: Systematic uncertainties for the $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction measurements and $A_{FB}^{\pi\pi}$ observable in partial 74 fb^{-1} Belle data.

Source	$A_{FB}^{\pi\pi}$	$\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0)$	$\mathcal{B}(B^0 \rightarrow D^- \rho^+)$
Charged track efficiency	1.40%	1.40%	1.40%
π^0 Reconstruction Efficiency	—	1.49%	1.49%
Number of $B\bar{B}$ pairs	—	1.5%	1.5%
Value of $R_{B^0 \bar{B}^0}$	—	1.23%	1.23%
PID efficiency (Total)	—	1.56%	1.56%
PID efficiency ($A_{FB}^{\pi\pi}$)	1.43%	—	—
Continuum Suppression	—	1.5%	1.5%
ΔE PDF modelling	2.8%	4.2%	3.6%
ΔE fitter bias	2.1%	0.95%	0.95%
Rare $B\bar{B}$	$< 0.1\%$	$< 0.1\%$	$< 0.1\%$
Bin migration	1.05%	0.26%	0.43%
Efficiency matrix	3.2%	0.43%	1.7%
$M_{\pi\pi}$ fitter bias	—	—	1.35%
Total systematics	5.08%	5.61%	5.60%

5.12 $M_{\pi\pi}$ Fitter Bias

For the fit to $M_{\pi\pi}$, a linearity test was performed on a sample of toy MC, the details of which are given in Section 3.6.4. The systematic uncertainty is conservatively taken to be half the size of the bias found. This amounts to 1.35% for the $B^0 \rightarrow D^- \rho^+$ branching fraction.

5.13 Summary of Systematic Uncertainties

This section summarises all sources of systematic uncertainties for the $B^0 \rightarrow D^- \pi^+ \pi^0$ and $B^0 \rightarrow D^- \rho^+$ decay branching fractions as well as the $A_{FB}^{\pi\pi}$ helicity asymmetry. Table XX and Table XXI lists the systematics for partial 74 fb^{-1} and full 570 fb^{-1} data, respectively, while Table XXII lists them for the $B^+ \rightarrow \bar{D}^0 \pi^+$ control mode. The total systematics are calculated as the quadratic sum over all individual sources.

The differences in the total systematics between the partial and full data sets can be attributed to the bin migration and efficiency matrix uncertainties which are determined separately for each data set.

TABLE XXI: Systematic uncertainties for the $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction measurements and $A_{FB}^{\pi\pi}$ observable in 570 fb^{-1} SVD2 Belle data.

Source	$A_{FB}^{\pi\pi}$	$\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0)$	$\mathcal{B}(B^0 \rightarrow D^- \rho^+)$
Charged track efficiency	1.40%	1.40%	1.40%
π^0 Reconstruction Efficiency	—	1.49%	1.49%
Number of $B\bar{B}$ pairs	—	1.5%	1.5%
Value of $R_{B^0 \bar{B}^0}$	—	1.23%	1.23%
PID efficiency (Total)	—	1.56%	1.56%
PID efficiency ($A_{FB}^{\pi\pi}$)	1.43%	—	—
Continuum Suppression	—	1.5%	1.5%
ΔE PDF modelling	2.8%	4.2%	3.6%
ΔE fitter bias	2.1%	0.95%	0.95%
Rare $B\bar{B}$	$< 0.1\%$	$< 0.1\%$	$< 0.1\%$
Bin migration	1.5%	2.8%	0.28%
Efficiency matrix	1.8%	1.3%	2.6%
$M_{\pi\pi}$ fitter bias	—	—	1.35%
Total systematics	4.66%	6.38%	5.93%

TABLE XXII: Systematic uncertainties for the $B^+ \rightarrow \bar{D}^0 \pi^+$ branching fraction measurement in 570 fb^{-1} SVD2 Belle data.

Source	$\mathcal{B}(B^+ \rightarrow \bar{D}^0 \pi^+)$
Charged track efficiency	1.05%
π^0 Reconstruction Efficiency	1.49%
Number of $B\bar{B}$ pairs	1.5%
Value of $R_{B^+ B^-}$	1.17%
PID efficiency	1.3%
Continuum Suppression	1.5%
ΔE PDF modelling	1.35%
ΔE fitter bias	0.4%
Rare $B\bar{B}$	$< 0.1\%$
Bin migration	$< 0.1\%$
Efficiency matrix	0.8%
Total systematics	3.67%

Chapter 6

Results and Conclusions

The results of the full $B^0 \rightarrow D^-\pi^+\pi^0$ and $B^0 \rightarrow D^-\rho^+$ analysis on real data collected by the Belle experiment are given in this chapter. The fitting strategy outlined in Section 3.5.1 was applied to the 570 fb^{-1} SVD2 Belle dataset corresponding to $(620 \pm 9.44) \times 10^6$ $B\bar{B}$ pairs.

The analysis was initially performed on a partial 74 fb^{-1} data set in order to verify its validity before moving ahead with the full dataset. The results of this partial unblinding are given alongside the full results.

6.1 Box Open Result - $B^0 \rightarrow D^-\pi^+\pi^0$ Branching Fraction

The final $B^0 \rightarrow D^-\pi^+\pi^0$ extracted yield is the linear sum of all fitted yields over all bins in $\cos\theta_h$ and $M_{\pi\pi}$, divided by the efficiency matrix. For the partial 74 fb^{-1} dataset, the extracted total yield is

$$N_{D^-\pi^+\pi^0} = 64\,513 \pm 2863(stat.) \quad (6.1)$$

From this, the branching fraction can be calculated using the formula

$$\mathcal{B} = \frac{N_{D^-\pi^+\pi^0}}{2 \times R_{B^0\bar{B}^0} \times N_{B\bar{B}} \times \mathcal{B}(D^- \rightarrow K^+\pi^-\pi^-) \times C} \quad (6.2)$$

where $N_{D^-\pi^+\pi^0}$ is the final extracted yield, $N_{B\bar{B}}$ is the number of $B\bar{B}$ pairs in the Belle dataset, $R_{B^0\bar{B}^0}$ is the fraction of those $B\bar{B}$ pairs that are B^0 s and C is a correction factor

that takes into account the fit bias (Section 3.6.3), π^0 efficiency correction (Section 5.2) and PID correction (Section 5.5). This results in the following measured branching fractions for both partial and full SVD2 Belle data:

$$\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0) = (9.95 \pm 0.43 \pm 0.56) \times 10^{-3} \quad (6.3)$$

$$\mathcal{B}(B^0 \rightarrow D^- \pi^+ \pi^0) = (8.92 \pm 0.23 \pm 0.57) \times 10^{-3} \quad (6.4)$$

where the first uncertainty is statistical and the second is systematic. The systematic values are obtained by multiplying the central value by the percent totals listed in Table XX and Table XXI. The sum of all fitted PDFs across all $\cos \theta_h$ and $M_{\pi\pi}$ bins is shown in Figure 6.2 for both partial and full datasets. This constitutes a first measurement of the total $B^0 \rightarrow D^- \pi^+ \pi^0$ branching fraction.

6.2 Box Open Result - $A_{FB}^{\pi\pi}$ Helicity Asymmetry

The extracted value of $A_{FB}^{\pi\pi}$ is calculated in the region $1 \text{ GeV}/c^2$ and $|\cos \theta_h| < 0.8$ according to the formula

$$A_{FB}^{\pi\pi} = \frac{N_F - N_B}{N_F + N_B} \quad (6.5)$$

where N_F and N_B are the total efficiency corrected yields across all bins in the forward and backward regions with $\cos \theta_h > 0$ or $\cos \theta_h < 0$, respectively. The calculation of the statistical uncertainty is given in Appendix D. This results in the following measured asymmetries for both partial and full SVD2 Belle data:

$$A_{FB}^{\pi\pi} = -0.028 \pm 0.005 \pm 0.051 \quad (6.6)$$

$$A_{FB}^{\pi\pi} = -0.048 \pm 0.002 \pm 0.047 \quad (6.7)$$

where the first uncertainty is statistical and the second is systematic. The systematic values are given by the percent totals listed in Table XX and Table XXI. The projection onto $\cos \theta_h$ in the considered region is shown in Figure 6.1 for both partial and full datasets. The partial result is consistent with the theoretical expectation of zero and constitutes a first measurement of the forward-backward asymmetry in $B^0 \rightarrow D^- \pi^+ \pi^0$.

6.3 Box Open Result - $B^0 \rightarrow D^- \rho^+$ Branching Fraction

The final $B^0 \rightarrow D^- \rho^+$ extracted yield is found from the fit to $M_{\pi\pi}$ below $1 \text{ GeV}/c^2$. For the partial 74 fb^{-1} dataset, the extracted total yield is

$$N_{D^- \rho^+} = 42\,165 \pm 1893(\text{stat.}). \quad (6.8)$$

From this, the branching fraction can be calculated using the same formula described previously (Equation 6.2) except for an additional factor of 0.82 in the denominator that takes into account the partial fit below $1 \text{ GeV}/c^2$. This results in the following measured branching fractions for both partial and full SVD2 Belle data:

$$\mathcal{B}(B^0 \rightarrow D^- \rho^+) = (7.87 \pm 0.37 \pm 0.44) \times 10^{-3} \quad (6.9)$$

$$\mathcal{B}(B^0 \rightarrow D^- \rho^+) = (7.91 \pm 0.21 \pm 0.47) \times 10^{-3} \quad (6.10)$$

where the first uncertainty is statistical and the second is systematic. The systematic values are obtained by multiplying the central value by the percent totals listed in Table XX and Table XXI. The full fit to $M_{\pi\pi}$ is shown in Figure 6.3 for both partial and full datasets. The partial result is consistent with the PDG world average of $(7.6 \pm 1.2) \times 10^{-3}$ [24] and both results constitute an improvement in its precision. By comparing Figure 6.3 to Figure 3.24 in MC data, a shift towards lower $M_{\pi\pi}$ in the underlying non-resonant $D^- \pi^+ \pi^0$ component becomes apparent. This may be an indication of a small S-wave contribution to the $\pi^+ \pi^0$ final state at around $0.5 \text{ GeV}/c^2$ and may be worth further examination in future work.

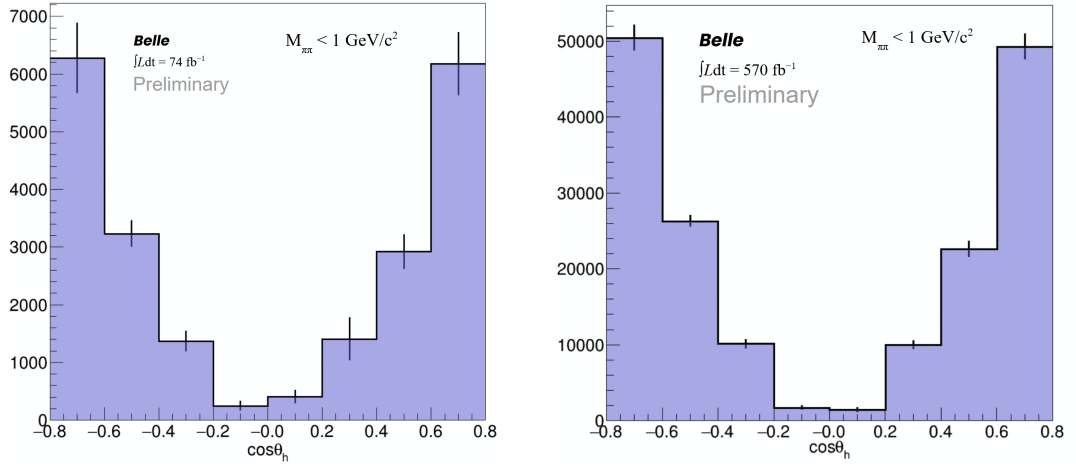


Figure 6.1: $\cos \theta_h$ distributions for (left) partial Belle data and (right) full Belle data.

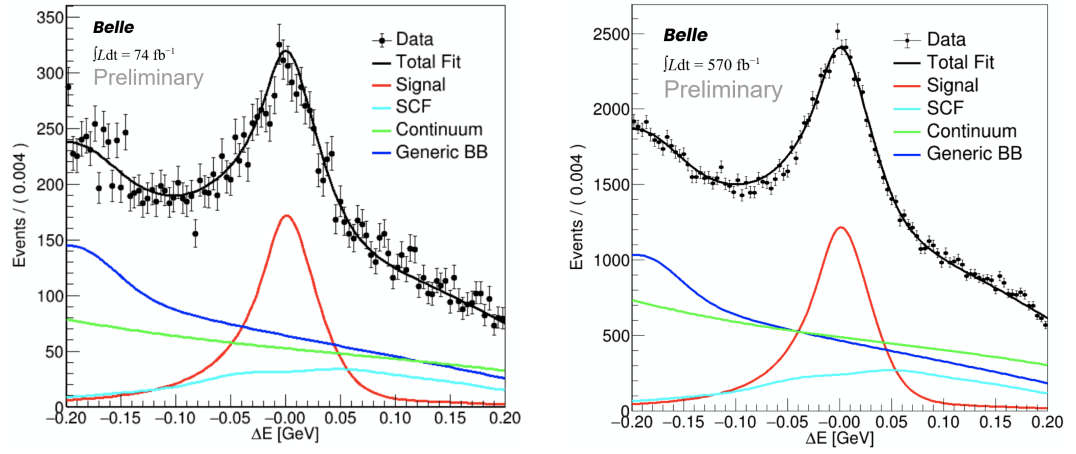


Figure 6.2: ΔE distributions for (left) partial Belle data and (right) full Belle data. Data points and PDFs are the sum across all bin fits in $\cos \theta_h$ and $M_{\pi\pi}$.

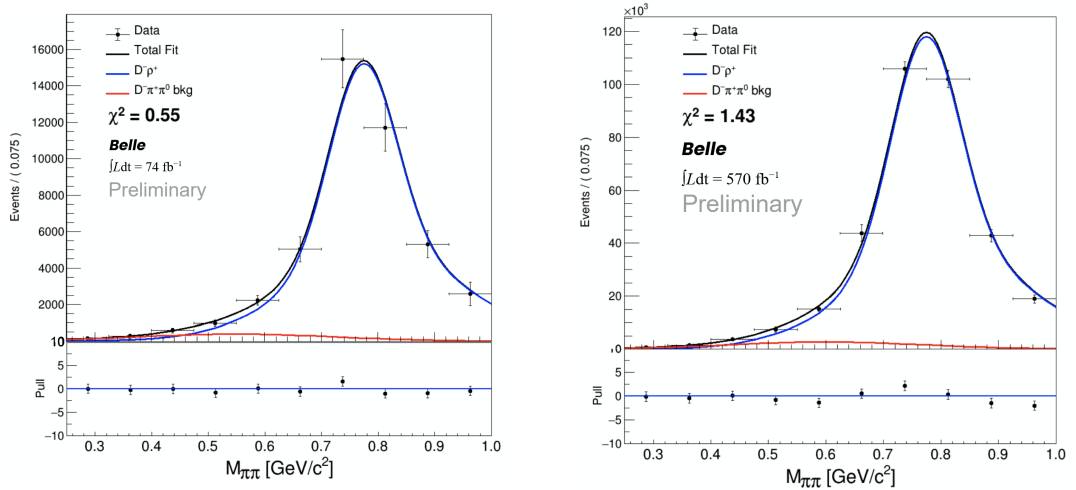


Figure 6.3: $M_{\pi\pi}$ distributions for (left) partial Belle data and (right) full Belle data. The χ^2 values given are per degree of freedom.

Appendix A

Binned ΔE PDF Modelling for $B^0 \rightarrow D^- \pi^+ \pi^0$

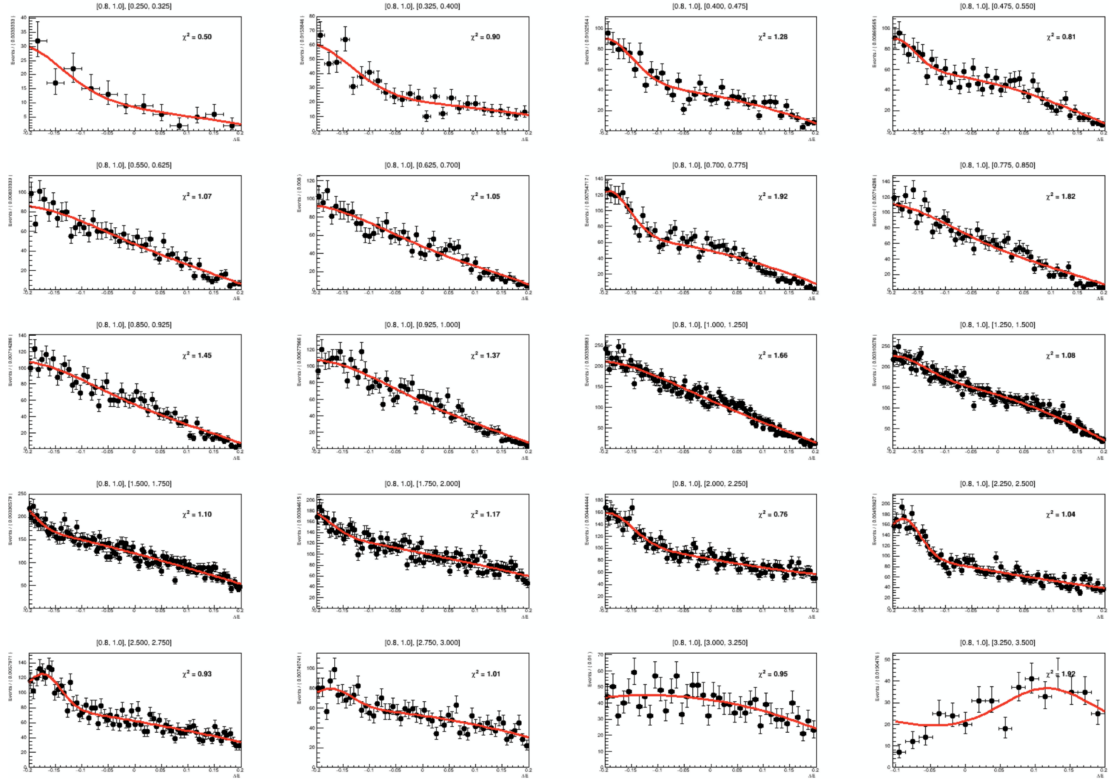


Figure A.1: Sample of 20 ΔE PDFs for generic $B\bar{B}$ MC in the $[0.8, 1.0]$ $\cos\theta_h$ bin.

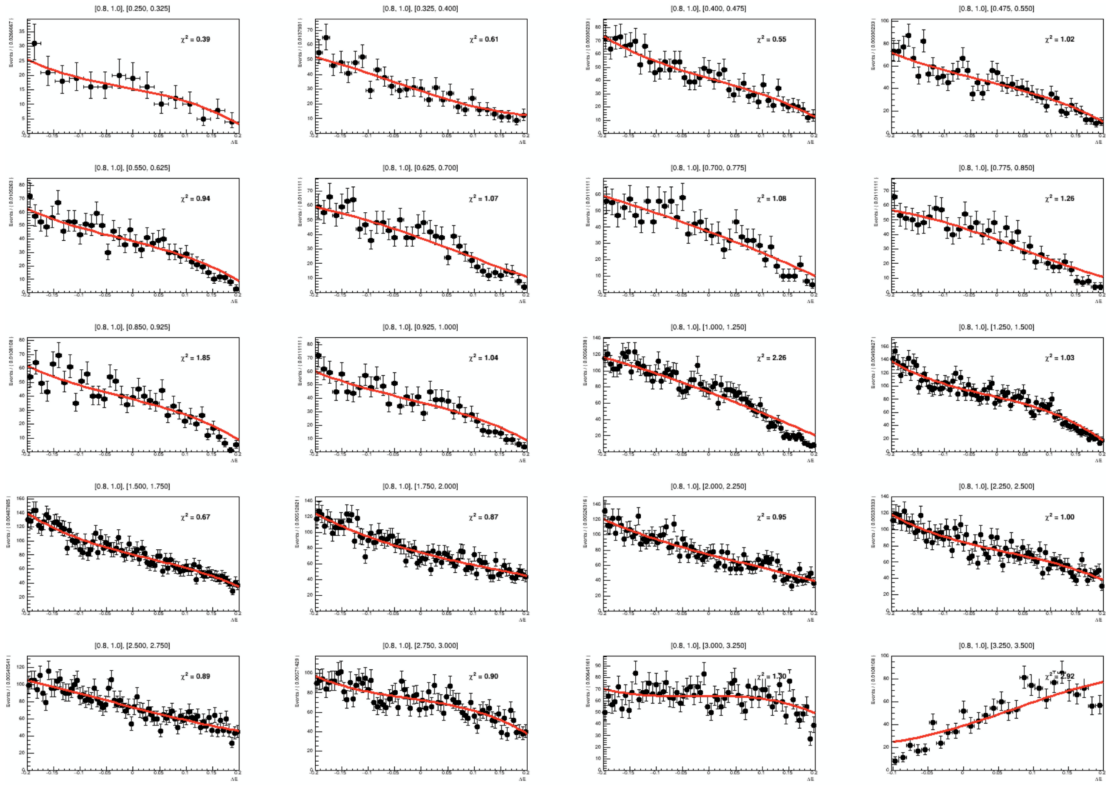


Figure A.2: Sample of 20 ΔE PDFs for continuum MC in the $[0.8, 1.0] \cos \theta_h$ bin.

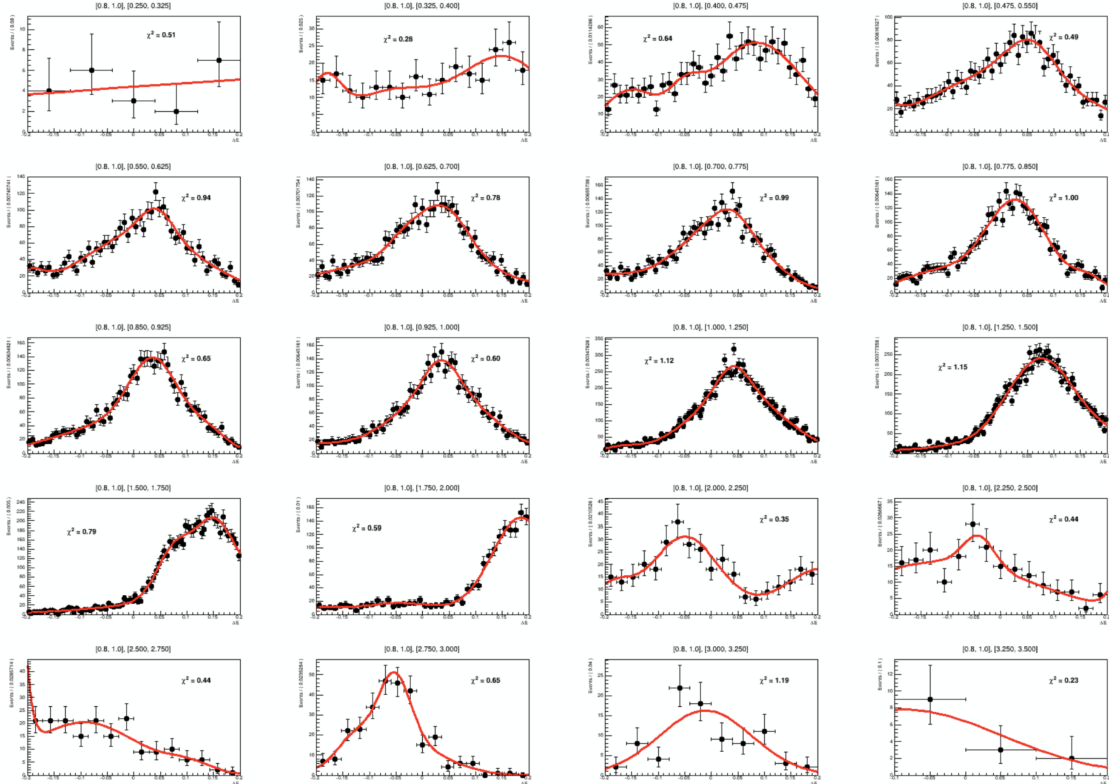


Figure A.3: Sample of 20 ΔE PDFs for self-cross feed MC in the $[0.8, 1.0] \cos \theta_h$ bin.

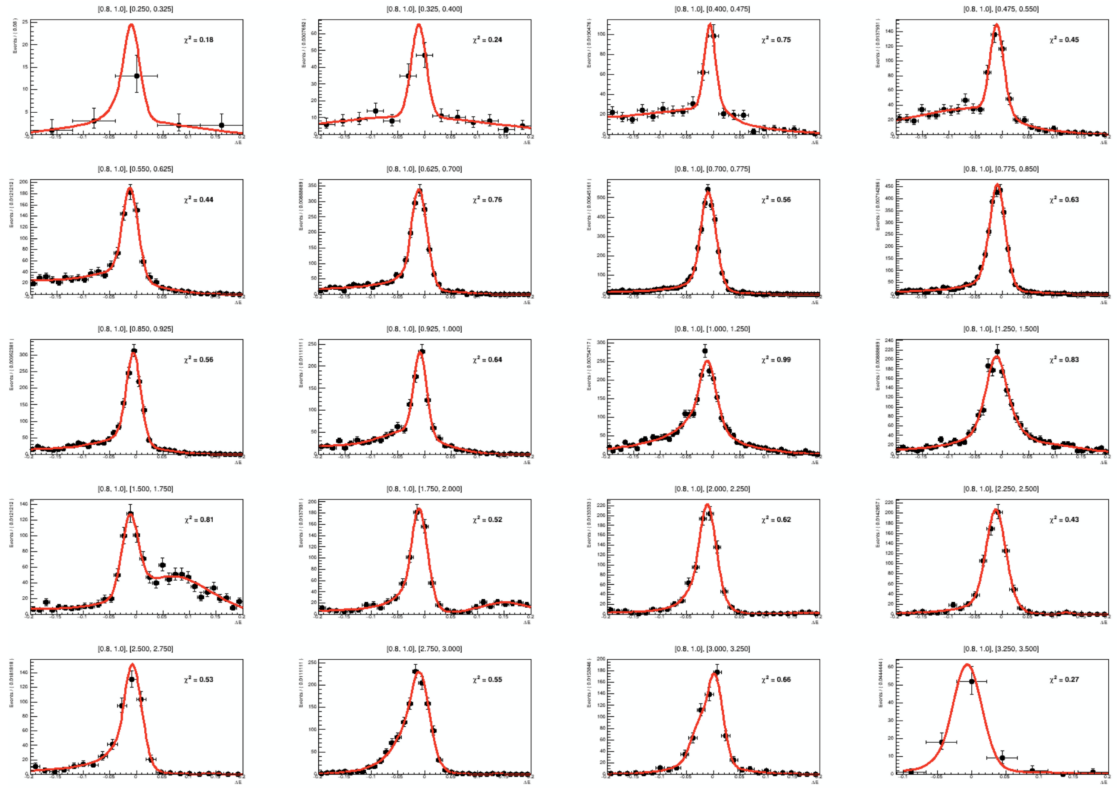


Figure A.4: Sample of 20 ΔE PDFs for $B^0 \rightarrow D^-\pi^+\pi^0$ signal MC in the $[0.8, 1.0]$ $\cos\theta_h$ bin.

Appendix B

Binned ΔE PDF Modelling for $B^+ \rightarrow \bar{D}^0 \pi^+$

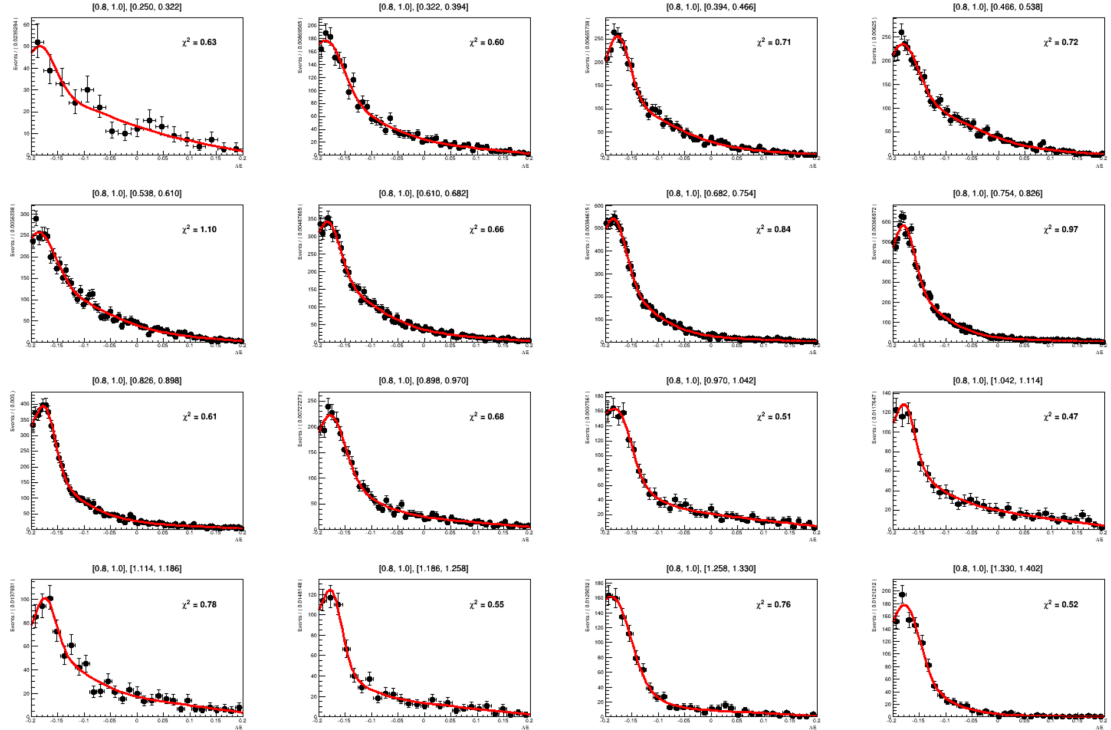


Figure B.1: Sample of 20 ΔE PDFs for generic $B\bar{B}$ MC in the $[0.8, 1.0] \cos \theta_h$ bin.

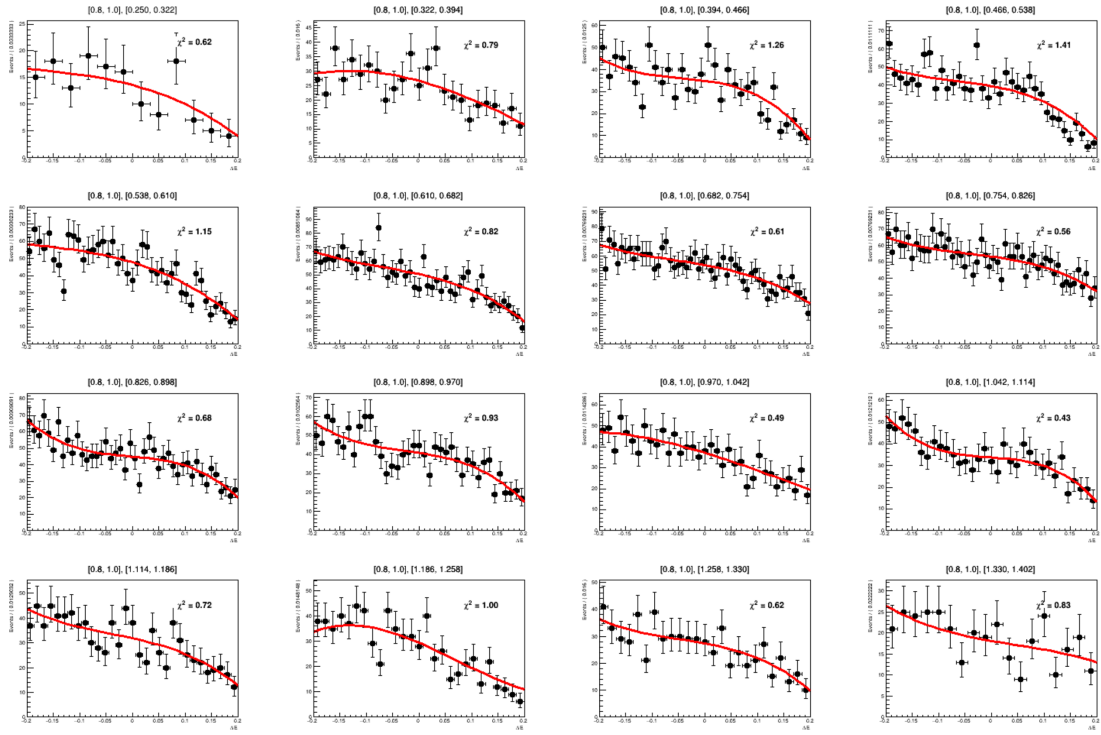


Figure B.2: Sample of 20 ΔE PDFs for continuum MC in the $[0.8, 1.0] \cos \theta_h$ bin.

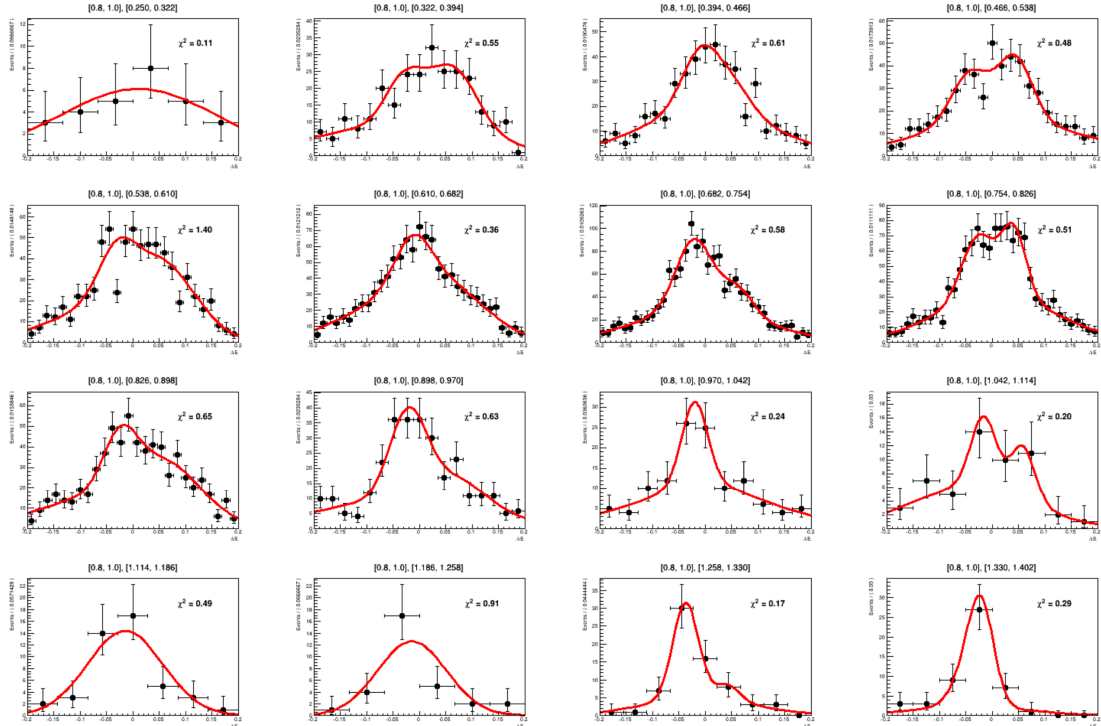


Figure B.3: Sample of 20 ΔE PDFs for self-cross feed MC in the $[0.8, 1.0] \cos \theta_h$ bin.

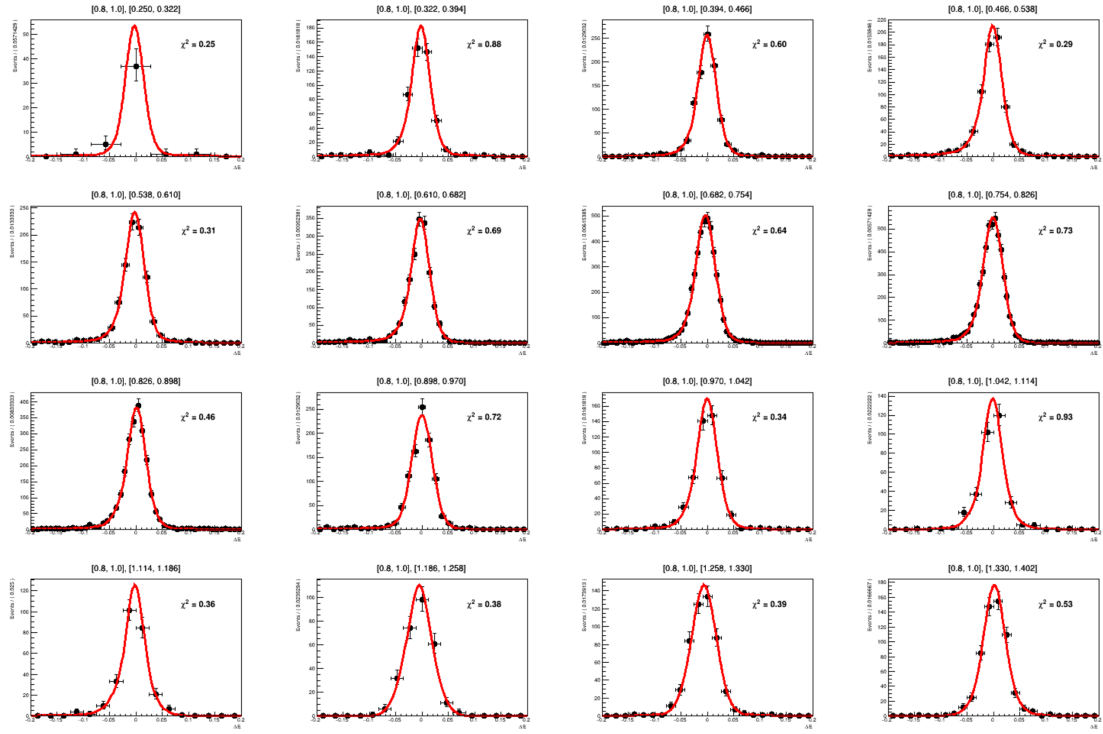


Figure B.4: Sample of 20 ΔE PDFs for $B^0 \rightarrow D^- \pi^+ \pi^0$ signal MC in the $[0.8, 1.0]$ $\cos \theta_h$ bin.

Appendix C

Reconstruction Efficiency Update Formula

The reconstruction efficiency found in MC signal samples can be expressed in terms of the fraction of self-crossfeed (SCF),

$$\epsilon_{sig}^{MC} = \frac{N_{sig}^{MC}}{N_{gen}} = \frac{(1 - f_{SCF}^{MC})N_{total}^{MC}}{N_{gen}} \quad (C.1)$$

where N_{total}^{MC} is the total reconstructed yield, inclusive of SCF, while N_{gen} is the generated yield in MC. Rearranging we get,

$$\frac{N_{total}^{MC}}{N_{gen}} = \frac{\epsilon_{sig}^{MC}}{1 - f_{SCF}^{MC}}. \quad (C.2)$$

However, during the fit f_{SCF} is floated and is thus extracted from data allowing for the estimation of ϵ_{sig}^{MC} to be updated. One can now write Equation C.1 in terms of data variables,

$$\epsilon_{sig}^{data} = \frac{(1 - f_{SCF}^{data})N_{total}^{MC}}{N_{gen}}. \quad (C.3)$$

Substituting Equation C.2 and rearranging for ϵ_{sig}^{data} gives

$$\epsilon_{sig}^{data} = \left(\frac{1 - f_{SCF}^{data}}{1 - f_{SCF}^{MC}} \right) \epsilon_{sig}^{MC}. \quad (C.4)$$

This provides a formula for updating the signal reconstruction efficiency based on the fraction of SCF found in data.

Appendix D

$A_{FB}^{\pi\pi}$ Statistical Uncertainty

What follows is a derivation of the statistical uncertainty for the forward-backward asymmetry, $A_{FB}^{\pi\pi} = \frac{N_F - N_B}{N_F + N_B}$.

One can rewrite this as the difference of two fractions:

$$A_{FB}^{\pi\pi} = \frac{N_F}{N_F + N_B} - \frac{N_B}{N_F + N_B} \quad (\text{D.1})$$

$$= \frac{N_F}{N_T} - \frac{N_B}{N_T} \quad (\text{D.2})$$

where $N_T = N_F + N_B$ is the total number of events. Taking the variance of both sides gives

$$\text{Var}(A_{FB}^{\pi\pi}) = \text{Var}\left(\frac{N_F}{N_T} - \frac{N_B}{N_T}\right) \quad (\text{D.3})$$

$$= \frac{1}{N_T^2}(\text{Var}(N_F) + \text{Var}(N_B)). \quad (\text{D.4})$$

One can treat N_F and N_B as random variables that follow binomial distributions $B(N_T, p)$ and $B(N_T, q)$ where p and q are the probabilities of each event type and are given by N_F/N_T and N_B/N_T , respectively. For a binomial distribution,

$$\text{Var}(N_F) = N_T p(1 - p) \quad (\text{D.5})$$

$$\text{Var}(N_B) = N_T q(1 - q). \quad (\text{D.6})$$

However, we know $p + q = 1$ and so $\text{Var}(N_F) = \text{Var}(N_B) = N_T pq$. Substituting into Equation D.4 gives

$$\text{Var}(A_{FB}^{\pi\pi}) = \frac{2pq}{N_T} \quad (\text{D.7})$$

$$\Rightarrow \sigma = \sqrt{\frac{2pq}{N_T}} \quad (\text{D.8})$$

where σ is the standard deviation of $A_{FB}^{\pi\pi}$. Expressed in terms of the forward and backward yields this gives

$$\sigma = \sqrt{\frac{2N_F N_B}{(N_F + N_B)^3}}. \quad (\text{D.9})$$

Bibliography

- [1] A. Abashian et al. The Belle detector. *Nucl. Instrum. Meth. A*, 479(1):117–232, 2002.
- [2] A. J. Bevan, B. Golob, Th. Mannel, and S. Prell. The Physics of the B Factories. *The European Physical Journal C*, 74(11), 2014.
- [3] S. Ryu et al. Measurements of branching fractions of τ lepton decays with one or more K_S^0 . *Physical Review D*, 89(7):072009, 2014.
- [4] Yo Sato, Sam Cunliffe, Frank Meier, and Anze Zupanc. Monte Carlo matching in the Belle II software. *EPJ Web of Conferences*, 251:03021, 2021.
- [5] G. Aad, T. Abajyan, and B. Abbott. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Physics Letters B*, 716(1):1–29, 2012.
- [6] S. Chatrchyan, V. Khachatryan, and A. M. Sirunyan. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Physics Letters B*, 716(1):30–61, 2012.
- [7] Peter W. Higgs. Broken Symmetries and the Masses of Gauge Bosons. *Physical Review Letters*, 13:508–509, 1964.
- [8] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Global Conservation Laws and Massless Particles. *Physical Review Letters*, 13:585–587, 1964.
- [9] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Physical Review Letters*, 13:321–323, 1964.
- [10] F. Abe et al. Observation of Top Quark Production in $\bar{p}p$ Collisions with the Collider Detector at Fermilab. *Physical Review Letters*, 74(14):2626–2631, 1995.

- [11] S. Kabe et al. F. J. Hasert. Observation of neutrino-like interactions without muon or electron in the gargamelle neutrino experiment. *Physics Letters B*, 46(1):138–140, 1973.
- [12] X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse. Measurement of the Electron Magnetic Moment. *Physical Review Letters*, 130(7):071801, 2023.
- [13] B. Aubert et al. The BABAR detector. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 479(1):1–116, 2002.
- [14] I. Belyaev, G. Carboni, N. Harnew, C. Matteuzzi, and F. Teubert. The history of LHCb. *The European Physical Journal H*, 46(1), 2021.
- [15] Hideki Miyake. Belle II Experiment: Status and Physics Prospects. *Nuclear and Particle Physics Proceedings*, 303-305, 2018.
- [16] A. B. Carter and A. I. Sanda. CP Violation in B Meson Decays. *Physical Review D*, 23:1567, 1981.
- [17] A. D. Sakharov. Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe. *Pisma Zh. Eksp. Teor. Fiz.*, 5:32–35, 1967. [Usp. Fiz. Nauk161,61(1991)].
- [18] P. A. R. Ade et al. Planck 2015 results: XIII. Cosmological parameters. *Astronomy & Astrophysics*, 594:A13, 2016.
- [19] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. Evidence for the 2π Decay of the K_2^0 Meson. *Phys. Rev. Lett.*, 13:138–140, 1964.
- [20] Makoto Kobayashi and Toshihide Maskawa. CP-Violation in the Renormalizable Theory of Weak Interaction. *Progress of Theoretical Physics*, 49(2):652–657, 1973.
- [21] Nicola Cabibbo. Unitary Symmetry and Leptonic Decays. *Physical Review Letters*, 10(12):531–533, 1963.
- [22] CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: <http://ckmfitter.in2p3.fr>.
- [23] M. S. Alam et al. Exclusive hadronic B decays to charm and charmonium final states. *Physical Review D*, 50(1):43–68, 1994.

- [24] R. L. Workman et al. Review of Particle Physics. *PTEP*, 2022:083C01, 2022.
- [25] J. et al. Zhang. Observation of $B^\mp \rightarrow \rho^\mp \rho^0$ Decays. *Physical Review Letters*, 91:221801, 2003.
- [26] T. Huber, J. Virto, and K. Keri Vos. Three-Body Non-Leptonic Heavy-to-heavy B Decays at NNLO in QCD. *JHEP*, 11:103, 2020.
- [27] Gunnar Källén and Eugen Merzbacher. Elementary Particle Physics. *Physics Today*, 18, 1965.
- [28] S. Kurokawa and E. Kikutani. Overview of the KEKB accelerators. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 499(1):1–7, 2003.
- [29] H. Bethe and W. Heitler. On the Stopping of Fast Particles and on the Creation of Positive Electrons. *Proceedings of the Royal Society of London Series A*, 146(856):83–112, 1934.
- [30] R. Brun et al. *GEANT 3: user's guide Geant 3.10, Geant 3.11*. Geneva, 1987.
- [31] David J. Lange. The EvtGen particle decay simulation package. *Nucl. Instrum. Meth. A*, 462(1):152–155, 2001. BEAUTY2000, Proceedings of the 7th Int. Conf. on B-Physics at Hadron Machines.
- [32] R. Itoh. BASF - BELLE Analysis Framework. In *9th International Conference on Computing in High-Energy and Nuclear Physics*, 1997.
- [33] T. Kuhr, C. Pulvermacher, M. Ritter, T. Hauth, and N. Braun. The Belle II Core Software. *Computing and Software for Big Science*, 3(1), 2018.
- [34] V. Bertacchi et al. Track finding at Belle II. *Computer Physics Communications*, 259:107610, 2021.
- [35] M. Gelb et al. B2BII: Data Conversion from Belle to Belle II. *Computing and Software for Big Science*, 2(1):9, 2018.
- [36] E. Richter-Was. QED bremsstrahlung in semileptonic B and leptonic τ decays. *Physics Letters B*, 303(1–2):163–169, 1993.

- [37] E. Nakano. Belle PID. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 494(1–3):402–408, 2002.
- [38] Jerome H. Friedman. Greedy Function Approximation: A Gradient Boosting Machine. *The Annals of Statistics*, 29(5):1189–1232, 2001.
- [39] Jerome H. Friedman. Stochastic gradient boosting. *Computational Statistics & Data Analysis*, 38(4):367–378, 2002.
- [40] Thomas Keck. FastBDT: A speed-optimized and cache-friendly implementation of stochastic gradient-boosted decision trees for multivariate classification. Technical report, Belle Collaboration, 2016.
- [41] E. Kou et al. The Belle II Physics Book. *Progress of Theoretical and Experimental Physics*, 2019(12), 2019.
- [42] D. M. Asner et al. Search for Exclusive Charmless Hadronic B Decays. *Physical Review D*, 53(3):1039–1050, 1996.
- [43] Geoffrey C. Fox and Stephen Wolfram. Observables for the analysis of event shapes in e^+e^- annihilation and other processes. *Physical Review Letters*, 41(23):1581–1585, dec 1978.
- [44] G. C. Fox and S. Wolfram. Event shapes in e^+e^- annihilation. *Nuclear Physics B*, 149(3):413–496, 1979.
- [45] T. Bloomfield et al. Measurement of the branching fraction and CP asymmetry for $B \rightarrow \bar{D}^0 \pi$ decays. *Physical Review D*, 105:072007, 2022.
- [46] R. H. Daltiz. CXII. On the analysis of τ -meson data and the nature of the τ -meson. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 44(357):1068–1080, 1953.
- [47] E. Fabri. A study of τ -meson decay. *Il Nuovo Cimento*, 11(5):479–491, 1954.
- [48] J.-F. Krohn et al. Global decay chain vertex fitting at Belle II. *Nucl. Instrum. Meth. A*, 976:164269, 2020.

-
- [49] M. Pivk. sPlot: A Quick Introduction. In *Statistical Problems in Particle Physics, Astrophysics and Cosmology*. PUBLISHED BY IMPERIAL COLLEGE PRESS AND DISTRIBUTED BY WORLD SCIENTIFIC PUBLISHING CO., 2006.
- [50] John Erthal Gaiser. *Charmonium Spectroscopy from Radiative Decays of the J/ψ and ψ'* . PhD thesis, Stanford University, 1982. Pg 178.
- [51] W. Verkerke and D. Kirkby. The RooFit Toolkit for Data Modeling. Technical report, University of California, 2005.
- [52] S. Nishida. Study of kaon and pion identification using inclusive D^* sample. Belle Note 779, Belle Collaboration, 2005.