

Measurement of the branching fraction and CP -violating asymmetry of the decay $B^0 \rightarrow \pi^0\pi^0$ using 387 million $\Upsilon(4S)$ decays in Belle II data

I. Adachi^{1D}, L. Aggarwal^{1D}, H. Ahmed^{1D}, H. Aihara^{1D}, M. Alhakami^{1D}, A. Aloisio^{1D}, N. Althubiti^{1D}, N. Anh Ky^{1D}, D. M. Asner^{1D}, H. Atmacan^{1D}, V. Aushev^{1D}, M. Aversano^{1D}, R. Ayad^{1D}, V. Babu^{1D}, H. Bae^{1D}, N. K. Baghel^{1D}, S. Bahinipati^{1D}, P. Bambade^{1D}, Sw. Banerjee^{1D}, S. Bansal^{1D}, M. Barrett^{1D}, M. Bartl^{1D}, J. Baudot^{1D}, A. Baur^{1D}, A. Beaubien^{1D}, F. Becherer^{1D}, J. Becker^{1D}, J. V. Bennett^{1D}, F. U. Bernlochner^{1D}, V. Bertacchi^{1D}, M. Bertemes^{1D}, E. Bertholet^{1D}, M. Bessner^{1D}, S. Bettarini^{1D}, V. Bhardwaj^{1D}, B. Bhuyan^{1D}, F. Bianchi^{1D}, T. Bilka^{1D}, D. Biswas^{1D}, A. Bobrov^{1D}, D. Bodrov^{1D}, A. Bolz^{1D}, A. Bondar^{1D}, J. Borah^{1D}, A. Boschetti^{1D}, A. Bozek^{1D}, M. Bračko^{1D}, P. Branchini^{1D}, R. A. Briere^{1D}, T. E. Browder^{1D}, A. Budano^{1D}, S. Bussino^{1D}, Q. Campagna^{1D}, M. Campajola^{1D}, L. Cao^{1D}, G. Casarosa^{1D}, C. Cecchi^{1D}, J. Cerasoli^{1D}, M.-C. Chang^{1D}, P. Chang^{1D}, R. Cheaib^{1D}, P. Cheema^{1D}, B. G. Cheon^{1D}, K. Chilikin^{1D}, K. Chirapatpimol^{1D}, H.-E. Cho^{1D}, K. Cho^{1D}, S.-J. Cho^{1D}, S.-K. Choi^{1D}, S. Choudhury^{1D}, J. Cochran^{1D}, L. Corona^{1D}, J. X. Cui^{1D}, E. De La Cruz-Burelo^{1D}, S. A. De La Motte^{1D}, G. de Marino^{1D}, G. De Nardo^{1D}, G. De Pietro^{1D}, R. de Sangro^{1D}, M. Destefanis^{1D}, S. Dey^{1D}, R. Dhamija^{1D}, A. Di Canto^{1D}, F. Di Capua^{1D}, J. Dingfelder^{1D}, Z. Doležal^{1D}, I. Domínguez Jiménez^{1D}, T. V. Dong^{1D}, X. Dong^{1D}, M. Dorigo^{1D}, D. Dossett^{1D}, S. Dubey^{1D}, K. Dugic^{1D}, G. Dujany^{1D}, P. Ecker^{1D}, D. Epifanov^{1D}, J. Eppelt^{1D}, P. Feichtinger^{1D}, T. Ferber^{1D}, T. Fillinger^{1D}, C. Finck^{1D}, G. Finocchiaro^{1D}, A. Fodor^{1D}, F. Forti^{1D}, A. Frey^{1D}, B. G. Fulsom^{1D}, A. Gabrielli^{1D}, E. Ganiev^{1D}, M. Garcia-Hernandez^{1D}, R. Garg^{1D}, G. Gaudino^{1D}, V. Gaur^{1D}, A. Gaz^{1D}, A. Gellrich^{1D}, G. Ghevondyan^{1D}, D. Ghosh^{1D}, G. Giakoustidis^{1D}, R. Giordano^{1D}, A. Giri^{1D}, P. Gironella Gironell^{1D}, A. Glazov^{1D}, B. Gobbo^{1D}, R. Godang^{1D}, O. Gogota^{1D}, P. Goldenzweig^{1D}, W. Gradl^{1D}, S. Granderath^{1D}, E. Graziani^{1D}, D. Greenwald^{1D}, Z. Gruberová^{1D}, Y. Guan^{1D}, K. Gudkova^{1D}, I. Haide^{1D}, S. Halder^{1D}, Y. Han^{1D}, T. Hara^{1D}, C. Harris^{1D}, K. Hayasaka^{1D}, H. Hayashii^{1D}, S. Hazra^{1D}, C. Hearty^{1D}, M. T. Hedges^{1D}, A. Heidelbach^{1D}, I. Heredia de la Cruz^{1D}, M. Hernández Villanueva^{1D}, T. Higuchi^{1D}, M. Hoek^{1D}, M. Hohmann^{1D}, R. Hoppe^{1D}, P. Horak^{1D}, C.-L. Hsu^{1D}, T. Humair^{1D}, T. Iijima^{1D}, K. Inami^{1D}, N. Ipsiita^{1D}, A. Ishikawa^{1D}, R. Itoh^{1D}, M. Iwasaki^{1D}, P. Jackson^{1D}, D. Jacobi^{1D}, W. W. Jacobs^{1D}, E.-J. Jang^{1D}, Q. P. Ji^{1D}, S. Jia^{1D}, Y. Jin^{1D}, A. Johnson^{1D}, K. K. Joo^{1D}, H. Junkerkalefeld^{1D}, D. Kalita^{1D}, A. B. Kaliyar^{1D}, J. Kandra^{1D}, K. H. Kang^{1D}, S. Kang^{1D}, G. Karyan^{1D}, T. Kawasaki^{1D}, F. Keil^{1D}, C. Ketter^{1D}, C. Kiesling^{1D}, C.-H. Kim^{1D}, D. Y. Kim^{1D}, K.-H. Kim^{1D}, J.-Y. Kim^{1D}, Y. J. Kim^{1D}, Y.-K. Kim^{1D}, K. Kinoshita^{1D}, P. Kodyš^{1D}, T. Koga^{1D}, S. Kohani^{1D}, K. Kojima^{1D}, A. Korobov^{1D}, S. Korpar^{1D}, E. Kovalenko^{1D}, R. Kowalewski^{1D}, P. Križan^{1D}, P. Krovovny^{1D}, T. Kuhr^{1D}, Y. Kulii^{1D}, D. Kumar^{1D}, M. Kumar^{1D}, R. Kumar^{1D}, K. Kumara^{1D}, T. Kunigo^{1D}, A. Kuzmin^{1D}, Y.-J. Kwon^{1D}, S. Lacaprara^{1D}, Y.-T. Lai^{1D}, K. Lalwani^{1D}, T. Lam^{1D}, L. Lanceri^{1D}, J. S. Lange^{1D}, T. S. Lau^{1D}, M. Laurens^{1D}, K. Lautenbach^{1D}, R. Leboucher^{1D}, F. R. Le Diberder^{1D}, M. J. Lee^{1D}, C. Lemettais^{1D}, P. Leo^{1D}, D. Levit^{1D}, P. M. Lewis^{1D}, C. Li^{1D}, L. K. Li^{1D}, Q. M. Li^{1D}, W. Z. Li^{1D}, Y. B. Li^{1D}, Y. P. Liao^{1D}, J. Libby^{1D}, J. Lin^{1D}, S. Lin^{1D}, M. H. Liu^{1D}, Q. Y. Liu^{1D}, Y. Liu^{1D}, Z. Q. Liu^{1D}, D. Liventsev^{1D}, S. Longo^{1D}, T. Lueck^{1D}, C. Lyu^{1D}, Y. Ma^{1D}, C. Madaan^{1D}, M. Maggiore^{1D}, S. P. Maharana^{1D}, R. Maiti^{1D}, S. Maity^{1D}, G. Mancinelli^{1D}, R. Manfredi^{1D}, E. Manoni^{1D}, M. Mantovano^{1D}, D. Marcantonio^{1D}, S. Marcello^{1D}, C. Marinas^{1D}, C. Martellini^{1D}, A. Martens^{1D}, A. Martini^{1D}, T. Martinov^{1D}, L. Massaccesi^{1D}, M. Masuda^{1D}, T. Matsuda^{1D}, D. Matvienko^{1D}, S. K. Maurya^{1D}, M. Maushart^{1D}, J. A. McKenna^{1D}, R. Mehta^{1D}, F. Meier^{1D}, D. Meleshko^{1D}, M. Merola^{1D}, C. Miller^{1D}, M. Mirra^{1D}, S. Mitra^{1D}, K. Miyabayashi^{1D}, H. Miyake^{1D}, R. Mizuk^{1D}, G. B. Mohanty^{1D}, S. Mondal^{1D}, S. Moneta^{1D}, H.-G. Moser^{1D}, R. Mussa^{1D}, I. Nakamura^{1D}, M. Nakao^{1D}, Y. Nakazawa^{1D}, M. Naruki^{1D}, Z. Natkaniec^{1D}, A. Natochii^{1D}, M. Nayak^{1D}, G. Nazaryan^{1D}, M. Neu^{1D}, C. Niebuhr^{1D}, S. Nishida^{1D}, S. Ogawa^{1D}, H. Ono^{1D}, Y. Onuki^{1D}, F. Otani^{1D}, P. Pakhlov^{1D}, G. Pakhlova^{1D}, E. Paoloni^{1D}, S. Pardi^{1D}, K. Parham^{1D}, H. Park^{1D}, J. Park^{1D}, K. Park^{1D}, S.-H. Park^{1D}, B. Paschen^{1D}, A. Passeri^{1D}, S. Patra^{1D}, T. K. Pedlar^{1D}, I. Peruzzi^{1D}, R. Peschke^{1D}, R. Pestotnik^{1D}, M. Piccolo^{1D}, L. E. Piilonen^{1D}, P. L. M. Podesta-Lerma^{1D}, T. Podobnik^{1D}, S. Pokharel^{1D}, C. Praz^{1D}, S. Prell^{1D}, E. Prencipe^{1D}, M. T. Prim^{1D}, I. Prudie^{1D}, H. Purwar^{1D}, P. Rados^{1D}, G. Raeuber^{1D}, S. Raiz^{1D}, V. Raj^{1D}, N. Rauls^{1D}, K. Ravindran^{1D}, M. Reif^{1D}, S. Reiter^{1D}, M. Remnev^{1D}, L. Reuter^{1D}, D. Ricalde Herrmann^{1D}, I. Ripp-Baudot^{1D}, G. Rizzo^{1D}, M. Roehrken^{1D}, J. M. Roney^{1D}, A. Rostomyan^{1D}, N. Rout^{1D}, D. A. Sanders^{1D}, S. Sandilya^{1D}, L. Santelj^{1D}, Y. Sato^{1D}, V. Savinov^{1D}, B. Scavino^{1D}, C. Schmitt^{1D}, S. Schneider^{1D}, M. Schnepf^{1D}, C. Schwanda^{1D}, A. J. Schwartz^{1D}, Y. Seino^{1D}, A. Selce^{1D}, K. Senyo^{1D}, J. Serrano^{1D}, M. E. Sevior^{1D}, C. Sfienti^{1D}, W. Shan^{1D}, C. Sharma^{1D}, C. P. Shen^{1D}, X. D. Shi^{1D}, T. Shillington^{1D}, T. Shimasaki^{1D}, J.-G. Shiu^{1D}, D. Shtol^{1D}, A. Sibidanov^{1D}, F. Simon^{1D}, J. B. Singh^{1D}, J. Skorup^{1D}, R. J. Sobie^{1D}, M. Sobotzik^{1D}, A. Soffer^{1D}, A. Sokolov^{1D}, E. Solovieva^{1D}, W. Song^{1D}, S. Spataro^{1D}, B. Spruck^{1D}, M. Starič^{1D}, P. Stavroulakis^{1D}, R. Stroili^{1D}, J. Strube^{1D}, Y. Sue^{1D}, M. Sumihama^{1D}, K. Sumisawa^{1D}, W. Sutcliffe^{1D}, N. Suwonjandee^{1D}, H. Svidras^{1D}, M. Takahashi^{1D}, M. Takizawa^{1D}, U. Tamponi^{1D}, K. Tanida^{1D}, F. Tenchini^{1D}, A. Thaller^{1D}, O. Tittel^{1D}, R. Tiwary^{1D}, D. Tonelli^{1D}, E. Torassa^{1D}, K. Trabelsi^{1D}, I. Tsaklidis^{1D}, M. Uchida^{1D}, I. Ueda^{1D}, K. Unger^{1D}, Y. Unno^{1D}, K. Uno^{1D}, S. Uno^{1D}, P. Urquijo^{1D}, J. Ur Rehman^{1D}, Y. Ushiroda^{1D}, S. E. Vahsen^{1D}, R. van Tonder^{1D}, K. E. Varvell^{1D}, M. Veronesi^{1D}, V. S. Vismaya^{1D}, L. Vitale^{1D}, V. Vobbillisetti^{1D}, R. Volpe^{1D}, A. Vossen^{1D}, M. Wakai^{1D}, S. Wallner^{1D}

E. Wang¹⁰, M.-Z. Wang¹⁰, Z. Wang¹⁰, A. Warburton¹⁰, M. Watanabe¹⁰, S. Watanuki¹⁰, C. Wessel¹⁰, E. Won¹⁰, X. P. Xu¹⁰, B. D. Yabsley¹⁰, S. Yamada¹⁰, W. Yan¹⁰, S. B. Yang¹⁰, J. Yelton¹⁰, J. H. Yin¹⁰, K. Yoshihara¹⁰, J. Yuan¹⁰, L. Zani¹⁰, F. Zeng¹⁰, B. Zhang¹⁰, V. Zhilich¹⁰, J. S. Zhou¹⁰, Q. D. Zhou¹⁰, and V. I. Zhukova¹⁰

(Belle II Collaboration)



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We measure the branching fraction and CP -violating flavor-dependent rate asymmetry of $B^0 \rightarrow \pi^0 \pi^0$ decays reconstructed using the Belle II detector in an electron-positron collision sample containing $387 \times 10^6 \Upsilon(4S)$ mesons. Using an optimized event selection, we find 125 ± 20 signal decays in a fit to background-discriminating and flavor-sensitive distributions. The resulting branching fraction is $(1.25 \pm 0.23) \times 10^{-6}$ and the CP -violating asymmetry is 0.03 ± 0.30 .

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The decay of the neutral bottom meson into a pair of neutral pions, $B^0 \rightarrow \pi^0 \pi^0$ [1], plays an important role in the study of the flavor-changing weak interactions of quarks. The decay properties can be used to test and refine phenomenological models of hadronic bottom-meson amplitudes and to provide important inputs for the determination of ϕ_2 , a relevant parameter in weak quark dynamics. Measurements of these properties are expected to significantly impact current constraints on potential processes not described by the standard model (SM).

Theoretical predictions for the branching fraction $\mathcal{B}(B^0 \rightarrow \pi^0 \pi^0)$ are challenging because the calculation of hadronic amplitudes involves low-energy, nonperturbative gluon exchanges. Currently available approximate methods often fail to reproduce data. Predictions based on QCD factorization [2–5] and perturbative QCD [6,7] are approximately 5 times smaller than the experimental results. In addition, the ratio of color-suppressed to color-allowed $b \rightarrow u$ amplitudes, as inferred from other charmless two-body decay modes, does not agree with expectations [8], possibly indicating anomalously large contributions [9,10]. An improved understanding of the $B^0 \rightarrow \pi^0 \pi^0$ decay amplitudes could be relevant to the so-called $B \rightarrow K\pi$ puzzle [11–13].

The study of CP -violating asymmetries in the rates of two-body decays involving the $b \rightarrow u$ transition, such as $B^0 \rightarrow \pi^0 \pi^0$, is also relevant. These asymmetries currently offer reliable and precise access to the least-well-known angle of the unitarity triangle, $\phi_2 \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$ [14], where V_{ij} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [15,16]. Improved measurements of ϕ_2 increase the power of tests of

CKM-matrix unitarity and impose more stringent bounds on possible SM extensions. Both $b \rightarrow u$ (W emission, or tree) and $b \rightarrow d$ (W exchange, or penguin) transitions contribute to the decay amplitude. The determination of ϕ_2 requires measurements of the branching fractions and CP asymmetries of all isospin-related $B \rightarrow \pi\pi$ decay modes, i.e., $B^0 \rightarrow \pi^+ \pi^-$, $B^0 \rightarrow \pi^0 \pi^0$, and $B^+ \rightarrow \pi^+ \pi^0$, or of analogous $B \rightarrow \rho\rho$ decays, to separate penguin and tree contributions [17,18].

Currently, the uncertainty in ϕ_2 is dominated by the uncertainties in the $B^0 \rightarrow \pi^0 \pi^0$ branching fraction and CP -violating flavor-dependent decay-rate asymmetry,

$$\mathcal{A}_{CP}(B^0 \rightarrow \pi^0 \pi^0) = \frac{\Gamma(\bar{B}^0 \rightarrow \pi^0 \pi^0) - \Gamma(B^0 \rightarrow \pi^0 \pi^0)}{\Gamma(\bar{B}^0 \rightarrow \pi^0 \pi^0) + \Gamma(B^0 \rightarrow \pi^0 \pi^0)}, \quad (1)$$

where Γ is the decay width. The world-average values $\mathcal{B}(B^0 \rightarrow \pi^0 \pi^0) = (1.55 \pm 0.17) \times 10^{-6}$ and $\mathcal{A}_{CP}(B^0 \rightarrow \pi^0 \pi^0) = 0.25 \pm 0.20$ [19] combine measurements reported by the *BABAR* [20] and *Belle* [21] Collaborations.

Improved determinations would bring the $B \rightarrow \pi\pi$ bounds on ϕ_2 closer in precision to those from $B \rightarrow \rho\rho$, thus improving knowledge of ϕ_2 . They may also help discriminate among the scenarios proposed to achieve a consistent picture of the relevant dynamics [22–25].

In this Letter, we present a measurement [26] of the branching fraction and CP asymmetry for the $B^0 \rightarrow \pi^0 \pi^0$ decay using a 365 fb^{-1} sample of electron-positron collision data. The sample contains $387 \times 10^6 \Upsilon(4S)$ mesons, that in turn decay almost exclusively to $B\bar{B}$ pairs, produced near threshold from 2019 through 2022. A 42.3 fb^{-1} sample collected at 60 MeV lower energy and hence not containing any $B\bar{B}$ pairs (off resonance) is also used for background modeling. The samples are collected with the Belle II detector [27], located at the SuperKEKB asymmetric-energy collider [28].

The principal experimental challenge is the reconstruction of a rare decay in a final state with no

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charged particles, $B^0 \rightarrow \pi^0(\rightarrow \gamma\gamma)\pi^0(\rightarrow \gamma\gamma)$. The reconstruction of photons is based solely on calorimeter information and therefore it is much less precise than the reconstruction of charged particles. In addition, it is affected by energy leakage and beam-induced backgrounds due to beam interactions with the beam pipe or residual gas. The analysis is developed using simulated samples and signal-free data control regions. The signal region in data is examined only when all procedures are established. Data are enriched in $B^0 \rightarrow \pi^0\pi^0$ events by means of an optimized selection based on two statistical-learning classifiers that suppress backgrounds, which are dominated by light-quark production. The flavor of the neutral B meson, needed for the measurement of the CP -violating asymmetry, is determined using information associated with the other, non-signal B meson produced in the $\Upsilon(4S)$ decay, since the $\pi^0\pi^0$ final state is common to B^0 and \bar{B}^0 . A multidimensional fit of sample composition determines the signal yield and CP -violating asymmetry. The fit results, combined with acceptance and efficiency corrections obtained from simulation and validated using control data, determine the quantities of interest. We use $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+\pi^0)\pi^+$ decays to validate photon reconstruction. In addition, we validate the analysis using $B^+ \rightarrow K^+\pi^0$ decays, which yield a final-state π^0 with kinematic properties similar to the signal π^0 mesons, and $B^0 \rightarrow \bar{D}^0(\rightarrow K^+\pi^-\pi^0)\pi^0$ decays, which, like the signal decay, yield two π^0 mesons. These B decay modes are 10 times more abundant than our expected signal.

Belle II consists of several subdetectors arranged in a cylindrical structure around the beam pipe [27]. The z axis is the symmetry axis of a superconducting solenoid, which generates a 1.5 T uniform field along the beam direction. The positive z direction corresponds approximately to the electron-beam direction and defines the origin of the polar angle θ . The detector structure is divided into three polar regions in increasing order of θ , the forward end cap, barrel, and backward end cap, which correspond to the polar-angle ranges $[12.4, 31.4]^\circ$, $[32.2, 128.7]^\circ$, and $[130.7, 155.1]^\circ$, respectively. The inner volume contains a two-layer silicon pixel detector surrounded by a four-layer double-sided silicon strip detector and a drift chamber. Of the outer pixel layer, only a 15% azimuthal sector was installed for the data used in this work. These subdetectors are primarily used to reconstruct charged-particle trajectories, and therefore their origin, momentum, and electric charge. These subdetectors also provide charged-particle identification through measurement of specific ionization. A time-of-propagation counter and an aerogel ring-imaging Cherenkov detector covering the barrel and forward end cap regions, respectively, are used for primary charged-particle identification. In this work, the electromagnetic calorimeter is particularly important. It is a segmented array of 8736 thallium-doped cesium iodide crystals arranged in a quasiprojective geometry toward the interaction point. They occupy the remaining volume inside the superconducting solenoid and cover

about 90% of the solid angle in the center-of-mass frame. The calorimeter identifies electrons and photons in the range of energies 0.02–10 GeV with fractional resolutions of 7.7% at 100 MeV or 2.2% at 1 GeV [29]. Resistive-plate chambers and scintillating bars are installed in the flux return of the magnet and identify muons and K_L^0 mesons. This detector, along with the calorimeter, also contributes charged-particle-identification information.

We use simulated samples to optimize event selection, select fit models, calculate signal efficiencies, and study sources of background. To study signal, we use $2 \times 10^6 \Upsilon(4S) \rightarrow B^0\bar{B}^0$ decays in which one B meson decays into the $\pi^0\pi^0$ final state and the other B -meson decay is unbiased. These samples are generated with the EVTGEN [30] and PYTHIA [31] software packages. To study backgrounds, we use simulated samples at least four times larger than the data sample. These samples consist of $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ processes generated with EVTGEN and PYTHIA along with $e^+e^- \rightarrow \tau^+\tau^-$ and continuum $e^+e^- \rightarrow q\bar{q}$ background, where q denotes a u , d , s , or c quark, generated with the KKMC [32], PYTHIA, and TAUOLA [33] software packages. Beam-induced backgrounds sampled from data are included in the simulation [34]. The detector response is simulated using the GEANT4 [35] software package. Simulated and experimental data are processed with the Belle II software [36,37].

The online event selection of our sample requires events to satisfy criteria based on total energy and neutral-particle multiplicity to preferentially retain hadronic events and has full efficiency for the signal decay mode.

In the offline analysis, we identify photon candidates as calorimeter-energy deposits (clusters) larger than 30 MeV that involve more than one crystal to reject calorimeter noise. Since CsI(Tl) scintillation light has a relatively long decay time, random photons from hadronic events combined with overlapping residual energy not related to the relevant collision may be misreconstructed as π^0 candidates. Hence, the time associated with cluster formation is required to be within 200 ns of the collision time. Multiplicative photon-energy corrections ranging between 0.990 and 1.010, with 0.15%–0.50% uncertainties, are derived from control samples in data and used to correct for calorimeter-energy miscalibration.

Selected photons are paired to form π^0 candidates, which are further selected to suppress combinatorial background from low-energy photons. We require the π^0 momentum in the laboratory frame be greater than 1.5 GeV/c and that the three-dimensional angle between the momenta of final-state photons in the laboratory frame be less than 0.4 rad. In the π^0 rest frame, the absolute value of the cosine of the angle between the photon direction and the boost direction from the laboratory frame is required to be less than 0.98 as misreconstructed π^0 mesons tend to peak near 1.00. The diphoton mass is required to be between 0.115 and 0.150 GeV/c^2 , a range of approximately -2.5 and $+2.0$

units of resolution around the known π^0 mass [19]. The range is asymmetric to compensate for energy leakage from the calorimeter. The high momentum of signal π^0 's offers a natural suppression of misreconstructed and non-signal photons. To reduce contributions from these sources further, we use a boosted decision tree [38] called “Photon-BDT.” This decision tree is trained on simulated signal photons against beam-background and misreconstructed photons using nine input observables. In approximate order of decreasing discriminating power, they are the energy detected in the crystal having the highest signal, three observables that describe the energy sharing among crystals [26], the photon's momentum transverse to the beam direction, the distance between the cluster and the trajectory of the nearest charged particle, the number of crystals in the cluster, the cluster polar direction, and one observable that describes the fraction of cluster energy detected in the central crystal. We choose the threshold on the Photon-BDT output that maximizes the yield of signal photons over the square root of the sum of misreconstructed- and beam-induced photon yields as expected from simulation. In simulated samples, this selection removes 83% of misreconstructed and beam-induced photons and retains 96% of signal photons. Studies of $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+\pi^0)\pi^+$ and $B^+ \rightarrow K^+\pi^0$ decays validate the Photon-BDT classifier's performance by showing that signal efficiency and Photon-BDT output distributions are consistent between simulations and data.

We improve the momentum resolution of the π^0 candidates by performing a kinematic fit that constrains the diphoton mass to the known π^0 mass [19]. Signal candidates are reconstructed by combining two π^0 candidates.

We use topological observables that exploit the jetlike nature of $q\bar{q}$ events and the isotropic distribution of final-state particles from $B\bar{B}$ events to reduce the large continuum background, along with observables specifically sensitive to the presence of the nonsignal B meson in the event. We train a boosted-decision-tree classifier C to discriminate signal from continuum background by analyzing 29 observables comprising, in approximate order of decreasing discriminating power, modified Fox-Wolfram moments [39], sphericity-related quantities [40], thrust-related quantities [41], and energy detected in sets of concentric cones with various opening angles centered around the thrust axis [42]. Observables showing correlations larger than 10% with those used in the sample-composition fit are excluded. The training uses simulated signal samples and off-resonance data. An optimization identifies the selection on C that minimizes the average variance on the branching fraction and CP asymmetry as expected from repeating the measurement on simplified simulated samples. The resulting selection on C rejects 98% of continuum background while retaining 64% of signal. For convenient modeling, we apply the properties of

the probability integral transform [43] to the classifier obtaining C_t , which has a Gaussian-like shape that peaks at 2.0 for signal and at 0.0 for background [44]. In addition, we restrict the sample using two kinematic variables that exploit four-momentum conservation in near-threshold $B\bar{B}$ pair production and are particularly convenient to isolate signal candidates, $M_{bc} = \sqrt{E_{beam}^2/c^4 - |\vec{p}_B|^2/c^2}$ and $\Delta E = E_B - E_{beam}$, where E_{beam} is the beam energy and (E_B, \vec{p}_B) is the four-momentum of the B candidate, both calculated in the $\Upsilon(4S)$ rest frame. The M_{bc} and ΔE distributions for signal decays peak at the B mass and close to zero, respectively. Candidate B mesons are required to have $M_{bc} > 5.2 \text{ GeV}/c^2$ and $-0.3 < \Delta E < 0.5 \text{ GeV}$.

Multiple signal candidates (typically just two) are reconstructed in 0.3% of events, in which case we choose one candidate at random. Following all selections, 27% of signal events remain, of which 99% are correctly reconstructed. Such a low misreconstruction rate is due to the low fraction of signal events in which the accompanying B meson yields a high-momentum π^0 candidate.

To measure the CP asymmetry, the B -signal flavor is assigned in each event by determining statistically the flavor of the accompanying neutral B meson (tagging). Tagging information is encoded in the $q = +1$ or -1 (for \bar{b} or b) flavor of the nonsignal B and in $r = 1-2w$, where w is the probability for wrongly tagging an event. The probability is calibrated on control data by measuring $B^0\bar{B}^0$ flavor oscillations with $B^0 \rightarrow D^{(*)-}\pi^+$ decays. We use a novel algorithm [45], which efficiently uses information from nonsignal charged particles and from their relationships. The algorithm improves the average value of r^2 , the statistical tagging efficacy, by 18% with respect to the algorithm used in the previous Belle II measurement [46]. Unlike in previous analyses where data were analyzed separately in independent contiguous qr intervals (binned flavor tagging), here the unbinned distribution of w_t , obtained using the properties of the probability integral transform of w , is used as a fit observable. It has a Gaussian-like shape that simplifies the fit model.

The resulting event sample consists of three components, i.e., signal (s), continuum (c), and background from non-signal B decays ($B\bar{B}$). The 1% fraction of genuine signal events that are misreconstructed is classified as signal. Simulated samples show that 81% of $B\bar{B}$ background is from $B^+ \rightarrow \rho^+\pi^0$ decays. We determine the branching fraction and CP asymmetry with a likelihood fit of the unbinned distributions of M_{bc} , ΔE , C_t , and w_t . The signal distribution peaks in each fit observable. The continuum background produces smooth distributions in M_{bc} and ΔE , and peaks at zero (positive) values in C_t (w_t). The $B\bar{B}$ background peaks at similar values of M_{bc} and C_t as the signal, but its ΔE distribution is shifted to negative values due to missing energy associated with the unreconstructed decay products.

The likelihood function is constructed as

$$\begin{aligned} \mathcal{L} \propto \prod_{i=1}^N & \left[f_s(1+q^i(1+q^i a_{\text{tag}})(1-2\chi_d)(1-2k(q^i)w^i)\mathcal{A}_{CP} \right. \\ & \times p_s(\Delta E^i, M_{bc}^i) p_s(C_t^i, w_t^i) \\ & + f_{B\bar{B}}(1+q^i(1+q^i a_{\text{tag}})(1-2k(q^i)w^i)\mathcal{A}_{B\bar{B}}) \\ & \times p_{B\bar{B}}(\Delta E^i) p_{B\bar{B}}(M_{bc}^i) p_{B\bar{B}}(C_t^i, w_t^i) \\ & + (1-f_s-f_{B\bar{B}})(1+q^i(1-2w^i)\mathcal{A}_c) \\ & \left. \times p_c(\Delta E^i, C_t^i) p_c(M_{bc}^i) p_c(w_t^i) \right], \end{aligned} \quad (2)$$

where i is the candidate, N is the known total number of candidates in the sample, f_j is the sample fraction for component j , and $p_j(\Delta E^i, M_{bc}^i, C_t^i, w_t^i, q^i)$ is the probability density function (PDF) for the i th candidate to belong to the j th component. Here q^i is the predicted flavor of the partner B meson of the i th candidate; w^i is the probability for incorrectly identifying the candidate flavor, which dilutes the true asymmetry; $k(q^i)$ is a multiplicative correction for the flavor-specific difference between predicted and observed w^i ; and a_{tag} is the B^0 – \bar{B}^0 asymmetry in tagging efficiency. The factorization of the PDFs in Eq. (2) accounts for observed dependences between observables. The values of $k(q^i)$ and a_{tag} are Gaussian-constrained from a flavor-oscillation fit to $B^0 \rightarrow D^{(*)-}h^+$ decays in simulation and data, where h^+ stands for π^+ or K^+ [45]. These and all other Gaussian constraints included in our model are omitted from the above likelihood expression to simplify notation. The CP asymmetry in data is further diluted by a factor of $(1-2\chi_d)$ due to B^0 – \bar{B}^0 oscillations. The known time-integrated B^0 – \bar{B}^0 -oscillation probability $\chi_d = 0.1858 \pm 0.0011$ [19] is Gaussian-constrained in the fit. The effective asymmetry of the $B\bar{B}$ background is constrained to the value $\mathcal{A}_{B\bar{B}} = 0.056 \pm 0.095$ as determined in the $\Delta E < -0.3$ GeV and $M_{bc} > 5.26$ GeV/ c^2 sideband. The effective asymmetry of the continuum background \mathcal{A}_c is freely determined by the fit.

Models for all sample components are chosen based on large samples of simulated events. The signal M_{bc} distribution is modeled using two Gaussian functions. The signal ΔE distribution is modeled, in intervals of M_{bc} , using the sum of a Gaussian function and a bifurcated Gaussian function. All functions have independent means and widths. The $B\bar{B}$ M_{bc} distribution is described with the sum of a Gaussian function and a Johnson function [47]. The continuum M_{bc} distribution is modeled with the sum of two ARGUS functions [48] as the beam energy, which determines the upper end point of the distribution, varies during data taking. The $B\bar{B}$ ΔE distribution is described with the sum of three Gaussian functions and a bifurcated Gaussian function, while continuum is described with a

straight line, with independent slopes in three intervals of w_t . The C_t distribution of continuum is modeled using the sum of a Gaussian function and a bifurcated Gaussian function. The C_t distributions of signal and $B\bar{B}$ are each modeled using the same functional form in three intervals of w_t , with independent parameters. The signal, $B\bar{B}$, and continuum w_t distributions are each modeled using the sum of three Gaussian functions with independent parameters. Signal and $B\bar{B}$ model parameters are determined using fits to simulated events. Additional degrees of freedom related to the position and width of the peaking structures are included in the signal-data fit as Gaussian constraints determined from fits to $B^+ \rightarrow K^+\pi^0$ decays. These account for residual data-simulation discrepancies such as the 12 ± 4 MeV difference in ΔE peak position. Continuum model parameters are freely determined by the fit.

We validate the analysis by applying it to $B^+ \rightarrow K^+\pi^0$ and $B^0 \rightarrow \bar{D}^0(\rightarrow K^+\pi^-\pi^0)\pi^0$ decays. The photon and π^0 criteria applied in the $B^+ \rightarrow K^+\pi^0$ selection are the same as for the $B^0 \rightarrow \pi^0\pi^0$ analysis. The kaon candidate is a charged particle that satisfies a loose requirement on the ratio $\mathcal{L}_K/(\mathcal{L}_\pi + \mathcal{L}_K)$, where the likelihood $\mathcal{L}_{\pi,K}$ for a pion or kaon hypothesis combines particle-identification information from all subdetectors except the pixel detector. For the $B^0 \rightarrow \bar{D}^0(\rightarrow K^+\pi^-\pi^0)\pi^0$ control channel, all photon and π^0 selections are the same as for signal except the 1.5 GeV/ c threshold on π^0 momentum, which is removed to accommodate the significantly lower momentum spectrum, and an additional $1.84 < m(K^+\pi^-\pi^0) < 1.88$ GeV/ c^2 restriction on the D mass. Figure 1 shows signal-enhanced data distributions with fit projections overlaid for the $B^+ \rightarrow K^+\pi^0$ channel. The signal-enhancing selection is defined as $5.275 < M_{bc} < 5.285$ GeV/ c^2 , $-0.10 < \Delta E < 0.05$ GeV, and $C_t > 0$ and applied to all relevant variables except the one displayed. We determine $\mathcal{B}(B^+ \rightarrow K^+\pi^0) = (14.3 \pm 0.5) \times 10^{-6}$, $\mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0) = 0.078 \pm 0.076$, $\mathcal{B}(B^0 \rightarrow \bar{D}^0(\rightarrow K^+\pi^-\pi^0)\pi^0) = (41.4 \pm 2.4) \times 10^{-6}$, and $\mathcal{A}_{CP}(B^0 \rightarrow \bar{D}^0\pi^0) = 0.01 \pm 0.10$, which all agree with known values [19] within our statistical-only uncertainties.

We maximize the likelihood on the sample of 7140 $B^0 \rightarrow \pi^0\pi^0$ candidates. The branching fraction, CP asymmetry, $B\bar{B}$ yield, and continuum yield are freely determined by the fit. Figure 2 shows signal-enhanced data distributions with fit projections overlaid. The signal-enhancing selection is the same as for the control modes and rejects approximately 91%–98% of continuum. A signal is observed in all distributions, overlapping continuum and $B\bar{B}$ background. The branching fraction is calculated using

$$\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = \frac{Nf_s}{2\epsilon f^{00}N_{Y(4S)}\mathcal{B}(\pi^0 \rightarrow \gamma\gamma)^2}, \quad (3)$$

where $Nf_s = 125 \pm 20$ is the observed signal yield, $\epsilon = (27.28 \pm 0.03)\%$ is the signal reconstruction and selection

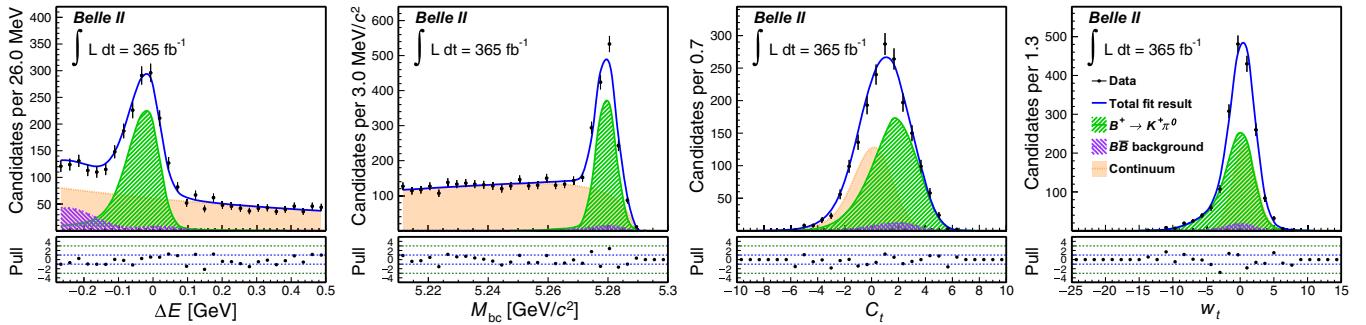


FIG. 1. Signal-enhanced (see text) distributions of (left to right) ΔE , M_{bc} , C_t , and w_t for $B^+ \rightarrow K^+ \pi^0$ control-sample candidates reconstructed in data, with fit projections overlaid. Lower panels show differences between observed and best-fit values divided by the fit uncertainties (pulls).

efficiency from simulation, $N_{\Upsilon(4S)} = (387 \pm 6) \times 10^6$ is the number of $\Upsilon(4S)$ mesons in the sample, $f^{00} = 0.486 \pm 0.008$ is the fraction of $\Upsilon(4S)$ mesons that decay into $B^0 \bar{B}^0$ pairs [49], and $\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = (98.82 \pm 0.03)\%$ [19] is the relevant π^0 branching fraction. The Poisson fluctuation on N is included in the branching-fraction statistical uncertainty to account for fluctuations of the total sample size. We obtain $\mathcal{B}(B^0 \rightarrow \pi^0 \pi^0) = (1.25 \pm 0.20) \times 10^{-6}$ and $\mathcal{A}_{CP}(B^0 \rightarrow \pi^0 \pi^0) = 0.03 \pm 0.30$ where uncertainties are statistical.

We consider sources of systematic uncertainties associated with assumptions made in the analysis, with possible biases due to discrepancies between relevant distributions in data and simulation, or with intrinsic uncertainties of external inputs (Table I). Whenever a systematic source is associated with a modeling choice in the analysis,

we determine its impact by using ensembles of simplified simulated experiments. To account for possible data-simulation discrepancies, we use control samples reconstructed in data and in simulation to estimate correction factors and assess their associated uncertainties, which are propagated in the results as systematic uncertainties. We also propagate uncertainties on the external inputs to the quantities of interest.

A systematic uncertainty of 8.1% associated with the π^0 pair reconstruction efficiency is determined from data using the decays $D^{*-} \rightarrow \bar{D}^0(\rightarrow K^+ \pi^- \pi^0) \pi^-$ and $D^{*-} \rightarrow \bar{D}^0(\rightarrow K^+ \pi^-) \pi^-$. The correction factor that matches the efficiency of the continuum-classifier selection in simulation with data is determined using $B^+ \rightarrow \bar{D}^0(\rightarrow K^+ \pi^-) \pi^+$ decays. The uncertainty on the correction factor is assigned as a systematic uncertainty. The systematic

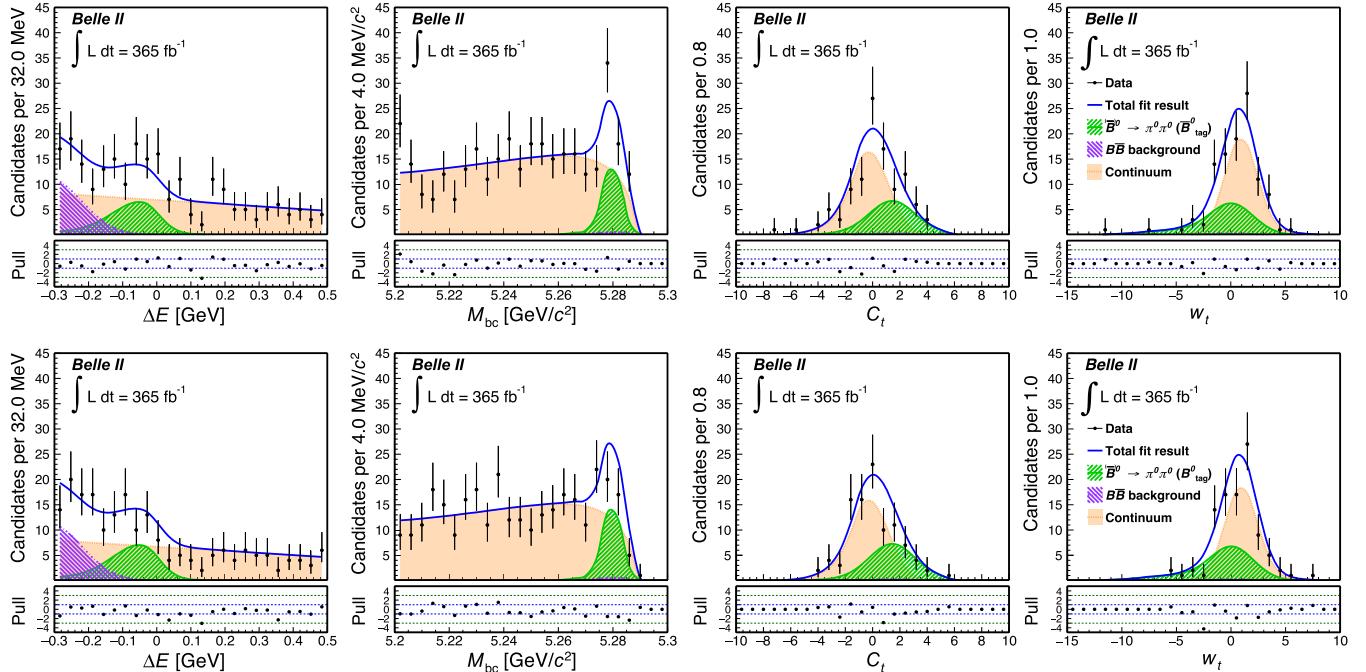


FIG. 2. Signal-enhanced distributions of (left to right) ΔE , M_{bc} , C_t , and w_t for $B^0 \rightarrow \pi^0 \pi^0$ signal candidates with (top) positive and (bottom) negative q tags reconstructed in data, with fit projections overlaid. Lower panels show pulls.

TABLE I. Fractional systematic uncertainties on the branching fraction and absolute systematic uncertainties on the CP asymmetry. Total systematic uncertainties, resulting from their sums in quadrature, are also given and compared with statistical uncertainties.

Source	\mathcal{B} (%)	\mathcal{A}_{CP}
π^0 efficiency	8.1	
Continuum-suppression efficiency	1.9	
$B\bar{B}$ -background model	1.7	0.01
Signal model	1.2	0.02
Continuum-background model	0.9	0.03
$\Upsilon(4S)$ branching fraction f^{00}	1.6	
Sample size $N_{\Upsilon(4S)}$	1.5	
$B^0\bar{B}^0$ -oscillation probability		<0.01
Wrong-tag probability calibration		0.01
Total systematic uncertainty	8.9	0.04
Statistical uncertainty	15.9	0.30

uncertainty associated with modeling choices, and with the uncertainties on the associated PDF parameters, is the maximum difference observed between averages of results obtained from fitting our model to simulated data generated with alternative functions and the average obtained with the nominal parameter and model choices or with parameters varied according to their covariances. The continuum component has no contribution from alternate parameter choices, as its parameters are directly determined in the fit. The systematic uncertainty on f^{00} accounts for the experimental uncertainty on the measurement and the uncertainty due to the assumption of isospin symmetry. The uncertainty in the number of $N_{\Upsilon(4S)}$ events is due to the uncertainty in the integrated-luminosity determination, the small amount of off-resonance data, and efficiency mismodeling in simulation. The uncertainty on the time-integrated $B^0\bar{B}^0$ -oscillation probability χ_d is assessed by repeating the fit after fixing the parameter to its best-fit value and subtracting in quadrature the uncertainties on the parameters of interest. The systematic uncertainty associated with the choice of calibration function used to obtain the factors $k(q)$ is determined by fitting simulated data generated with an alternative function, which includes additional corrections proportional to w^2 . The total systematic uncertainty is the sum in quadrature of the individual contributions.

The final results are

$$\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = (1.25 \pm 0.20 \pm 0.11) \times 10^{-6} \quad (4)$$

and

$$\mathcal{A}_{CP}(B^0 \rightarrow \pi^0\pi^0) = 0.03 \pm 0.30 \pm 0.04, \quad (5)$$

where the first contributions to the uncertainties are statistical and the second systematic. The (statistical) linear

correlation between these two quantities is +1.5%. This work supersedes a previous Belle II result based on about one-half of the data sample [44] and incorporates a number of enhancements. We improve background suppression by increasing the discriminating power of classifiers; we simplify the sample-composition fit by including as an observable the predicted signal flavor obtained by new algorithms with higher efficacy; we include in the fit additional control data to constrain backgrounds directly from data, thus reducing systematic uncertainties. These improvements reduce the fractional systematic uncertainty on the branching fraction by 40% and both the absolute statistical and systematic uncertainty on the CP asymmetry by 3%, for a given sample size. Combining these improvements with the increased sample size produces results competitive with the current best values, which are based on larger samples. We average our results with previous measurements of $B^0 \rightarrow \pi^0\pi^0$ branching fraction and CP asymmetry apart from the previous Belle II results [44] and include them, along with recent $B^0 \rightarrow \pi^+\pi^-$ and $B^+ \rightarrow \pi^+\pi^0$ inputs [19,50], in an isospin analysis that follows Ref. [17] to assess impact on $B \rightarrow \pi\pi$ -based ϕ_2 constraints. Our results reduce by 10° the 68% CL exclusion interval surrounding the CKM-favored solution, corresponding to a 30% fractional increase in ϕ_2 precision (see the Appendix). This makes the precision of ϕ_2 determinations based on $B \rightarrow \pi\pi$ decays competitive with the precision of $B \rightarrow \rho\rho$ determinations, resulting in a global improvement on the ϕ_2 knowledge.

In summary, we report an improved Belle II measurement of the branching fraction and direct CP asymmetry of $B^0 \rightarrow \pi^0\pi^0$ decays reconstructed in the full electron-positron collision sample at the $\Upsilon(4S)$ collected through 2022. The results are $\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = (1.25 \pm 0.23) \times 10^{-6}$ and $\mathcal{A}_{CP}(B^0 \rightarrow \pi^0\pi^0) = 0.03 \pm 0.30$. These measurements achieve a precision superior to, or comparable with, the precision of previous measurements, based on larger samples, and advance our knowledge of two-body charmless B decays and of the angle ϕ_2 .

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

Appendix: Impact on ϕ_2 determination. We assess the impact of the results in terms of changes in the constraints on the CKM angle ϕ_2 determined solely from $B \rightarrow \pi\pi$ isospin relations. We combine the averaged branching fractions of $B^0 \rightarrow \pi^+\pi^-$, $B^+ \rightarrow \pi^+\pi^0$, and $B^0 \rightarrow \pi^0\pi^0$ decays, together with the lifetimes of the B^+ and B^0 mesons, and the observables $S_{\pi^+\pi^-}$, $C_{\pi^+\pi^-}$, and $C_{\pi^0\pi^0}$, following Ref. [17]. Figure 3 compares the p-value as a function of ϕ_2 resulting from the combination of all $B \rightarrow \pi\pi$ results available until 2023 [51] with the 2024 combination in which the existing Belle II $B^0 \rightarrow \pi^0\pi^0$ measurements [44] are replaced with the results of this work.

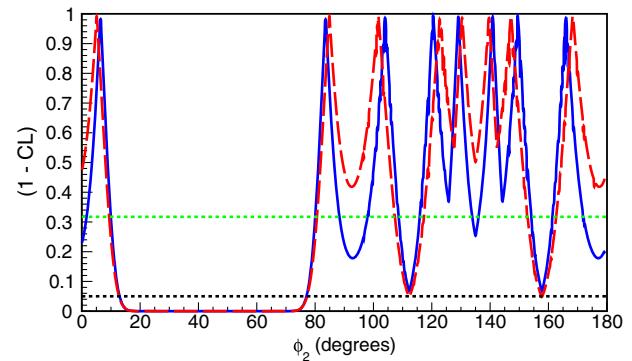


FIG. 3. P-value as a function of the CKM angle ϕ_2 from an isospin-based combination of all $B \rightarrow \pi\pi$ results (red dashed) without and (blue solid) with the inclusion of the results of this work.

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