

1 Search for the decay $B^+ \rightarrow K^+ \tau^+ \tau^-$ using data from the Belle and Belle II experiments

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We report a search for the rare decay $B^+ \rightarrow K^+ \tau^+ \tau^-$ using 1.2×10^9 $\Upsilon(4S)$ mesons produced near threshold in electron-positron collisions and collected by the Belle and Belle II experiments. We fully reconstruct the hadronic decay of one B meson produced in the $\Upsilon(4S) \rightarrow B^+ B^-$ decay, and search for $B^\pm \rightarrow K^\pm \tau^+ \tau^-$ candidates among the remaining collision products, reconstructing a charged kaon and leptonic decays of the τ leptons. We optimize the selection for best sensitivity and look for an excess over background at low values of the residual energy detected in the calorimeter after full event reconstruction. We observe no significant excess and set the limit $\mathcal{B}(B^+ \rightarrow K^+ \tau^+ \tau^-) < 0.56 \times 10^{-3}$ at the 90% confidence level, improving on the only previous result by a factor of four.

Identifying the particles and interactions that extend the standard model (SM) is currently a principal objective of particle physics. Weak-interaction processes that change quark flavor without changing the electric charge, known as flavor-changing neutral currents, are promising probes for non-SM physics. In the SM, the amplitudes for these processes are suppressed, because they cannot occur via single $W^\pm$ emission and require a loop-mediated $W^\pm$ exchange. However, virtual non-SM particles of high mass can replace the exchanged SM particles in those loop amplitudes. This could alter the process rates, indicating non-SM physics. Among flavor-changing neutral current processes, the $B^+ \rightarrow K^+ \tau^+ \tau^-$ decay is particularly interesting. The SM branching fraction, subtracting $B^+ \rightarrow K^+[c\bar{c} \rightarrow] \tau^+ \tau^-$ contributions, is expected to be $(1.0\text{--}2.0) \times 10^{-7}$ [1–3]. However, non-SM contributions proposed to accommodate the observed excesses in $B^{+,0} \rightarrow \bar{D}^{0,-(*)} \tau^+ \nu_\tau$ [4] and $B^+ \rightarrow K^+ \nu \bar{\nu}$ [5] decay rates could enhance it by a factor of up to 10^3 [6–10], bringing it near the current experimental sensitivity.

The only search for $B^+ \rightarrow K^+ \tau^+ \tau^-$ decays to date was reported by the BABAR experiment, which set a 90%-confidence-level (CL) upper limit on the branching fraction at 2.25×10^{-3} [11]. Null searches for related processes have also been reported by the Belle, Belle II, and LHCb experiments [12–15]. All the resulting bounds exceed the SM predictions by four three to four orders of magnitude, leaving opportunities for further experimental exploration.

This Letter reports a search for the rare decay $B^+ \rightarrow K^+ \tau^+ \tau^-$ [16, 17]. Charge-conjugate processes are implied throughout. The search uses 1.2×10^9 $\Upsilon(4S)$ mesons produced near threshold in electron-positron collisions and collected by the Belle and Belle II experiments. The search uses 772 $\times 10^6$ $\Upsilon(4S)$ meson decays produced near threshold in electron-positron collisions, and reconstructed from the full 711 fb^{-1} data set collected by the Belle experiment, and 387 $\times 10^6$ $\Upsilon(4S)$ decays from the 365 fb^{-1} data set collected by the Belle II experiment between 2019 and 2023. The collisions are produced by the KEKB and SuperKEKB asymmetric-energy colliders at KEK near the center-of-mass (c.m.) energy 10.58 GeV, which corresponds to the peak of the $\Upsilon(4S)$ resonance.

The principal challenge in this analysis arises from the presence of multiple final-state neutrinos from the τ -lepton decays. The neutrinos are not reconstructed, precluding the availability of distinctive signal kinematic distributions needed to suppress 10^8 -times larger backgrounds. The analysis therefore exploits the advantages of near-threshold $B\bar{B}$ pair production to identify signal. We infer the signal four-momentum and suppress background by fully reconstructing a hadronic decay of the partner B^- meson, pair-produced with the signal B^+ meson in the $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^+ B^-$ process. We reconstruct signal candidates in events containing a kaon candidate with charge opposite to the partner- B charge and two opposite-charge lepton candidates. Using only decays $\tau^+ \rightarrow \ell^+ \nu_\ell \bar{\nu}_\tau$ of the signal τ leptons, where ℓ denotes a muon or an electron, further reduces background and simplifies the analysis without degrading sensitivity. A restriction to events with opposite-charge kaon-lepton mass larger than the charmed-meson mass suppresses large backgrounds dominated by $b \rightarrow c(\rightarrow s\ell^{(\prime)}\nu)\ell\bar{\nu}$ quark-level processes. Near-threshold $B^+ B^-$ production implies also that no additional particles, and therefore no additional energy in the calorimeter, are expected after $B^+ B^-$ reconstruction. Hence, we determine the signal yield by counting the excess events, over the expected background yield, at low values of the calorimeter en-

ergy detected after subtraction of the observed energies of both B meson candidates. The analysis is executed using simulated and control-data samples before examining the data in the signal-search region. The search is conducted independently in the Belle and Belle II data and the results are combined.

The Belle detector [18] collected data from collisions of 8-GeV electrons with 3.5-GeV positrons produced by the KEKB collider [19]. An upgraded detector, Belle II [20], studies collisions of 7-GeV electrons on 4-GeV positrons produced by the SuperKEKB collider [21]. Each detector is a nearly 4π hermetic solenoidal magnetic spectrometer comprising inner silicon-vertex subdetectors and an outer drift chamber, surrounded by Cherenkov-based charged-particle identification subdetectors, a crystal electromagnetic calorimeter, and outer subdetectors for penetration-based detection of muons and K_L^0 mesons.

Detailed Monte Carlo simulation is used to design the analysis, decide selection criteria and determine their efficiencies, study backgrounds, and assess the analysis sensitivity. Simplified simulations are used to study estimator properties. The software packages EVTGEN [22], PYTHIA6 [23] (PYTHIA8) [24], KKMC [25], and TAUOLA [26] simulate particle production and decay in Belle (Belle II); PHOTOS [27] simulates photon radiation from final-state charged particles, and GEANT3 [28] (GEANT4 [29]) simulates interactions with the Belle (Belle II) detector and its response. We generate 5×10^7 $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^+(\rightarrow K^+\tau^+\tau^-)B^-$ signal events for each experiment. The decay of the partner bottom meson is unbiased. The signal decay follows the dynamical model of Ref. [30]. Simulated background samples are four times larger than the collision-data sample. They consist of B^+B^- events, $B^0\bar{B}^0$ events, and continuum $e^+e^- \rightarrow q\bar{q}$ events, where q stands for a u , d , s , or c quark. None of the simulated $e^+e^- \rightarrow \tau^+\tau^-$ events passes the selection. We use the Belle II software [31, 32] to reconstruct simulated and collision events in both Belle [33] and Belle II data.

A trigger selects events online based on total energy and neutral-particle multiplicity to preferentially retain collisions that yield hadrons, with full efficiency for signal events.

In the offline analysis, we fully reconstruct the hadronic decay of the candidate partner B meson using the full-event interpretation algorithm (FEI) [34]. The FEI selects final-state particles originating from the interaction space-point through standard quality criteria and combines them bottom-up according to known hadronic decay chains to achieve full B meson reconstruction. Backgrounds are suppressed by means of two kinematic observables that exploit four-momentum conservation in near-threshold $B\bar{B}$ pair production, $M_{bc} = \sqrt{(E_{\text{beam}}^*)^2/c^4 - |\vec{p}_B^*|^2/c^2}$ and $\Delta E = E_B^* - E_{\text{beam}}^*$, where E_{beam}^* is the beam energy and $(E_B^*/c, \vec{p}_B^*)$ is the four-momentum of the B candidate, both calculated in the

c.m. frame. The M_{bc} and ΔE distributions for correctly reconstructed B decays peak at the B mass and close to zero, respectively. Hence, partner B mesons are required to have $M_{bc} > 5.27$ GeV/ c^2 and $-0.15 < \Delta E < 0.10$ GeV. Furthermore, we require that only three charged particles are reconstructed in addition to the partner B and impose a loose restriction on the FEI-reconstruction quality. We use topological observables that exploit the differences between the jet-like distribution of continuum products and the isotropic distribution of $B\bar{B}$ decay products to reduce the large continuum background [35]: the spatial distributions of particle momenta must be isotropic [36] and the thrust direction of the partner B not opposite to the thrust direction of all other particles in the event. Signal-candidate selections reduce continuum background by 68% (65%) in Belle (Belle II) data while reducing signal efficiency by 15% (13%). If multiple partner B candidates are reconstructed in an event, the candidate with highest FEI quality is chosen. Partner B reconstruction retains 0.8% (0.5%) of Belle (Belle II) simulated signal events and the partner B decay is correctly reconstructed in 28.7% (43.9%) of the simulated signal events.

We use the three charged particles not associated with the partner B to reconstruct signal B candidates. We select only charged-particle trajectories (tracks) loosely originating from the interaction space-point to suppress spurious tracks from beam-induced background. Furthermore, we require tracks to be reconstructed within the polar acceptance of the drift chambers and to either be associated with a sufficient number of chamber measurement points, in Belle II data, or to correspond to particles with momentum $p_T > 200$ MeV/ c transverse to the beam direction, in Belle data. This ensures reliable measurements of specific-ionization energy losses for charged-particle identification (PID). Kaons, pions, muons, and electrons are identified using PID information from several subdetectors combined into likelihood-ratio-like observables. Muon identification is performed using only information from the muon subdetector for Belle data, and using all subdetectors for Belle II data. This implies that Belle muon candidates have transverse momenta larger than 0.6 GeV/ c , as needed to reach the outer detectors, resulting in smaller efficiency and misidentification rate relative to Belle II data. The PID criteria yield average efficiencies of 92.1% (90.0%) for kaons, 74.7% (83.0%) for electrons, and 43.6% (65.0%) for muons, and fractions of 4.6% (4.5%), 0.2% (2.0%), and 0.8% (2.7%) of misidentified candidates in Belle (Belle II) data, as determined in control samples as functions of momentum and polar angle [37].

The reconstruction of calorimeter energy deposits (clusters) generated by neutral particles is critical, as signal extraction relies on those clusters. To suppress beam-induced background photons, we select clusters with energies exceeding, respectively, 100.0 (100.0) MeV, 50.0

(55.0) MeV, and 150.0 (150.0) MeV in the forward, central, and backward (with respect to the electron-beam direction) regions of the calorimeter for Belle (Belle II) data. In Belle II data, we restrict the cluster polar angle to the drift-chamber acceptance to suppress electrons misreconstructed as photons. Cluster selections in Belle II are more stringent than in Belle to suppress higher beam-induced backgrounds. We also require clusters to be reconstructed at least 30 cm from the closest extrapolated track to suppress background due to secondary particles generated in nuclear interactions of charged hadrons with the calorimeter. These criteria minimize data-to-simulation differences in cluster-related quantities as observed in several control samples [17].

A signal B -meson candidate is formed by combining a kaon candidate of charge opposite to that of the partner B meson with two opposite-charge leptons in e^+e^- , $e^\pm\mu^\mp$, or $\mu^+\mu^-$ combinations. We analyze these three dilepton combinations together, as simulation shows no significant sensitivity gain from separate analyses. We suppress $B^+ \rightarrow J/\psi(\rightarrow \ell^+\ell^-)K^+$ background by restricting the dilepton mass to be less than $3.0 \text{ GeV}/c^2$, which is the region relevant for our signal. We also suppress $\gamma \rightarrow e^+e^-$ conversion backgrounds by requiring the electron-positron mass to exceed $100 \text{ MeV}/c^2$.

We then combine signal and partner B candidates to form $\Upsilon(4S)$ candidates. For each $\Upsilon(4S)$ candidate, we construct the squared momentum transfer (or beam-constrained squared τ -pair mass) $q^2 = (p_{e^+e^-} - p_{B_{\text{tag}}} - p_K)^2$, where $p_{e^+e^-}$, $p_{B_{\text{tag}}}$, and p_K are the sum of the four-momenta of both beams, the four-momentum of the partner B , and that of the kaon, respectively. We impose $q^2 > 14.18 \text{ GeV}^2/c^4$, corresponding to the kinematic acceptance used in SM predictions [1]. This kinematically suppresses contributions from $B^+ \rightarrow \psi(2S)(\rightarrow \tau^+\tau^-)K^+$ decays, whose rate is ten-times larger than the expected SM-signal. We further reject background by vetoing events that have π^0 candidates unassociated with the $\Upsilon(4S)$ reconstruction. These candidates are identified as pairs of neutral clusters detected in the central calorimeter region with diphoton masses between 131 and 138 MeV/c^2 . These selections remove 99% (99%) of the residual background after the partner B selection with 3.1% (5.8%) signal efficiency in Belle (Belle II) data.

At this stage, remaining backgrounds are primarily B^+B^- events in which a properly reconstructed partner B^- is accompanied by a $B^+ \rightarrow \bar{D}^0(\rightarrow K^+\ell^{(\prime)-}\bar{\nu}_{\ell^{(\prime)}})\ell^+\nu_{\ell^{(\prime)}}$ decay or the analogous decay involving a $\bar{D}^{*0}$ instead of a $\bar{D}^0$ meson, which are both signal-like. The opposite-charge kaon-lepton mass $m(K^+\ell^-)$ exhibits a major transition in background size and composition at the D^0 meson mass, as shown in Fig. 1. The narrow peak is from $D^0 \rightarrow K^-\pi^+$ decays with the pion misidentified as a muon. The large component of B^+ mesons decaying into D mesons that in turn decay into π^0 's unassociated with the candidate populates masses lower than the charm

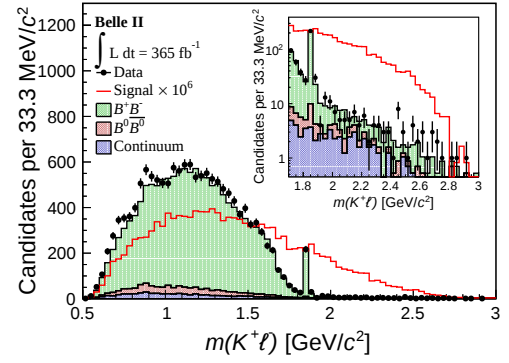


Figure 1: Distribution, with simulation overlaid, of opposite-charge kaon-lepton mass in Belle II events. Logarithmic vertical scale in the inset assists visualization of the high kaon-lepton mass analysis region.

mass. This results in a significantly higher signal-to-background ratio at higher $m(K^+\ell^-)$ values. Restricting the search to events with $m(K^+\ell^-) > 1.9 \text{ GeV}/c^2$ reduces the remaining background by 98.7% (99.2%) while keeping 20% (16%) of signal in Belle (Belle II) data, thus constituting an important analysis choice.

The principal remaining backgrounds are $B \rightarrow D^{(*)}\ell^+\nu$ decays followed by semileptonic D decays and continuum events. These are further suppressed through an optimized selection on the following three additional discriminating quantities. The squared missing mass $M_{\text{miss}}^2 = (p_{e^+e^-} - p_{\text{tot}})^2/c^2$, where p_{tot} is the total four-momentum of all particles reconstructed in the event, is sensitive to the higher number of neutrinos in signal events compared to background events [38]. The c.m. momentum $p_{\ell^+}^*$ of the signal lepton with same charge as the signal kaon also has a distinctive spectrum for signal. The third variable is the search-region width in the signal-extraction observable E_{extra} , which is the summed energy of all neutral clusters not used in $\Upsilon(4S)$ reconstruction. The selection requirements for these are optimized to provide the best average expected (statistical only) upper limit in background-only simulation, yielding $M_{\text{miss}}^2 > 3.0$ (1.6) GeV^2/c^4 , $p_{\ell^+}^* > 0.4$ (0.5) GeV/c , and $E_{\text{extra}} < 100$ (250) MeV for Belle (Belle II). No event contains more than one reconstructed signal B candidate after the selection.

We determine the signal yield by counting the excess events over the background expectation, $N_{\text{sig}} = N_{\text{obs}} - N_{\text{bkg}}$, in the signal-search E_{extra} region. The background expectation is obtained by extrapolating, in simulation, the background yields observed in three independent signal-depleted sidebands. These are events with $q^2 < 14.18 \text{ GeV}^2/c^4$, events in the $5.20 < M_{\text{bc}} < 5.27 \text{ GeV}/c^2$ sideband of the partner B , and events with $E_{\text{extra}} > 100$ (250) MeV for Belle (Belle II) data. For each of these sidebands, the selection on the other two observables is the same as for the signal-search region.

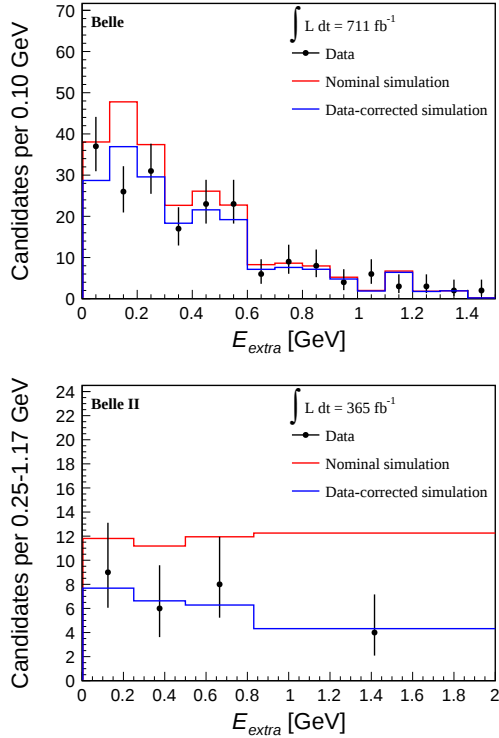


Figure 2: Distributions of E_{extra} for (circles) data and (histograms) simulation (red) before and (blue) after the linear-scaling fit for (top) Belle and (bottom) Belle II events populating the partner- B M_{bc} sideband.

We validate the simulated E_{extra} distribution by comparing it with experimental data in the three sidebands. Good agreement is observed, but within large statistical uncertainties due to small sample sizes. Hence, we further ensure reliability of the background extrapolation into the signal-search region by correcting the simulated E_{extra} shape and normalization in those sidebands to match the data. A likelihood-ratio test [39] favors a scaling between data and simulation based on a linear function of E_{extra} . The scaling parameters are determined with a likelihood fit to the binned ratios of data and simulation distributions, simultaneous across the three sidebands (Fig. 2 Ref. [38]). The distribution is Poisson in each bin, for both data and simulated-event counts. We then extrapolate the corrected simulation into the signal-search E_{extra} region to determine the expected background yields, 14.1 ± 1.6 events for Belle and 3.5 ± 0.7 events for Belle II, where the uncertainties are only statistical.

Figure 3 shows the distributions of E_{extra} in the data after final selections, with background expectations from simulation overlaid. The first bin of each distribution is the signal-search region. The event yields observed in the Belle and Belle II signal-search regions, 11 and 6, respectively, show no significant excesses with respect to the expectations.

The signal branching fraction is determined as

$$\mathcal{B}(B^+ \rightarrow K^+ \tau^+ \tau^-) = \frac{N_{\text{sig}}}{2\epsilon f^{+-} N_{\Upsilon(4S)}}, \quad (1)$$

where ϵ is the efficiency for reconstructing signal, which includes the relevant product of $\mathcal{B}(\tau \rightarrow \ell \nu_\tau)$ factors that are $(17.81 \pm 0.04)\%$ for electrons and $(17.39 \pm 0.04)\%$ for muons, and $f^{+-} = 0.511^{+0.007}_{-0.011}$ is the $B^+ B^-$ fraction [4] of the $N_{\Upsilon(4S)} = [(772 \pm 11) + (387 \pm 6)] \times 10^6$ $\Upsilon(4S)$ decays recorded in the combined data set. Efficiencies for signal are determined from simulation and corrected using control data. We use $B^+ \rightarrow \bar{D}^0 \pi^+$ decays to match to data the efficiency for the partner- B FEI reconstruction in simulation. We reconstruct partner B candidates using FEI and select 1.5–3.0-GeV/ c -momentum charged pions in the rest of the event. We fit the yields of charmed-meson signals appearing in the spectrum of recoil mass, $\sqrt{(E_{\text{beam}}^* - E_\pi^*)^2/c^4 - (\vec{p}_B^* + \vec{p}_\pi^*)^2/c^2}$, where E_π^* and $\vec{p}_\pi^*$ are the energy and momentum of the pion and $\vec{p}_B^*$ is the momentum of the partner B in the c.m. frame [16]. We then compare the resulting yields with those observed in simulation and use their ratio $r_{\text{FEI}} = 0.76 \pm 0.03$ (0.74 ± 0.05) to correct the Belle (Belle II) FEI efficiency observed in simulation. We use analysis candidates populating the 1.8–1.9 GeV/ c^2 range in the opposite-charge kaon-lepton mass to determine the π^0 veto efficiency. This sample is dominated by $B^+ \rightarrow \bar{D}^0(\rightarrow K^+ \pi^-) \ell^+ \nu$ decays in which the pion is misreconstructed as a lepton, yielding a narrow peak. A simultaneous fit to the events that pass and those that fail the π^0 veto determines the efficiency ratio between data and simulation. This ratio, 1.03 ± 0.02 (1.03 ± 0.03) for Belle (Belle II) corrects the π^0 veto efficiency in simulation, assuming similar π^0 multiplicities in data and simulation.

The primary source of systematic uncertainty is the additive uncertainty on the estimate of the background yield in the signal region, which is 17.4% (33.6%), relative to the central value of the background estimate, for Belle (Belle II). This combines the 9% (18%) statistical uncertainty on the extrapolation, due to the finite sizes of the sideband samples, and the systematic uncertainty associated with the shape assumed for the background E_{extra} scaling. The latter is assessed by repeating the extrapolation with a uniform or a quadratic scaling with respect to E_{extra} and taking the largest difference in signal yield with respect to the default linear scaling. The remaining systematic uncertainties are all multiplicative. The second-largest contribution is the uncertainty on the partner- B reconstruction efficiency. This combines the uncertainty on the data-to-simulation efficiency ratio and the efficiency change observed when using an alternative approach to calibrate the FEI reconstruction efficiency, based on inclusive $B^+ \rightarrow X_c \ell^+ \nu$ decays reconstructed using only the lepton. The systematic uncertainty on the signal branching fraction is

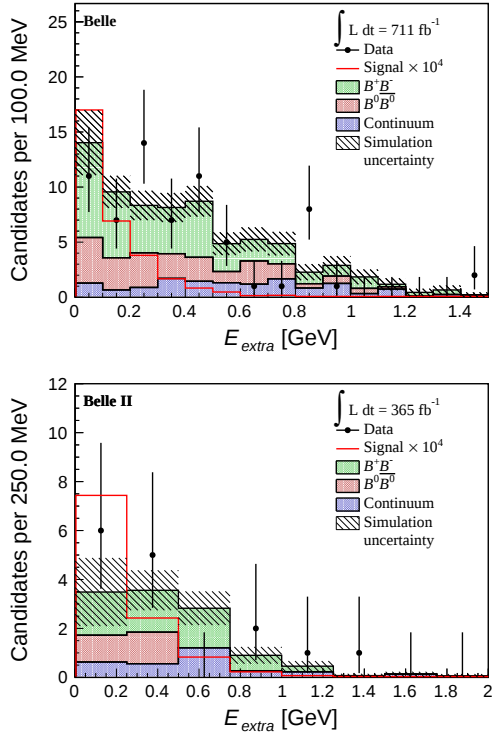


Figure 3: Distributions of E_{extra} after final selections in (top) Belle and (bottom) Belle II data. Expectations based on corrected simulation for backgrounds and for 10^4 -enhanced SM signal [1] are overlaid.

10.0% (13.3)% for Belle (Belle II). The uncertainties on PID-related efficiencies in Belle data are determined using $J/\psi \rightarrow \ell^+\ell^-$ and $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ decays to be 2.4%, 2.5%, and 1.6% for muons, electrons, and kaons, respectively. The corresponding Belle II uncertainties are 0.6%, 0.3%, and 0.9%, obtained using the same processes, as well as $e^+e^- \rightarrow \ell^+\ell^-e^+e^-$ and $e^+e^- \rightarrow \ell^+\ell^-\gamma$. The uncertainty for the π^0 veto efficiency is 2.2% (2.5%) in Belle (Belle II) data, as determined from $B^+ \rightarrow \bar{D}^0(\rightarrow K^+\pi^-)\ell^+\nu$ decays reconstructed as a narrow peak in the opposite-charge kaon-lepton mass. An uncertainty of 0.35% [40] (0.27%) per charged particle is assigned to account for possible data-simulation differences in the Belle (Belle II) tracking efficiency. This uncertainty is fully correlated between the three signal tracks, leading to a 1.05% (0.81%) uncertainty on the signal branching fraction. We use a simulated $B^+ \rightarrow K^+\tau^+\tau^-$ sample generated with an alternative model for the hadronic form-factors [3], to determine a 3.8% (4.0%) uncertainty due to the SM signal modeling. The systematic uncertainties due to the limited sizes of simulated samples used to determine efficiencies are 3.1% (3.3%) for Belle (Belle II). The uncertainties arising from the values of $N_{\mathcal{Y}(4S)}$ and f^{+-} are 1.4% (1.6%) and $^{+1.4}_{-2.1}\%$, respectively, for Belle (Belle II).

We determine exclusion limits using the CLs method

	Belle	Belle II
N_{bkg}	$14.1 \pm 1.6 \pm 1.9$	$3.5 \pm 0.7 \pm 0.9$
N_{obs}	11	6
$\epsilon \times 10^5$	$1.45 \pm 0.05 \pm 0.18$	$1.26 \pm 0.04 \pm 0.18$
$\mathcal{B}(B^+ \rightarrow K^+\tau^+\tau^-) \times 10^4$	$-2.7^{+3.2}_{-2.6} \pm 2.2$	$5.1^{+5.6}_{-4.3} \pm 2.5$
Obs. (exp.) limit (10^{-3})	0.5 (0.7)	1.4 (0.9)

Table I: Expected background yields, observed event yields, signal efficiencies, observed branching fractions, and observed (expected) 90% CL exclusion limits.

based on a profile-likelihood-ratio test [41, 42]. Systematic uncertainties are incorporated with Gaussian constraints. Coverage tests show no significant undercoverage. Table I shows the results of the search, which is conducted independently in the Belle and Belle II samples. The resulting branching fractions are averaged yielding $\mathcal{B}(B^+ \rightarrow K^+\tau^+\tau^-) = (0.04^{+0.27+0.17}_{-0.23-0.18}) \times 10^{-3}$. The corresponding combined exclusion limit is $\mathcal{B}(B^+ \rightarrow K^+\tau^+\tau^-) < 0.56 \times 10^{-3}$ at the 90% confidence level.

In summary, we search for the rare decay $B^+ \rightarrow K^+\tau^+\tau^-$ using $772 \times 10^6 \mathcal{Y}(4S)$ meson decays from the full data set of electron-positron collisions at the $\mathcal{Y}(4S)$ resonance collected by the Belle experiment and $387 \times 10^6 \mathcal{Y}(4S)$ decays collected by the Belle II experiment between 2019 and 2023. The analysis achieves world-leading results by means of an optimized selection applied to a large data set. We observe no significant signal and set a 90% CL upper limit at $\mathcal{B}(B^+ \rightarrow K^+\tau^+\tau^-) < 0.56 \times 10^{-3}$. This limit is four times more stringent than the only previous bound and contributes constraints to the parameters of several SM extensions [6–10].

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SUPPLEMENTAL MATERIAL

We report here additional figures that help illustrating analysis procedures and their rationales. Figure 4 shows the distribution of missing mass squared, which is used in the optimized selection. The higher prevalence of neutrinos in the signal decay with respect to the most prominent backgrounds offers distinctive discrimination. Similarly, Fig. 5 shows the distribution of the c.m. momentum of the candidate signal lepton that has the same charge as the candidate signal kaon. Distinctive kinematic-charge correlations produce a higher mean-valued spectrum for signal events compared to backgrounds. Figures 6 to 7 show the distributions of E_{extra} before and after the linear-scaling fit that matches simulation to data for events populating the q^2 and E_{extra} sidebands. These distributions, along with the analogous distribution of events populating the partner- B sideband reported in the main text body, are simultaneously fit to achieve the final background expectation. Figure 8 shows the distributions of simulated signal efficiency as a function of q^2 after final selections and corrections.

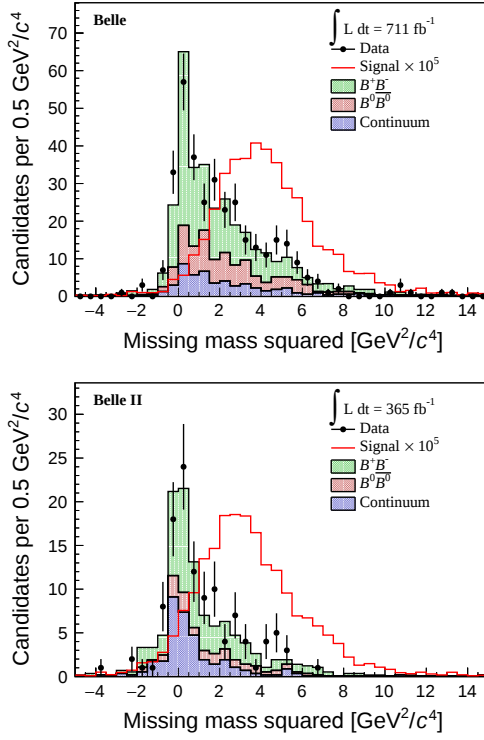


Figure 4: Distribution, with simulation overlaid, of missing mass squared in (top) Belle and (bottom) Belle II events restricted to the $m(K^+\ell^-) > 1.9 \text{ GeV}/c^2$ region.

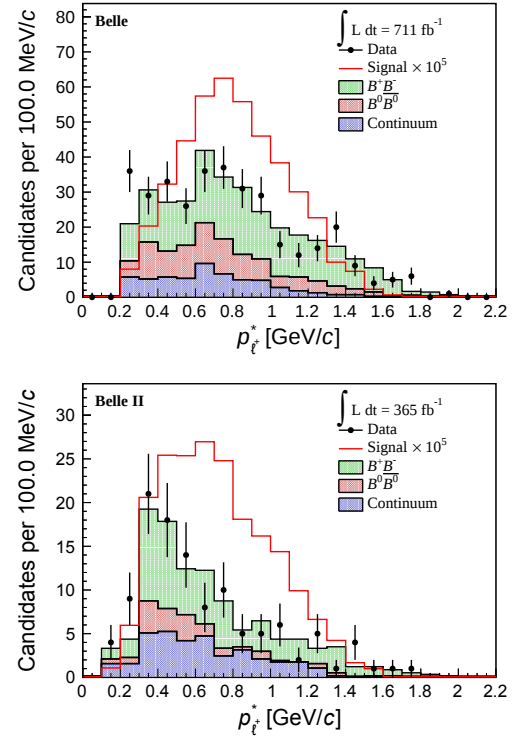


Figure 5: Distribution, with simulation overlaid, of c.m. momentum of the signal lepton with same charge as the signal kaon in (top) Belle and (bottom) Belle II events restricted to the $m(K^+\ell^-) > 1.9 \text{ GeV}/c^2$ region.

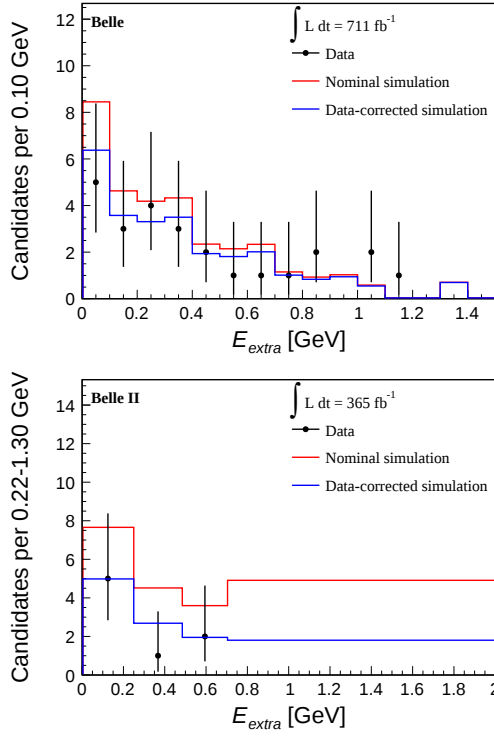


Figure 6: Distributions of E_{extra} for (circles) data and (histograms) simulation (red) before and (blue) after the linear-scaling fit for (top) Belle and (bottom) Belle II events populating the q^2 sideband.

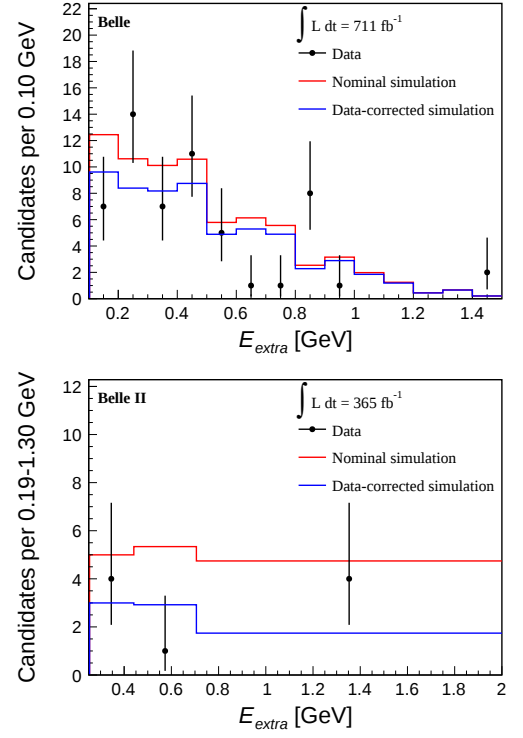


Figure 7: Distributions of E_{extra} for (circles) data and (histograms) simulation (red) before and (blue) after the linear-scaling fit for (top) Belle and (bottom) Belle II events populating the E_{extra} sideband.

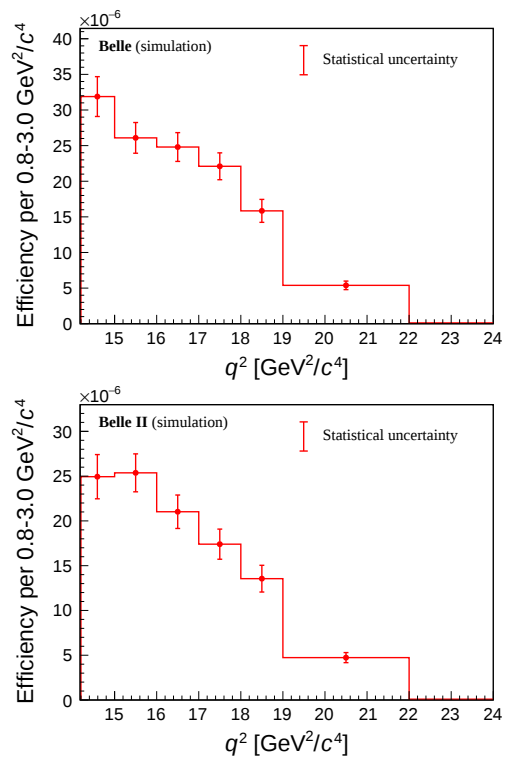


Figure 8: Distributions of efficiency as a function of q^2 for reconstructing simulated (top) Belle and (bottom) Belle II signal events. The binning follows Ref. [3].