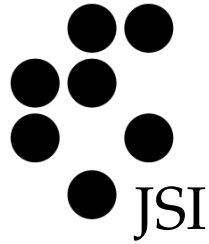




# Experimental mini-review on exclusive $|V_{ub}|$ at $B$ factories

Matic Lubej

CKM 2016, Mumbai, India

Monday, November 28$^{th}$, 2016

# Outline

# Exclusive $|V_{ub}|$

- Charmless hadronic final state
- $B \to \pi \ell \nu$ most precise, but only 7 % of all $B \to X_u \ell \nu$
- Others include $X_u = \rho, \omega, \eta$, multi-particle states, ...

Pseudoscalar*

$$\frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = |V_{ub}|^2 \frac{G_F^2 \tau_B}{24\pi^3} p_\pi^3 |f_+^{B\pi}(q^2)|^2$$

Vector (helicity basis)*:

$$\frac{d\mathcal{B}(B \to V \ell \nu)}{dq^2} = |V_{ub}|^2 \frac{G_F^2 p_V q^2 \tau_B}{96\pi^3 m_B^2} \left[ |H_0(q^2)|^2 + |H_+(q^2)|^2 + |H_-(q^2)|^2 \right]$$

Experimental measurement of the branching fraction and theoretical input on form factors needed to determine $|V_{ub}|$.

---

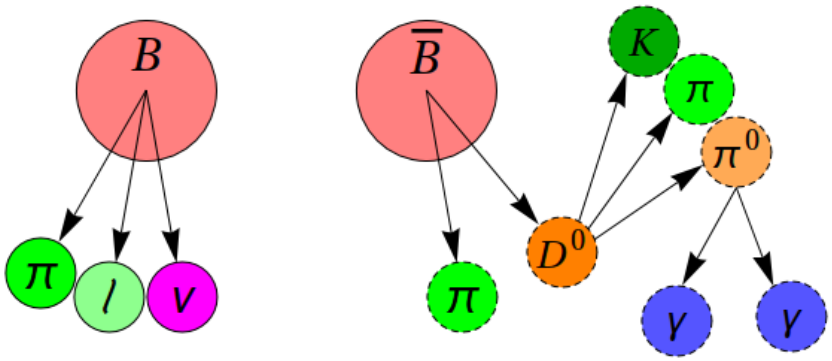
* Simplified for low mass charged leptons ($e$ and $\mu$)

# Measurements at $B$ factories

- Initial state well known: $e^+e^- \to \Upsilon(4S)$ @ $E_{CMS} \approx M_{\Upsilon(4S)}$
- $\Upsilon(4S)$ at rest $\to B\overline{B}$
- Neutrino 4-momentum inferred from missing momentum in event (assuming only one neutrino missing)

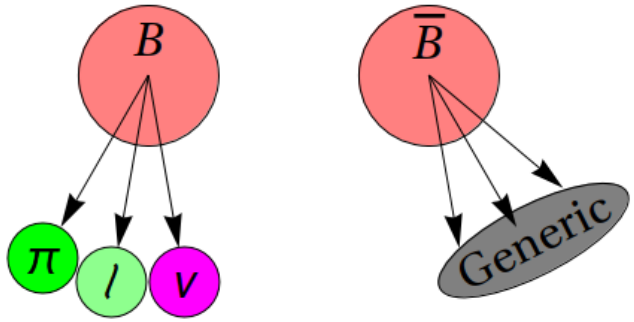
## Reconstruction methods

### Tagged measurement



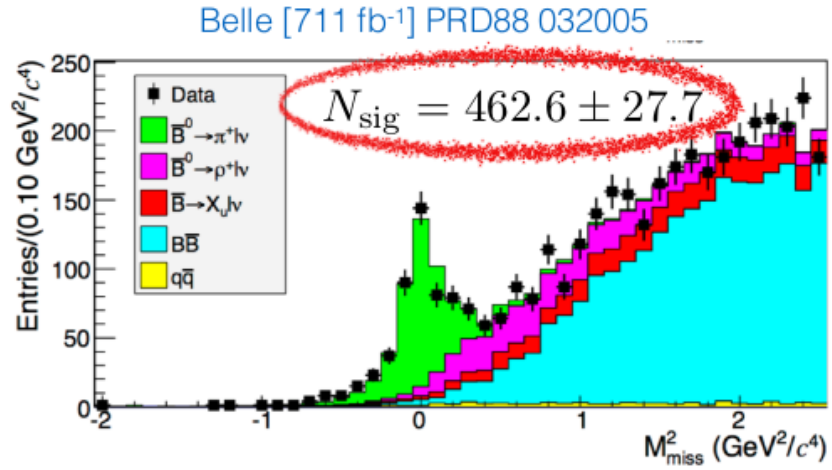

Example scheme of a tagged mode

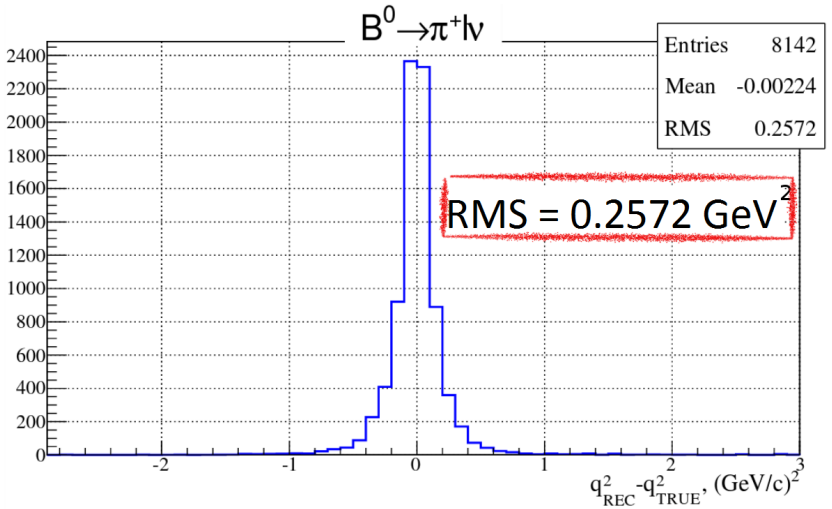
### Untagged measurement

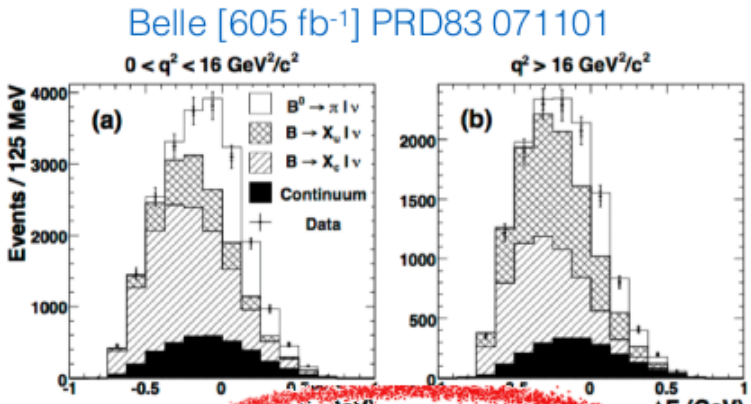


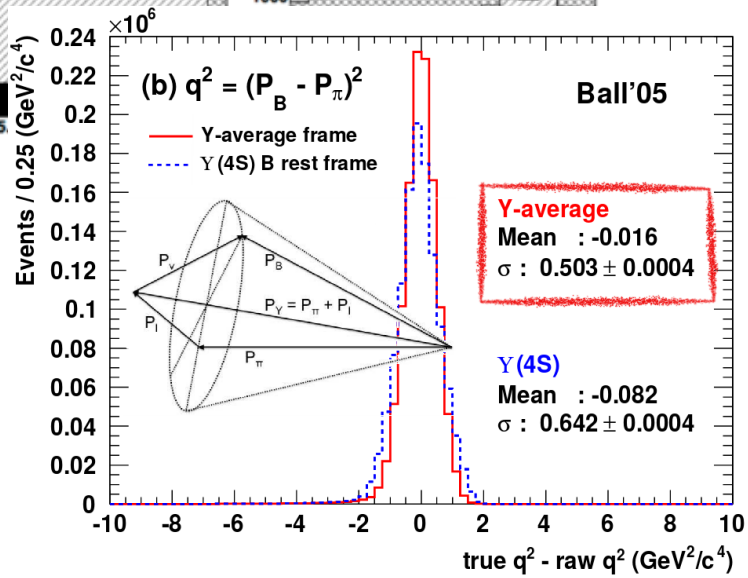

Companion $B$ not reconstructed

# Tagged vs. Untagged



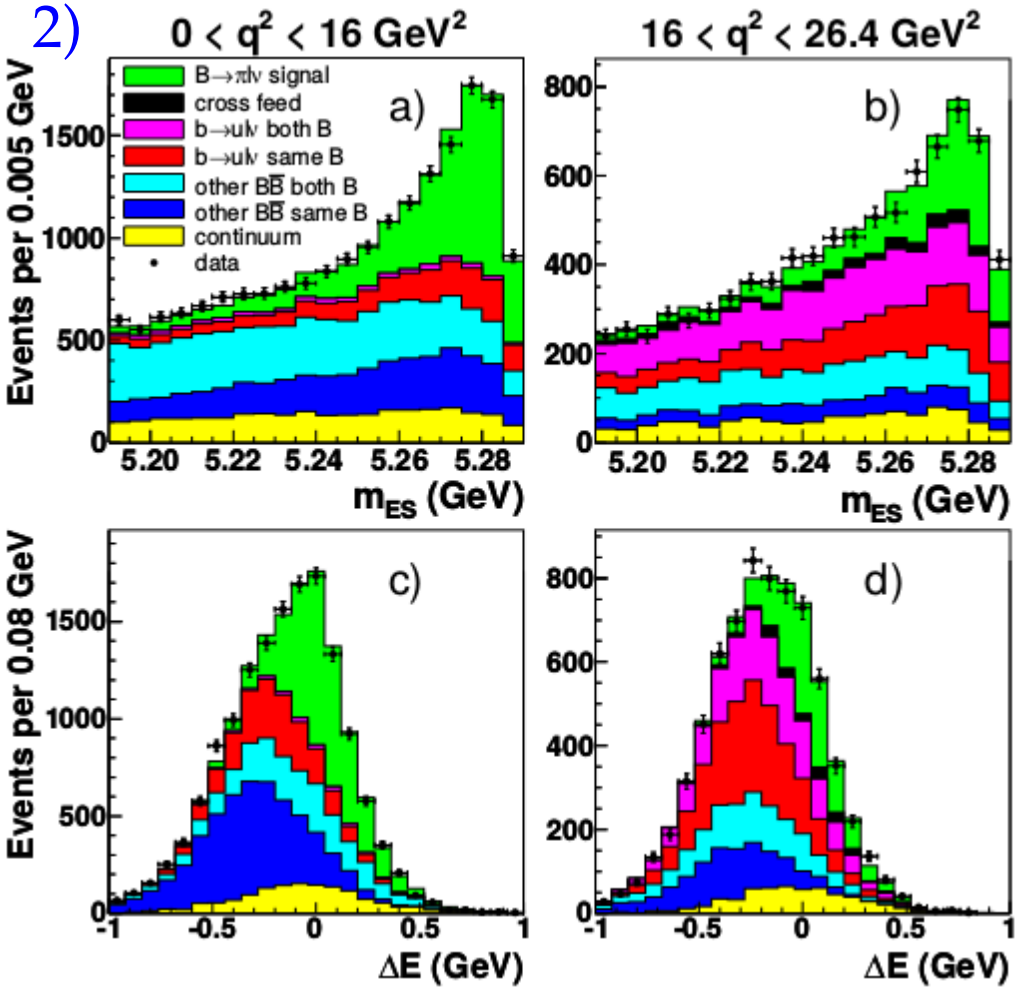




Tagged → Efficiency → Untagged

Tagged ← $q^2$ res. ← Untagged

# $B \to \pi \ell^+ \nu$ untagged at *BaBar*

2)




Other $b \to u$ BKG at high $q^2$

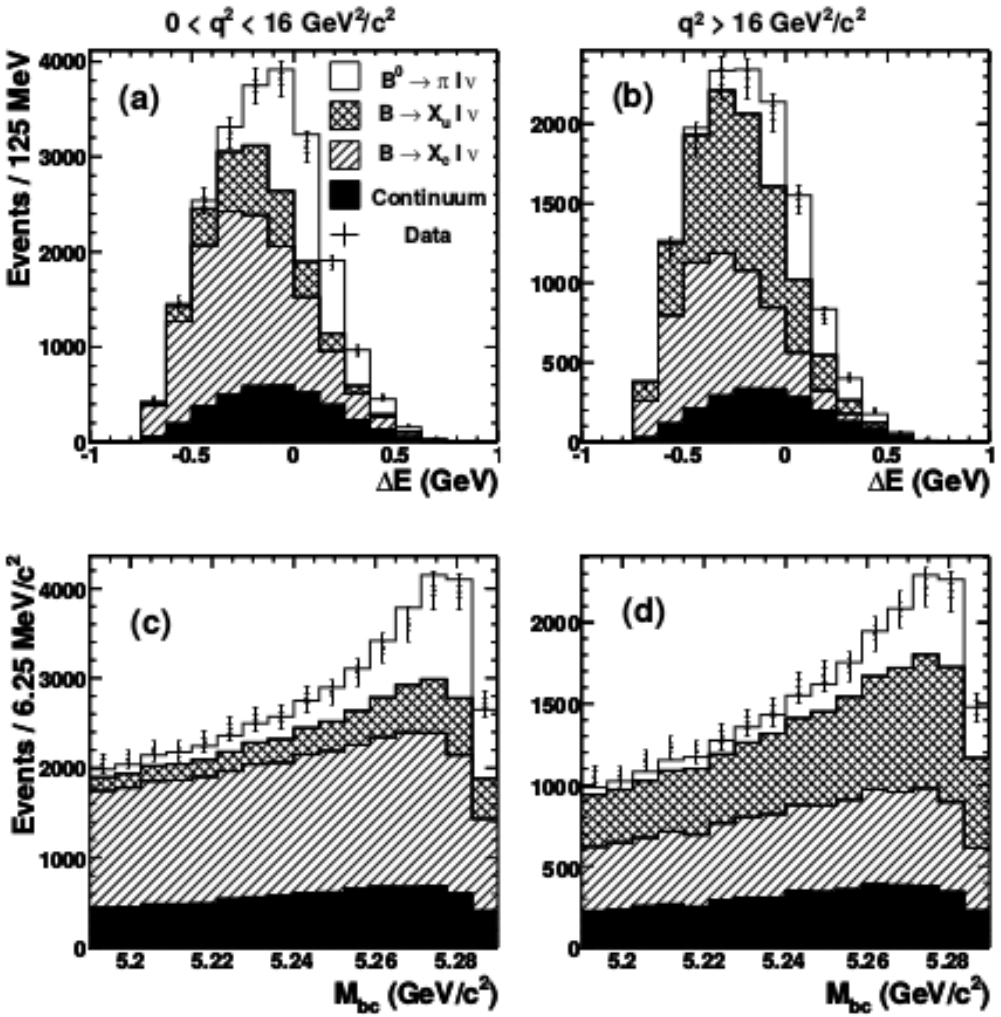
- 1) Phys.Rev. D83 (2011) 032007
- Data sample: 349 fb$^{-1}$
- Combined signal $N_{sig} \approx 10.5\text{k} \pm 400$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 6 bins

- 2) Phys.Rev. D86 (2012) 092004
- Data sample: 416.1 fb$^{-1}$
- Combined signal $N_{sig} = 12.5\text{k} \pm 400$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 12 bins

$B^+, B^0$ combined result from 2):

$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.45 \pm 0.04_{\text{stat}} \pm 0.06_{\text{sys}}) \times 10^{-4}$

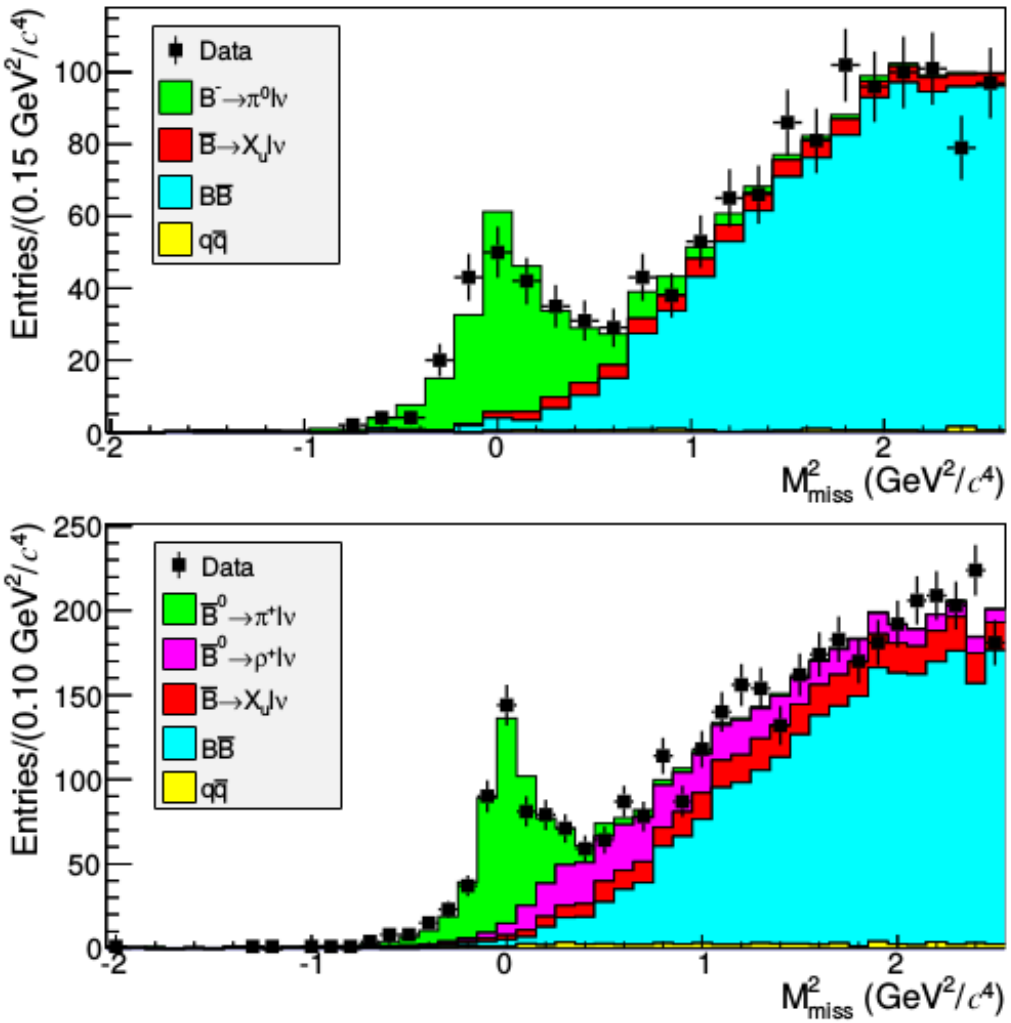
# $B^0 \to \pi^- \ell^+ \nu$ untagged at *Belle*




- Phys.Rev. D83 (2011) 071101
- Data sample: 605 fb$^{-1}$
- Combined signal $N_{sig} \approx 21.5\text{k} \pm 500$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 13 bins

$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.49 \pm 0.04_{\text{stat}} \pm 0.07_{\text{sys}}) \times 10^{-4}$$

# $B \to \pi \ell^+ \nu$ tagged at *Belle*

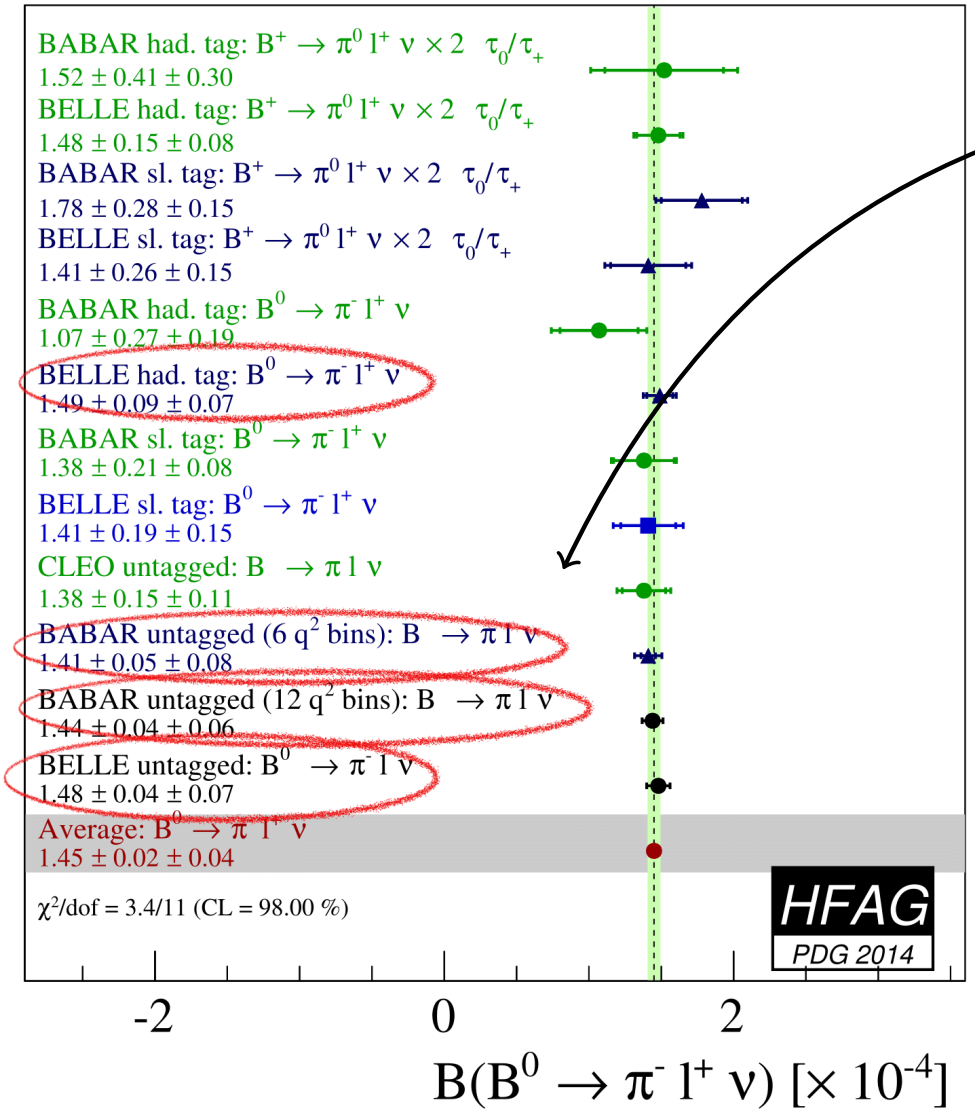



- Phys.Rev. D88 (2013) no.3, 032005
- Data sample: 711 fb$^{-1}$
- Signal candidates
  $N_{B^0 \to \pi^- \ell^+ \nu} \approx 500 \pm 30$
  $N_{B^+ \to \pi^0 \ell^+ \nu} \approx 200 \pm 20$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 13 (7) bins for $\pi^-$ ($\pi^0$)

$B^+, B^0$ combined result:

$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.49 \pm 0.08_{\mathrm{stat}} \pm 0.07_{\mathrm{sys}}) \times 10^{-4}$$

# Other measurements of $B \to \pi \ell \nu$



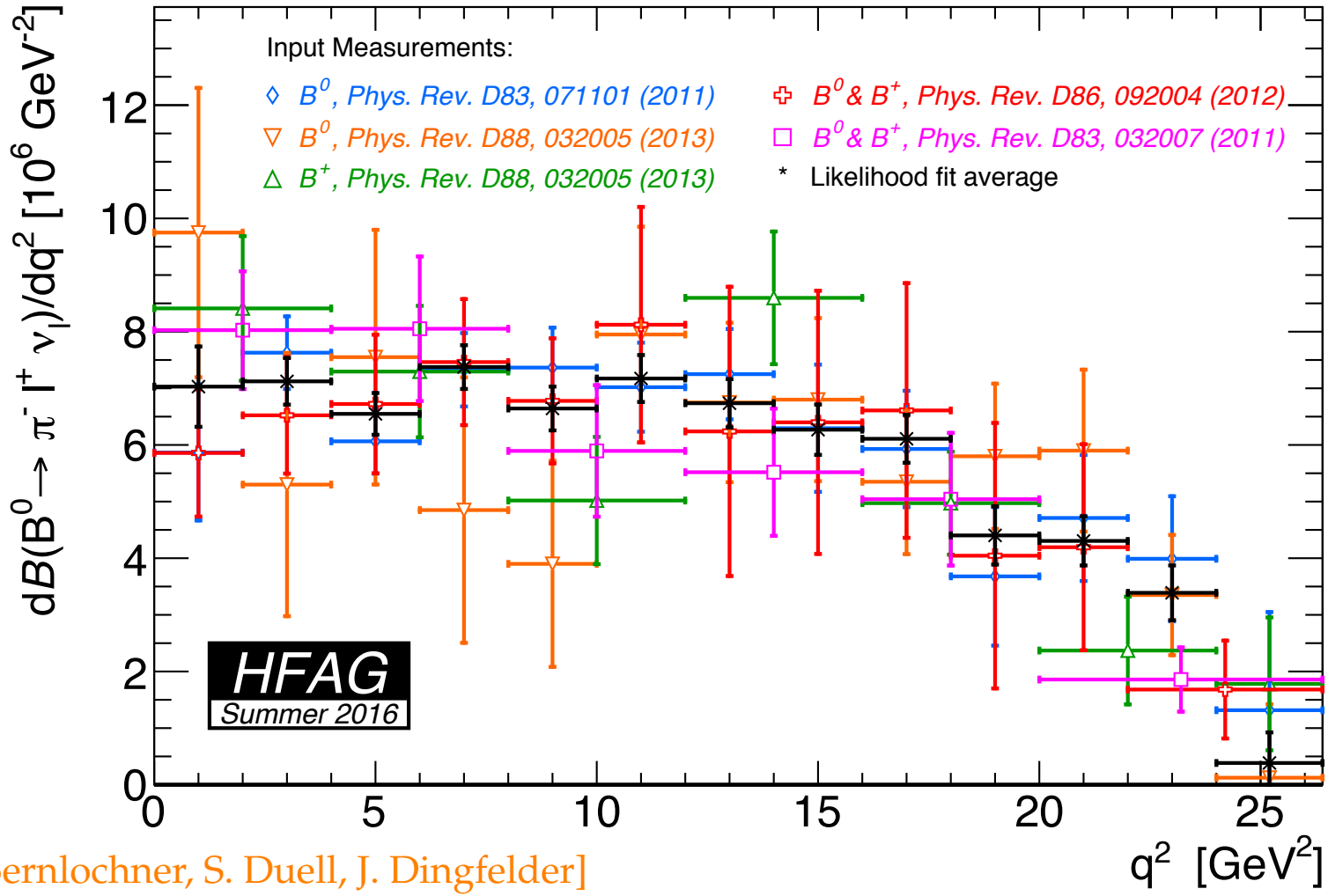

Presented today

Possible to average these inputs

Optimal $|V_{ub}|$ extraction with simultaneous fit to: averaged data (full $q^2$) + averaged lattice + LCSR with model independent FF parametrization

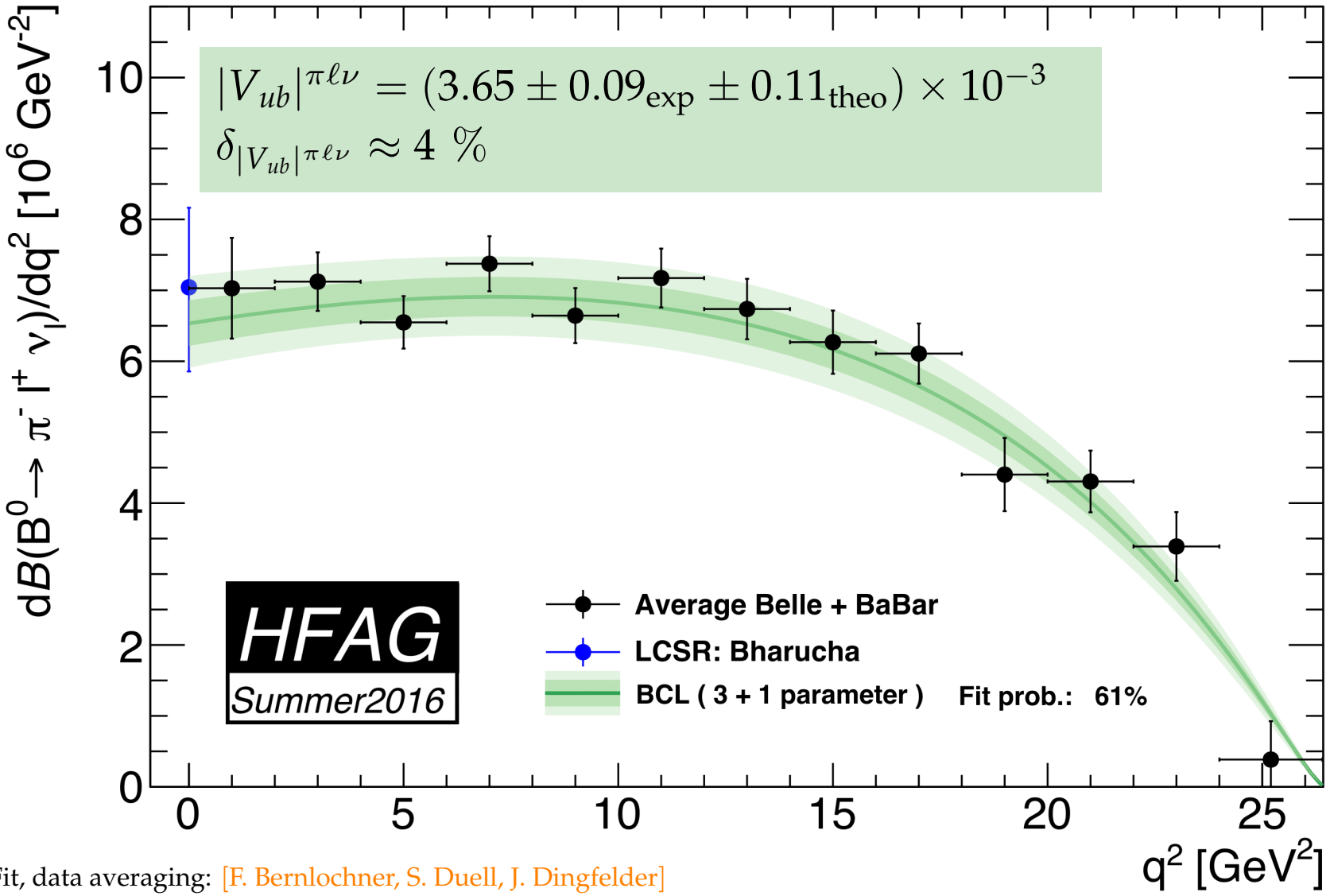
# $B \to \pi \ell \nu$

The most precise measurements can be averaged with a likelihood fit.



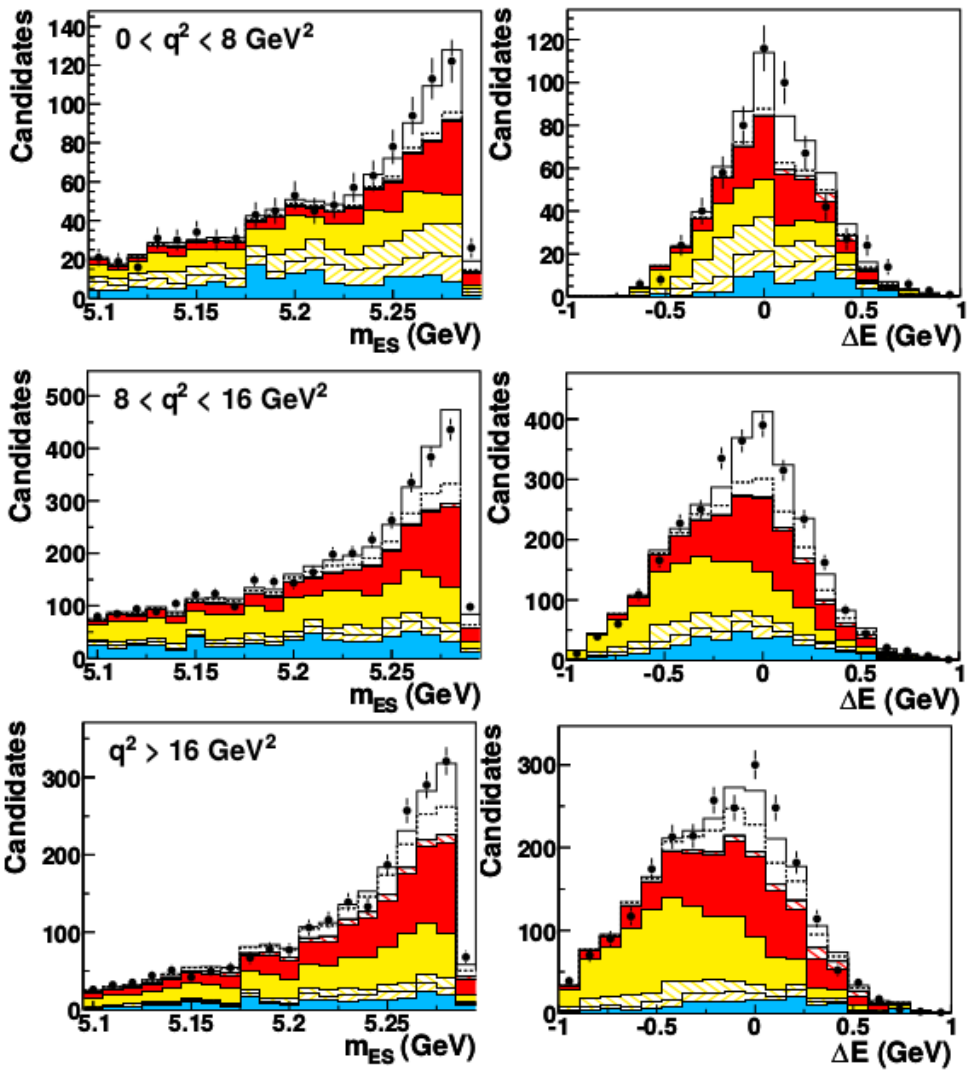

[F. Bernlochner, S. Duell, J. Dingfelder]

# $B \to \pi \ell \nu$

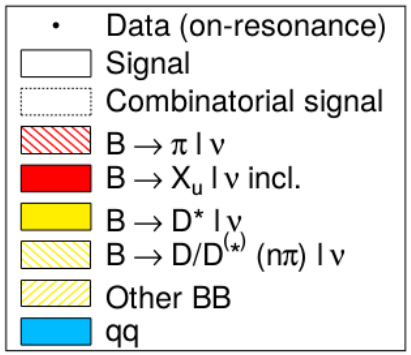



Fit, data averaging: [F. Bernlochner, S. Duell, J. Dingfelder]
LQCD averaging: [FLAG-3 review (arXiv:1607.00299)]
LQCD: [Fermilab/MILC, Phys.Rev. D92 (2015) no.1, 014024]
LQCD: [RBC/UKQCD, Phys.Rev. D91 (2015) no.7, 074510]
LCSR: [A. Bharucha, JHEP 1205 (2012) 092]

# $B \to \rho \ell^+ \nu$ untagged at *BaBar*



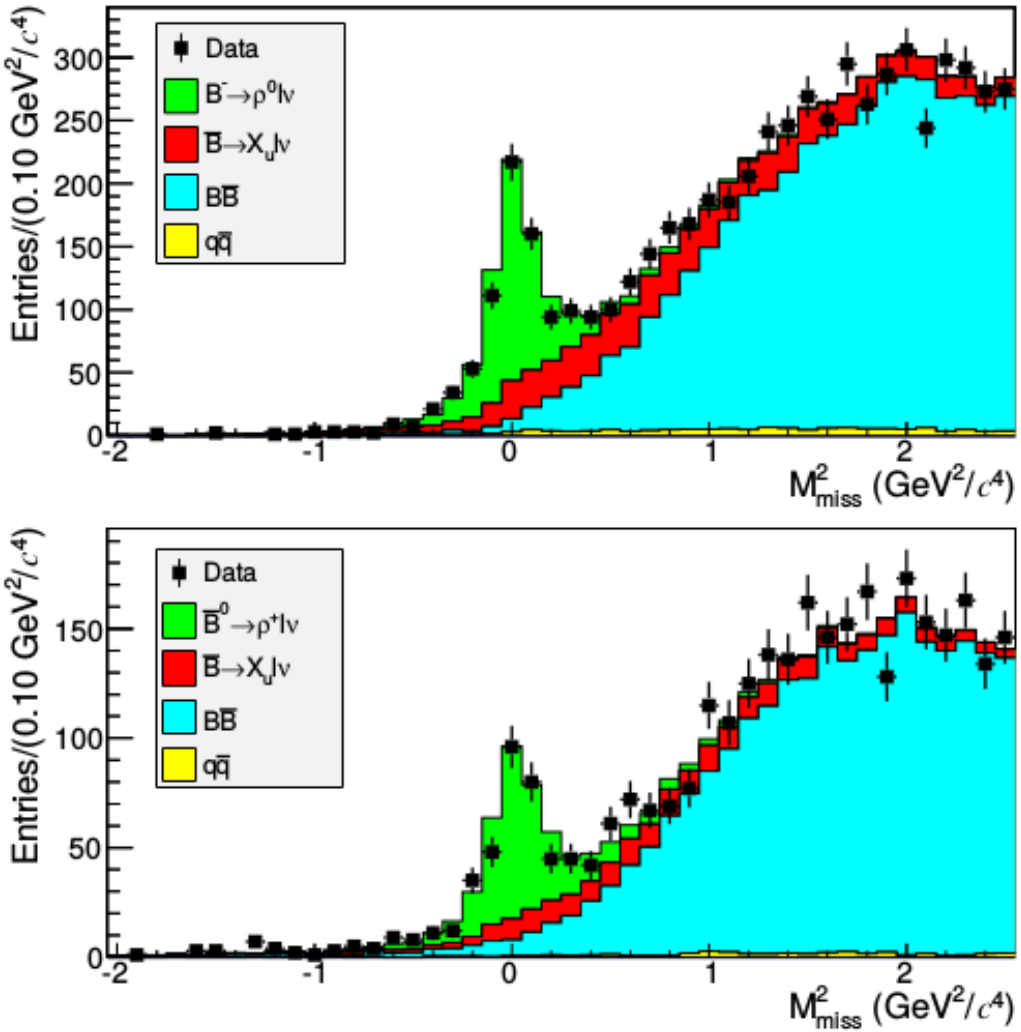

- Phys.Rev. D83 (2011) 032007
- Data sample: 349 fb$^{-1}$
- Combined signal $N_{sig} \approx 3.3\text{k} \pm 300$
- $d\mathcal{B}/dq^2$ spectrum in 3 bins




$B^+, B^0$ combined result:

$\mathcal{B}(B^0 \to \rho^- \ell^+ \nu) = (1.75 \pm 0.15_{\text{stat}} \pm 0.27_{\text{sys}}) \times 10^{-4}$

# $B \to \rho \ell^+ \nu$ tagged at *Belle*

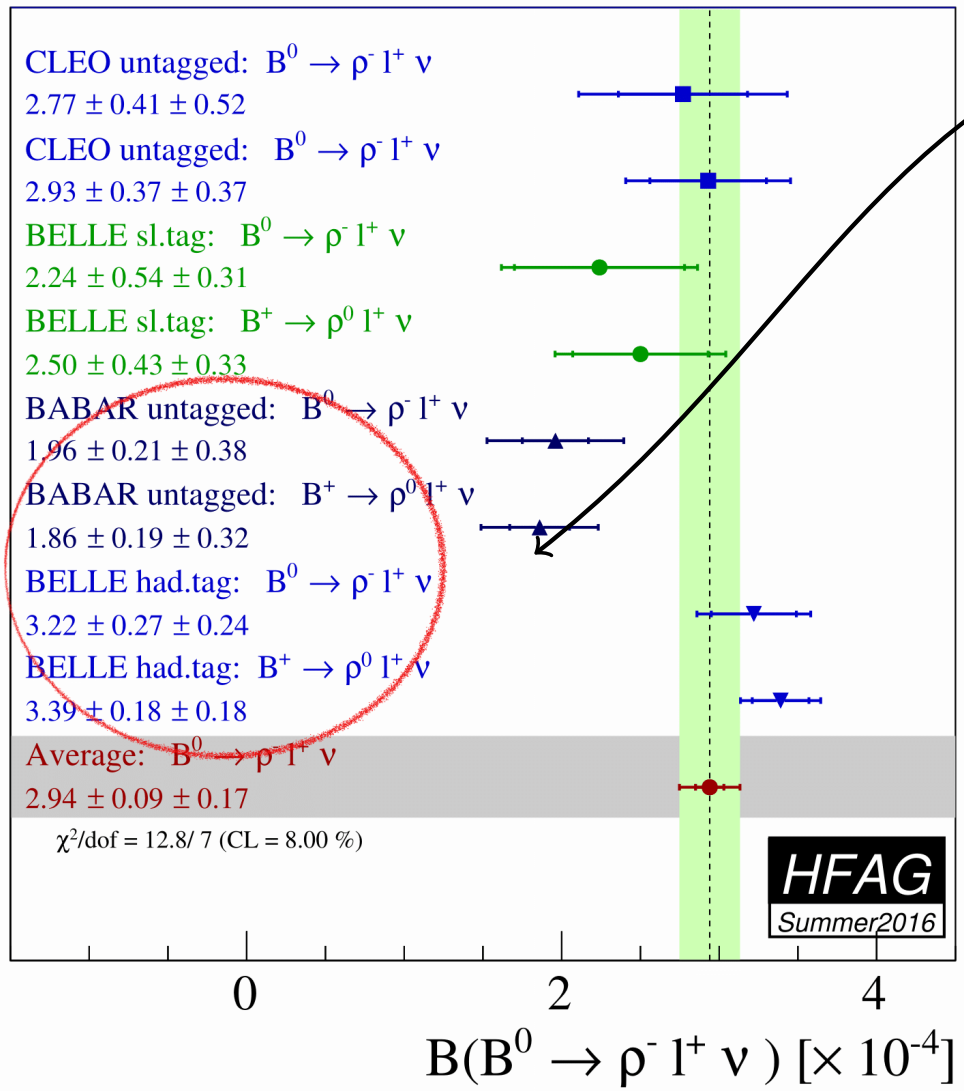



- Phys.Rev. D88 (2013) no.3, 032005
- Data sample: 711 fb$^{-1}$
- $N_{\rho^0} \approx 600 \pm 35.0$
- $N_{\rho^+} \approx 300 \pm 30$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 11 (6) bins for $\rho^0$ ($\rho^+$)

$B^+, B^0$ combined result:

$\mathcal{B}(B^0 \to \rho^- \ell^+ \nu) = (3.34 \pm 0.16_{\mathrm{stat}} \pm 0.17_{\mathrm{sys}}) \times 10^{-4}$

# Other measurements of $B \to \rho \ell \nu$




Presented today

No LQCD available, LCSR only

Presented data in regions
$q^2 < 8, 12$ GeV$^2$ (Belle)
$q^2 < 8$ GeV$^2$ (BaBar)
+LCSR

[A. Bharucha et al., JHEP 1608 (2016) 098]

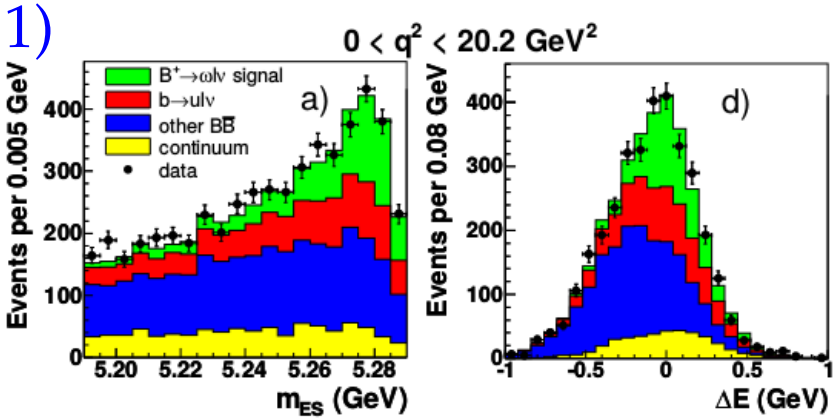
↓

Weighted avg. of $|V_{ub}|$ values (exp: 0 %, theo: 100 % correlation)

$$|V_{ub}|^{\rho\ell\nu} = (3.21 \pm 0.11_{\text{exp}} \pm 0.22_{\text{theo}}) \times 10^{-3}$$

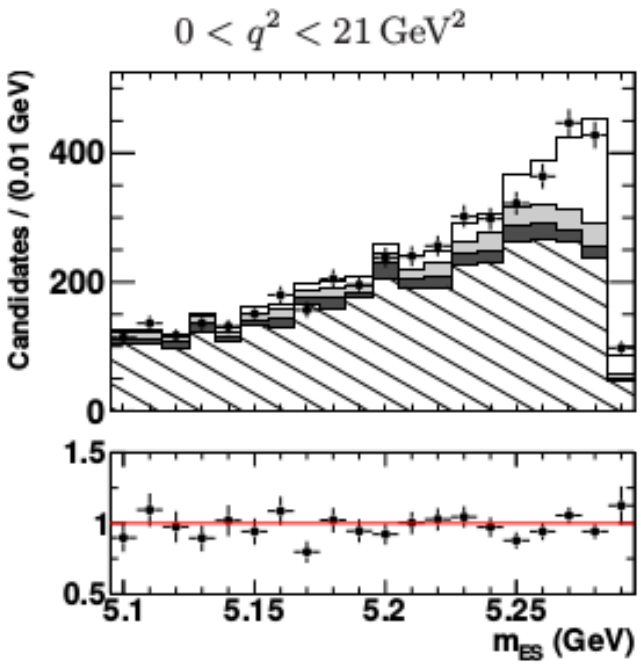
# $B^+ \to \omega \ell^+ \nu$ untagged at *BaBar*

1)




$\mathcal{B}(B^+ \to \omega \ell^+ \nu) = (1.19 \pm 0.16_{\text{stat}} \pm 0.09_{\text{sys}}) \times 10^{-4}$

2)

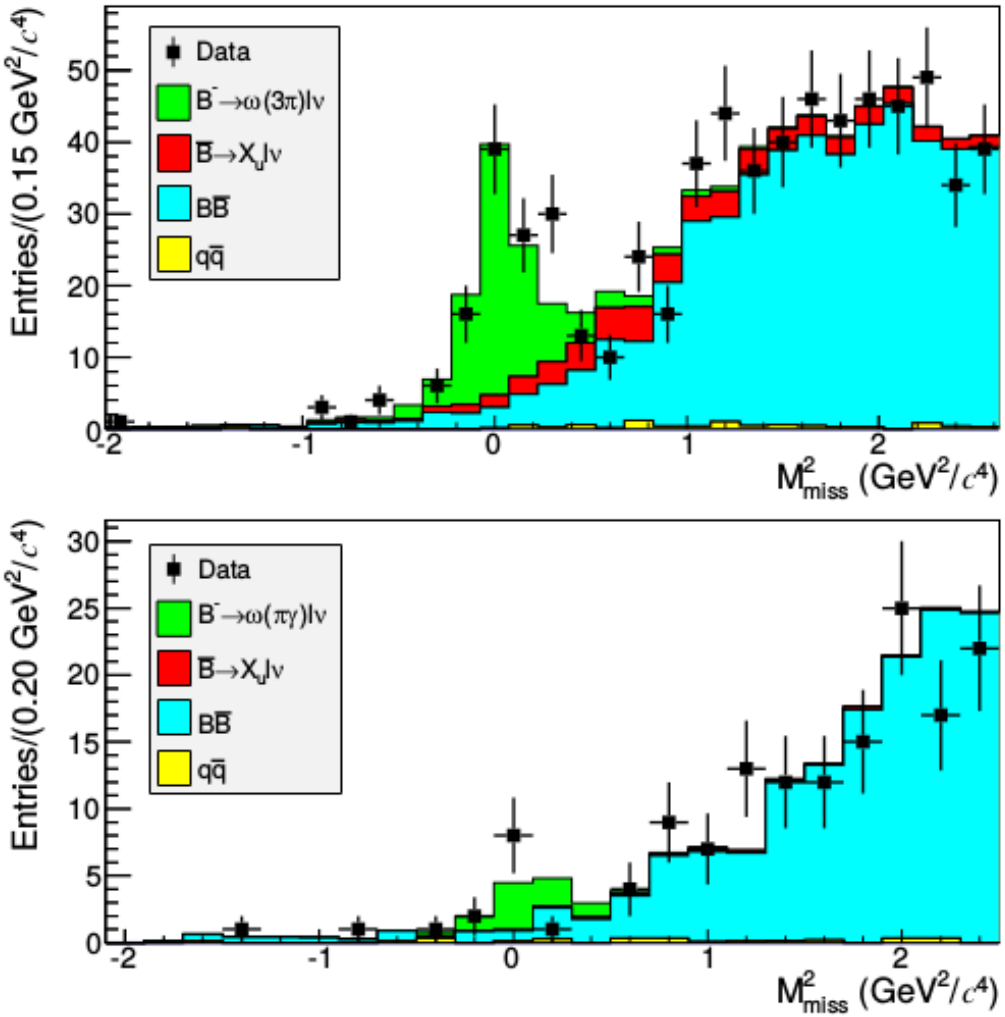



$\mathcal{B}(B^+ \to \omega \ell^+ \nu) = (1.21 \pm 0.14_{\text{stat}} \pm 0.08_{\text{sys}}) \times 10^{-4}$

- 1) Phys.Rev. D86 (2012) 092004
- Data sample: 416.1 fb$^{-1}$
- Combined signal $N_{sig} \approx 2\text{k} \pm 200$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 5 bins

- 2) Phys.Rev. D87 (2013) no.3, 032004
- Data sample: 426 fb$^{-1}$
- Combined signal $N_{sig} \approx 1.1\text{k} \pm 100$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 5 bins

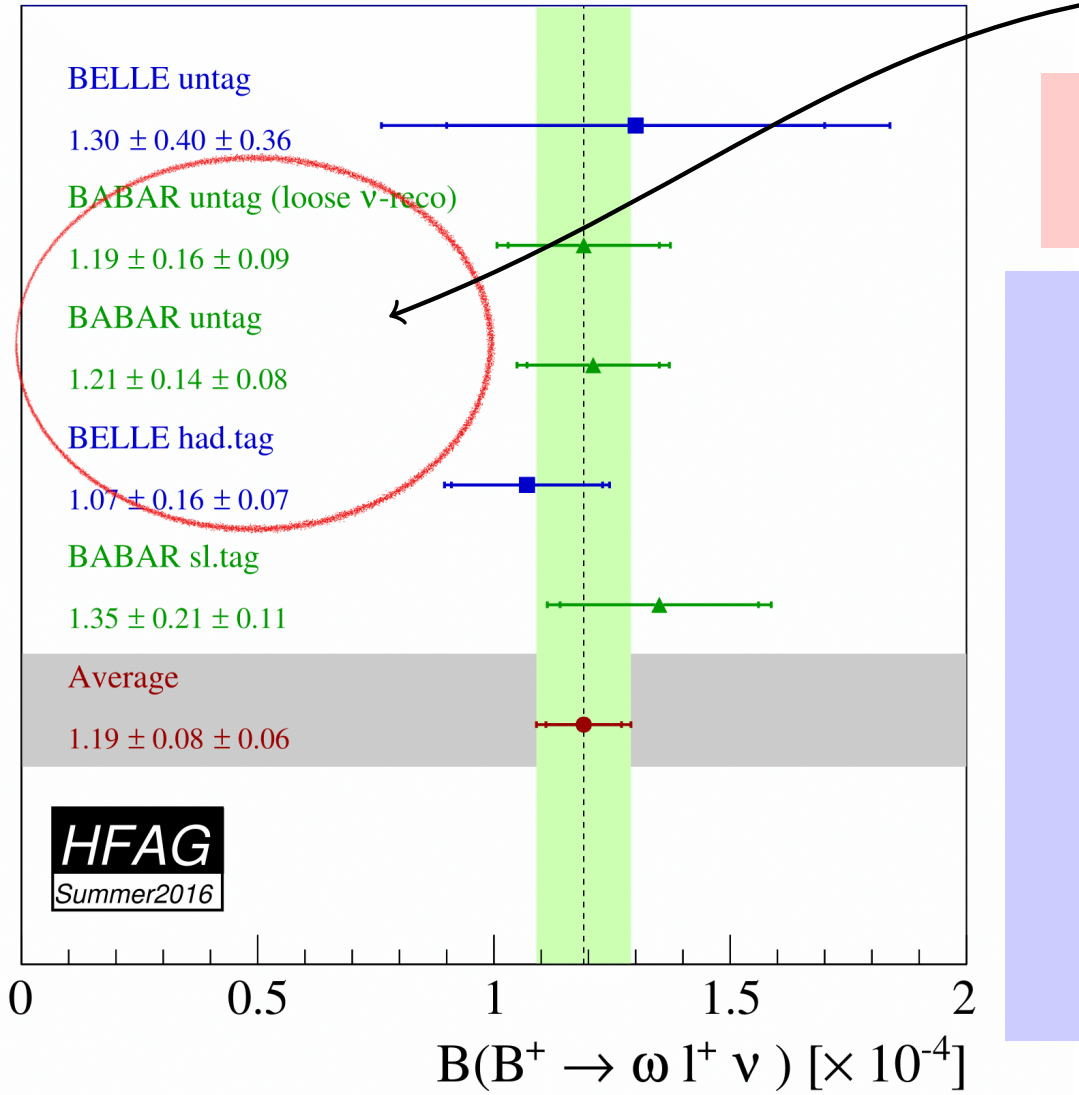
# $B^+ \to \omega \ell^+ \nu$ tagged at *Belle*




- Phys.Rev. D88 (2013) no.3, 032005
- Data sample: 711 fb$^{-1}$
- $N_{\omega(3\pi)} \approx 100 \pm 10$
- $N_{\omega(\pi\gamma)} \approx 10 \pm 5.0$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 3 bins

$$\mathcal{B}(B^+ \to \omega \ell^+ \nu) = (1.07 \pm 0.16_{\mathrm{stat}} \pm 0.07_{\mathrm{sys}}) \times 10^{-4}$$

# Other measurements of $B^+ \to \omega \ell^+ \nu$




No LQCD available, LCSR only

Presented data in regions
$q^2 < 7$ GeV$^2$ (Belle)
$q^2 < 8, 12$ GeV$^2$ (BaBar)
+LCSR

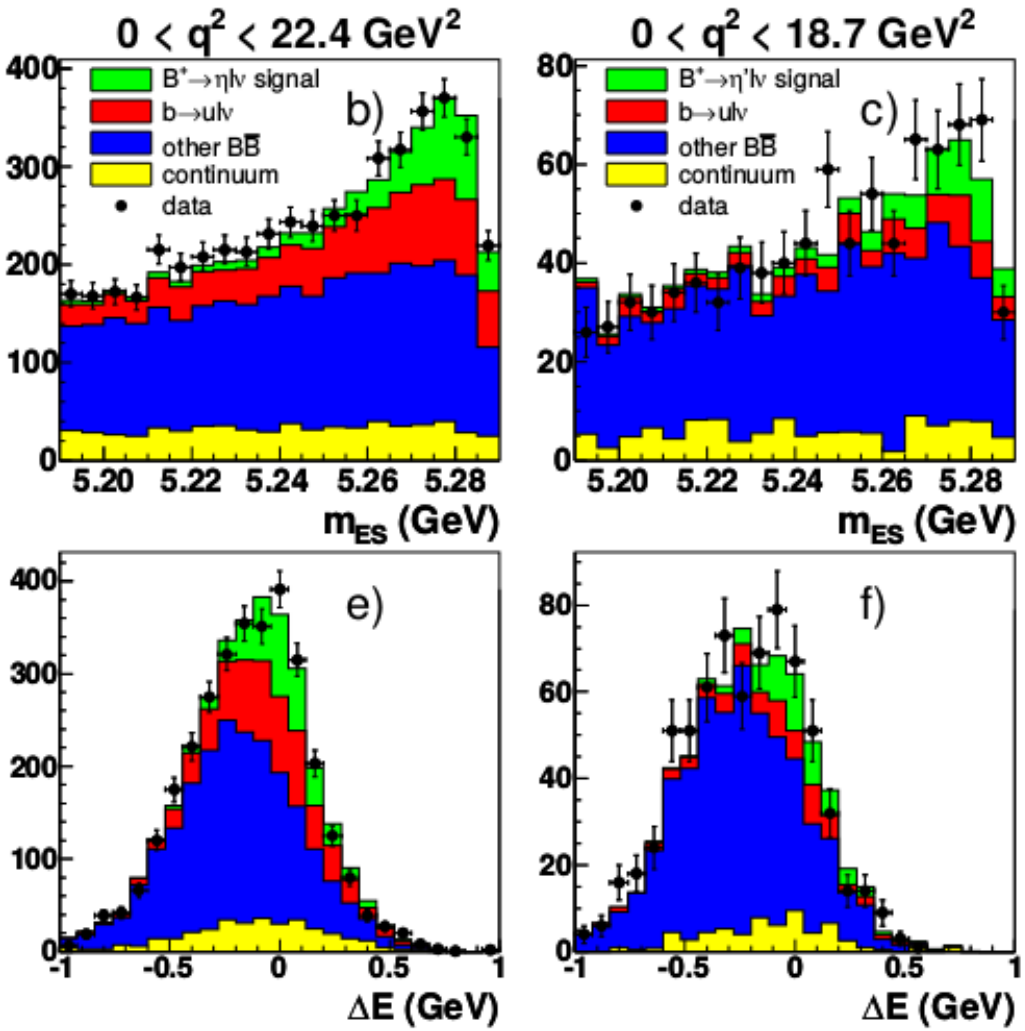
[A. Bharucha et al., JHEP 1608 (2016) 098]

↓

Weighted avg. of $|V_{ub}|$ values (exp: 0 %, theo: 100 % correlation)

$$|V_{ub}|^{\omega \ell \nu} = (2.90 \pm 0.20_{\text{exp}} \pm 0.24_{\text{theo}}) \times 10^{-3}$$

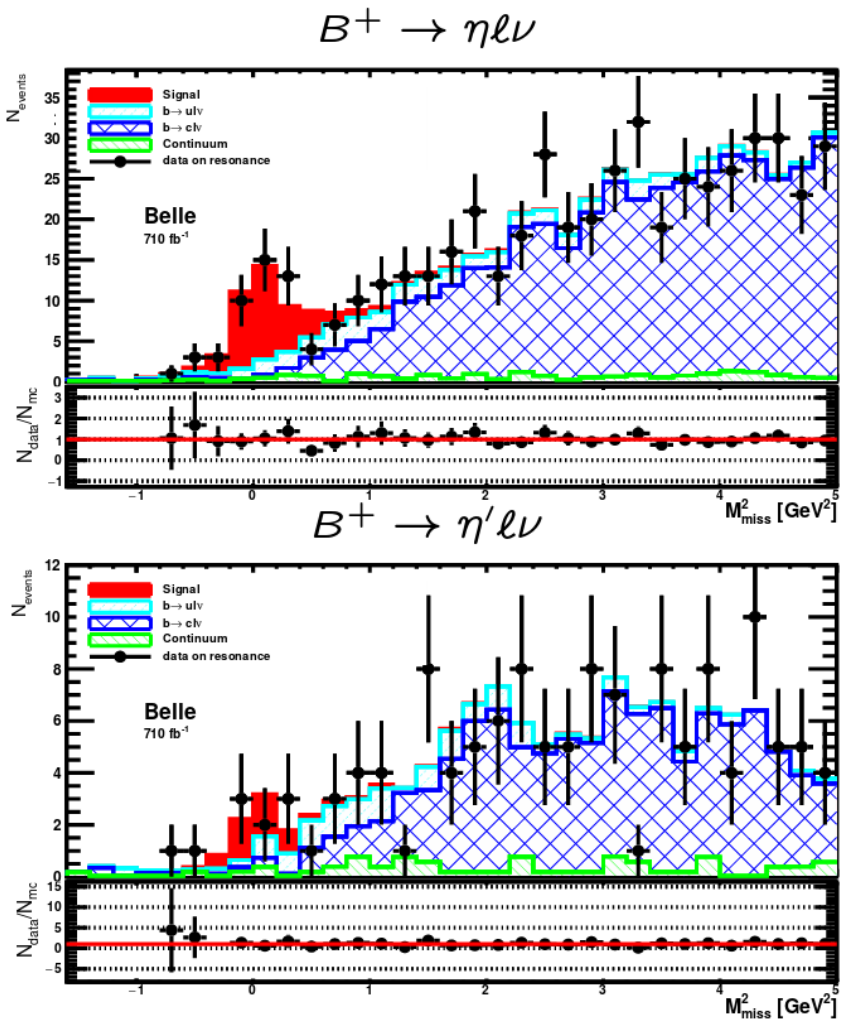
# $B^+ \to \eta^{(\prime)} \ell^+ \nu$ untagged at *BaBar*




- Phys.Rev. D86 (2012) 092004
- Data sample: 416.1 fb$^{-1}$
- $\eta$ in $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ modes
- $\eta'$ in $\eta(\gamma\gamma)\pi^+\pi^-$ modes
- $N_\eta \approx 900 \pm 100$
- $N_{\eta'} \approx 150 \pm 50$
- $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectrum in 5 bins for $\eta$ (full $q^2$)

$$\mathcal{B}(B^+ \to \eta \ell^+ \nu) = (0.38 \pm 0.05_{\mathrm{stat}} \pm 0.05_{\mathrm{sys}}) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to \eta' \ell^+ \nu) = (0.24 \pm 0.08_{\mathrm{stat}} \pm 0.03_{\mathrm{sys}}) \times 10^{-4}$$

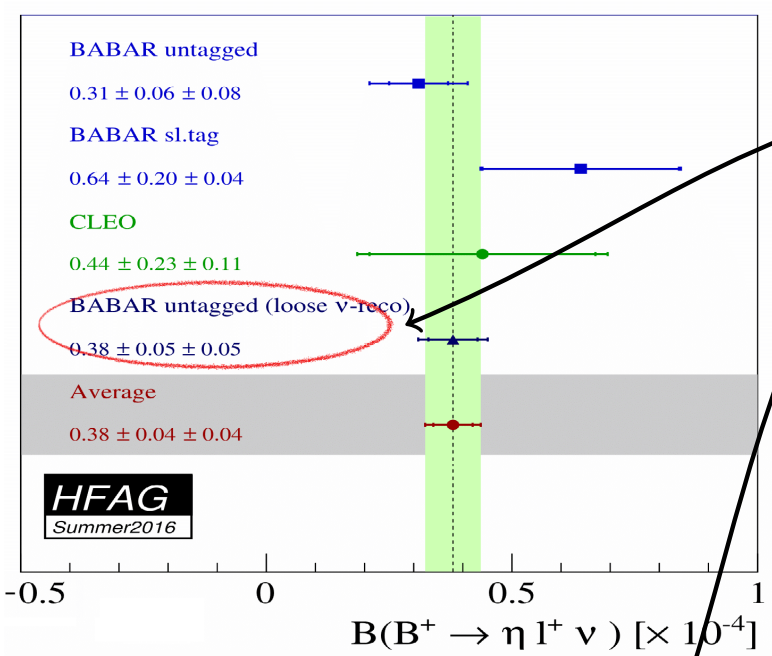
# $B^+ \to \eta^{(\prime)} \ell^+ \nu$ tagged at *Belle*

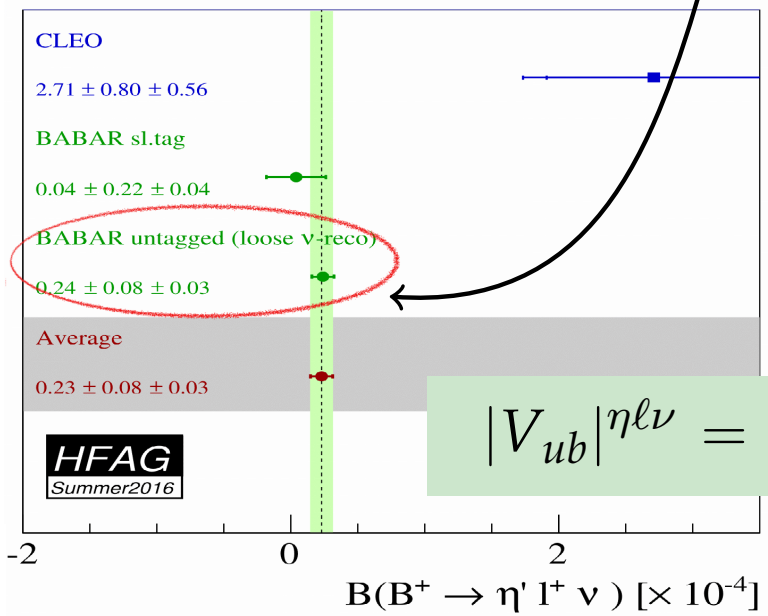



- In collaboration review, to be submitted soon
- Data sample: 711 fb$^{-1}$
- $N_\eta \approx 40 \pm 10$
- $N_{\eta'} \approx 6 \pm 4$

$$\mathcal{B}(B^+ \to \eta \ell^+ \nu) = (0.42 \pm 0.11_{\text{stat}} \pm 0.03_{\text{sys}}) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to \eta' \ell^+ \nu) = (0.36 \pm 0.27_{\text{stat}} \pm 0.03_{\text{sys}}) \times 10^{-4}$$

# Other measurements of $B^+ \to \eta^{(\prime)} \ell^+ \nu$




Presented today (BaBar)

No LQCD available, LCSR only

Presented BaBar untagged data in full $q^2$ region +LCSR

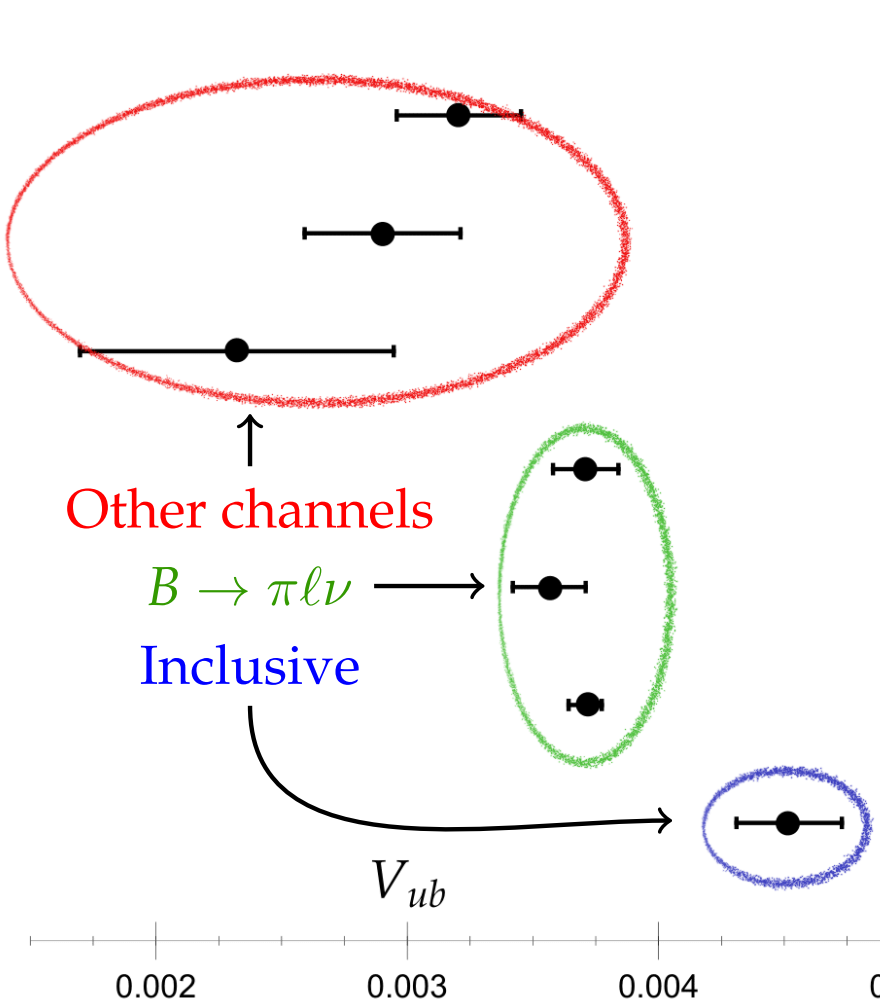
[P. Ball et al., JHEP 0708 (2007) 025]

$\downarrow$

$|V_{ub}|$ from measured (exp) and reduced (theo) $\Delta\mathcal{B}$

$$|V_{ub}|^{\eta\ell\nu} = (2.32 \pm 0.43_{\text{exp}} \pm {}^{+0.38}_{-0.49}\text{theo}) \times 10^{-3}$$

# Overview of various $|V_{ub}|$ values




$B \to \rho\, l\, \nu$ (data + LCSR)
[A. Bharucha et al. JHEP 1608 (2016) 098]

$B \to \omega\, l\, \nu$ (data + LCSR)
[A. Bharucha et al. JHEP 1608 (2016) 098]

$B \to \eta\, \mathrm{l}\, \nu$ (BaBar2012 + LCSR)
[P. Ball et al., JHEP 0708 (2007) 025]

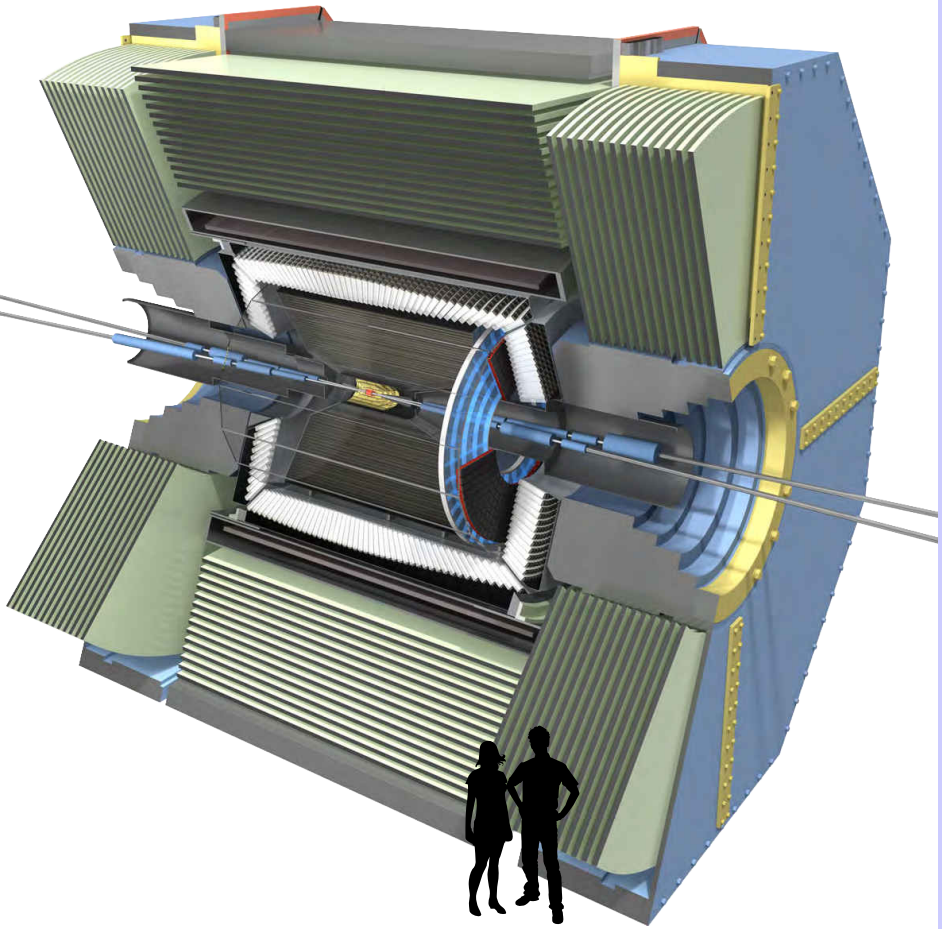
$B \to \pi\, \mathrm{l}\, \nu$ (data + LCSR + LQCD)
[F. Bernlochner, S. Duell, J. Dingfelder]

CKMFitter – indirect
[CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)]

CKMFitter – direct
[CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)]

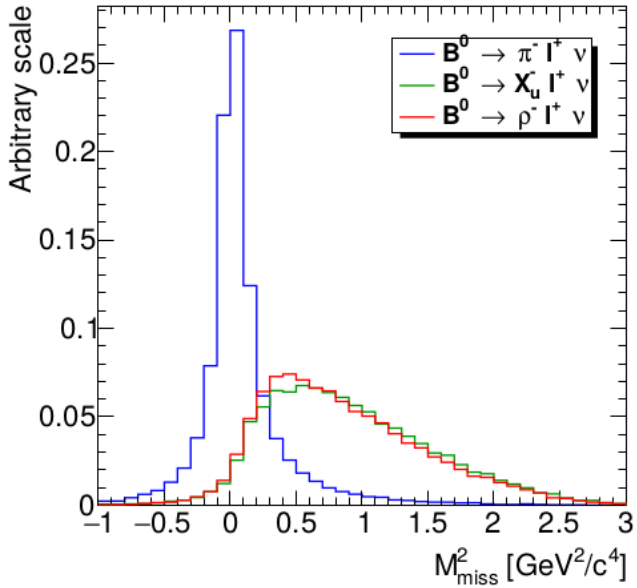
Inclusive (GGOU)
[P. Gambino, P. Giordano, G. Ossola, N. Uraltsev, JHEP 0710:058,2007]

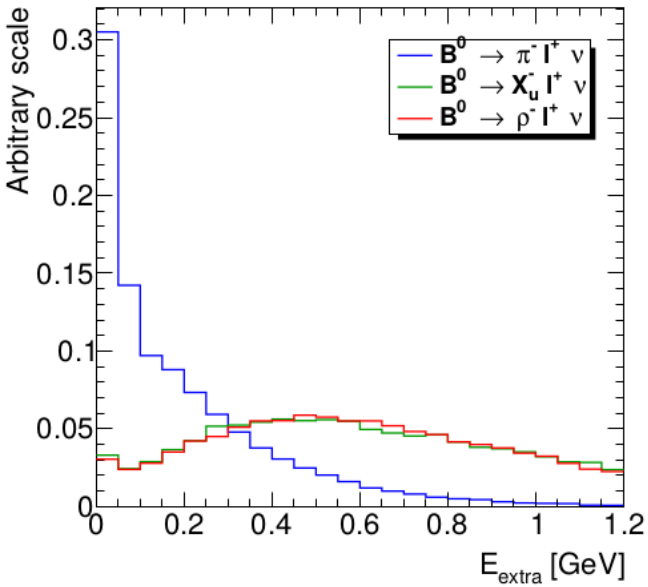
# Expected improvements at Belle II



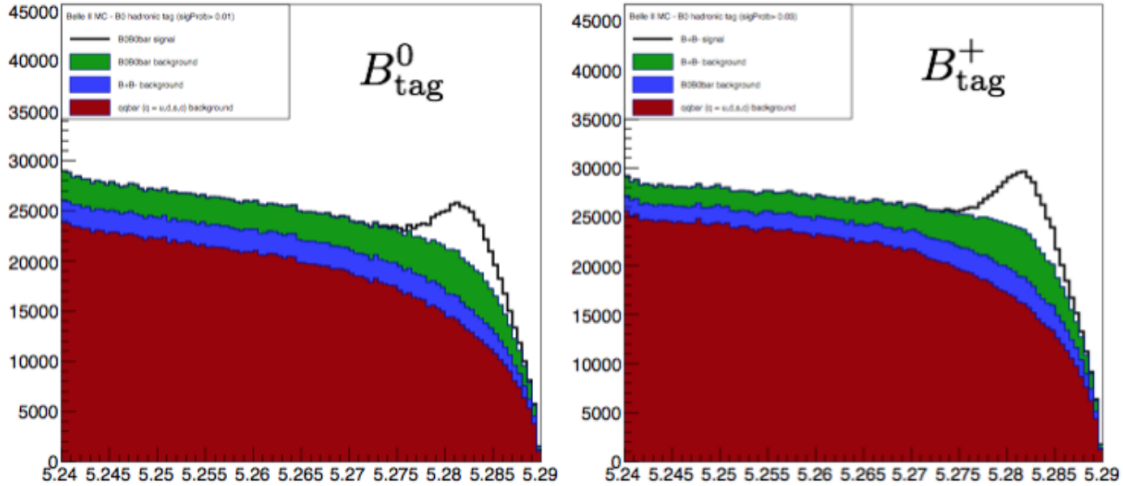
- Start in 2018
- Higher luminosity compared to Belle ($\times 40$)
- More data expected
  $\sim 50$ ab$^{-1}$ @ Belle II
  $\sim 1$ ab$^{-1}$ @ Belle
- Improved detector efficiency and purity (tracking, PID, $K/\pi$ separation, ...)
- Smarter software and more precise algorithms

# $B \to \pi \ell \nu$ tagged at Belle II (MC study)




[B2TiP, to be published]



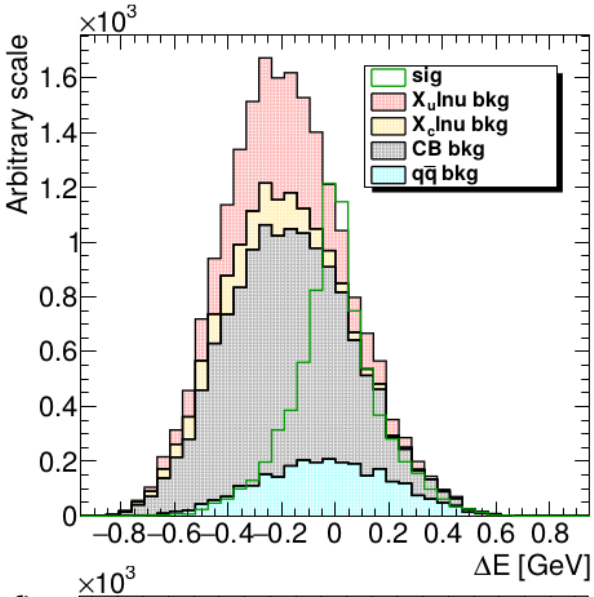

$\varepsilon(B^0_{tag}) = 0.33\%$ **Belle II** $\varepsilon(B^+_{tag}) = 0.36\%$

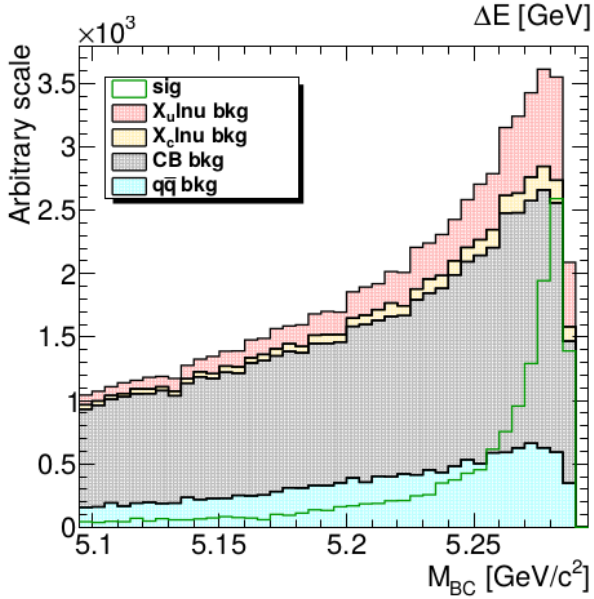
$\varepsilon(B^0_{tag}) = 0.19\%$ **Belle** $\varepsilon(B^+_{tag}) = 0.28\%$

Better tagging algorithm with significantly higher tagging reconstruction efficiency

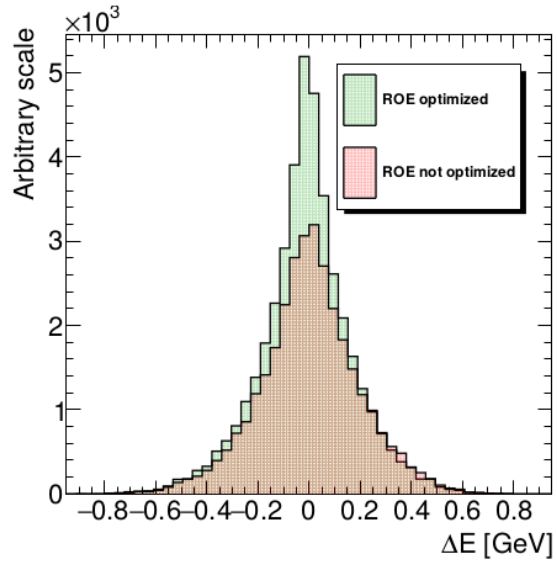
$B \to \pi \ell \nu$ efficiency compared to Belle tagged [Phys.Rev. D88 (2013) no.3, 032005]: 0.3 % $\to$ 0.55 %

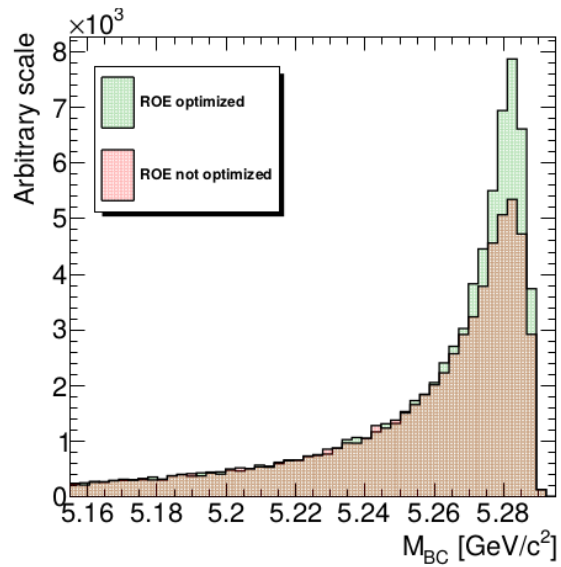
# $B \to \pi \ell \nu$ untagged at Belle II (MC study)




[B2TiP, to be published]




RestOfEvent (ROE): tracks and cluster not used in signal $B$ reco

Perform "clean-up" of ROE to discard duplicated tracks clusters from beam BKG

$B \to \pi \ell \nu$ efficiency compared to Belle untagged [Phys.Rev. D83 (2011) 071101]: 11 % $\to$ 20 %

# Error scaling

$$\sigma_{\text{tot}}(\mathcal{L}) = \sqrt{(\sigma^2_{\text{stat}}(\mathcal{L}_0) + \sigma^2_{\text{sysred}}(\mathcal{L}_0)) \times \frac{\mathcal{L}_0}{\mathcal{L}} + \sigma^2_{\text{sysirred}}(\mathcal{L}_0)}$$

| Source | Error (Limit) [%] Tagged [%] | Untagged |
|---|---|---|
| Tracking efficiency | 0.4 | 2.0 |
| Pion identification | – | 1.3 |
| Lepton identification | 1.0 | 2.4 |
| Kaon veto | 0.9 | – |
| Continuum description | 1.0 | 1.8 |
| Tag calibration and $N_{B\overline{B}}$ | 4.5 (2.0) | 2.0 (1.0) |
| $X_u\ell\nu$ cross-feed | 0.9 | 0.5 (0.5) |
| $X_c\ell\nu$ background | – | 0.2 (0.2) |
| Form factor shapes | 1.1 | 1.0 (1.0) |
| Form factor background | – | 0.4 (0.4) |
| Total (reducible, irreducible) | 5.0 (4.6, 2.0) | 4.5 (4.2, 1.6) |

Irreducible systematics

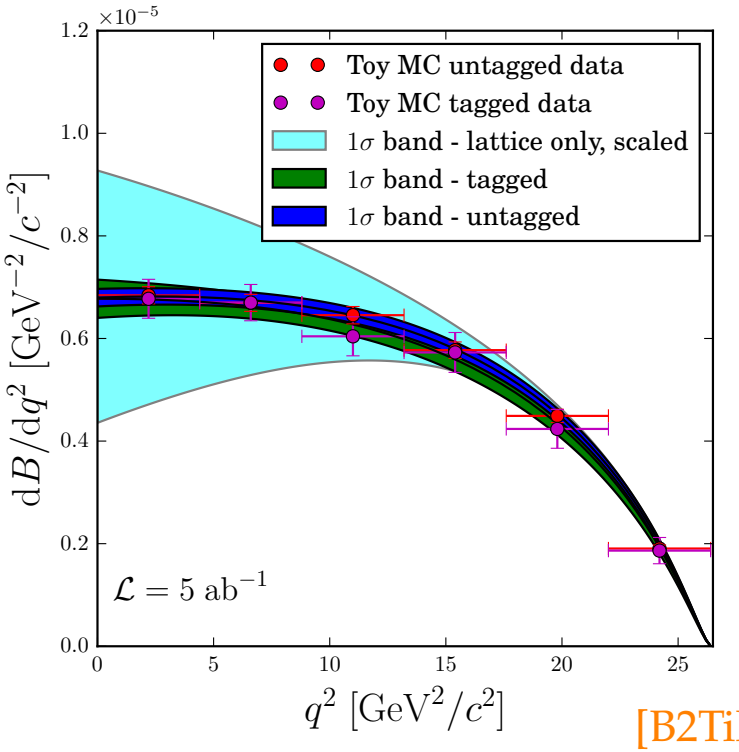
Belle II errors estimated from Belle

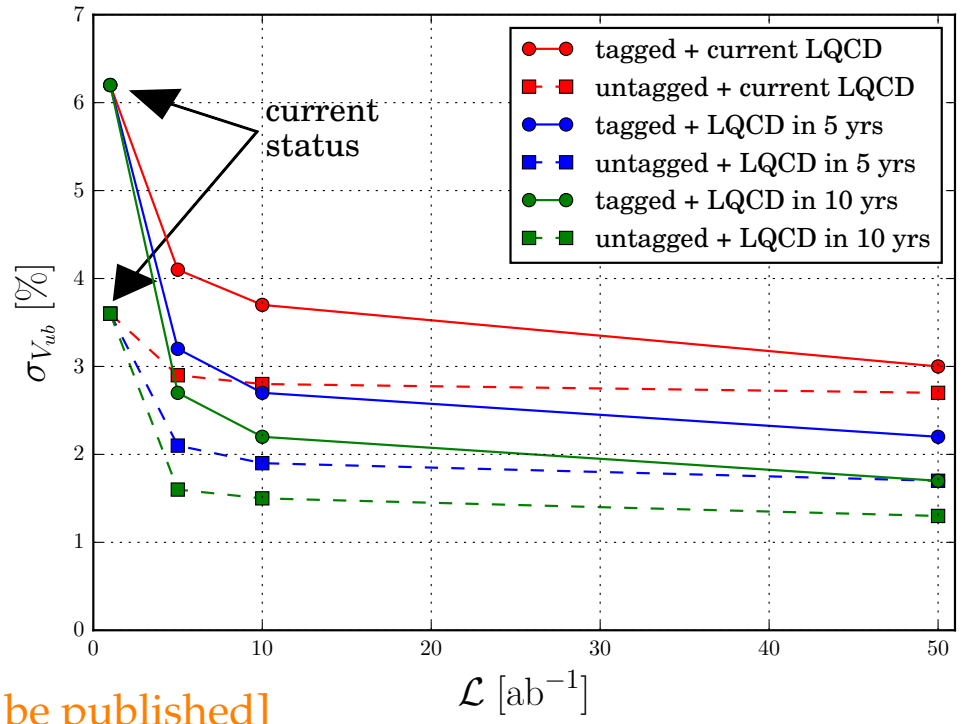
- perfect scaling of statistical errors
- systematics consist of reducible and irreducible
- limits on systematics
  2.0 % for tagged
  1.6 % for untagged

[B2TiP, to be published]

# $|V_{ub}|$ from $B \to \pi \ell \nu$ @ Belle II

Toy MC studies based on Belle II MC, LQCD forecasts estimated at 5 years (5, 10 ab$^{-1}$) and 10 years (50 ab$^{-1}$)




[B2TiP, to be published]

$|V_{ub}|^{\pi \ell \nu}$ from simultaneous fit for $\mathcal{L} = 5$ ab$^{-1}$, including lattice forecasts and error scaling.

$\delta_{|V_{ub}|^{\pi \ell \nu}}$ estimates for 5, 10 and 50 ab$^{-1}$:
Tagged: 3.2, 2.7 and 1.7 %
Untagged: 2.1, 1.9 and 1.3 %

LQCD forecasts: [A. Kronfeld, T. Kaneko, S. Simula]

# Belle II prospects for exclusive $|V_{ub}|$: $B \to (\rho, \omega)\ell\nu$

No extensive studies for these projections.
Possible to assume sample sizes in the future based on Belle (hadronic tag) @ 711 fb$^{-1}$ with efficiency improvements:

- $N_{\rho^0} = (621.7 \pm 35.0) \to\sim$ 80k ($\delta_{\text{stat}} \approx 0.5\%$) @ 50 ab$^{-1}$
- $N_{\rho^+} = (343.3 \pm 28.3) \to\sim$ 44k ($\delta_{\text{stat}} \approx 0.7\%$) @ 50 ab$^{-1}$
- $N_{\omega(3\pi)} = (96.7 \pm 14.5) \to\sim$ 12.5k ($\delta_{\text{stat}} \approx 1.3\%$) @ 50 ab$^{-1}$

- With such sample possible to do a full helicity angle analysis
- Also possible to check for right-handed currents
- Will contribute to better understanding of the $b \to u$ spectrum
- Can we expect lattice for these modes by then?

# Summary

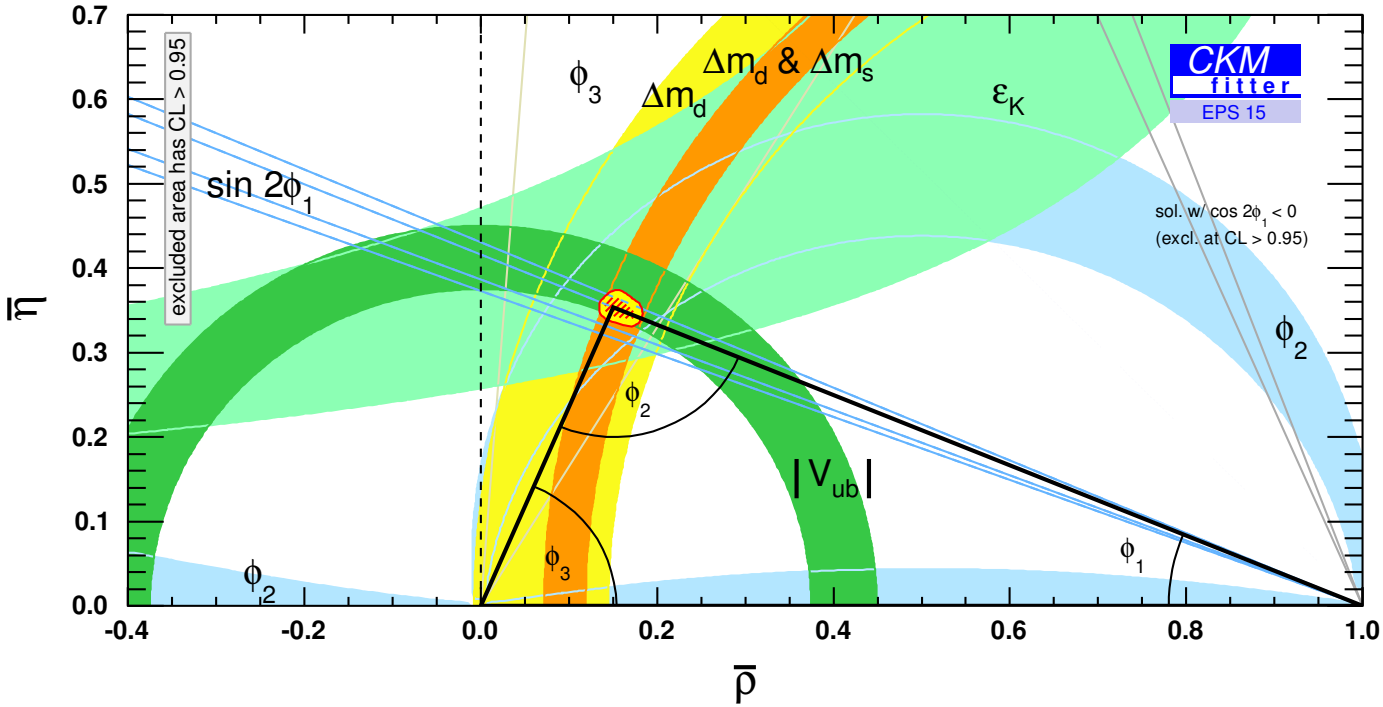
- Successful measurements of semileptonic $B$ decays at $B$ factories in the past
- Results for $B \to \pi\ell\nu$ consistent, current precision $\delta^{\pi}_{|V_{ub}|} \approx 4\ \%$
- Some tension between results for $B \to \rho\ell\nu$, results for $B \to \omega\ell\nu$ and $B \to \eta^{(\prime)}\ell\nu$ consistent
  - Very large errors for $|V_{ub}|$ in these channels: $\delta^{\rho,\omega,\eta}_{|V_{ub}|} > 10\ \%$
- Belle $B \to \eta^{(\prime)}\ell\nu$ measurement with had. tag to be submitted
- Expected $|V_{ub}|$ precision with Belle II dataset and LQCD forecasts for $B \to \pi\ell\nu$ channel:
  - Tagged: 1.7 %
  - Untagged: 1.3 %

Thank you!

# BACKUP

# Why $|V_{ub}|$?

## Best handle on NP!



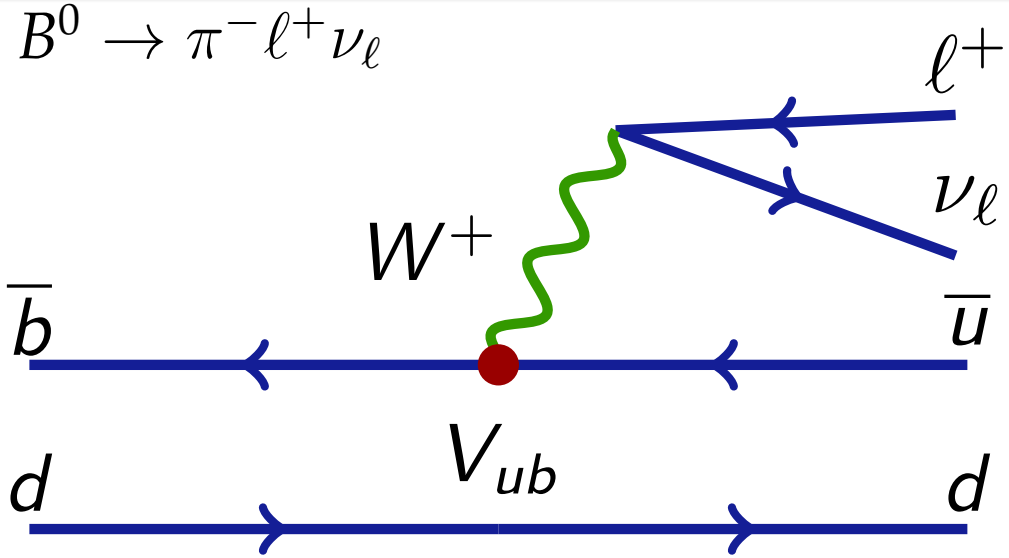

Precision from semi-leptonic decays: 3 – 4 %

- Era of searching for new physics (NP) → precision measurements
- $|V_{ub}|$ has **largest** error among unitarity triangle (UT) parameters

# Where to look for it?

- Answer:
  Charmless semileptonic $B$ meson decays ($b \to u\ell\nu$)
- Measurement:
  Partial branching ratio in $q^2$ region or spectrum needed
- Although the most precise, $B \to \pi\ell\nu$ only amounts to about 7 % of $B \to X_u\ell\nu$ decays

e.g. $B^0 \to \pi^-\ell^+\nu_\ell$




Until recently measurements of such decays only possible at $B$ factories

$$d\Gamma \propto G_F^2 |V_{ub}|^2 \, |L^\mu \langle \pi^- | u\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\bar{b}|B^0\rangle|^2$$

# Form factor calculations

Low $\longleftarrow$ $q^2$ region $\longrightarrow$ High

**LCSR**

- Low $q^2$ region ($q^2 <$ 6-7 GeV$^2$), mostly at $q^2 = 0$
- Unperturbative
- For pseudoscalar an vector decays

**LQCD**

- Intermediate to high $q^2$ region ($q^2 > 14$ GeV$^2$)
- Unquenched (quark-loops in QCD vacuum incorporated)
- For a limited set of decays, hard to figure out for complex states

# $|V_{ub}|$ extraction

- Need to extrapolate theory input to a certain or full $q^2$ region
- Model dependent/independent: Whether the model makes any assumptions regarding FF shape

## Calculation from $\Delta\mathcal{B}$

- Measure partial branching ratio in a $q^2$ region
- Calculate reduced branching ratio in same region

$$|V_{ub}|^2 = \frac{\Delta\mathcal{B}(q^2_{min}, q^2_{max})}{\tau_B \Delta\zeta(q^2_{min}, q^2_{max})}$$

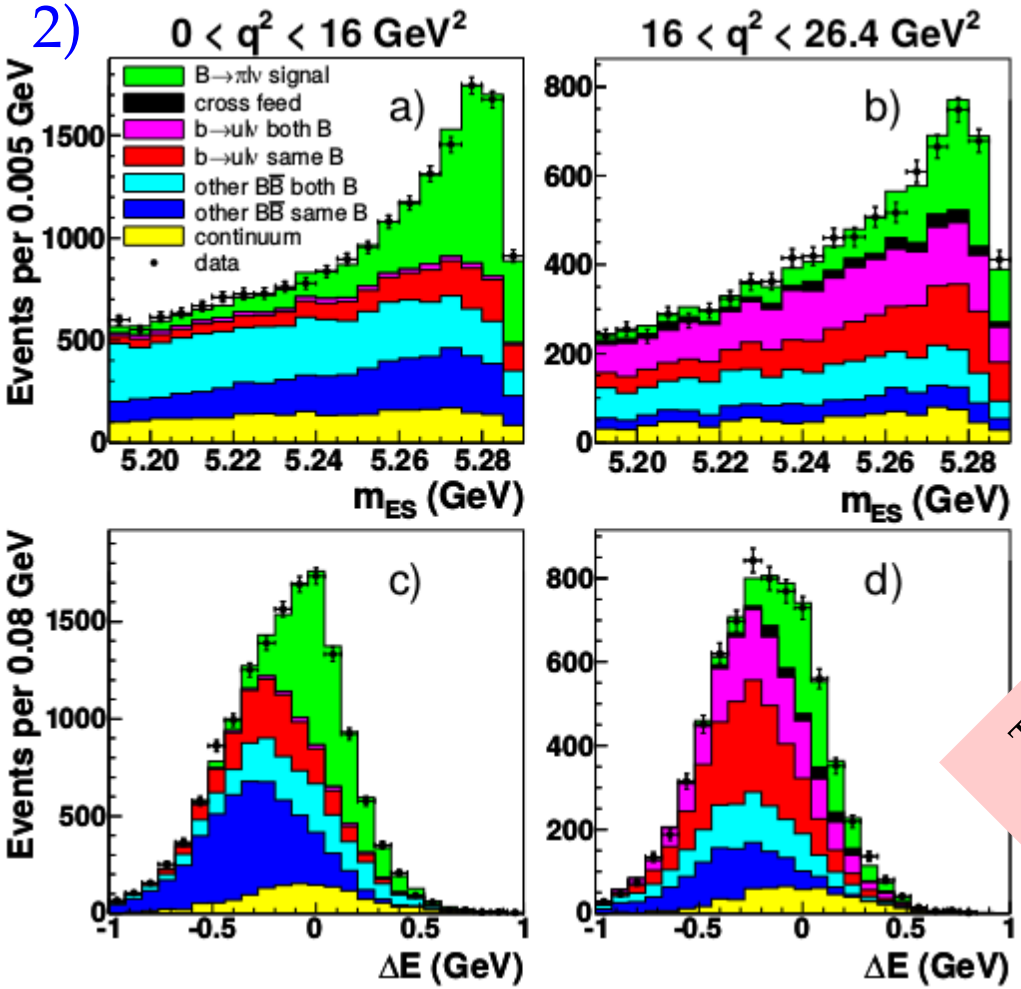
## Simultaneous fit to data and theory

- Measure $\Delta\mathcal{B}/\Delta q^2$ spectrum in bins of $q^2$
- Extract from simultaneous fit to data (shape + scale) and theory input (shape) by minimizing

$$\chi^2 = \chi^2_{data} + \chi^2_{theory}$$

# $B \to \pi \ell^+ \nu$ untagged at *BaBar*

2)




Other $b \to u$ BKG at high $q^2$

- 1) Phys.Rev. D83 (2011) 032007
- Data sample
- Combine
  $N_{sig} \approx$
- $d\mathcal{B}$ in 6
- ...ev. D86 (2012)
- ...ta sample: 416.1 fb$^{-1}$
- Combined signal
  $N_{sig} = 12.5\text{k} \pm 400$
- $d\mathcal{B}/dq^2$ spectrum in 12 bins

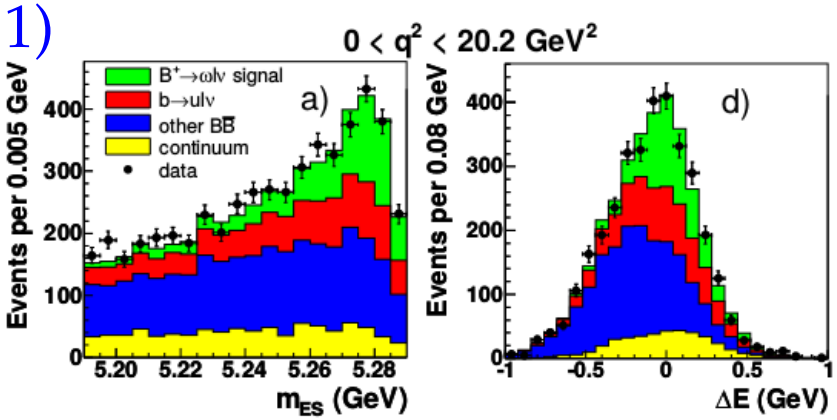
The present analysis of the $B^0 \to \pi^- \ell^+ \nu$ decay channel makes use of the full BaBar data set compared to only a subset in 1).

$B^+, B^0$ combined result from 2):

$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.45 \pm 0.04_{\text{stat}} \pm 0.06_{\text{sys}}) \times 10^{-4}$$

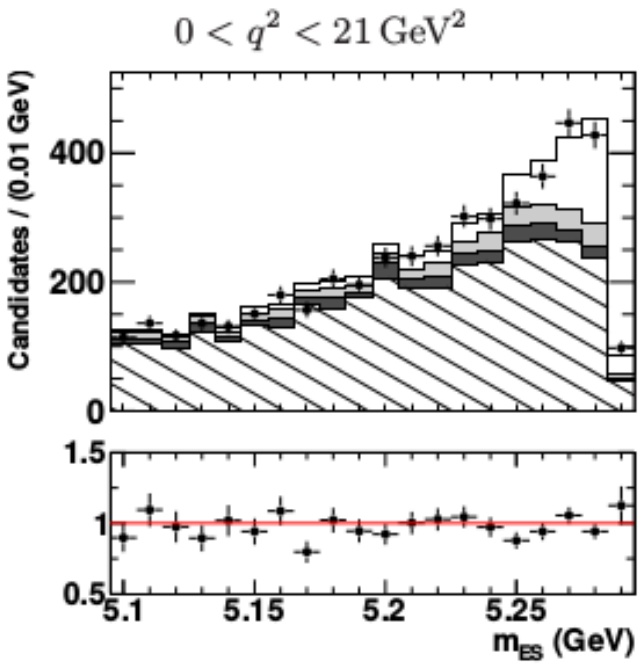
# $B^+ \to \omega \ell^+ \nu$ untagged at *BaBar*

1)




$\mathcal{B}(B^+ \to \omega \ell^+ \nu) = (1.19 \pm 0.16_{\text{stat}} \pm 0.09_{\text{sys}}) \times 10^{-4}$

2)




$\mathcal{B}(B^+ \to \omega \ell^+ \nu) = (1.21 \pm 0.14_{\text{stat}} \pm 0.08_{\text{sys}}) \times 10^{-4}$

- 1) Phys.Rev. D[illegible] (2012) 092004
- Data sar[illegible] $^{-1}$
- Comb[illegible] $N_$[illegible]
- [illegible] 5

The principal difference between this analysis and the previous ones is that the combinatorial-$\omega$ background is taken from the sideband of the data $m_{3\pi}$ distribution rather than from MC simulation.

- [illegible]. D87 (2013) [illegible]004
- [illegible]sample: 426 fb$^{-1}$
- [illegible]mbined signal $N_{sig} \approx 1.1\text{k} \pm 100$
- $d\mathcal{B}/dq^2$ spectrum in 5 bins

# Belle II prospects for exclusive $|V_{ub}|$: $B \to \pi\ell\nu$

## Tagged $B \to \pi\ell\nu$

- Efficiency: 0.3 % $\to$ 0.55 %
- $\sigma^{exp}_{\Delta\mathcal{B}/\Delta q^2}$ scaling with $\mathcal{L}$: 2.7, 2.4 and 2.1 % for 5, 10 and 50 ab$^{-1}$
- Irreducible systematics: 2.0 %
- $|V_{ub}|$ precision: $\delta_{|V_{ub}|} = 3.2$, 2.7 and 1.7 % for 5, 10 and 50 ab$^{-1}$

## Untagged $B \to \pi\ell\nu$

- Efficiency: 12 % $\to$ 20 %
- $\sigma^{exp}_{\Delta\mathcal{B}/\Delta q^2}$ scaling with $\mathcal{L}$: 2.2, 1.9 and 1.7 % for 5, 10 and 50 ab$^{-1}$
- Irreducible systematics: 1.6 %
- $|V_{ub}|$ precision: $\delta_{|V_{ub}|} = 2.1$, 1.9 and 1.3 % for 5, 10 and 50 ab$^{-1}$

Projections include lattice forecasts. [A. Kronfeld, T. Kaneko, S. Simula]