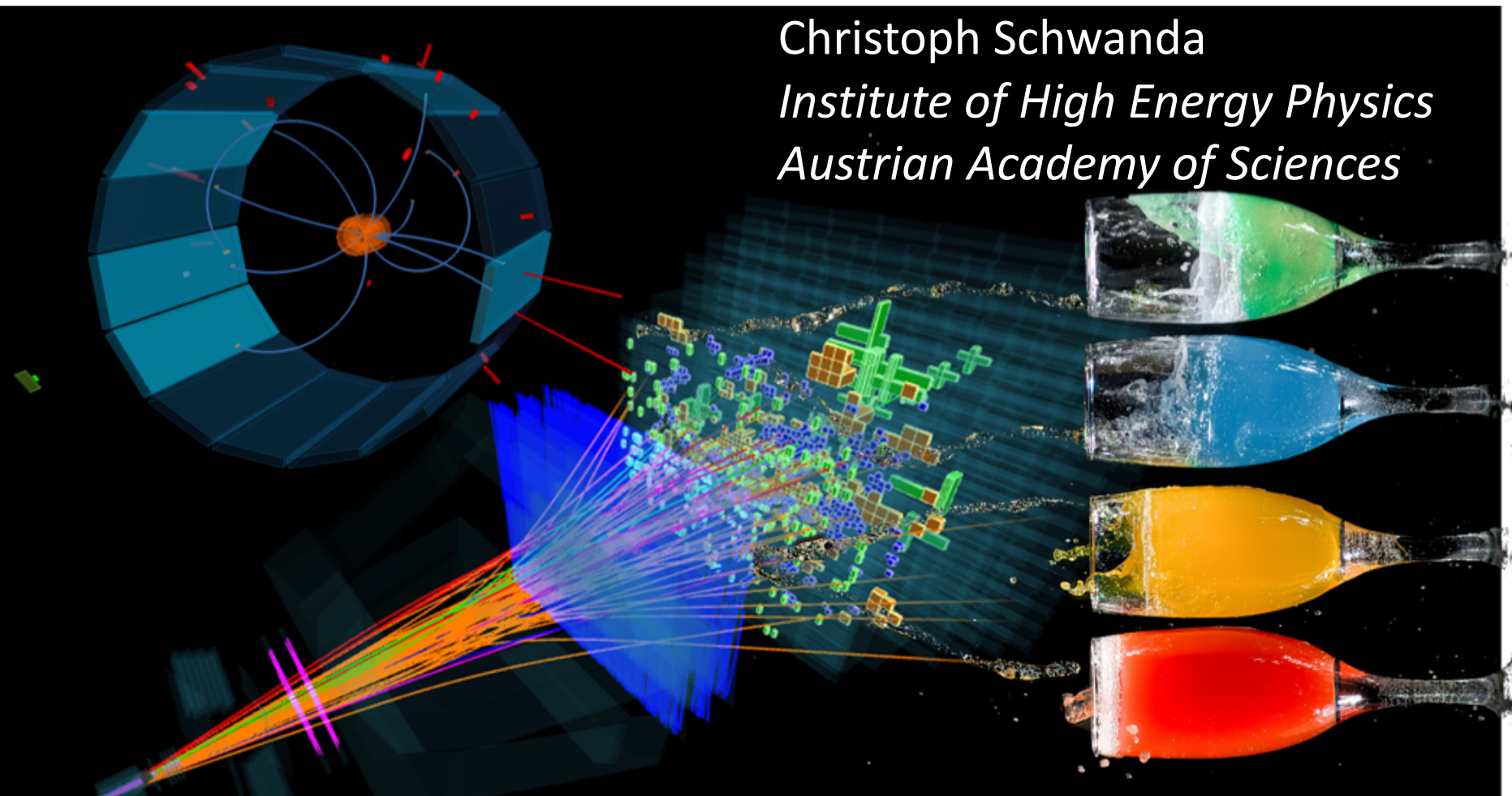


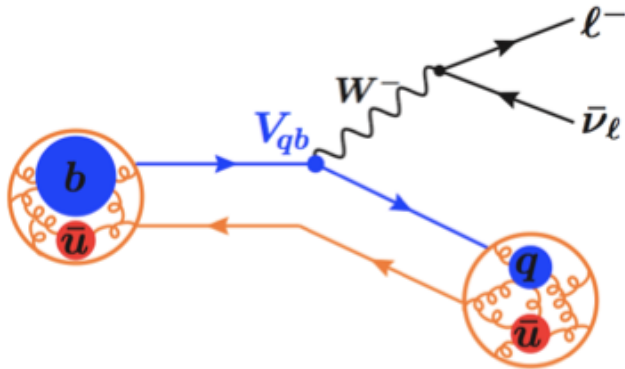
$|V_{cb}|$ at Belle/Belle II

Christoph Schwanda
*Institute of High Energy Physics
Austrian Academy of Sciences*



Flavor 2019: New Physics in Flavor from LHC to Belle II
Munich Institute for Astro- and Particle Physics, May 20-22, 2019

$|V_{cb}|$ from semileptonic B decays



$$d\Gamma \propto G_F^2 |V_{qb}|^2 |L_\mu \langle X | \bar{q} \gamma_\mu P_L b | B \rangle|^2$$

	Experiment	Theory
Exclusive V_{cb}	$B \rightarrow D\ell\nu, D^*\ell\nu$ (low backgrounds)	Lattice QCD, light cone sum rules
Inclusive V_{cb}	$B \rightarrow X\ell\nu$ (higher background)	Operator product expansion

HFLAV 2016:

$|V_{cb}|$ excl ($D^*\ell\nu$):
(39.05 +/- 0.75) x 10⁻³

$|V_{cb}|$ incl:
(42.19 +/- 0.78) x 10⁻³

Let's look at $B \rightarrow D^{(*)}l\nu$

$$w = \frac{P_B \cdot P_{D^{(*)}}}{m_B m_{D^{(*)}}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

$$B \rightarrow D^*l\nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_{D^*}^3}{48\pi^3} (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \chi(w) \mathcal{F}^2(w) |V_{cb}|^2$$

$$B \rightarrow Dl\nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} \mathcal{G}^2(w) |V_{cb}|^2$$

Form factor parameterizations

- Caprini, Lellouch, Neubert [Nucl.Phys. B530, 153(1998)]

$B \rightarrow D^* | \nu$

$$h_{A_1}(w) = h_{A_1}(1) [1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3],$$
$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

$B \rightarrow D | \nu$

$$\mathcal{G}(z) = \mathcal{G}(1) (1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3)$$

Parameters: $F(1), \rho^2, R_1(1), R_2(1)$
 $G(1), \rho^2$

- Boyd, Grinstein, Lebed [Phys. Rev. Lett. 74, 4603 (1995)]

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^N a_{i,n} z^n, \quad z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

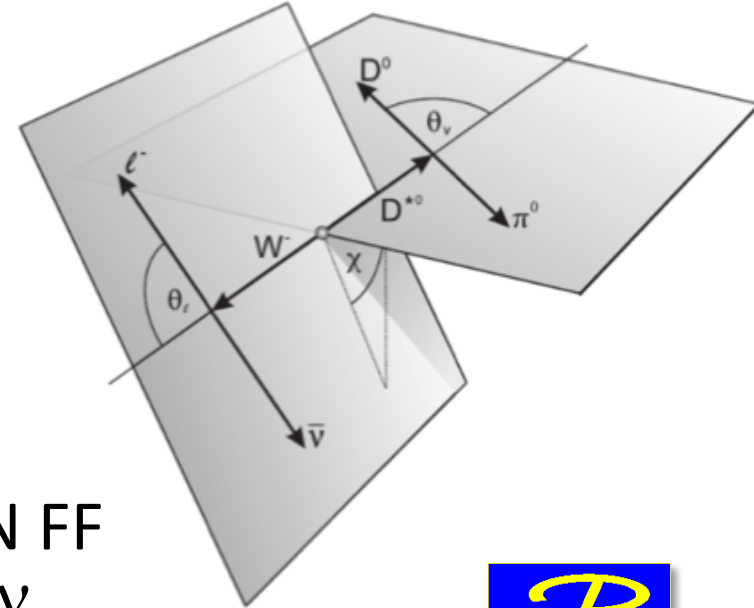
Parameters: coefficients $a_{i,n}$
Order N of the expansion a priori not defined by theory

Belle $B \rightarrow D^* l \nu$ hadronic tag [\[arXiv:1702.01521\]](https://arxiv.org/abs/1702.01521)

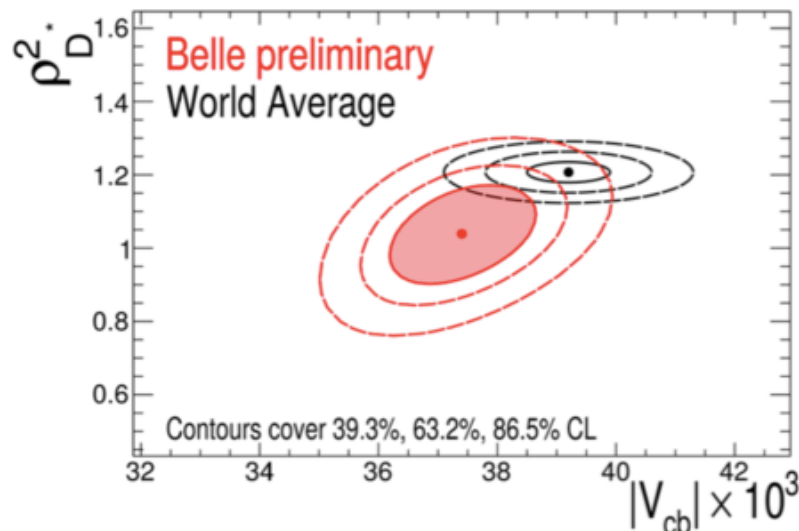
- CLN FF parameterization
[NPB 530, 153 (1998)]

$$\frac{d^4\Gamma(B \rightarrow D^* l \nu)}{dw d\cos\theta_v d\cos\theta_\ell d\chi} = f(|V_{cb}|^2, \underbrace{\rho_{D^*}^2, R_1, R_2}_{\text{form factor parameters}})$$

$$W = \frac{m_B^2 - m_{D^*}^2 - q^2}{m_B m_{D^*}}$$



- Belle measured $|V_{cb}|$ and the CLN FF parameters in the decay $B \rightarrow D^* l \nu$



Parameter	Measurement	World average
$ V_{cb} \cdot 10^3$	37.4 ± 1.2	39.2 ± 0.7
$\rho_{D^*}^2$	1.04 ± 0.13	1.20 ± 0.03
R_1	1.38 ± 0.07	1.40 ± 0.03
R_2	0.86 ± 0.10	0.85 ± 0.02

CLN parametrisation

Parameterization dependence?

- D. Bigi, P. Gambino, S. Schacht, Phys. Lett. B769 (2017) 441

BGL Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	27.9/32	31.4/35
$ V_{cb} $	0.0417 $\left(\begin{smallmatrix} +20 \\ -21 \end{smallmatrix}\right)$	0.0404 $\left(\begin{smallmatrix} +16 \\ -17 \end{smallmatrix}\right)$
a_0^f	0.01223(18)	0.01224(18)
a_1^f	-0.054 $\left(\begin{smallmatrix} +58 \\ -43 \end{smallmatrix}\right)$	-0.052 $\left(\begin{smallmatrix} +27 \\ -15 \end{smallmatrix}\right)$
a_2^f	0.2 $\left(\begin{smallmatrix} +7 \\ -12 \end{smallmatrix}\right)$	1.0 $\left(\begin{smallmatrix} +0 \\ -5 \end{smallmatrix}\right)$
$a_1^{\mathcal{F}1}$	-0.0100 $\left(\begin{smallmatrix} +61 \\ -56 \end{smallmatrix}\right)$	-0.0070 $\left(\begin{smallmatrix} +54 \\ -52 \end{smallmatrix}\right)$
$a_2^{\mathcal{F}1}$	0.12 (10)	0.089 $\left(\begin{smallmatrix} +96 \\ -100 \end{smallmatrix}\right)$
a_0^g	0.012 $\left(\begin{smallmatrix} +11 \\ -8 \end{smallmatrix}\right)$	0.0289 $\left(\begin{smallmatrix} +57 \\ -37 \end{smallmatrix}\right)$
a_1^g	0.7 $\left(\begin{smallmatrix} +3 \\ -4 \end{smallmatrix}\right)$	0.08 $\left(\begin{smallmatrix} +8 \\ -22 \end{smallmatrix}\right)$
a_2^g	0.8 $\left(\begin{smallmatrix} +2 \\ -17 \end{smallmatrix}\right)$	-1.0 $\left(\begin{smallmatrix} +20 \\ -0 \end{smallmatrix}\right)$

CLN Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	34.3/36	34.8/39
$ V_{cb} $	0.0382 (15)	0.0382 (14)
$\rho_{D^*}^2$	1.17 $\left(\begin{smallmatrix} +15 \\ -16 \end{smallmatrix}\right)$	1.16 (14)
$R_1(1)$	1.391 $\left(\begin{smallmatrix} +92 \\ -88 \end{smallmatrix}\right)$	1.372 (36)
$R_2(1)$	0.913 $\left(\begin{smallmatrix} +73 \\ -80 \end{smallmatrix}\right)$	0.916 $\left(\begin{smallmatrix} +65 \\ -70 \end{smallmatrix}\right)$
$h_{A_1}(1)$	0.906 (13)	0.906 (13)

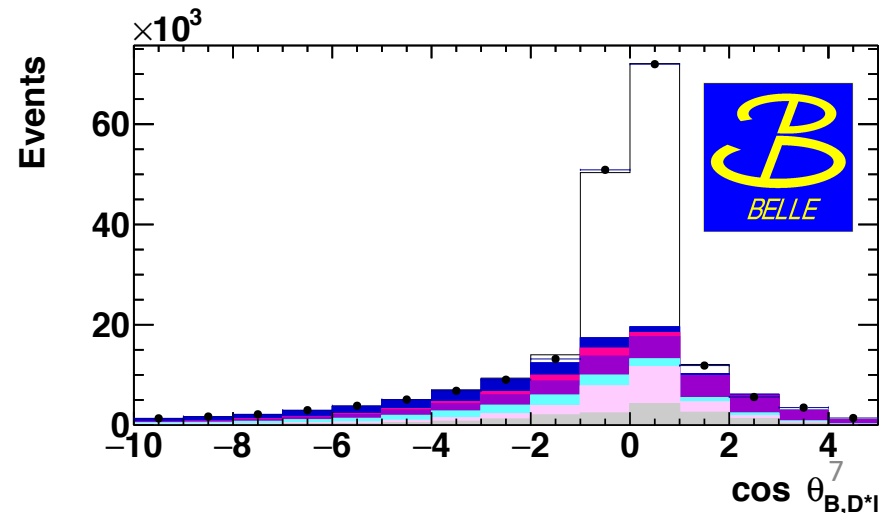
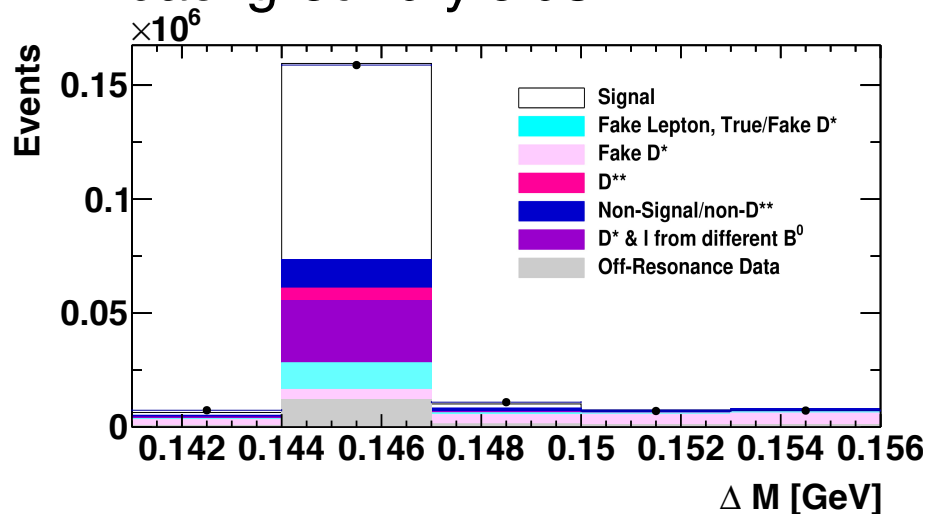
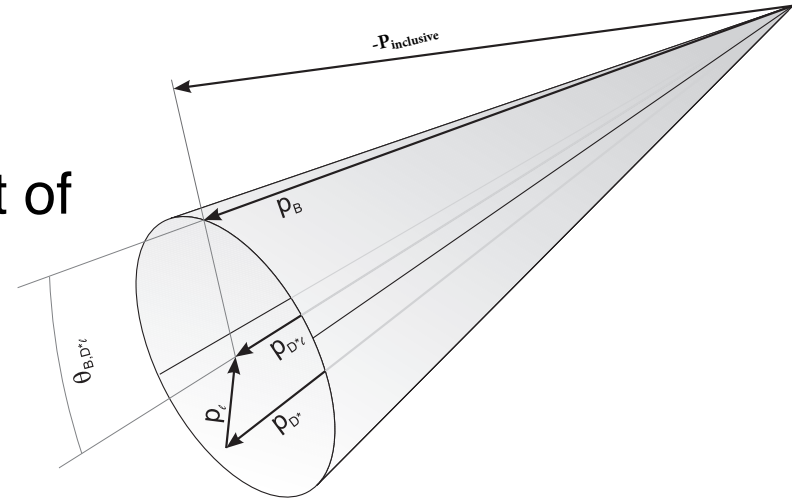
- B. Grinstein, A. Kobach, Phys. Lett. B771 (2017) 359

$$|V_{cb}| = (37.4 \pm 1.3) \times 10^{-3} \quad (\text{CLN})$$

$$|V_{cb}| = (41.9 \pm_{-1.9}^{+2.0}) \times 10^{-3} \quad (\text{BGL})$$

Belle $B^0 \rightarrow D^{*-} l^+ \nu$ untagged [arXiv:1809.03290] subm.to PRD

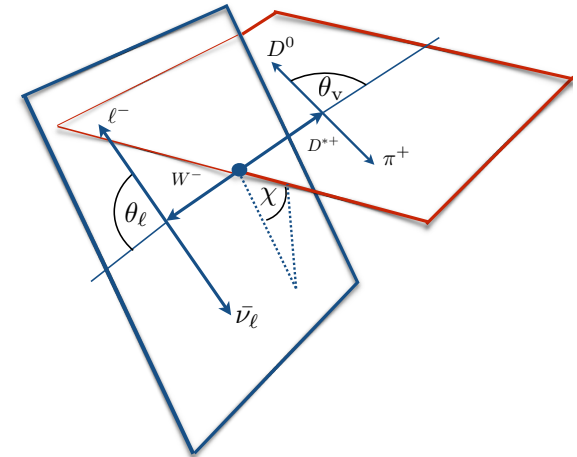
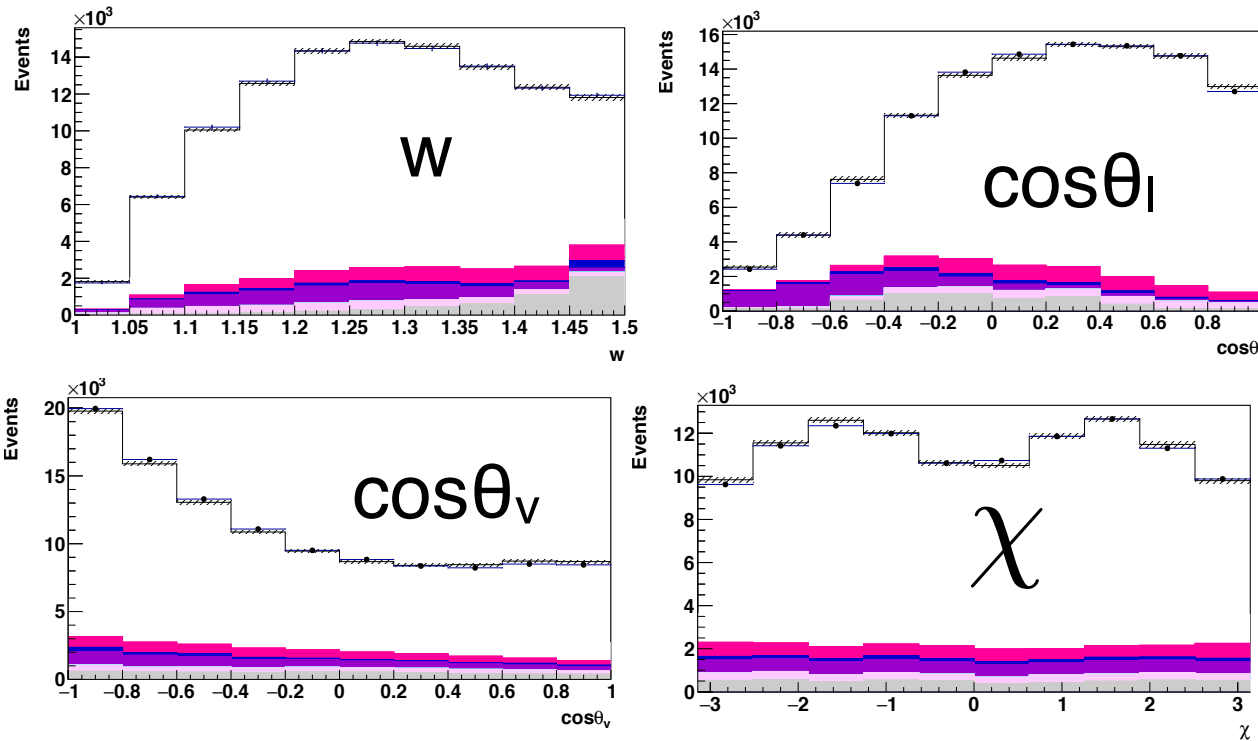
- Measure $|V_{cb}|$ using **Belle** 711fb^{-1} .
- Signal Selection using
 - 3D - Binned Maximum Likelihood fit of
 - $(\cos\theta_{B,D^*l})$
 - $\Delta M = \text{mass}(D^* - D^0)$
 - lepton momentum
- Float Signal & Backgrounds components from MC to extract background yields



CLN: FF parameters and $|V_{cb}|$



Simultaneous fit of 1D projections of w , $\cos\theta_l$, $\cos\theta_v$, χ to extract ρ^2 , $R_1(1)$, $R_2(1)$ and $F(1)|V_{cb}|$



- Signal
- Fake Lepton, True/Fake D^*
- Fake D^*
- D^{**}
- Non-Signal/non- D^{**}
- D^* & l from different B^0
- Off-Resonance Data

$$\rho^2 = 1.106 \pm 0.031 \pm 0.007$$

$$R_1(1) = 1.229 \pm 0.028 \pm 0.009$$

$$R_2(1) = 0.852 \pm 0.021 \pm 0.006$$

$$F(1)|V_{cb}|\eta_{EW} \times 10^3 = 35.1 \pm 0.2 \pm 0.6$$

BGL: FF parameters and $|V_{cb}|$

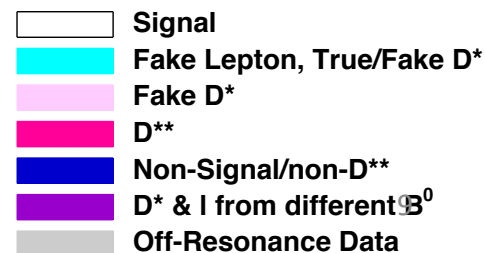
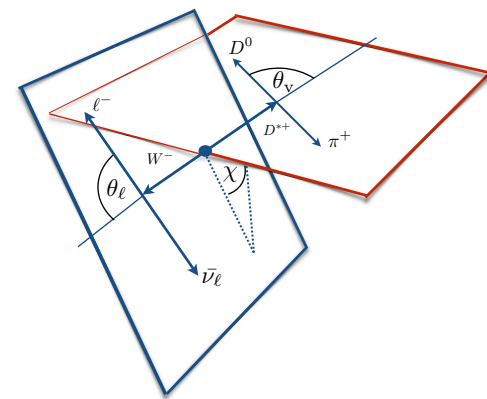
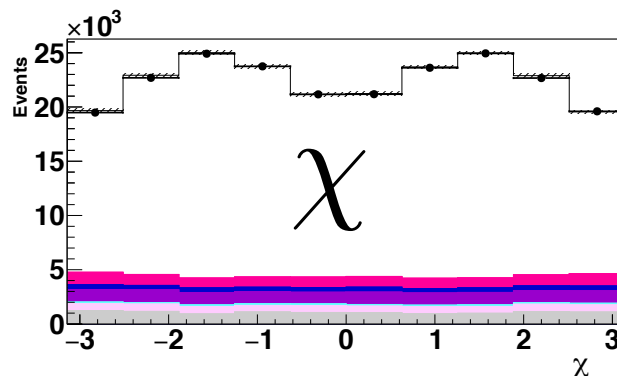
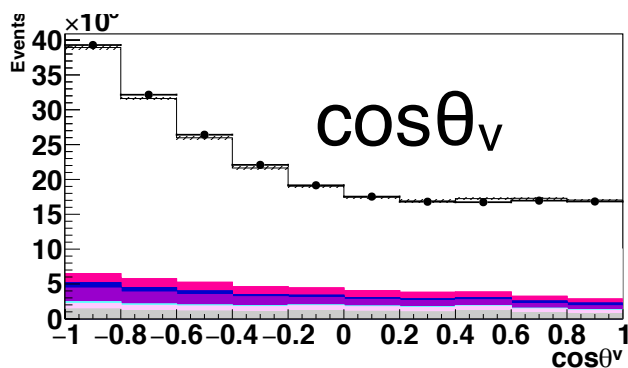
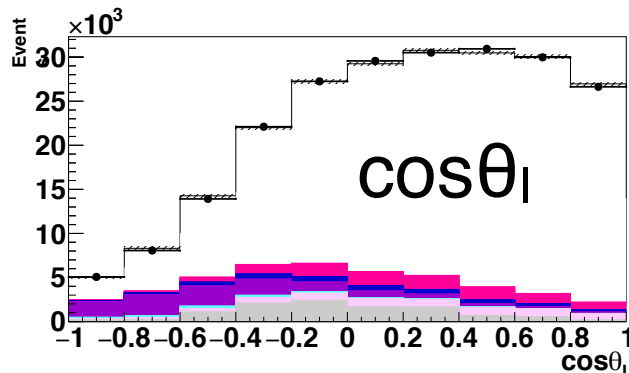
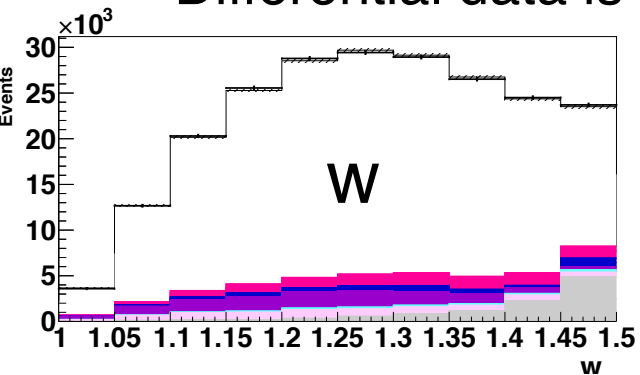


Simultaneous fit of 1D projections of w , $\cos\theta_l$, $\cos\theta_v$, χ to extract the coefficients of the BGL expansion (up to 3rd order) and $F(1)|V_{cb}|$

$$F(1)|V_{cb}|\eta_{EW} \times 10^3 = 34.9 \pm 0.2 \pm 0.6$$

- Consistent with CLN
- Differential data is provided

$$\begin{aligned} \tilde{a}_0^f \times 10^3 &= -0.506 \pm 0.004 \pm 0.008, \\ \tilde{a}_1^f \times 10^3 &= -0.65 \pm 0.17 \pm 0.09, \\ \tilde{a}_1^{F1} \times 10^3 &= -0.270 \pm 0.064 \pm 0.023, \\ \tilde{a}_2^{F1} \times 10^3 &= +3.27 \pm 1.25 \pm 0.45, \\ \tilde{a}_0^g \times 10^3 &= -0.929 \pm 0.018 \pm 0.013, \end{aligned}$$



Summary $B \rightarrow D^* l \nu$

CLN

	$ V_{cb} \times 10^3$	
Belle tagged 2018	37.4 +/- 1.2	arXiv:1702.01521
Belle untagged 2018	38.4 +/- 0.6	arXiv:1809.03290, subm. to PRD
HFLAV 2016	39.1 +/- 0.7	Eur.Phys.J. C77 (2017) 895

BGL

	$ V_{cb} \times 10^3$	
Belle tagged 2018	41.7 +/- 2.0	arXiv:1702.01521, PLB769 (2017) 441
Belle untagged 2018	38.3 +/- 0.8	arXiv:1809.03290, subm. to PRD
BaBar tagged 2019	38.4 +/- 0.9	arXiv:1903.10002, subm. to PRL

Talk by
Biplab Dey
yesterday

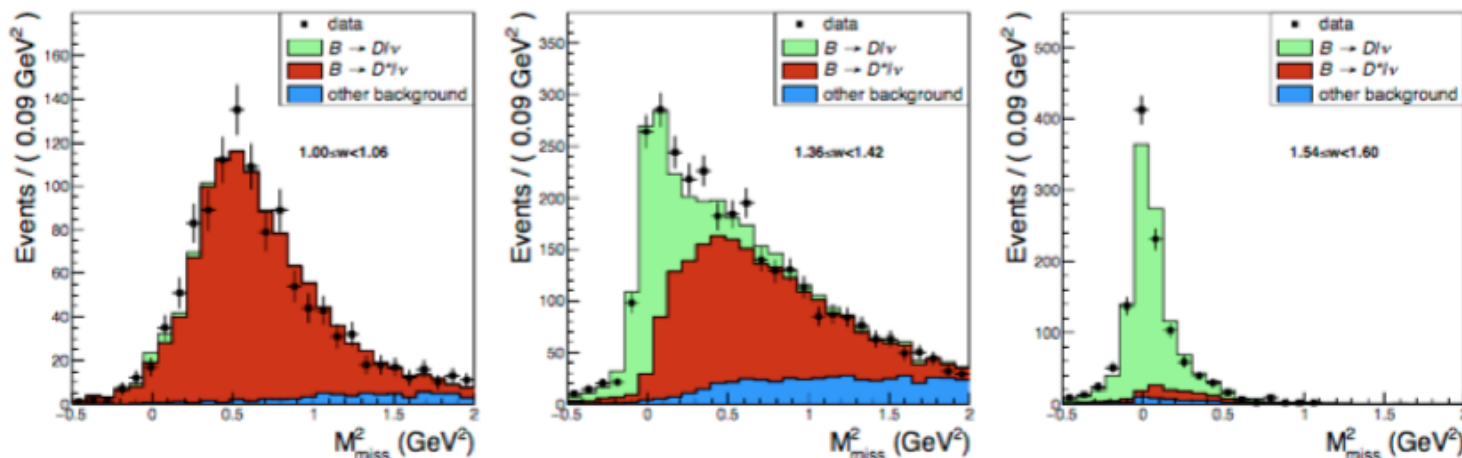
Recent data does not support the FF parameterization dependence of $|V_{cb}|$ excl!!!

B \rightarrow Dlv at Belle

[R. Glattauer, CS, Phys. Rev. D93, 032006 (2016)]

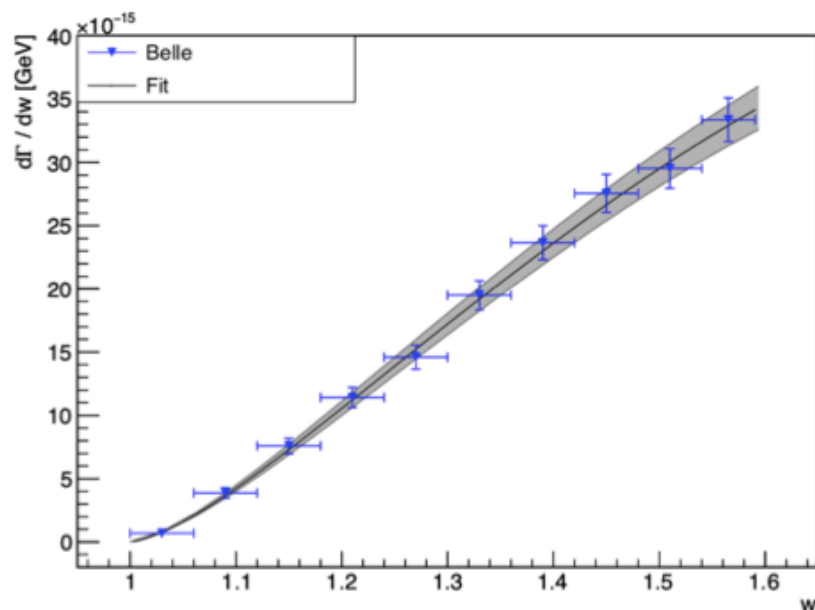


- 711/fb of Belle Y(4S) data
- Full reconstruction of one B (hadronic tag)
- 10 D^+ and 13 D^0 modes are used on the signal side, covering 28.9% and 40.1% of the width
- Signal extraction from M_{miss}^2 in 10 bins of w
- 16,992 \pm 192 signal events
(5150 \pm 95 neutral, 11,843 \pm 167 charged B events)



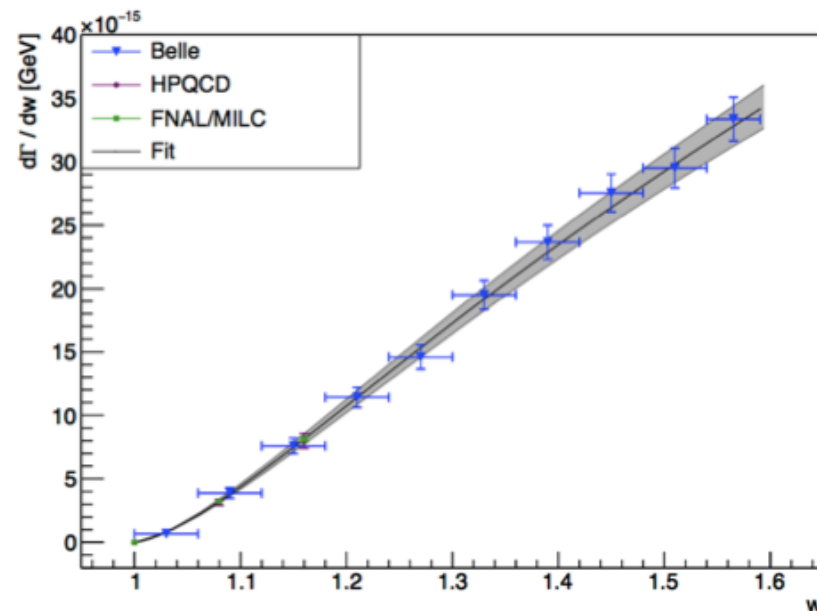
Belle 2016: CLN vs. BGL fit with lattice data

CLN



$$|V_{cb}| = (39.9 \pm 1.3) \times 10^{-3}$$

BGL



Vs.

$$|V_{cb}| = (40.8 \pm 1.1) \times 10^{-3}$$

$|V_{cb}|$ from inclusive decays

$$B \rightarrow Xlv \quad \Gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left(1 + \frac{c_5(\mu) \langle O_5 \rangle(\mu)}{m_b^2} + \frac{c_6(\mu) \langle O_6 \rangle(\mu)}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \right)$$

- Based on the Operator Product Expansion (OPE)
- $\langle O_i \rangle$: hadronic matrix elements (non-perturbative)
 c_i : coefficients (perturbative)
- **Parton-hadron duality** \rightarrow the hadronic ME depend only on the initial state

	Kinetic [JHEP 1109 (2011) 055]	1S [PRD70, 094017 (2004)]
$O(1)$	m_b, m_c	m_b
$O(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$O(1/m_b^3)$	ρ_D^3, ρ_{LS}^3	ρ_1, τ_{1-3}

Moments of the E_ℓ and M_X^2 spectrum

Also other observables in $B \rightarrow X\ell\nu$ can be expanded into an OPE with the same heavy quark parameters, e.g.,

- The n^{th} moment of the (truncated) lepton energy spectrum

$$R_n(E_{\text{cut}}, \mu) = \int_{E_{\text{cut}}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} dE_\ell, \quad \langle E_\ell^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)}$$

- The n^{th} moment of the (truncated) M_X^2 spectrum

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dm_X^2}$$

Master plan:

- Measure the quark masses and heavy quark parameters using moments
- Substitute them in the formula of the semileptonic width
- Determine $|V_{cb}|$ from the semileptonic branching fraction

Two sets of theoretical calculations

- “Kinetic running mass”
 - P. Gambino, N. Uraltsev, Eur. Phys. J. C34, 181 (2004)
 - P. Gambino, JHEP 1109 (2011) 055
 - A. Alberti, P. Gambino, K.J. Healey, S. Nandi, Phys. Rev. Lett. 114, 061802 (2015)
- “1S mass”
 - C. Bauer, Z. Ligeti, M. Luke, A. Manohar, M. Trott, Phys. Rev. D70, 094017 (2004)
- Non-perturbative parameters in the $1/m_b$ expansion

	Kinetic	1S
$O(1)$	m_b, m_c	m_b
$O(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$O(1/m_b^3)$	ρ_D^3, ρ_{LS}^3	ρ_1, τ_{1-3}

Data used in $b \rightarrow c$ inclusive analyses

BaBar	$\langle E^n_l \rangle$: $n=0,1,2,3$ [PRD 69, 111104 (2004), PRD 81, 032003 (2010)] $\langle M^{2n}_X \rangle$: $n=1,2, 3$ [PRD 81, 032003 (2010)]
Belle	$\langle E^n_l \rangle$: $n=0,1,2,3$ [PRD 75, 032001 (2007)] $\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 75, 032005 (2007)]
CDF	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 71, 051103 (2005)]
CLEO	$\langle M^{2n}_X \rangle$: $n=1,2$ [PRD 70, 032002 (2004)] $\langle E^n_\gamma \rangle$: $n=1$ [PRL 87, 251807 (2001)]
DELPHI	$\langle E^n_l \rangle$: $n=1,2,3$ $\langle M^{2n}_X \rangle$: $n=1,2$ [EPJ C45, 35 (2006)]

- 23 measurements from BaBar, 15 measurements from Belle, 12 from other experiments
- Newest measurement is from the year 2010!

Kinetic scheme analysis

HFLAV

Summer 2016

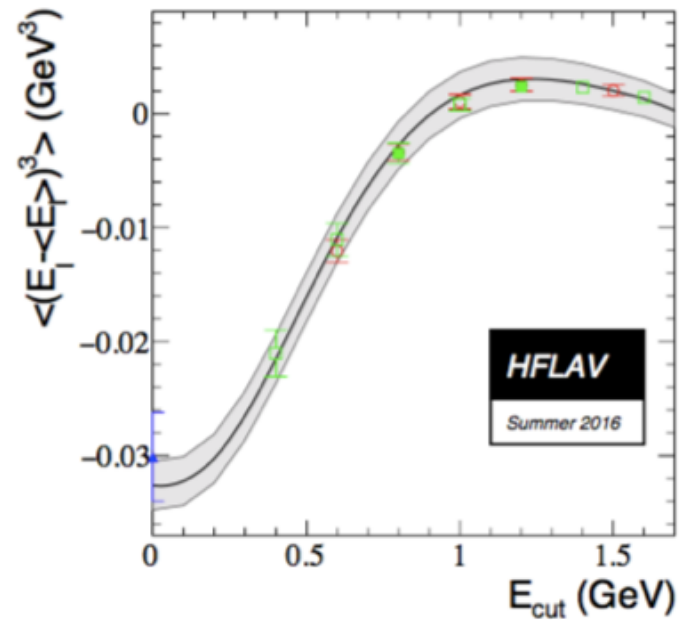
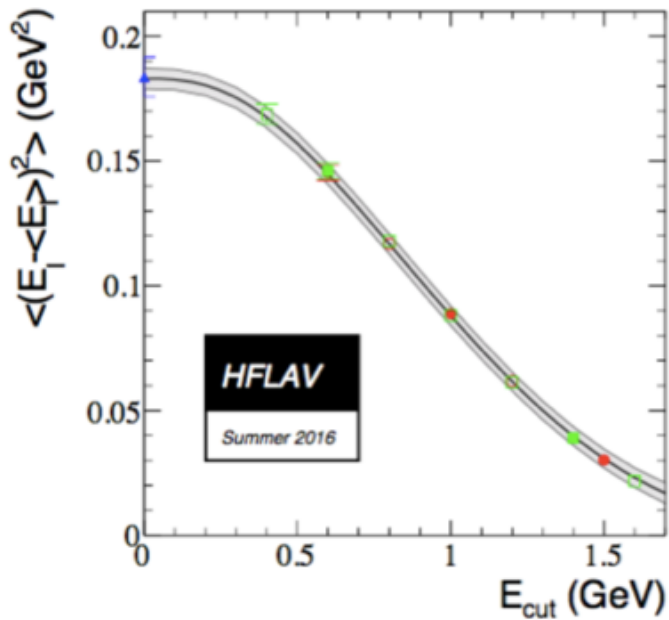
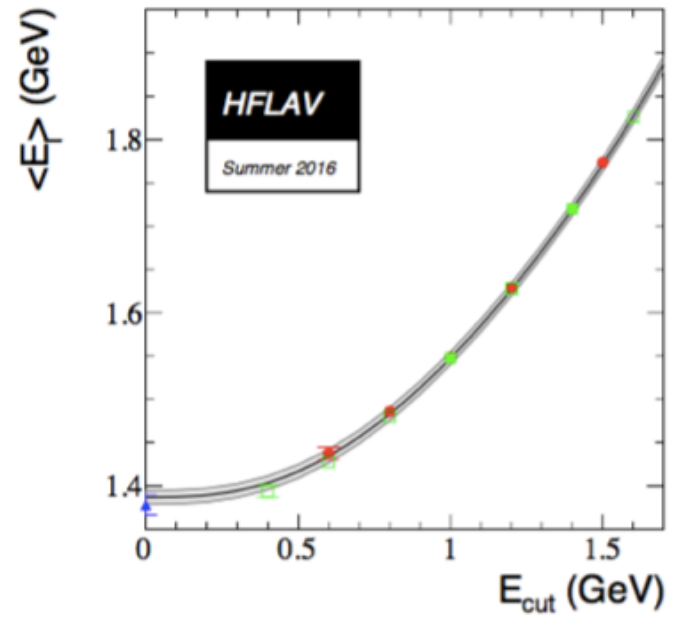
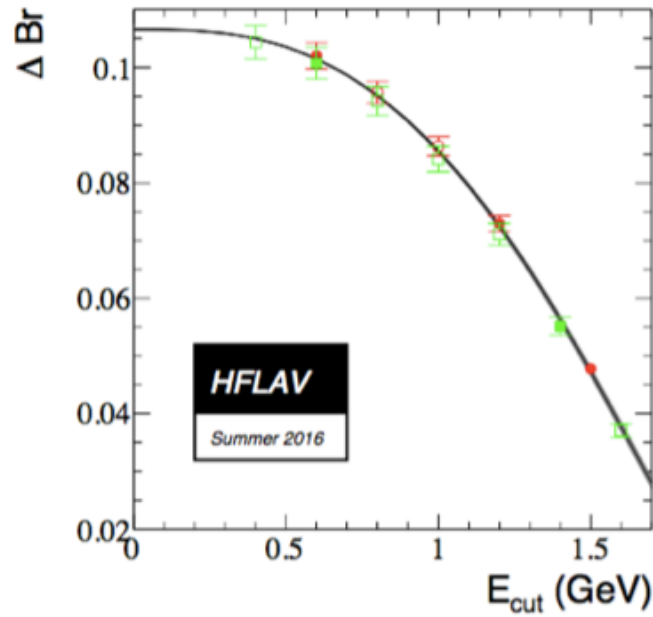
	$ V_{cb} $ [10^{-3}]	m_b^{kin} [GeV]	$m_c^{\overline{\text{MS}}}$ [GeV]	μ_π^2 [GeV^2]	ρ_D^3 [GeV^3]	μ_G^2 [GeV^2]	ρ_{LS}^3 [GeV^3]
value	42.19	4.554	0.987	0.464	0.169	0.333	-0.153
error	0.78	0.018	0.015	0.076	0.043	0.053	0.096
$ V_{cb} $	1.000	-0.257	-0.078	0.354	0.289	-0.080	-0.051
m_b^{kin}		1.000	0.769	-0.054	0.097	0.360	-0.087
$m_c^{\overline{\text{MS}}}$			1.000	-0.021	0.027	0.059	-0.013
μ_π^2				1.000	0.732	0.012	0.020
ρ_D^3					1.000	-0.173	-0.123
μ_G^2						1.000	0.066
ρ_{LS}^3							1.000

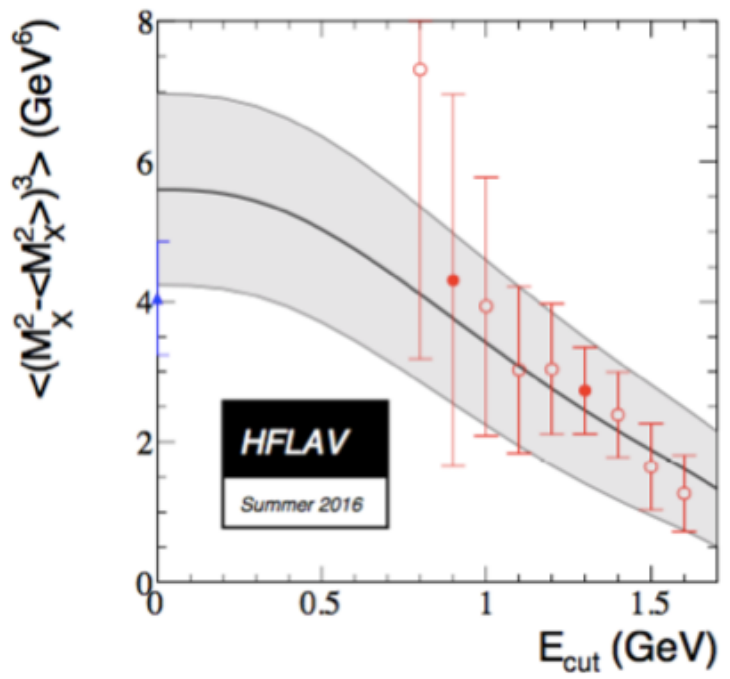
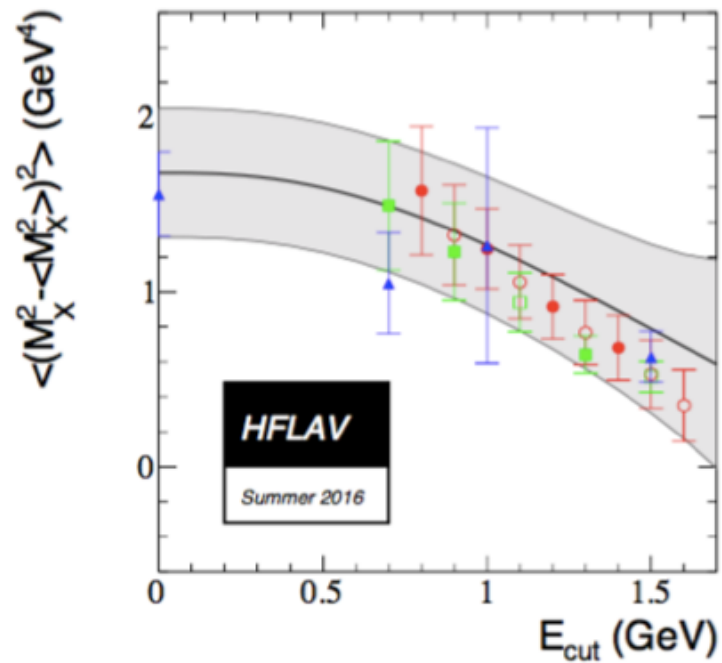
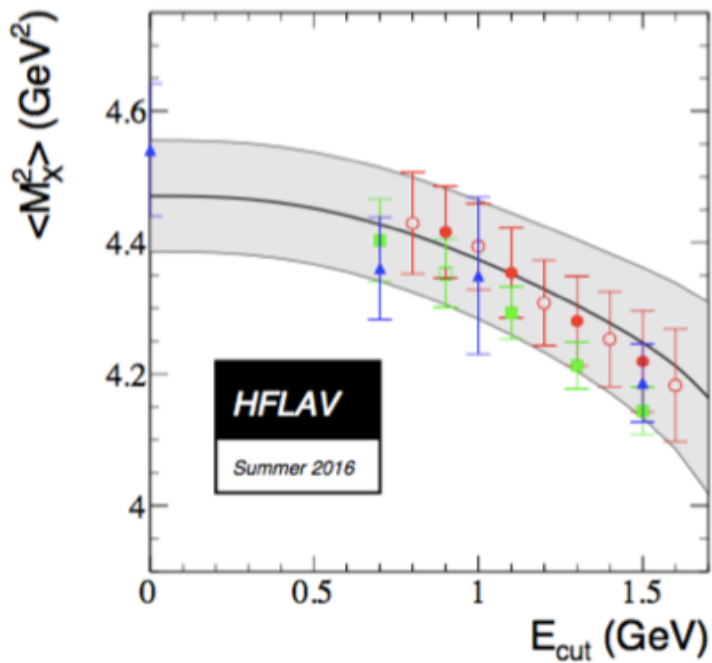
$$\mathcal{B}(\overline{B} \rightarrow X_c \ell^- \bar{\nu}_\ell) = (10.65 \pm 0.16)\%$$

χ^2 of 15.6 for 43 degrees of freedom.

- c quark mass constraints $m_c^{\overline{\text{MS}}}(3 \text{ GeV}) = 0.986 \pm 0.013 \text{ GeV}$
- Average B lifetime: (1.579 +/- 0.004) ps

- BaBar
- Belle
- ▲ Other



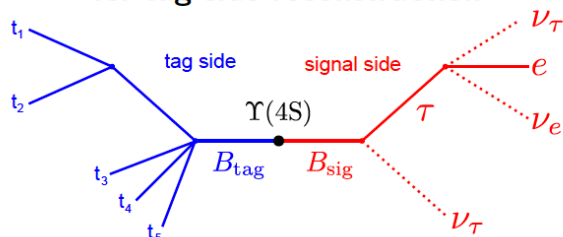


- BaBar
- Belle
- ▲ Other

Outlook to Belle II – improved tagging

1807.08680

New Full Event Interpretation (FEI) algorithm for tag-side reconstruction

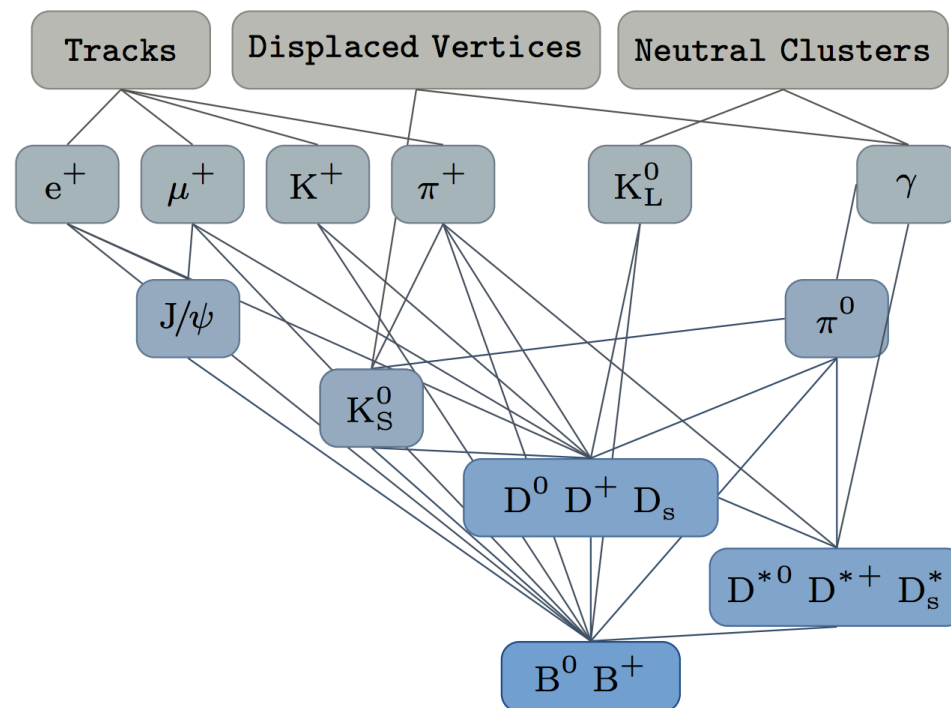


- > 5000 B decays modes reconstructed
- O(200) particle decay channels for training
- Output is candidate-wise **signal probability**

Tagging ε on MC

Tag	FR ¹	FEI Belle	FEI Belle II
Hadronic B^+	0.28%	0.76%	0.66%
SL B^+	0.67%	1.80%	1.45%
Hadronic B^0	0.18%	0.46%	0.38%
SL B^0	0.63%	2.04%	1.94%

¹Belle Full Reconstruction algorithm.



Talk by Lu Cao yesterday

Outlook to Belle II (2)

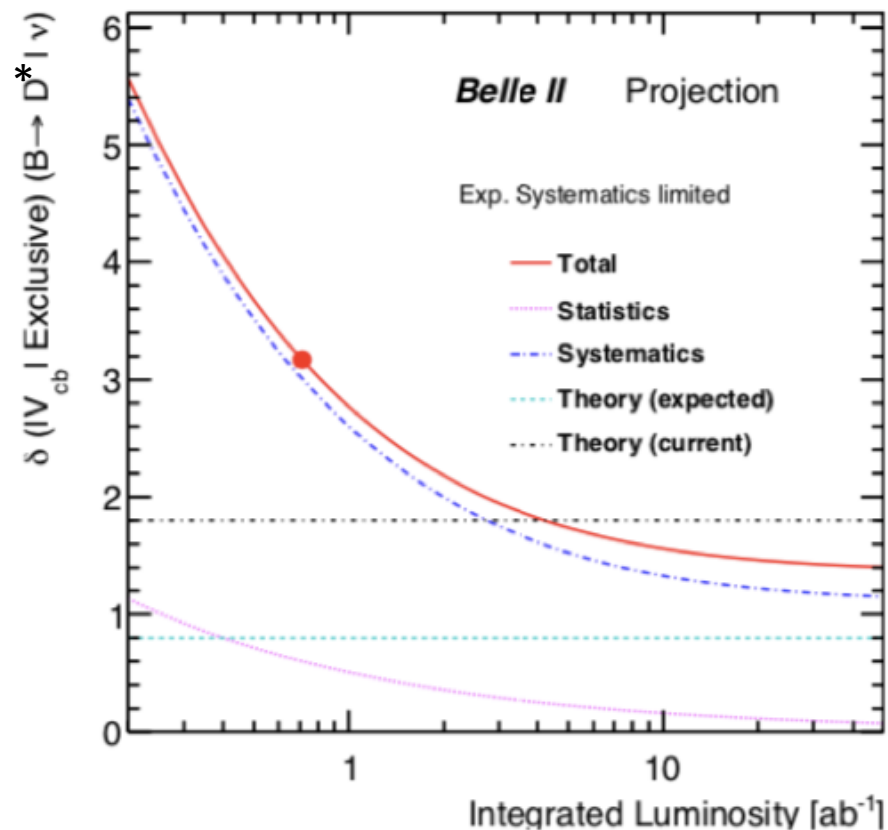
- $|V_{cb}|$ exclusive

	ab ⁻¹	Stat.	Syst		ΣExp	Theory	Σ
			(Red.,Irred.)				
$D^* \ell^- \bar{\nu}_\ell$	0.7	0.6%	(2.8, 1.1)%		3.1%	1.8%	3.6%
	5	0.2%	(1.1, 1.1)%		1.5%	1.0%	1.8%
	50	0.1%	(0.3, 1.1)%		1.2%	0.8%*	1.4%
$D \ell^- \bar{\nu}_\ell$	0.4	4.5%	(3.1, 1.2)%		5.6%	2.2%	6.0%
	5	1.3%	(0.9, 1.2)%		2.0%	1.5%*	2.7%
	50	0.6%	(0.4, 1.2)%		1.4%	1.0%*	1.7%

K. Lieret @ CKM 2018

- $|V_{cb}|$ inclusive

- Experimental correlations between input measurements (only partially accounted for in the current analysis)



Summary and outlook

- After the excitement in 2017/2018, recent data for $B \rightarrow D^* l \nu$ seem to indicate that there is no significant form FF parameterization dependence in the determination of $|V_{cb}|$ exclusive
 - $|V_{cb}|$ exclusive fit values reported by Belle [[arXiv:1809.03290](#)] and BaBar [[arXiv:1903.10002](#)] are consistent with the previous HFLAV 2016 average
 - The $2-3\sigma$ -ish discrepancy with $|V_{cb}|$ inclusive remains
- Experimental $D(^*)l\nu$ data does not constrain the FF slope around zero recoil well [[arXiv:1905.08209](#)]
→ lattice input for $D^*l\nu$ at non-zero recoil needed
- New experimental input to the $|V_{cb}|$ inclusive analysis might also provide insights

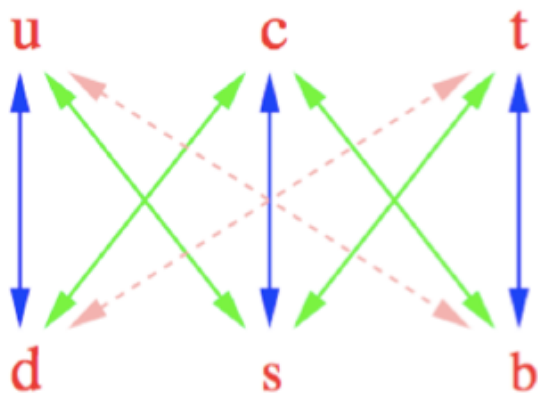
Backup

Cabibbo-Kobayashi-Maskawa quark mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \mathbf{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\mathbf{V} \mathbf{V}^\dagger = \mathbf{V}^\dagger \mathbf{V} = 1$$



- The unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix transforms the flavour eigenstates into the physical quark states
- The CKM element magnitudes determine the possible quark flavour transitions in charged current processes

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W_\mu^\pm + \text{h.c.}$$

CP violation

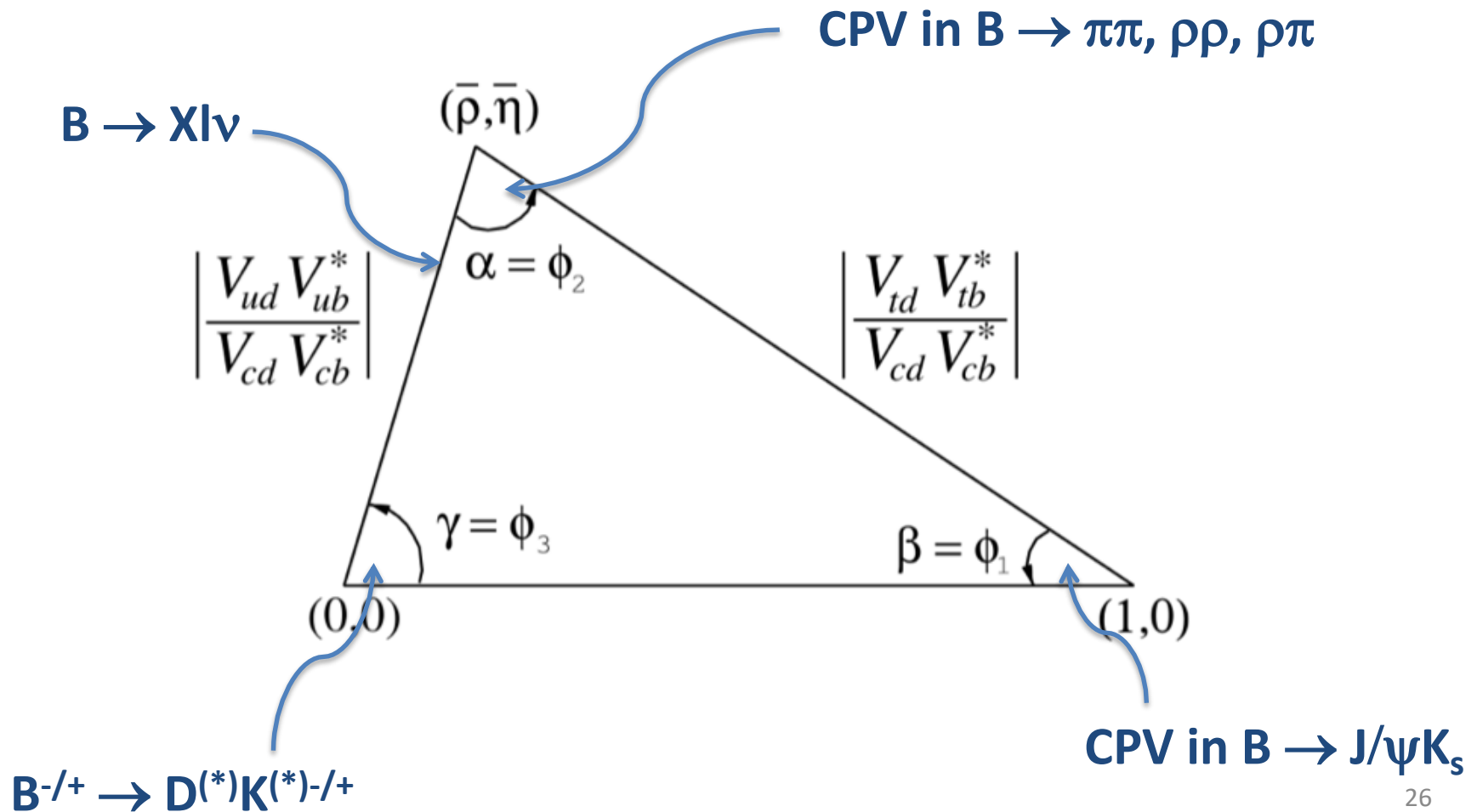
$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

Wolfenstein parametrization of V_{CKM}

- However, V_{CKM} also contains a complex phase, responsible for all CP-violating phenomena in the SM
- CPV established ($>5\sigma$) in 17 observables (in K and B physics)
→ extremely constrained system
- New physics would typically disturb the SM pattern of CPV

The CKM unitarity triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



1999 – 2010: B factory at KEK (Japan)

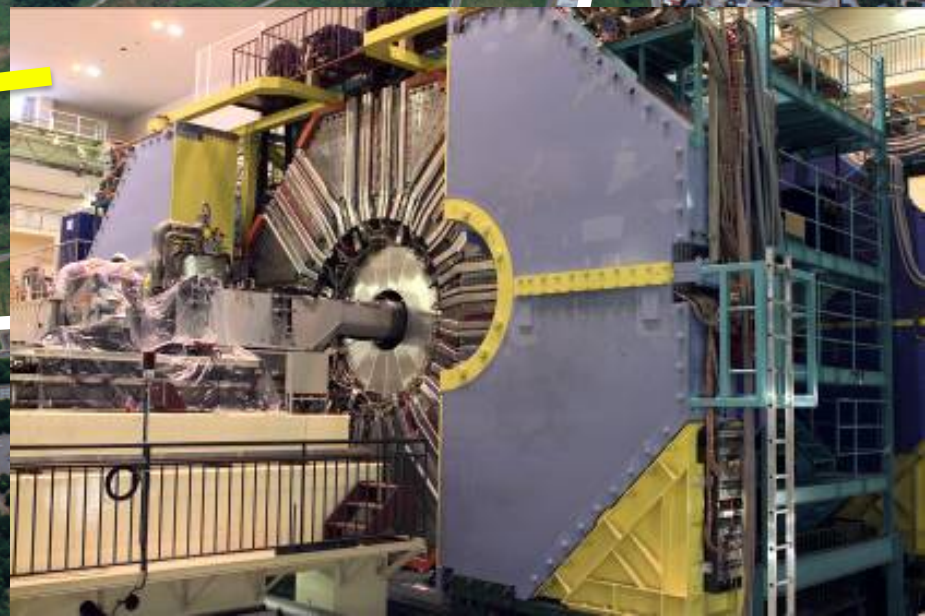


KEKB double
ring e^+e^- collider

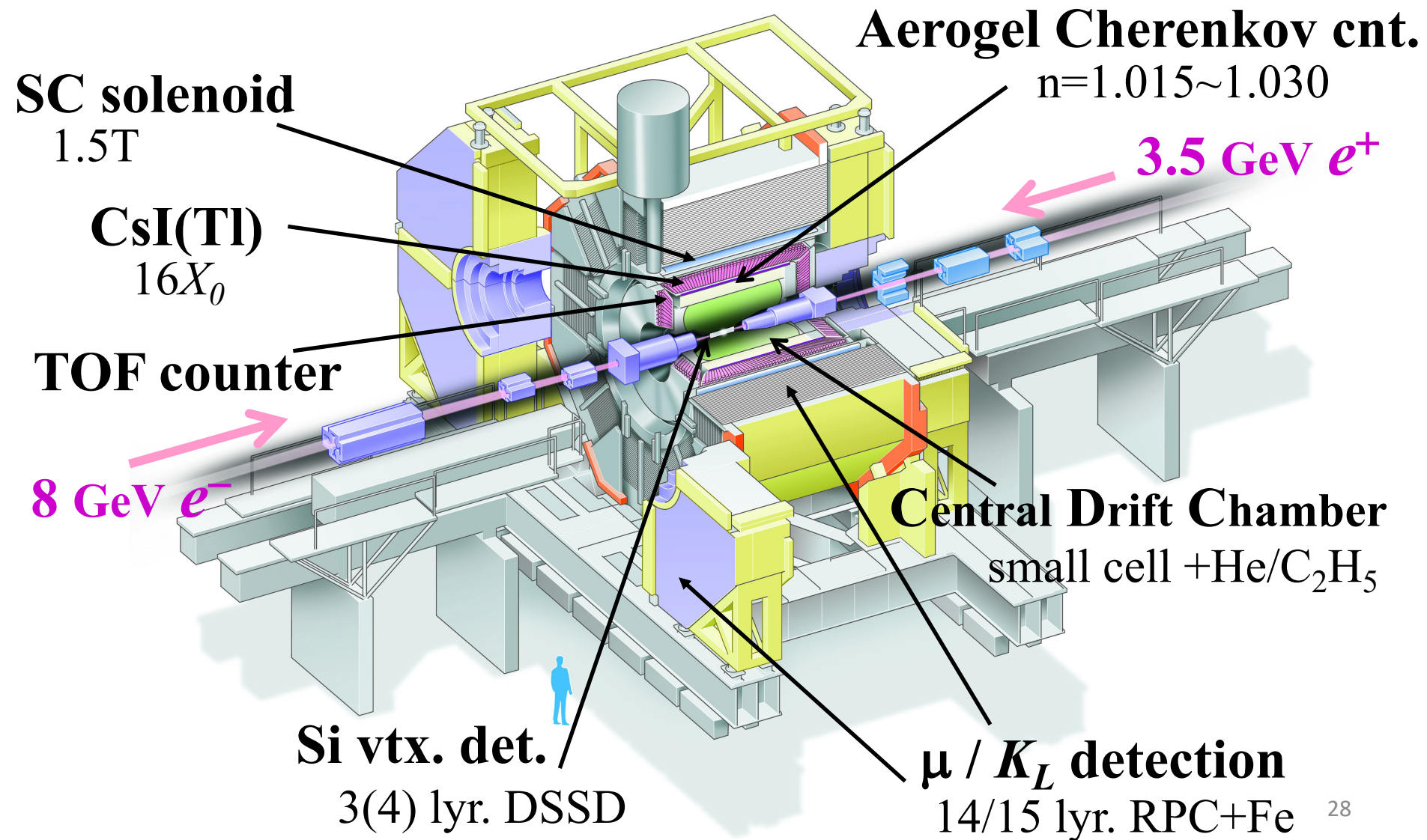
Linac

$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$

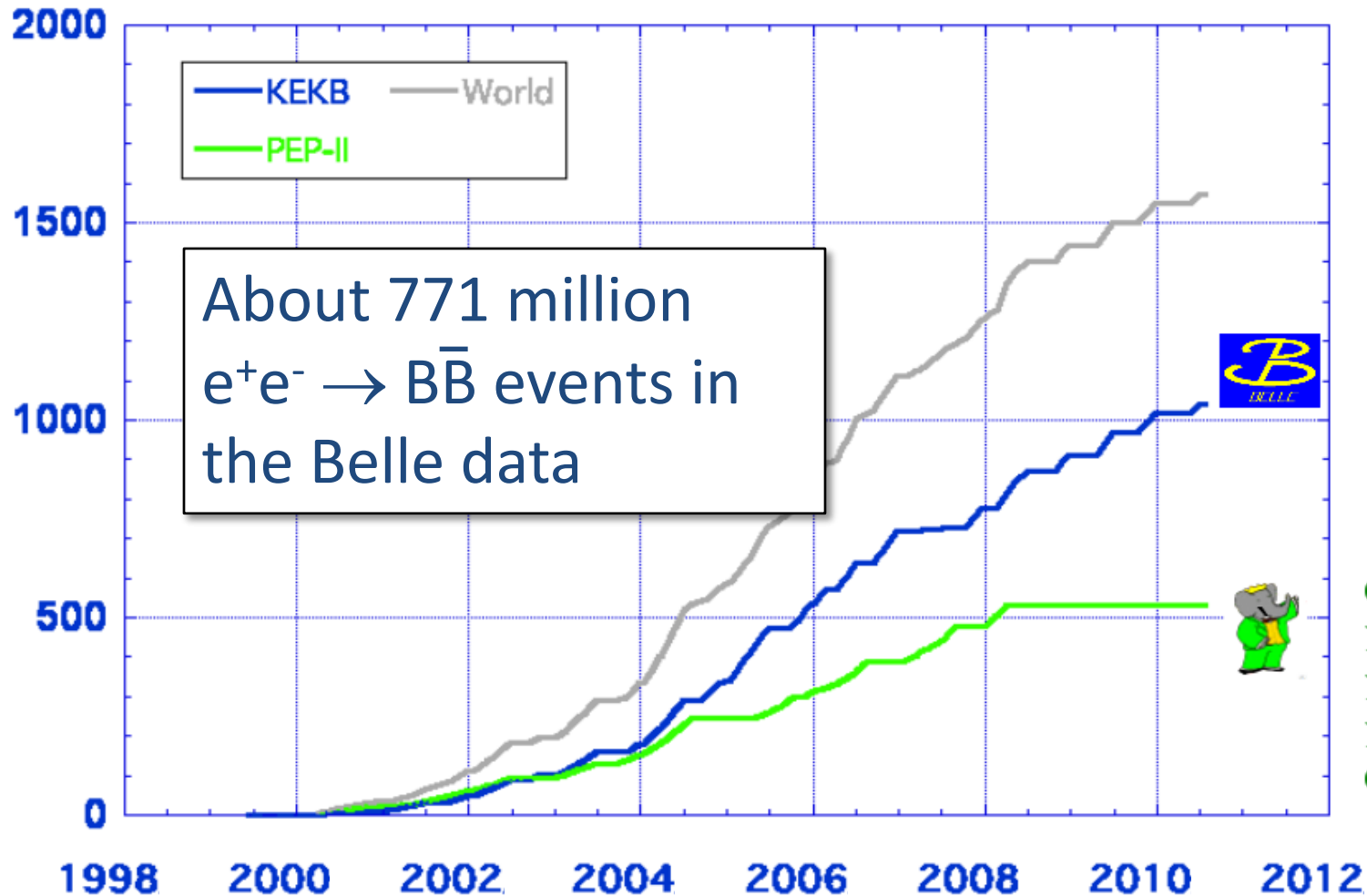
Belle detector



The Belle detector



Belle and BaBar luminosity



$> 1 \text{ ab}^{-1}$

On resonance:

$\Upsilon(5S): 121 \text{ fb}^{-1}$

$\Upsilon(4S): 711 \text{ fb}^{-1}$

$\Upsilon(3S): 3 \text{ fb}^{-1}$

$\Upsilon(2S): 24 \text{ fb}^{-1}$

$\Upsilon(1S): 6 \text{ fb}^{-1}$

Off reson./scan:

$\sim 100 \text{ fb}^{-1}$

$\sim 550 \text{ fb}^{-1}$

On resonance:

$\Upsilon(4S): 433 \text{ fb}^{-1}$

$\Upsilon(3S): 30 \text{ fb}^{-1}$

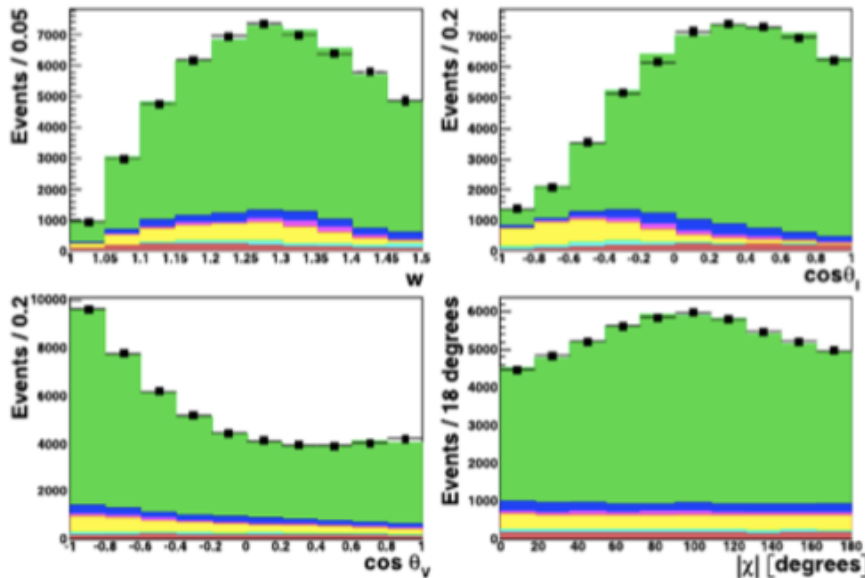
$\Upsilon(2S): 14 \text{ fb}^{-1}$

Off resonance:

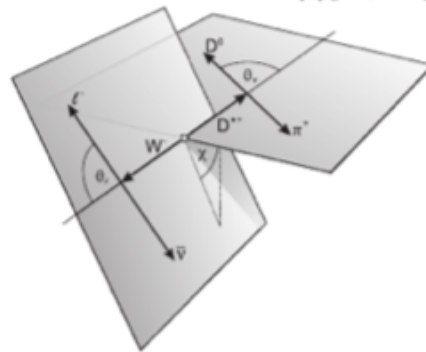
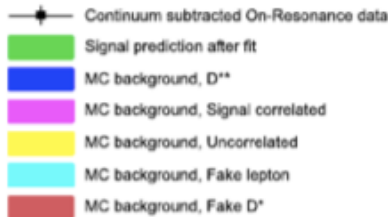
$\sim 54 \text{ fb}^{-1}$

$B^0 \rightarrow D^{*-}l^+\nu$ at Belle

[W. Dungen, CS, Phys. Rev. D 82, 112007 (2010)]



- 711/fb of Belle Y(4S) data
- About 120,000 reconstructed $B^0 \rightarrow D^{*-}l^+\nu$ decays
- Fit in 40 bins of w , $\cos \theta_l$, θ_ν and χ to obtain CLN F.F. parameters
- Dominant experimental systematics: tracking

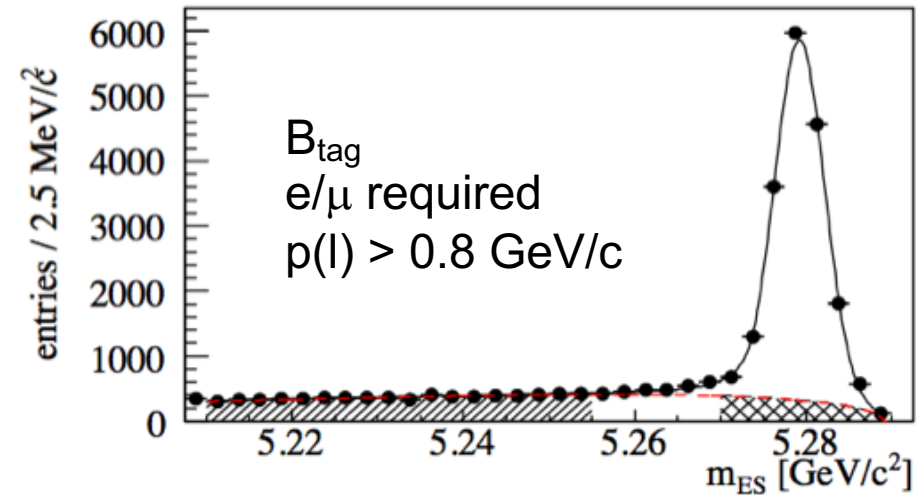
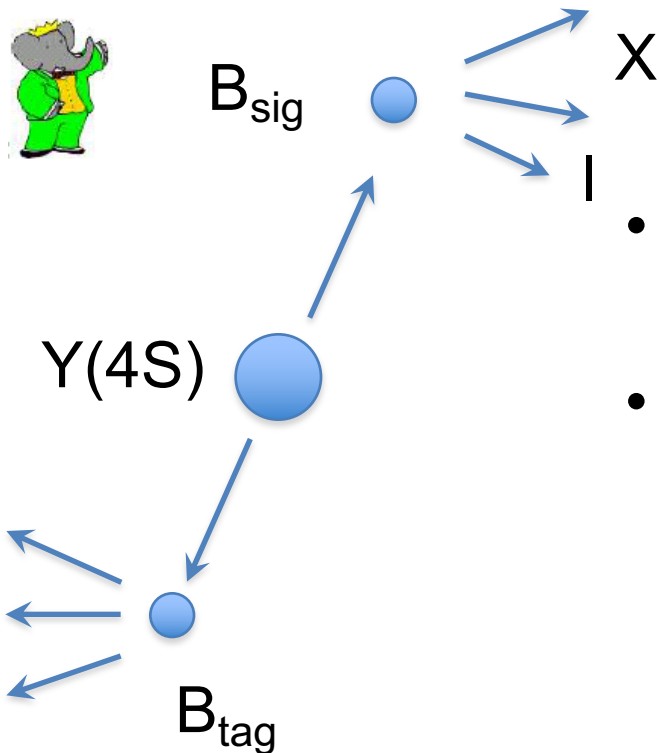


$$\begin{aligned}
 \mathcal{F}(1)|V_{cb}| &= (34.6 \pm 0.2 \pm 1.0) \times 10^{-3} \\
 \rho^2 &= 1.214 \pm 0.034 \pm 0.009 \\
 R_1(1) &= 1.401 \pm 0.034 \pm 0.018 \\
 R_2(1) &= 0.864 \pm 0.024 \pm 0.008 \\
 \chi^2/ndf &= 138.8/155
 \end{aligned}$$

BaBar hadronic moments

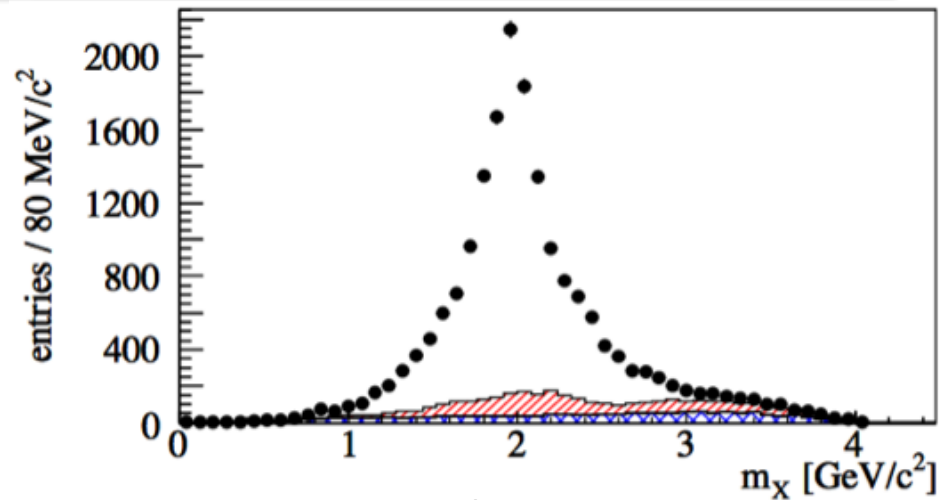
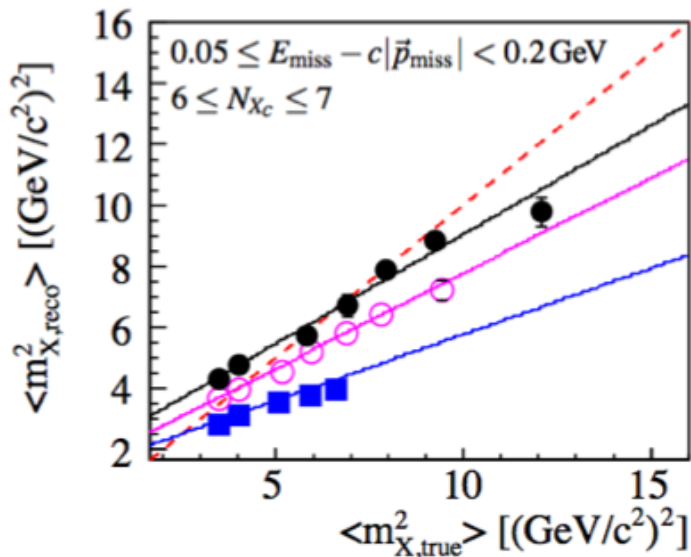
232M BB

- Fully reconstruct the hadronic decay of one B in $Y(4S) \rightarrow BB$ (efficiency $\sim 0.4\%$, purity $\sim 80\%$)



- Require one identified lepton amongst the signal-side particles ($p > 0.8 \text{ GeV}/c$)
- Combine all remaining particles to the X system and do a kinematic fit
 - 4-momentum conservation
 - Missing mass consistent with zero mass neutrino

- Hadronic mass spectrum after kinematic fit



- Linear correction of the measured moments in bins of X multiplicity, $E_{\text{miss}} - cp_{\text{miss}}$ and lepton momentum
- Moments of the hadronic mass spectrum up to M_X^6 for E_{cut} between 0.8 and 1.9 GeV are measured
- Also **mixed mass-energy moments** are determined and the **electron energy moments** from [PRD69, 111104] are re-evaluated

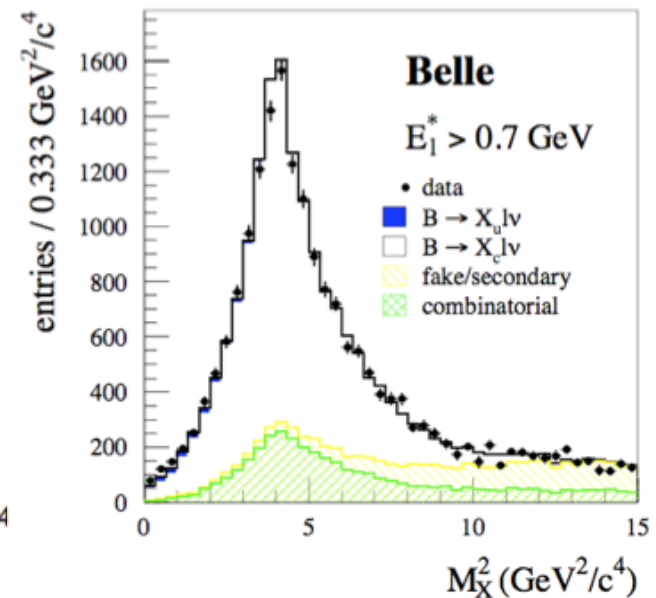
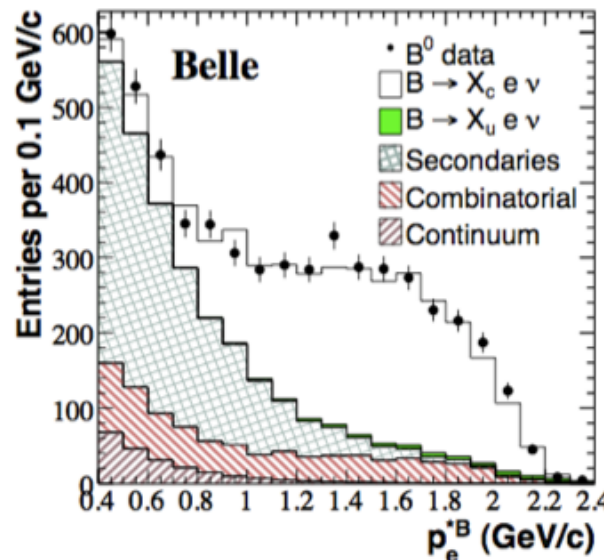
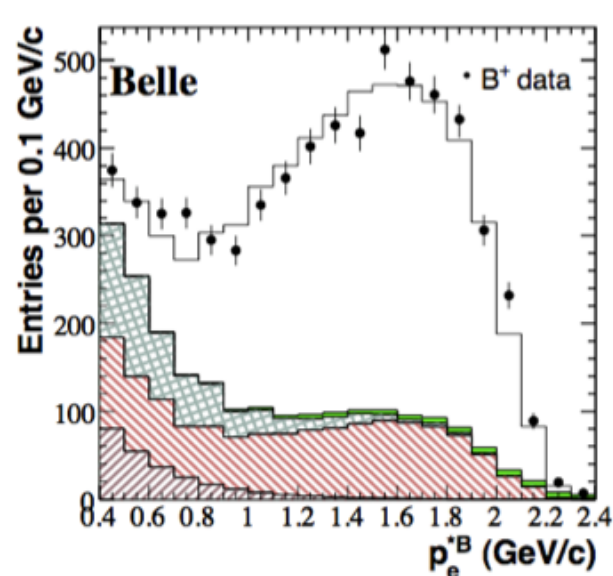
Belle E_1 and M_X^2 moments

PRD 75, 032001 (2007)

PRD 75, 032005 (2007)

152M BB

- For both the E_1 and M_X^2 measurements, similar experimental method using fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $\langle E_e^n \rangle$ measured for $n=0, \dots, 4$ and $E_{\text{cut}}=0.4-2.0$ GeV
- $\langle M_X^{2n} \rangle$ measured for $n=1, 2$ and $E_{\text{cut}}=0.7-1.9$ GeV



1S scheme analysis

	m_b^{1S} [GeV]	λ_1 [GeV ²]	ρ_1 [GeV ³]	τ_1 [GeV ³]	τ_2 [GeV ³]	τ_3 [GeV ³]	$ V_{cb} $ [10 ⁻³]
value	4.691	-0.362	0.043	0.161	-0.017	0.213	41.98
error	0.037	0.067	0.048	0.122	0.062	0.102	0.45
m_b^{1S}	1.000	0.434	0.213	-0.058	-0.629	-0.019	-0.215
λ_1		1.000	-0.467	-0.602	-0.239	-0.547	-0.403
ρ_1			1.000	0.129	-0.624	0.494	0.286
τ_1				1.000	0.062	-0.148	0.194
τ_2					1.000	-0.009	-0.145
τ_3						1.000	0.376
$ V_{cb} $							1.000

χ^2 of 23.0 for 59 degrees of freedom

- B quark mass constrained with $B \rightarrow X_s \gamma$ data
- Average B lifetime: (1.579 +/- 0.004) ps