

Belle II prospects for the measurements of $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$

Jitendra Kumar

On behalf of the Belle II collaboration

Carnegie
Mellon
University

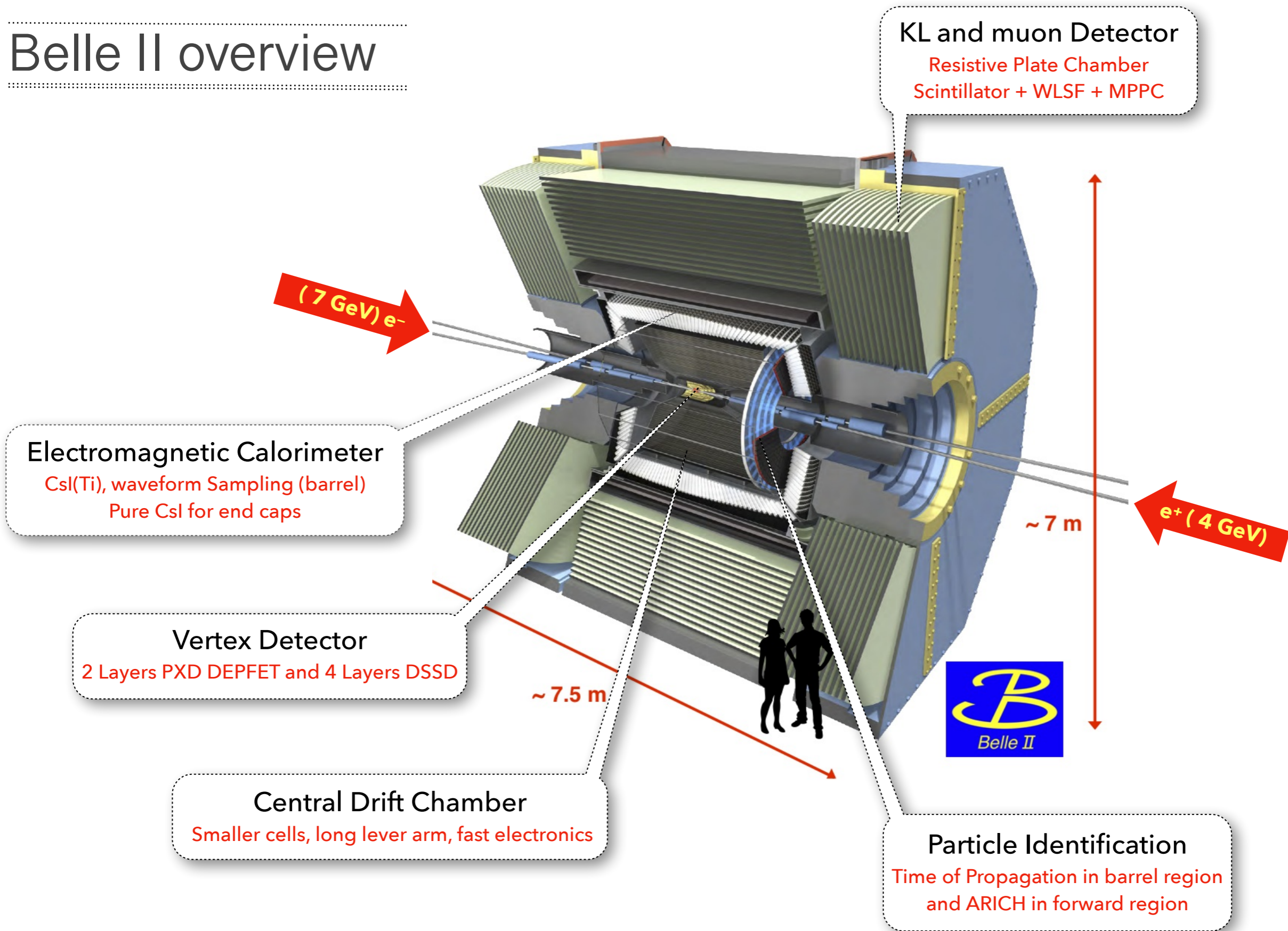


Outline

Overview of measurements, studies and future prospects

- ▶ Belle II detector and data taking status
- ▶ Charm/ τ potential and performance at Belle II
- ▶ Belle II prospects on $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$
- ▶ Summary

Belle II overview

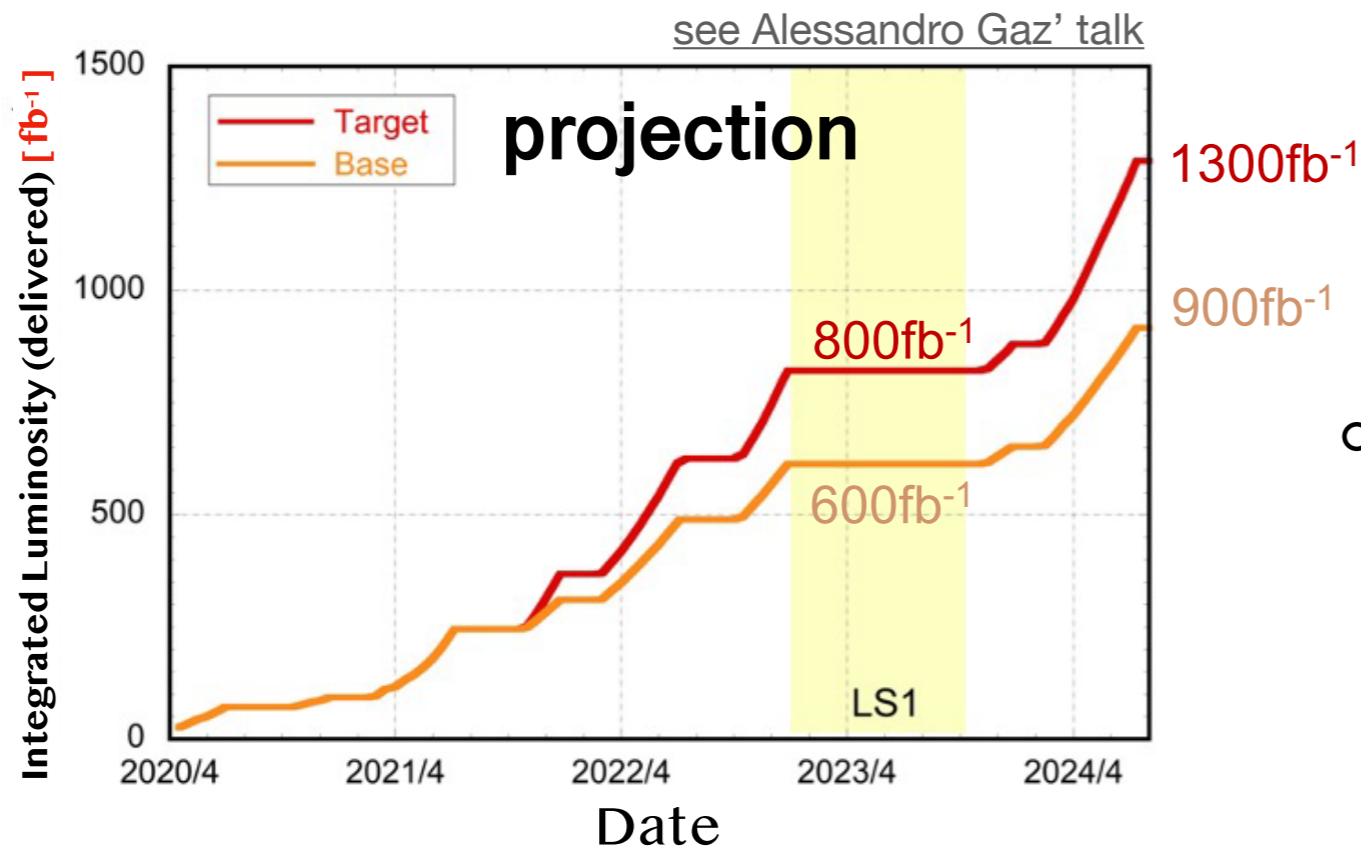
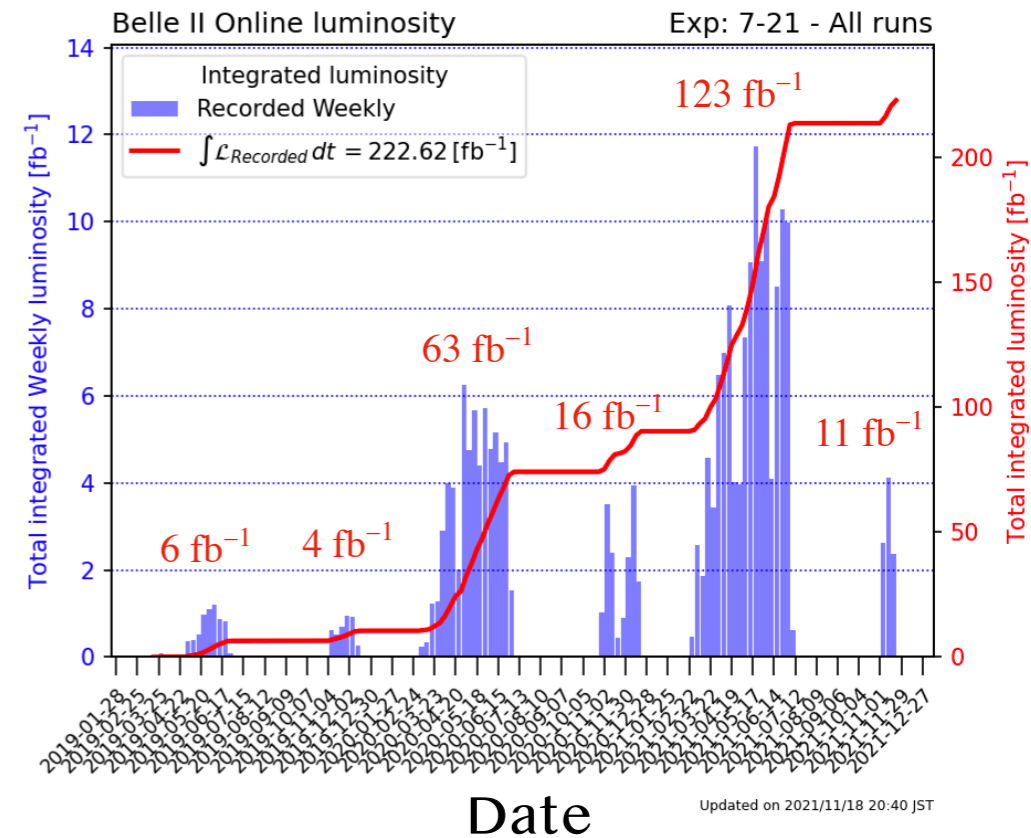


Ref: Belle2 TDR: arXiv: 1011.0352

Belle II data status

as of
now

- ▶ Continued data-taking through Covid-19 pandemic
- ▶ Integrated luminosity $L_{\text{int}} \sim 223 \text{ fb}^{-1}$ (Nov 18, 2021)
- ▶ Highest instantaneous luminosity $\sim 3.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
 - **New world record archived** in June 2021 🏆🌟
 - SuperKEKB design luminosity: $6.5 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$



○ **so far..**

BelleII charm/ τ studies focused on detector/reconstruction performance, resolutions, and systematic effects..

*Belle highest in June'09 : $2.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$

charm/ τ opportunities at Belle II

Powerful **SuperKEKB**

- ▶ $50 \text{ ab}^{-1} = \sim 50 \times \text{Belle}$
- ▶ e^+e^- collisions (asymmetric beam energies)
 - offer stringent kinematic constraints for reconstruction of final states with neutrinos
 - acceptance and trigger criteria that introduce much less bias on flight length and kinematic properties
 - ..more in [Physics Book](#)

Impact

▶ Charm Physics

- B-factory ▶ “charm-factory” (60×10^9 pairs of c with 50 ab^{-1})
 - excellent Dalitz plot analysis (uniform efficiency and non-biasing trigger)
 - better reconstruction of neutrinos

▶ τ Physics

- B-factory ▶ “ τ -factory” ($\sim 50 \times 10^9$ events with 50 ab^{-1})
 - measure wide range of observables (CP asymmetries, invariant mass spectra, lepton universality etc.)
 - precision measurements or indirect search of BSM (beyond SM) physics
 - direct search of forbidden decays

charm/ τ opportunities at Belle II

highlights of Belle II

- ▶ **New silicon vertex detector** provides better vertex resolution
- ▶ Good PID even with higher beam background environment
- ▶ More tracking volume[∅] \rightarrow higher K_s efficiency (w.r.t. LHCb)
- ▶ .. more in TDR and Physics Book

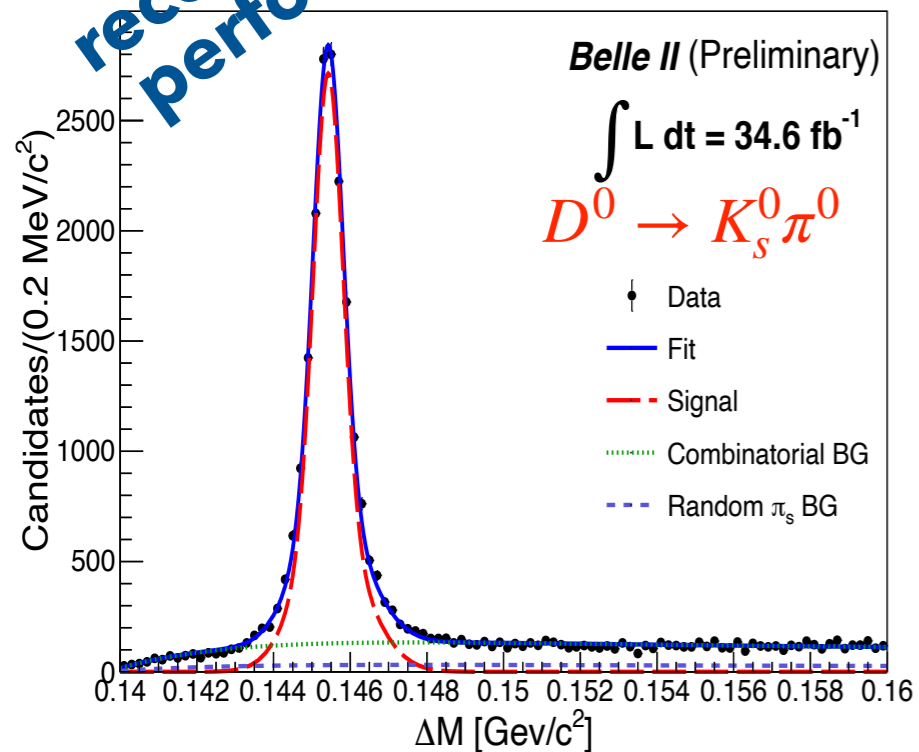
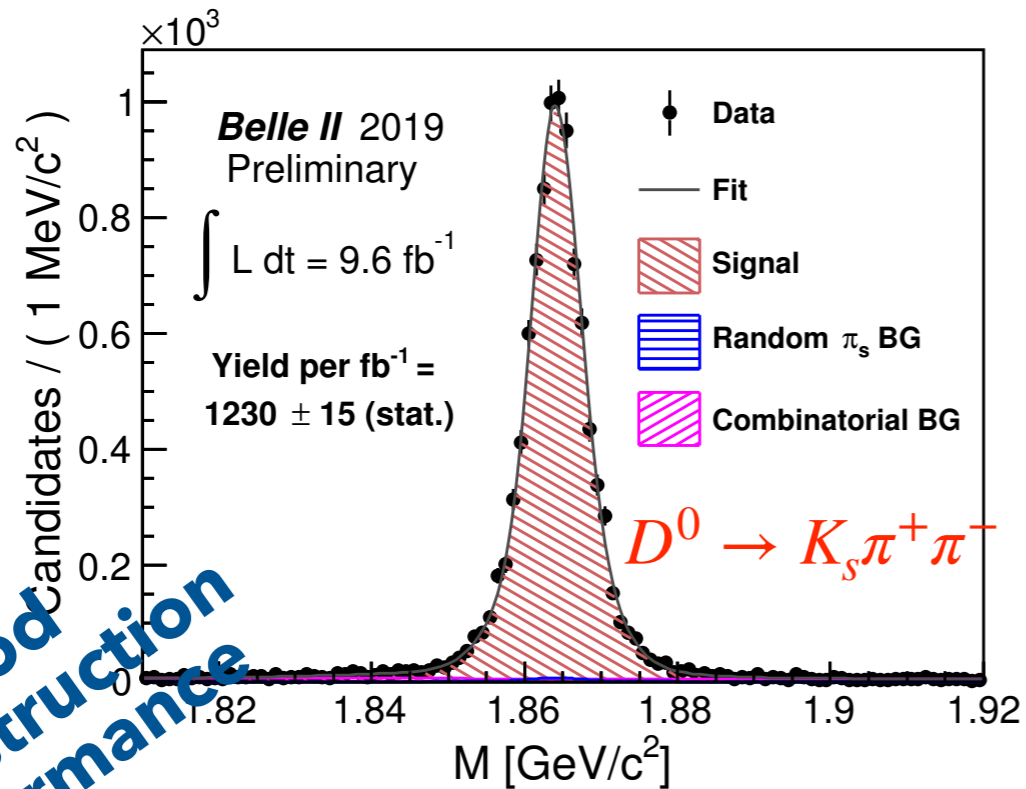
Impact

▶ Charm/ τ Physics

- Facilitate measurement of mixing parameters, and CP violations with neutrals in the final state
- Belle II performance is expected to improve w.r.t. to Belle;
 - improved IP resolution (e.g. x2 better D^0 proper time resolution)
 - reduced statistical uncertainties
 - ..and if systematic uncertainties are reduced

Current Belle II performance

charm results

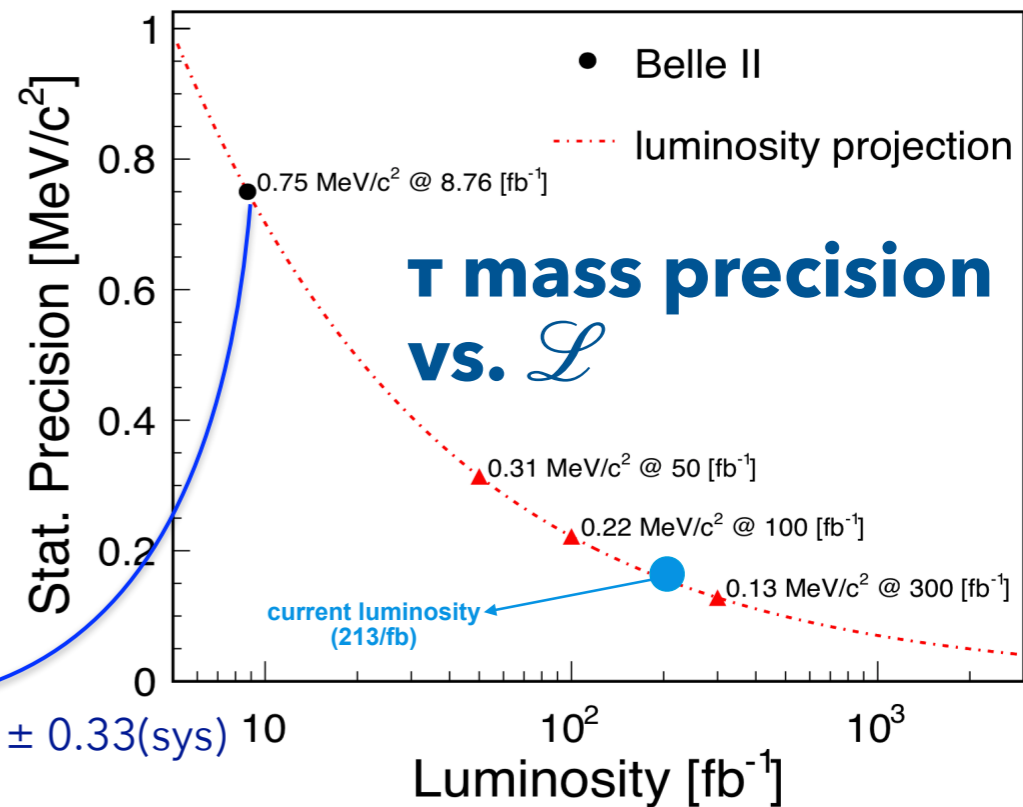
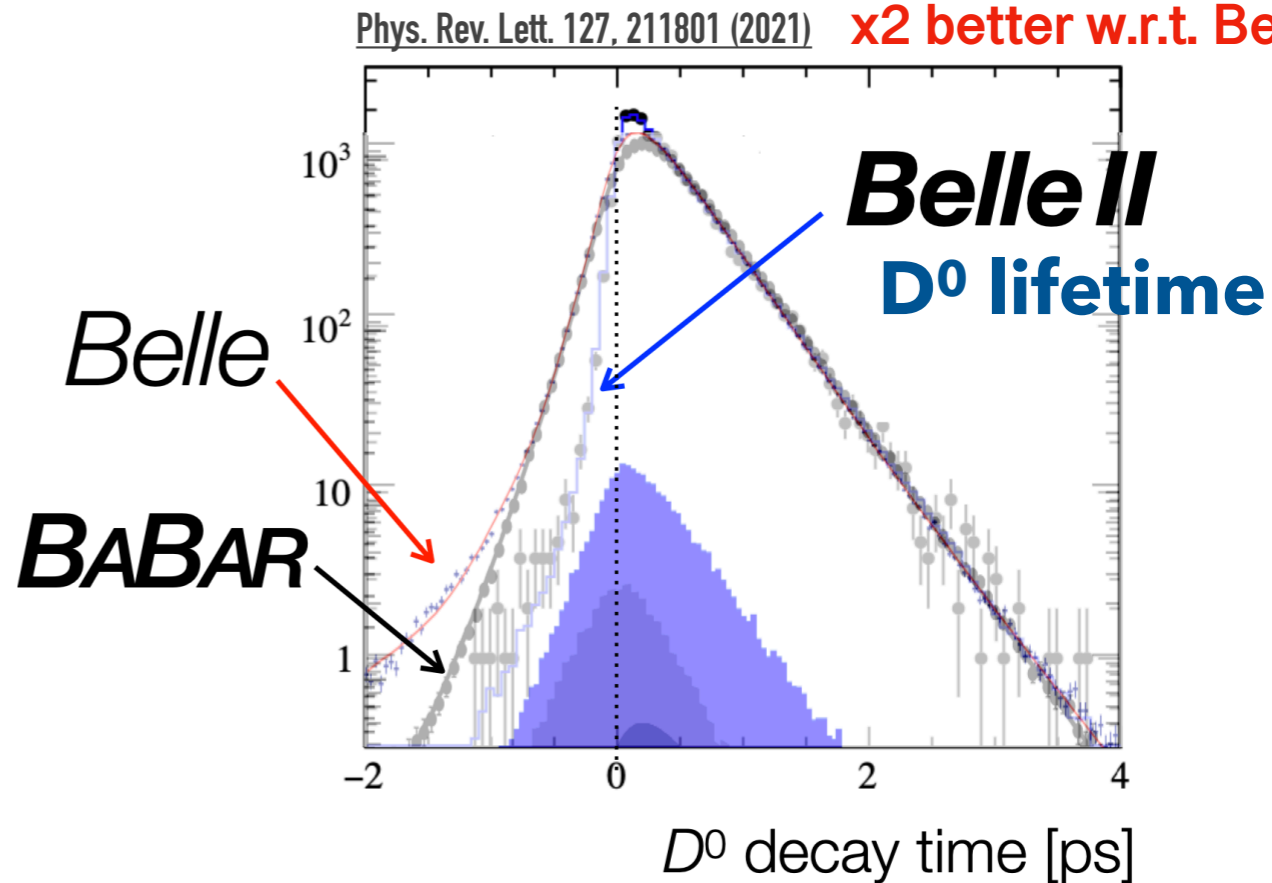


$m_\tau = 1777.28 \pm 0.75 \pm 0.33(\text{sys})$

τ results

arXiv:2008.04665
[Belle II 2020](#)

World's Best
x2 better w.r.t. Belle



1 | V_{us} | status and Belle II prospects

- ✱ $s \leftrightarrow u$ transition
- ✱ Experimental measurements
 - ▶ kaon decays
 - traditional (also for form factor) and most accurate among all
 - but precision is limited from theory (LCQD) uncertainties (on form factor $f_+(0)$ & f_k/f_π)
 - ▶ hyperon decays
 - ▶ τ decays with strangeness in the final state (**today's focus**)

$|V_{us}|$ from τ decays (methods)

Exclusive method

1. Compare BR ratio: $\tau^- \rightarrow K^- \nu_\tau$ and $\tau^- \rightarrow \pi^- \nu_\tau$

$$\frac{\Gamma_{\tau^- \rightarrow K^- \nu_\tau}}{\Gamma_{\tau^- \rightarrow \pi^- \nu_\tau}} = \frac{|V_{us}|^2 f_K^2 (1 - m_K^2/m_\tau^2)^2}{|V_{ud}|^2 f_\pi^2 (1 - m_\pi^2/m_\tau^2)^2} (1 - \delta_{LD})$$

long-distance radiative corrections

- ▶ $|V_{us}| = 0.2234 \pm 0.0015$ (HFLAV 2021 preliminary)
 - ⊙ -2.1σ from CKM unitarity
 - ⊙ but large uncertainties as compared to kaons
 - ▶ $|V_{us}|(K \rightarrow l3): 0.2231 \pm 0.0006$

electroweak corrections

2. via branching fraction: $\tau^- \rightarrow K^- \nu_\tau$

$$\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$$

- ▶ $|V_{us}| = 0.2226 \pm 0.0015$ (HFLAV 2021 preliminary)
 - ⊙ -2.6σ from CKM unitarity
 - ⊙ but large uncertainties as compared to kaons
 - ▶ $|V_{us}|(K \rightarrow l3): 0.2231 \pm 0.0006$

Inclusive method

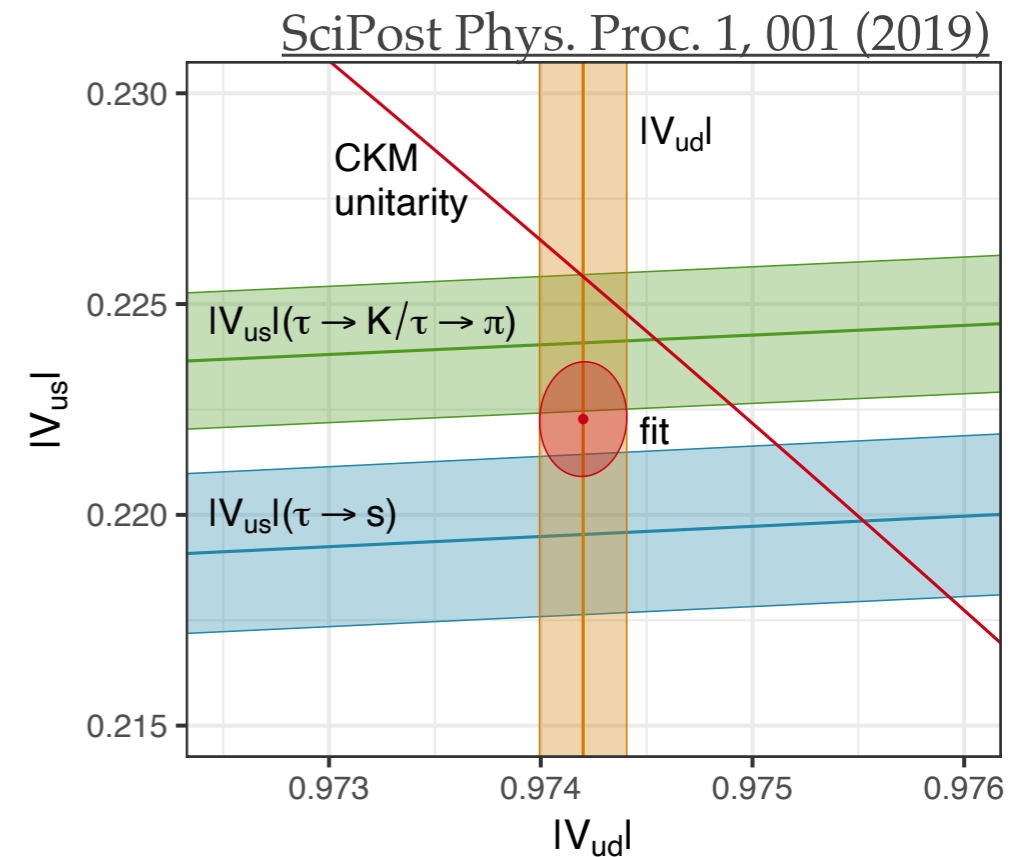
1. via spectral moments: $\tau \rightarrow s$ decays

$$|V_{us}| = \sqrt{\frac{R_s}{\frac{R_{NS}}{|V_{ud}|^2} - \delta_\tau}}$$

S (hadron with $S = 0$)
NS (hadron with $S \neq 0$)

- ▶ $|V_{us}| = 0.2192 \pm 0.0019$ (HFLAV 2021 preliminary)
 - ⊙ -3.6σ lower from CKM unitarity

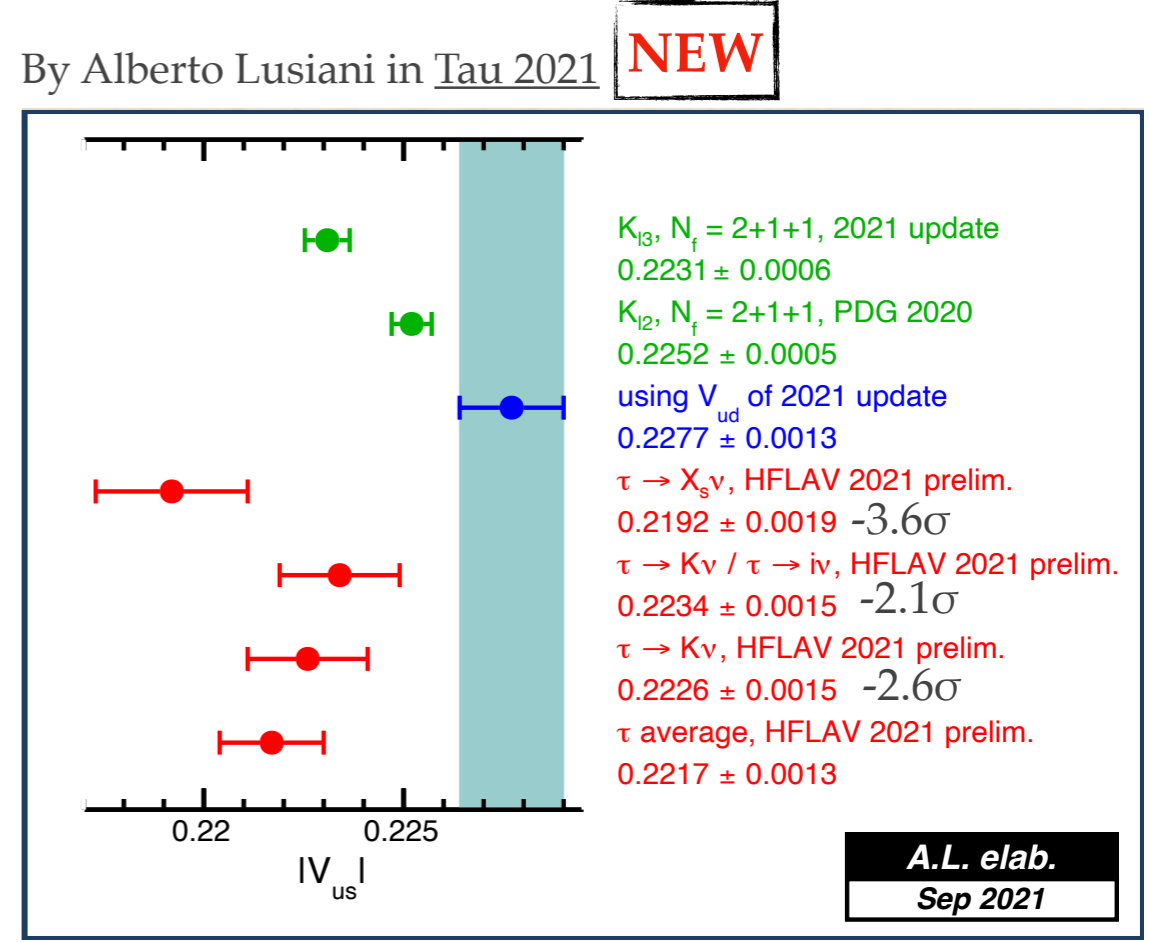
⊙ alternate methods [1], [2]: consistent with K and CKM unitarity



$|V_{us}|$ from τ decays (status)

Current status

- ▶ Kaon decays (HFLAV 2021 preliminary)
 - ▶ $|V_{us}|$: see latest numbers on plot for $l3$ & $l2$
 - ⊙ $|V_{ud}| - |V_{us}|_K$ anomaly $\sim 3\sigma$
 - ▶ $\sim 5\sigma$ without increased $|V_{ud}|$ systematics
- ▶ τ decays (average HFLAV 2021 preliminary)
 - ▶ $|V_{us}| = 0.2217 \pm 0.0013$
 - ⊙ $\tau \rightarrow s$: -3.6σ lower from CKM unitarity

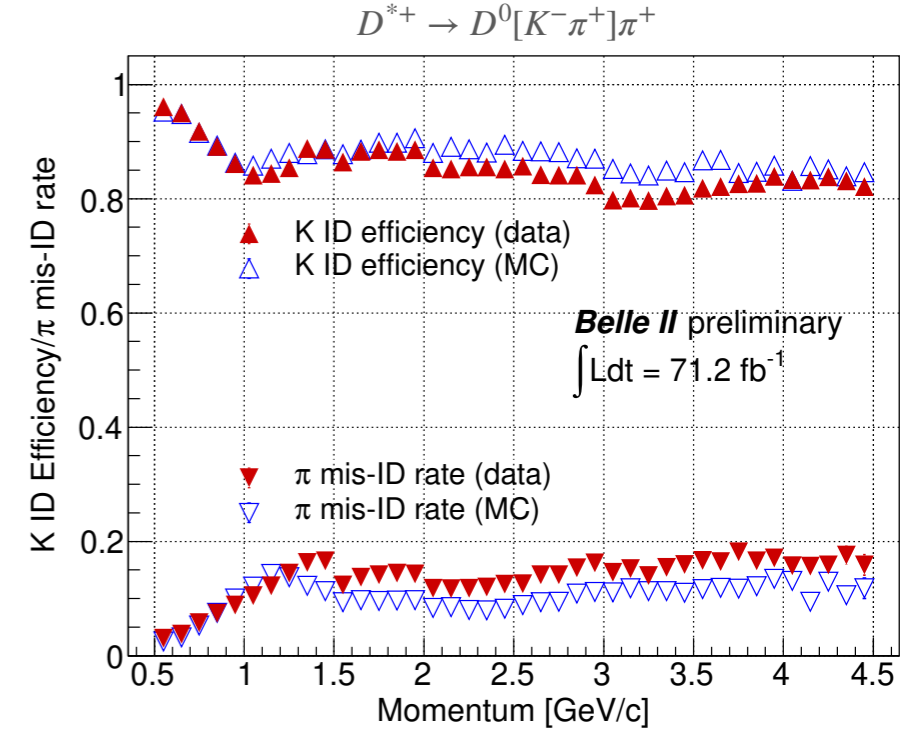


Belle II prospects

- will perform LFU like analysis (use 3x1 and 1x1 topologies)
- statistical uncertainties will be improved with larger data-set
- also improved systematics from
 - PID¹, trigger efficiency from detector upgrades
 - MC inputs (background estimation, modeling of decays²)

1. PID (scale factor uncertainty will scale inverse to the statistics of the data sets)
2. Modeling of decays in the generator (KKMC, Tauola)

Belle II PID performance (efficiency/fake rates)



2 | V_{cs} | status and Belle II prospects

- ✱ Cabibbo-favoured ($c \rightarrow s$ transition)
- ✱ Experimental measurements
 - with D and D_s meson decays (**today's focus**)
 - Leptonic ($D_s \rightarrow \ell \nu$) decay \leadsto **simplest and theoretically cleanest processes**
Decay constants f_D is required from Lattice QCD
 - Semi-leptonic decay ($D \rightarrow K \ell \nu$)
Form factor $f(q^2)$ is required from Lattice QCD
 - charm baryon and W^\pm decays

$|V_{cs}|$ via leptonic decay: $D_s \rightarrow \ell \nu$

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 |V_{cd(s)}|^2 M_{D_{(s)}^+} M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}^+}^2}\right)^2$$

Overview

- decay modes: $\rightarrow \mu\nu$, $\rightarrow e\nu$ & $\rightarrow \tau\nu$
- decay suppressed by helicity conservation hence decay rates $\propto m_l^2$
 - $\rightarrow e\nu$ branching fraction is very small $\sim 10^{-7}$
 - $\rightarrow \tau\nu$ is favored over $\mu\nu$

analysis method (Belle)

$$e^+e^- \rightarrow c\bar{c} \rightarrow D_{tag} X_{frag} K_{frag} D_s^{*-} (\rightarrow D_s^- \gamma)$$

$$D^+, D^0, \Lambda_c^+ \text{ \& \& } D^{*+}, D^{*0} \quad \pi, K(\text{even}), p$$

Tag: Tagged decays
Frag: Fragmented particles

Step1: reconstruct tag side D_{tag} , build X_{frag} and then extract D_s^- via missing mass analysis
 \rightarrow missing mass peak at $\sim D_s^-$ mass

Step2: used signal from step 1 and search/extract D_s^- yield for $\rightarrow \mu\nu$, $\rightarrow e\nu$ & $\rightarrow \tau\nu$

\rightarrow **measure branching fraction** ► $\mathcal{B}(D_s^+ \rightarrow f) = \frac{N(D_s^+ \rightarrow f)}{N_{D_s}^{\text{inc}} \cdot f_{\text{bias}} \cdot \varepsilon(D_s^+ \rightarrow f | \text{incl. } D_s^+)}$

Step3: calculate $f_{D_s} V_{us}$ from step 2, then two approach

1. take f_{D_s} from Lattice QCD
 \rightarrow **extract V_{us} and compare w/ CKM unitarity**

2. take V_{us} from CKM unitarity
 \rightarrow **extract f_{D_s} and compare w/ Lattice QCD**

$|V_{cs}|$ via leptonic decay: $D_s \rightarrow \ell\nu$

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+\nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 |V_{cd(s)}|^2 M_{D_{(s)}^+} M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}^+}^2}\right)^2$$

Current status $\triangleright D_s \rightarrow \mu\nu$

- several results in past years by BaBar, Belle, BESIII[1] [2][latest] and CLEO-c
- the most precise result from BESIII (2021) with 6.2 fb^{-1}
- Belle performed analysis with 913 fb^{-1}

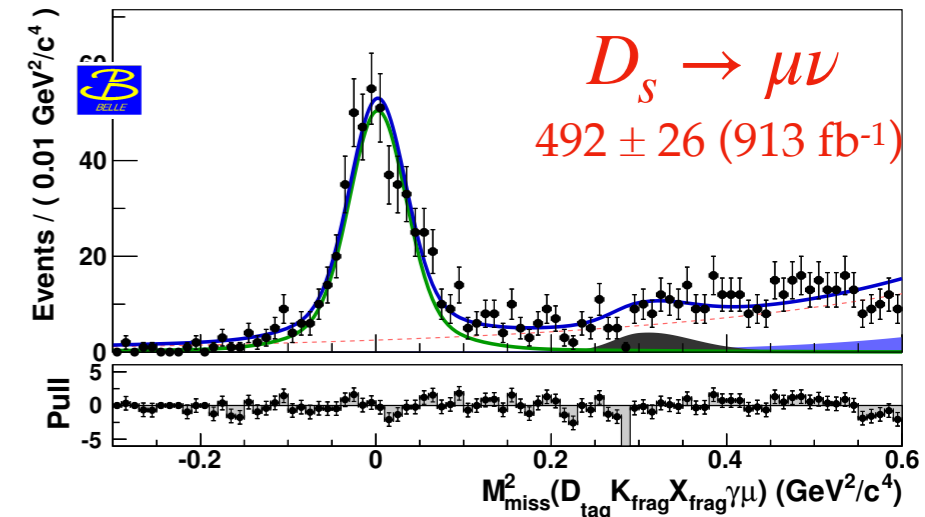
HFLAV 2021

● $|V_{cs}| = 0.9839 \pm 0.0115(\text{exp.}) \pm 0.0020(\text{LQCD})$

- average with $D_s \rightarrow \tau\nu$
- $f_{D_s} = 249.9 \pm 0.5 \text{ MeV}$ (LQCD) = 0.2% precision

● $f_{D_s} = 252.6 \pm 3.0 \text{ MeV}$

- global fit: $|V_{cs}| = 0.973394^{+0.000074}_{-0.000096}$



$D_s \rightarrow \mu\nu$ Yield

Belle II prospects 50 ab^{-1}

- \leadsto improved stats. uncertainty
 - $\delta(|V_{cs}|) = \pm 0.004$ (stat.)
 - $\delta(|f_{D_s}|) = \pm 0.9$ (stat.) MeV
- \leadsto systematics uncertainty (possible improvements)
 - with precision measurement of peaking backgrounds
 - in normalization (err. scaled with luminosity and are reducible with clean X_{tag})

inclusive Yield per luminosity

B-factory (Belle)	$100 \cdot 10^3 \text{ per } \text{ab}^{-1}$
Charm-factory (CLEO-c)	$73 \cdot 10^3 \text{ per } \text{fb}^{-1}$
$\sqrt{s} = 4.17 \text{ GeV}$	
Charm-factory (BESIII)	$22 \cdot 10^3 \text{ per } \text{fb}^{-1}$
$\sqrt{s} = 4.01 \text{ GeV}$	

Source	$\mu\nu$ [%]
Statistical	± 5.32
Normalization	± 1.95
Tag bias	± 1.37
Tracking	± 0.35
Efficiency	± 1.78
PID	± 1.96
D_s background	± 0.82
Comb. bkg. PDF	± 0.02
Signal PDF	—
τ cross-feed	—
$\mathcal{B}(\tau \rightarrow X)$	—
PDF stat.	—
Total syst.	± 3.67
Stat. + Syst.	± 6.46

$|V_{cs}|$ via leptonic decay: $D_s \rightarrow \ell \nu$

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 |V_{cd(s)}|^2 M_{D_{(s)}^+} M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}^+}^2}\right)^2$$

Current status $\triangleright D_s \rightarrow \tau \nu$

- several results in past years by BaBar, Belle, BESIII[1] (new: [2][3]) and CLEO-c
- Belle modes; $\rightarrow e^+ \nu \nu$, $\rightarrow \mu^+ \nu \nu$, $\rightarrow \pi^+ \nu$
- **signal** D_s extraction via fit to excess E_{ECL}

HFLAV 2021

● $|V_{cs}| = 0.9839 \pm 0.0115(\text{exp.}) \pm 0.0020(\text{LQCD})$

- average with $D_s \rightarrow \mu \nu$
- $f_{D_s} = 249.9 \pm 0.5 \text{ MeV}$ (LQCD)

● $f_{D_s} = 252.6 \pm 3.0 \text{ MeV}$

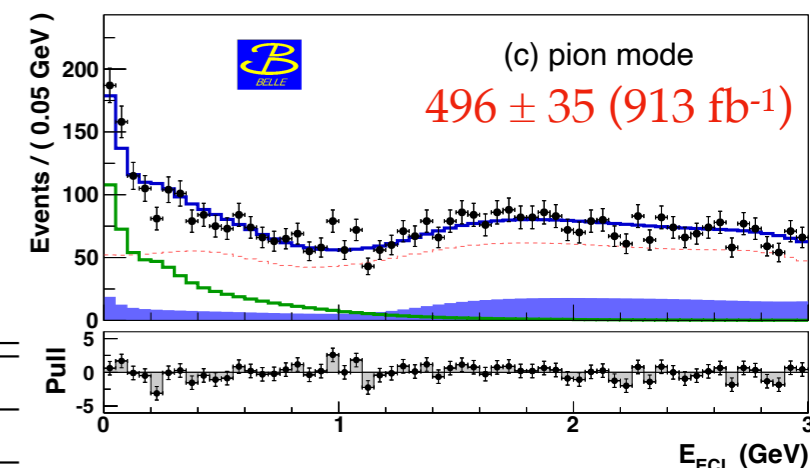
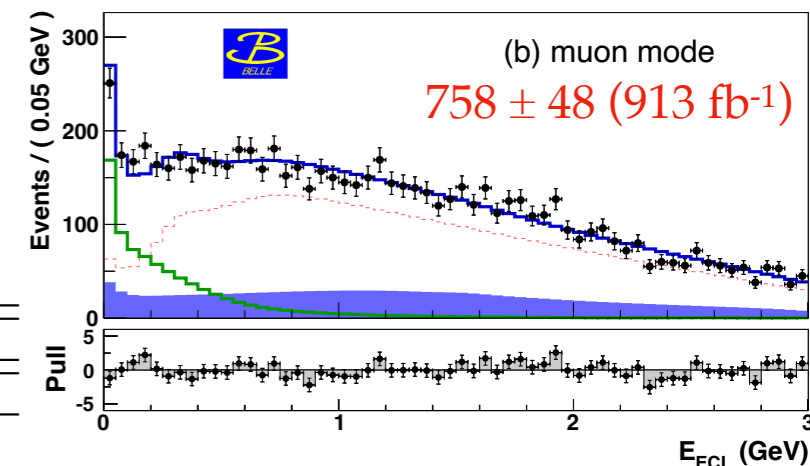
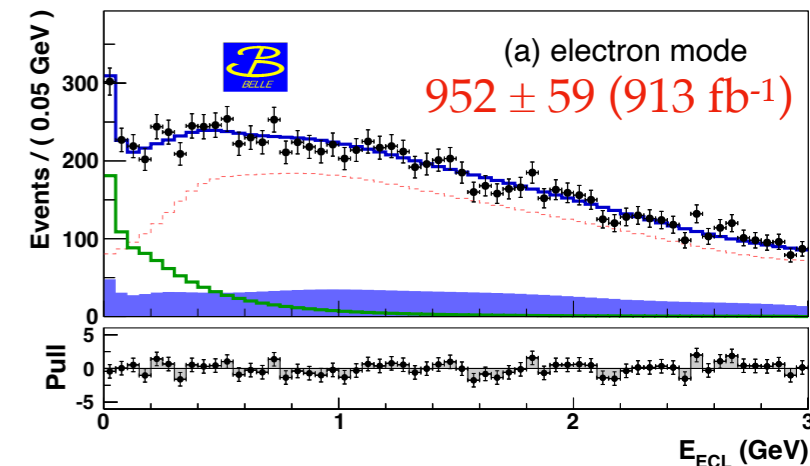
- global fit: $|V_{cs}| = 0.973394^{+0.000074}_{-0.000096}$

$D_s \rightarrow \tau \nu$ Yield

Source	$\tau \nu$ [%]
Statistical	± 3.75
Normalization	± 1.95
Tag bias	± 1.37
Tracking	± 0.35
Efficiency	± 0.84
PID	± 1.70
D_s background	± 2.84
Comb. bkg. PDF	-2.54
Signal PDF	$+2.95$
τ cross-feed	± 0.94
$\mathcal{B}(\tau \rightarrow X)$	± 0.19
PDF stat.	± 1.44
Total syst.	$+5.40$ -5.19
Stat. + Syst.	$+6.57$ -6.40

Belle II prospects 50 ab^{-1}

- \sim improved stats. uncertainty
 - $\delta(|V_{cs}|) = \pm 0.003$ (stat.) \sim comparable to theory err.
 - $\delta(|f_{D_s}|) = \pm 0.6$ (stat.) MeV \sim comparable to theory err.
- \sim systematics uncertainty (possible improvements)
 - with precision measurement of peaking backgrounds
 - in normalization (err. scaled with luminosity and are reducible with clean X_{tag})



$|V_{cs}|$ via semi-leptonic decay: $D \rightarrow K\ell\nu$

Current status $\triangleright D \rightarrow K\ell\nu$

- several results in past years by BaBar, Belle, BESIII and CLEO-c

HFLAV 2021

$\sim 0.4\%$ precision

- Form factors $f_+^{DK}(0) = 0.765 \pm 0.0031$ (ETM 17D, 18)
- $|V_{cs}| = 0.9447 \pm 0.0043(\text{exp.}) \pm 0.0137(\text{LQCD})$

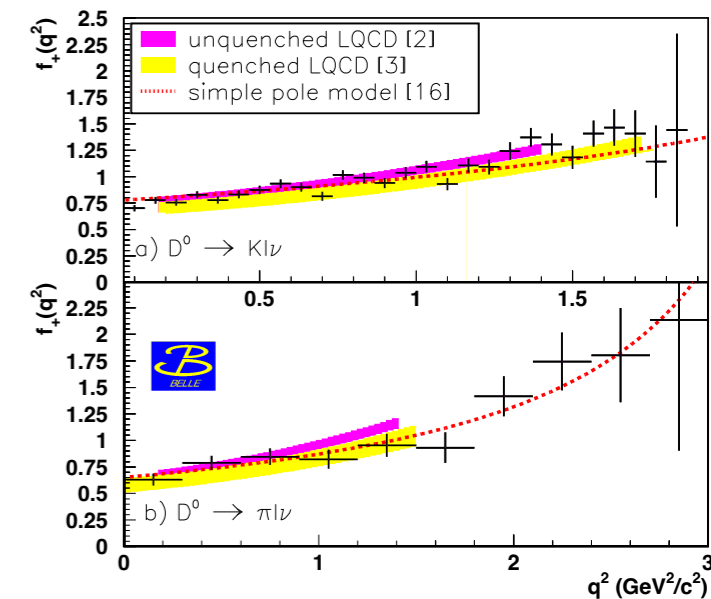
Belle II prospects

$$e^+e^- \rightarrow c\bar{c} \rightarrow D_{tag}^{(*)} X_{frag} D_{sig}^{*-} (\rightarrow \bar{D}_{sig}^0 \pi_s^-)$$

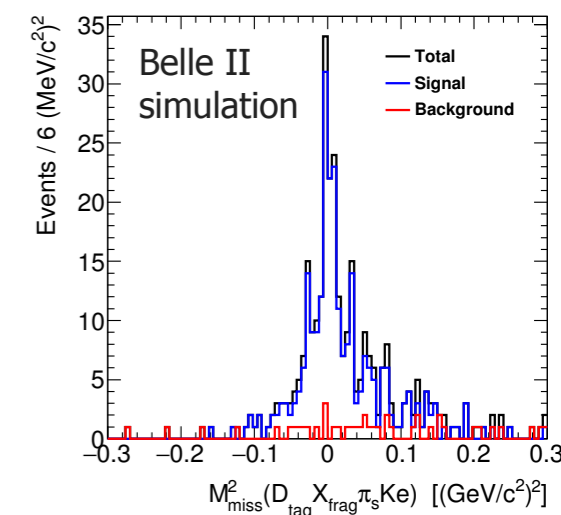
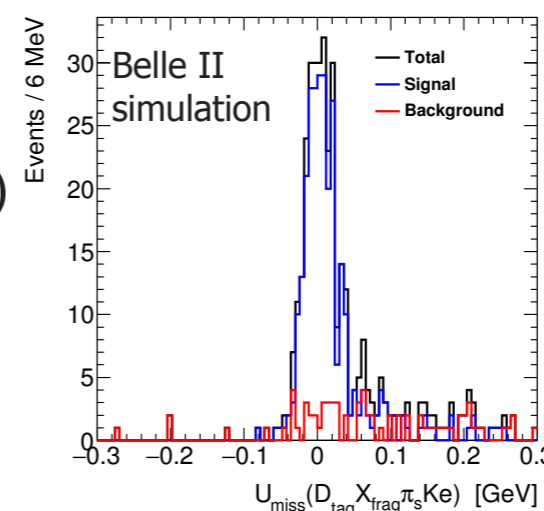
- MC studies with 1 ab^{-1}
 - based on $M_{miss}^2 = P_{miss}^2$ & $U_{miss} = E_{miss} - P_{miss}$
 - missing mass resolution is comparable with Belle
 - small continuum background contribution
 - with 50 ab^{-1} data
 - larger data ($\sim 4.55 \times 10^5 D \rightarrow K\ell\nu$)
 - \rightarrow reduced stat. uncertainties
- scenario with charm factory experiments (e.g. BESIII)
 - challenging to compete with BESIII (with 20 fb^{-1} data plans)
 - but Belle II will add important confirmation/constraints

$$P_{miss} = P_{e^+} + P_{e^-} - P_{D_{tag}} - P_{X_{frag}} - P_h - P_l$$

- Belle results @ 282 fb^{-1}
 $\sim 2567 D \rightarrow K\ell\nu$



Tag side:	D^0	D^+
	$K^- \pi^+$	$K^- \pi^+ \pi^+$
	$K^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^0$
Final state:	$K^- \pi^+ \pi^+ \pi^-$	$K_S^0 \pi^+$
	$K^- \pi^+ \pi^+ \pi^- \pi^0$	$K_S^0 \pi^+ \pi^0$
	$K_S^0 \pi^+ \pi^-$	$K_S^0 \pi^+ \pi^+ \pi^-$
	$K_S^0 \pi^+ \pi^- \pi^0$	$K^+ K^- \pi^+$
X_{frag} :	π^+	none
	$\pi^+ \pi^0$	π^0
	$\pi^+ \pi^+ \pi^-$	$\pi^+ \pi^-$
		$\pi^+ \pi^- \pi^0$



3 $|V_{cd}|$ status and prospects

- ✱ Cabibbo-suppressed ($c \rightarrow d$ transition)
- ✱ Experimental measurements
 - Early study via neutrino production of charm (νN)
 - More precise results using D meson decays (**today's focus**)
 - ▶ Leptonic ($D^+ \rightarrow \ell^+ \nu$) decay
Decay constant f_D is required from (e.g. Lattice QCD)
 - ▶ Semi-leptonic decay ($D \rightarrow \pi \ell \nu$)
Form factor $f(q^2)$ is required from theory (e.g. Lattice QCD)

$|V_{cd}|$ via leptonic decay: $D^+ \rightarrow \ell^+ \nu$

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 |V_{cd(s)}|^2 M_{D_{(s)}^+} M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}^+}^2} \right)^2$$

Overview

- ▶ decay modes: $\rightarrow \mu\nu$, $\rightarrow e\nu$ & $\rightarrow \tau\nu$
- ▶ Belle II analysis method will be similar to $D_s \rightarrow \ell\nu$ analysis

Current status

- ▶ $f_D \cdot |V_{cd}|$: so far from charm factories only

- $\mu^+ \nu_\mu$ ▶ [CLEO-c\(2008\)](#) and [BESIII \(2014\)](#)
- $\tau^+ \nu_\tau$ ▶ [CLEO-c\(2008\)](#) for upper limit on BR and [BESIII \(2019\)](#)
- $e^+ \nu_e$ ▶ [CLEO-c\(2008\)](#) for upper limit on BR

⊙ world average $f_D \cdot |V_{cd}| = 46.1 \pm 1.0 \pm 0.3 \pm 0.2$ (from $\mu^+ \nu_\mu$)

⊙ ratio of $BR(\mu^+ \nu_\mu) / BR(\tau^+ \nu_\tau)$ is compatible with SM prediction

- ▶ decay constants f_{D^+} from LQCD

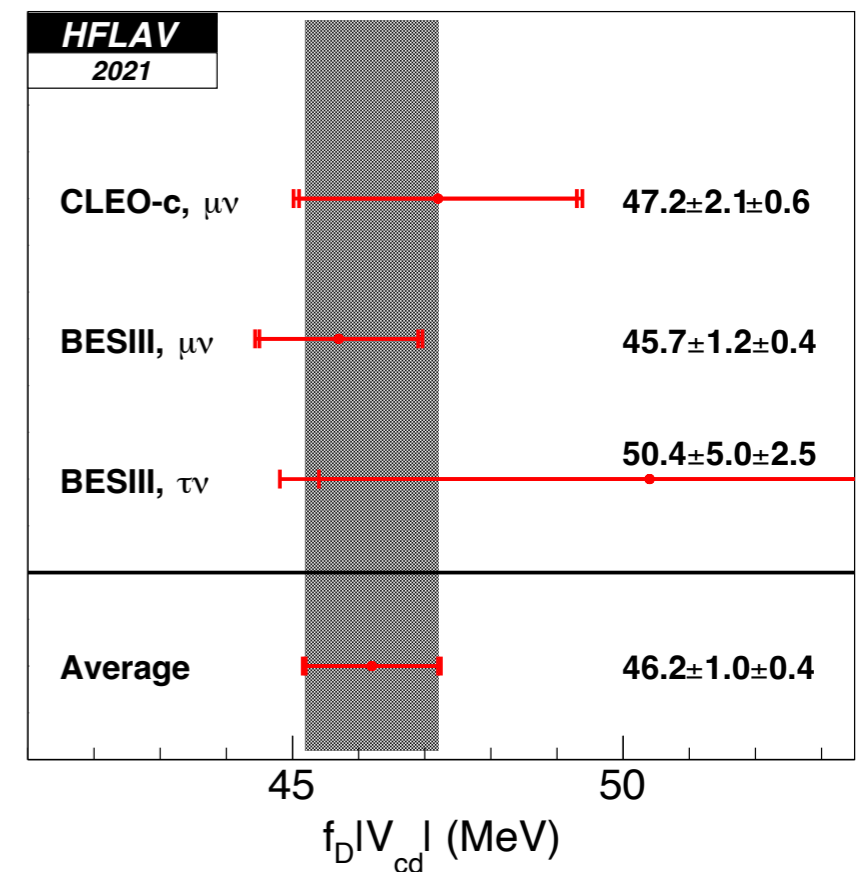
– $f_{D^+} \triangleright = 212.7 \pm 0.7$ MeV

⊙ average from [FNAL/MILC 17](#) and [ETM 14E](#)

- ▶ $|V_{cd}|_{D \rightarrow \ell\nu}$ HFLAV (June 2021)

⊙ $V_{cd} = 0.2181 \pm 0.0049(\text{exp.}) \pm 0.0007(\text{LQCD})$

⊙ also consistent with semi-leptonic measurement $D \rightarrow \pi \ell \nu$ decays (in slide #18)



$|V_{cd}|$ via leptonic decay $D^+ \rightarrow \ell^+ \nu$

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 |V_{cd(s)}|^2 M_{D_{(s)}^+} M_{\ell^+}^2 \left(1 - \frac{M_{\ell^+}^2}{M_{D_{(s)}^+}^2}\right)^2$$

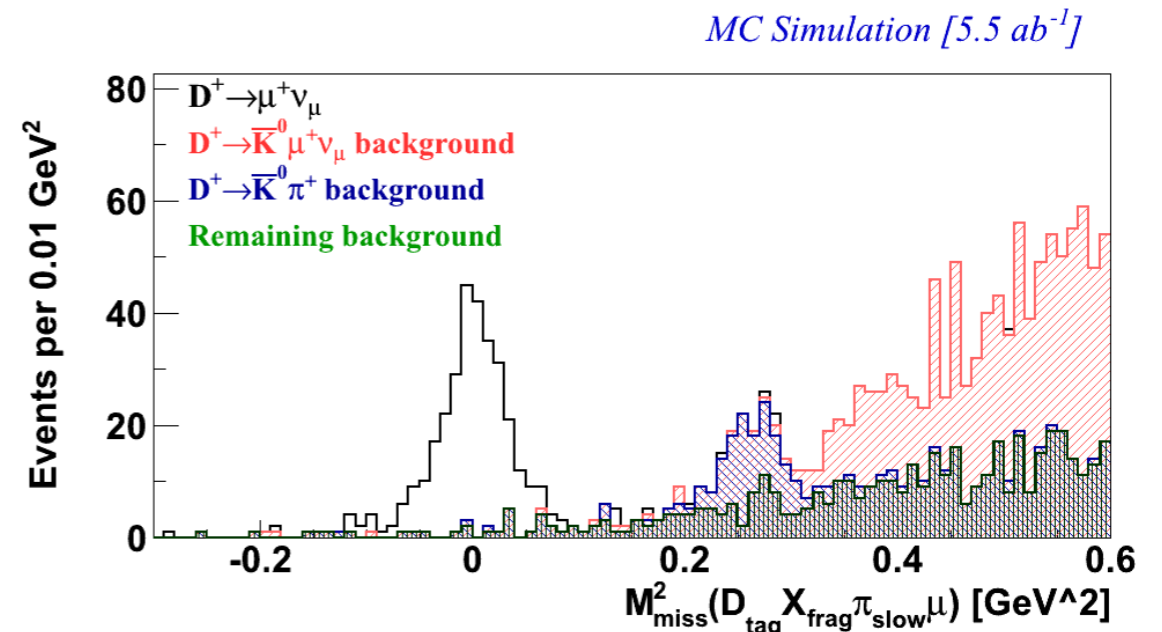
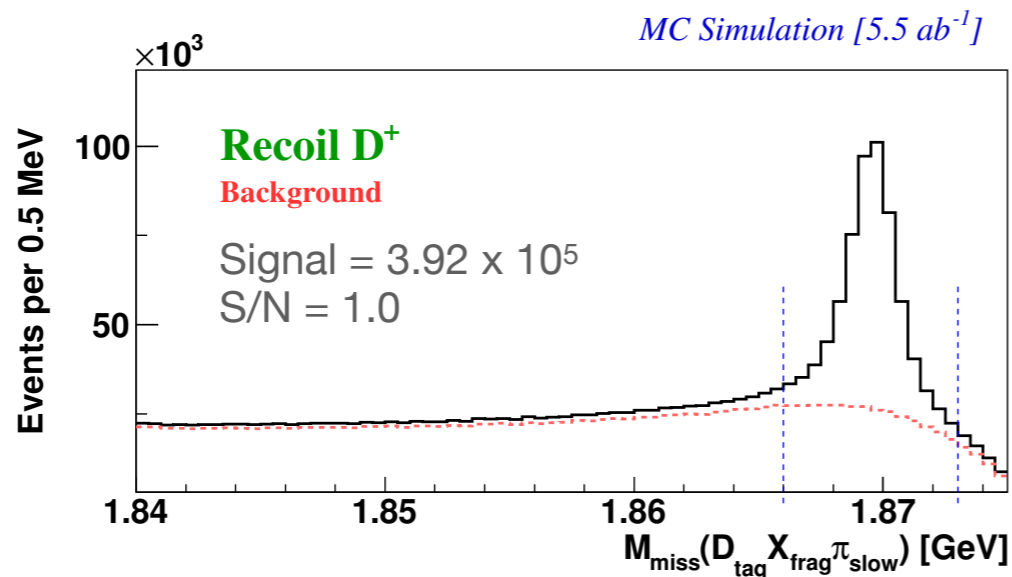
Belle II prospects $\triangleright D^+ \rightarrow \mu^+ \nu$

o MC studies \triangleright Belle II MC: 5.5 ab^{-1}

o Method $e^+e^- \rightarrow c\bar{c} \rightarrow \bar{D}X_{frag}D^{*+} (\rightarrow D^+\pi_{slow}^0) \mu^+\nu_\mu$

\triangleright Fit to missing mass $\bar{D}X_{frag}\mu\pi_{slow}^0$

\triangleright Require 1 charged track from IP and with μ -ID requirement



o Belle II (50 ab^{-1})

$\triangleright D^+ \rightarrow \mu^+ \nu_\mu$: inclusive (exclusive) decays $\sim 3.5 \times 10^6$ (1250)

\triangleright Statistical error on $\delta(f_D \cdot |V_{cd}|) = 0.65 \text{ MeV}$ (which currently dominates in WA)

\sim improved by factor of 2 w.r.t. to current measurement from CLEOc (1.9) and BESIII (1.2)

\sim also competitive to BESIII plans with 20 fb^{-1} (\sim current $\times 7$) planned over next two years

$|V_{cd}|$ via semi-leptonic decay: $D^0 \rightarrow \pi^- \ell^+ \nu$

► decay modes: $\pi \ell \nu$ & $\pi \mu \nu$

Current status

○ Several results in past years by BaBar, Belle, BESIII and CLEO-c

● Form factors $f_+^{\pi K}(0) = 0.612 \pm 0.035$ (ETM 17D, 18)

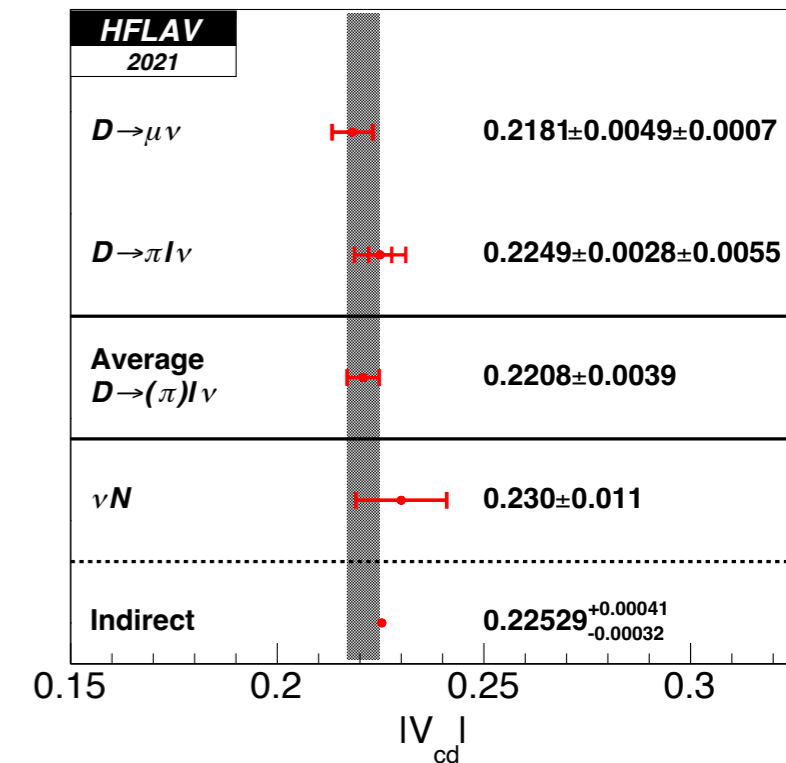
● $|V_{cd}| = 0.2249 \pm 0.0028(\text{exp.}) \pm 0.0055(\text{LQCD})$
less precision > 2%

Belle II prospects

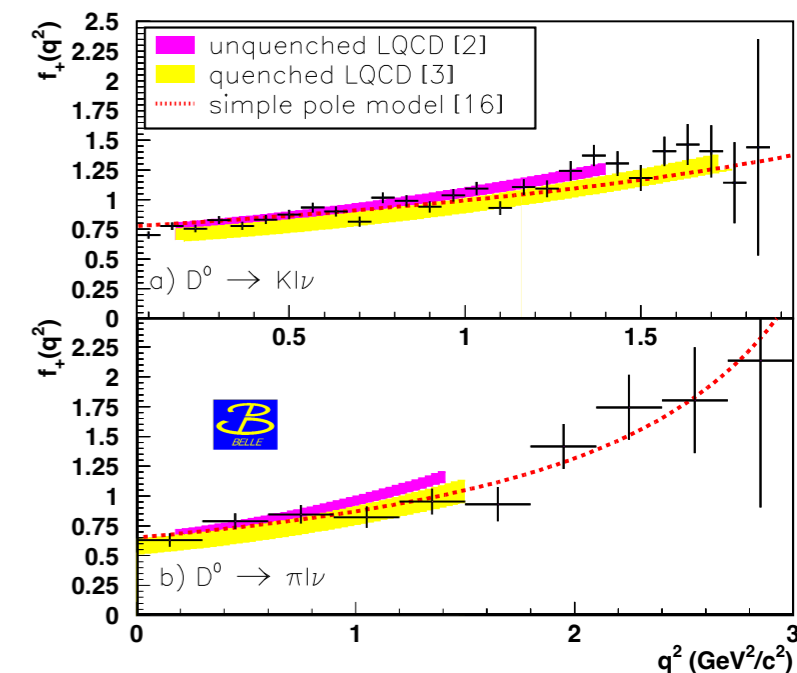
○ Belle II MC studies with 1 ab^{-1} (method discussed at slide: #11)
— missing mass resolution is comparable with Belle

○ with 50 ab^{-1} data-set
— larger sample $\sim 7 \times 10^5$ (projected w/ BaBar analysis) of $D_s \rightarrow \pi \ell \nu$
→ reduced stat. error

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 p_h^3}{24\pi^3} |V_{cd}|^2 |f_+(q^2)|^2$$



► Belle results @ 282 fb^{-1}



— Belle has higher purity (79% but $\sim 21\text{K}$ signal @ 50 ab^{-1}) as compare to BaBar (53% purity)
 — But Belle measurement have larger systematic err.

Summary

► SuperKEKB and Belle II provide an excellent platform for charm/ τ measurements

– a good start ..

► $\tau(D^0) = 410.5 \pm 1.1$ (stat) ± 0.8 (syst) fs [Phys. Rev. Lett. 127, 211801 \(2021\)](#)

○ World's best: D^0 decay time resolution (x2 better than that of Belle/BaBar)

– more exciting results to come soon with larger luminosity in coming years.

► **CKM parameters with full 50 ab⁻¹**

◎ **|V_{us}| (from τ)**

○ Belle II will provide an important insight to the current discrepancy of |V_{us}| from kaon decays and τ decays (also inclusive vs exclusive)

○ also will add important input to the current 3σ $|V_{ud}| - |V_{us}|_K$ anomaly

◎ **|V_{cs}| and |V_{cd}| (from charm)**

○ Statistically improved results from leptonic and semi-leptonic D/D_s decays

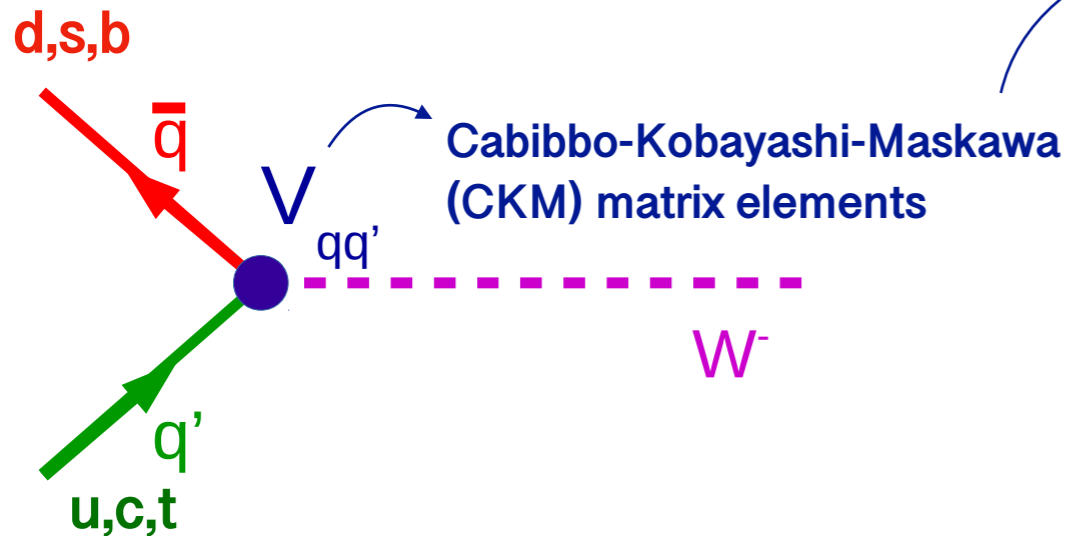
○ Belle II will also measure |V_{cd}| from $D^+ \rightarrow \mu^+ \nu$ decays (first attempt in B-factory)

Thank you

CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

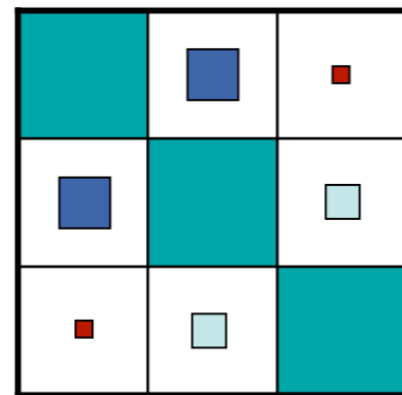
► In SM: the coupling of the quarks via the charged weak current is described by CKM matrix



First generation	Second generation	Third generation
u Up	c Charm	t Top
d Down	s Strange	b Bottom

$$q = +\frac{2}{3}$$

$$q = -\frac{1}{3}$$



◦ 3x3 unitarity complex matrix

► Unitarity constraints + freedom to redefine the complex phase (~ 4 parameters $\Rightarrow 3$ mixing angle and 1 phase \Rightarrow CPV)

◦ with Wolfenstein parameterization

► $\lambda = \sin(\theta_c) = 0.22$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

◦ unitarity triangles

► $V_{qq'} V_{qq'}^\dagger = V_{qq'}^\dagger V_{qq'} = 1$

► $q \neq q'$: 6 triangle relations ($\sum 3$ complex number = 0)

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

