Status of the CKM Matrix

with a Focus on V_{ub} and V_{cb}

Markus Prim
on behalf of the Belle and Belle II collaborations and
with material from the LHCb collaboration

08/11/2022 - DISCRETE 2022

BELLE Belle II UNIVERSITÄT BONN

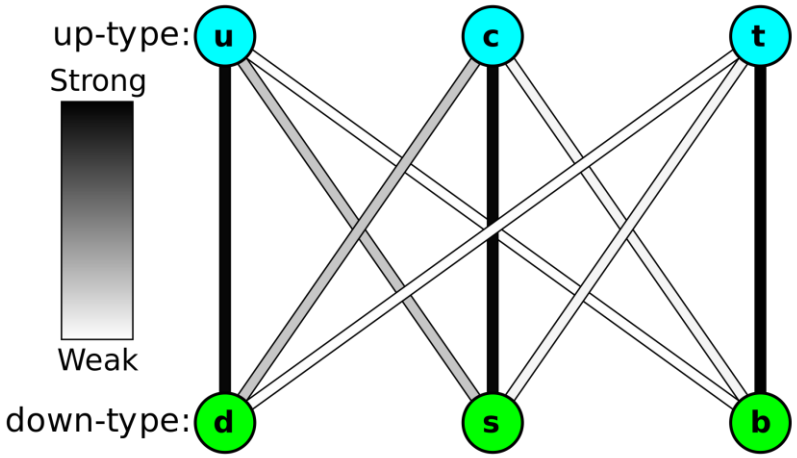
CKM Matrix

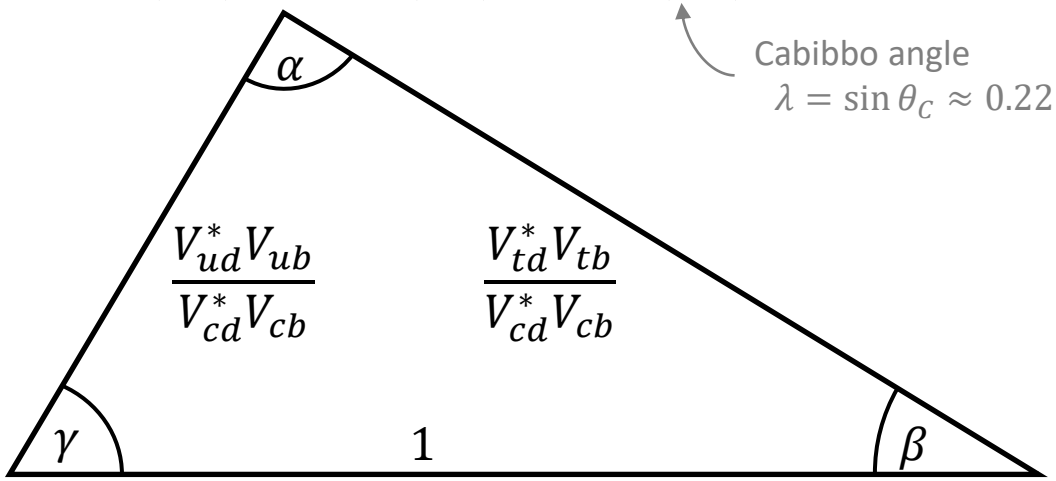
$$\begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}$$

Unitarity
$VV^{\dagger} = 1$

Over-constrain unitarity condition
→Potent test of the Standard Model

$$\underbrace{V_{ud}V_{ub}^{*}}_{O(\lambda^3)} + \underbrace{V_{cd}V_{cb}^{*}}_{O(\lambda^3)} + \underbrace{V_{td}V_{tb}^{*}}_{O(\lambda^3)} = 0$$

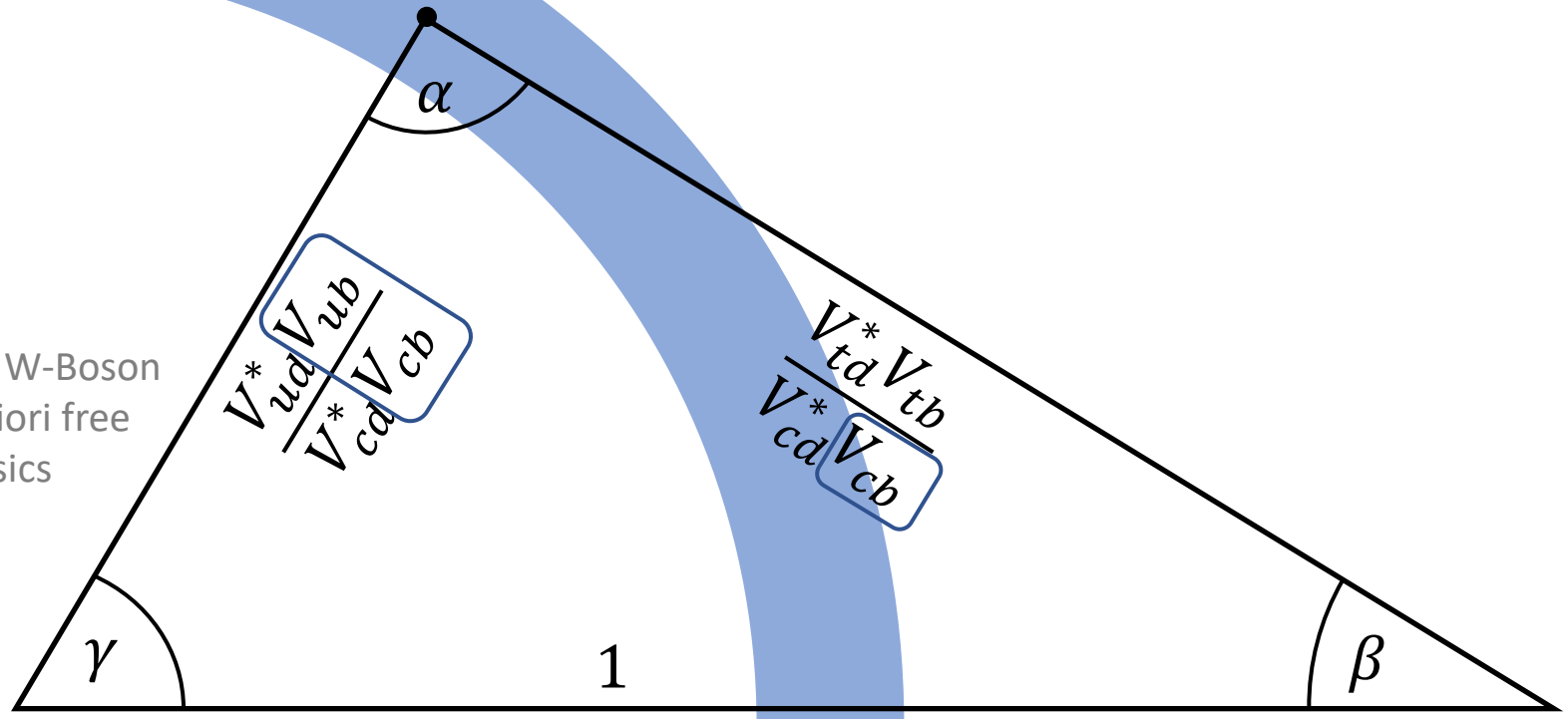




Cabibbo angle
$\lambda = \sin\theta_C \approx 0.22$

How do we measure the CKM triangle?

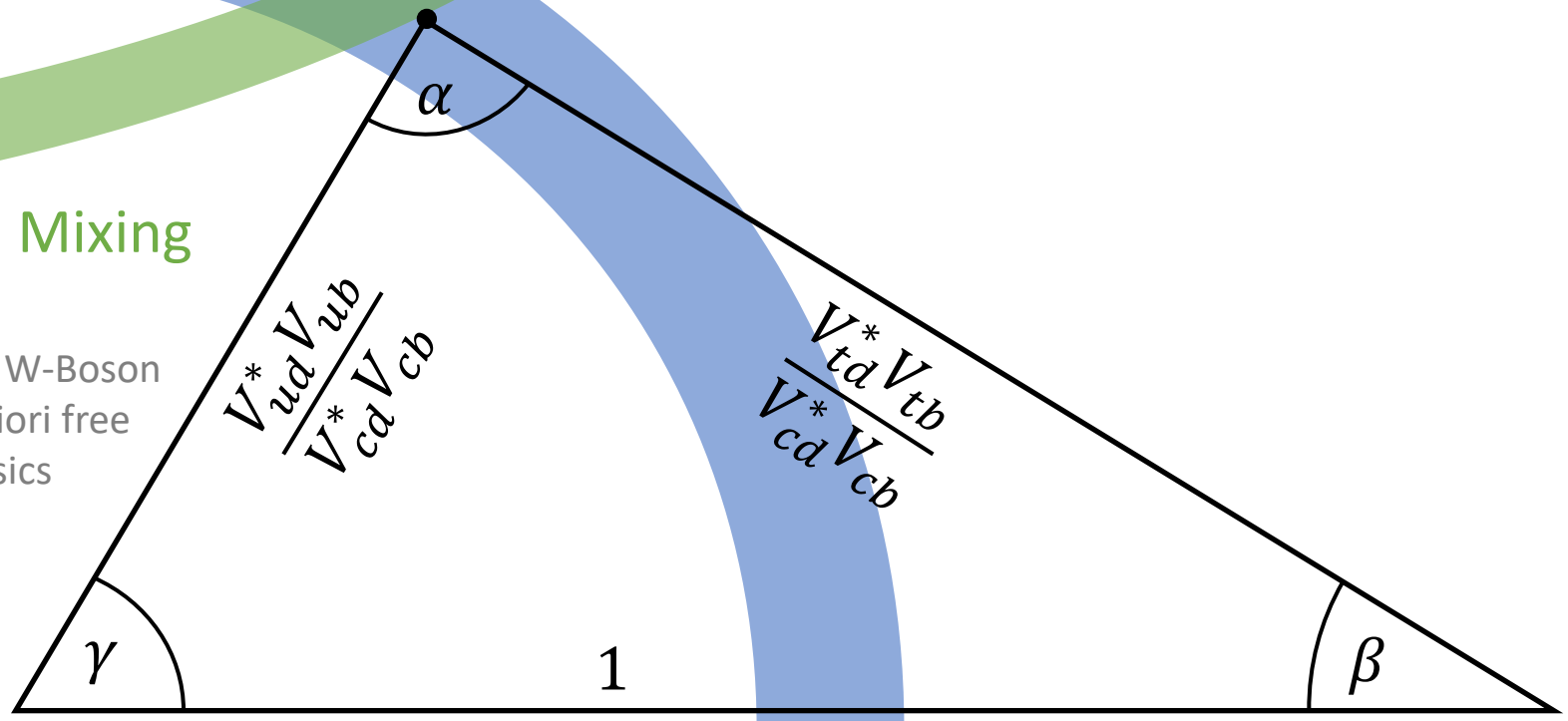
$\frac{|V_{ub}|}{|V_{cb}|}$ Dominated by W-Boson emission, a-priori free from new physics



How do we measure the CKM triangle?

CPV Kaon Mixing

$\frac{|V_{ub}|}{|V_{cb}|}$ Dominated by W-Boson emission, a-priori free from new physics

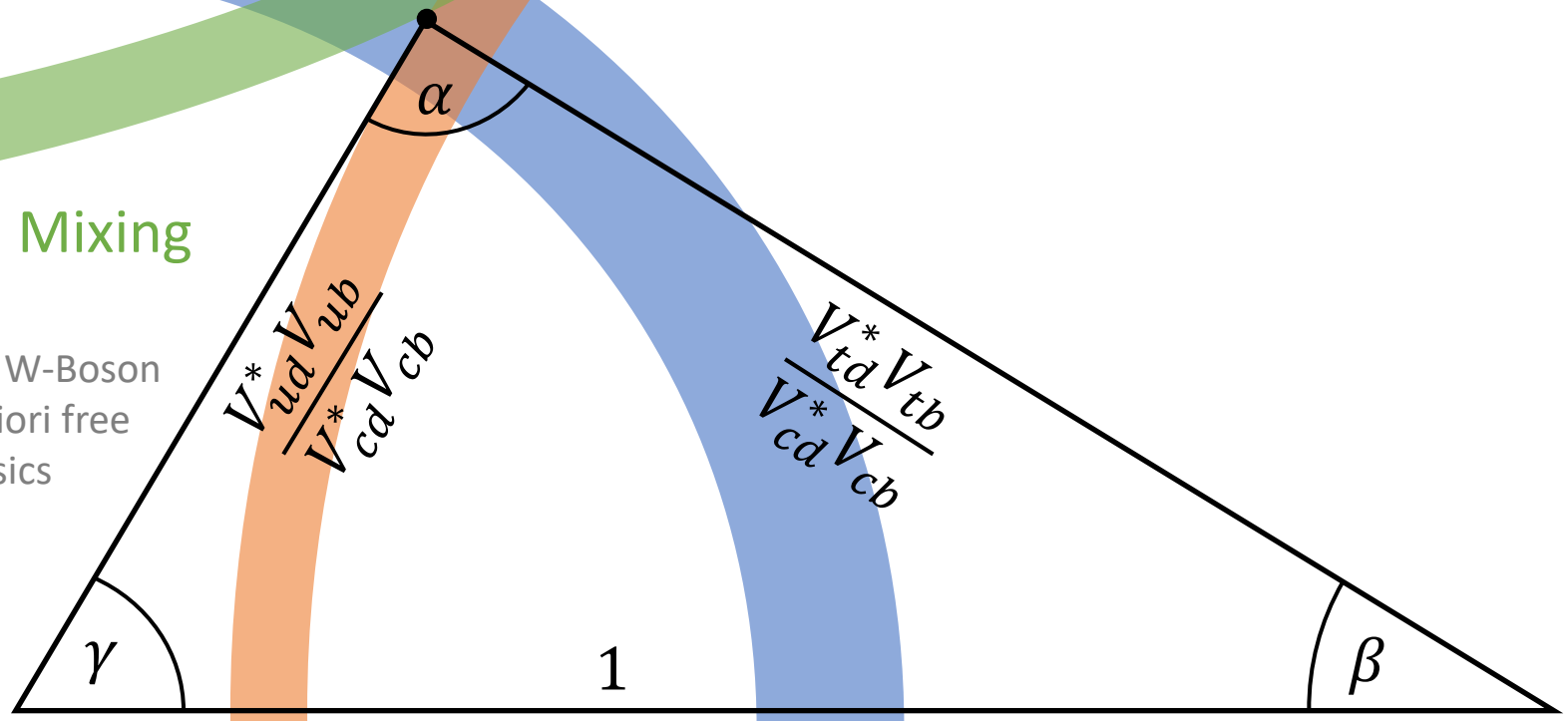


How do we measure the CKM triangle?

B-Meson Mixing

CPV Kaon Mixing

$\frac{|V_{ub}|}{|V_{cb}|}$ Dominated by W-Boson emission, a-priori free from new physics



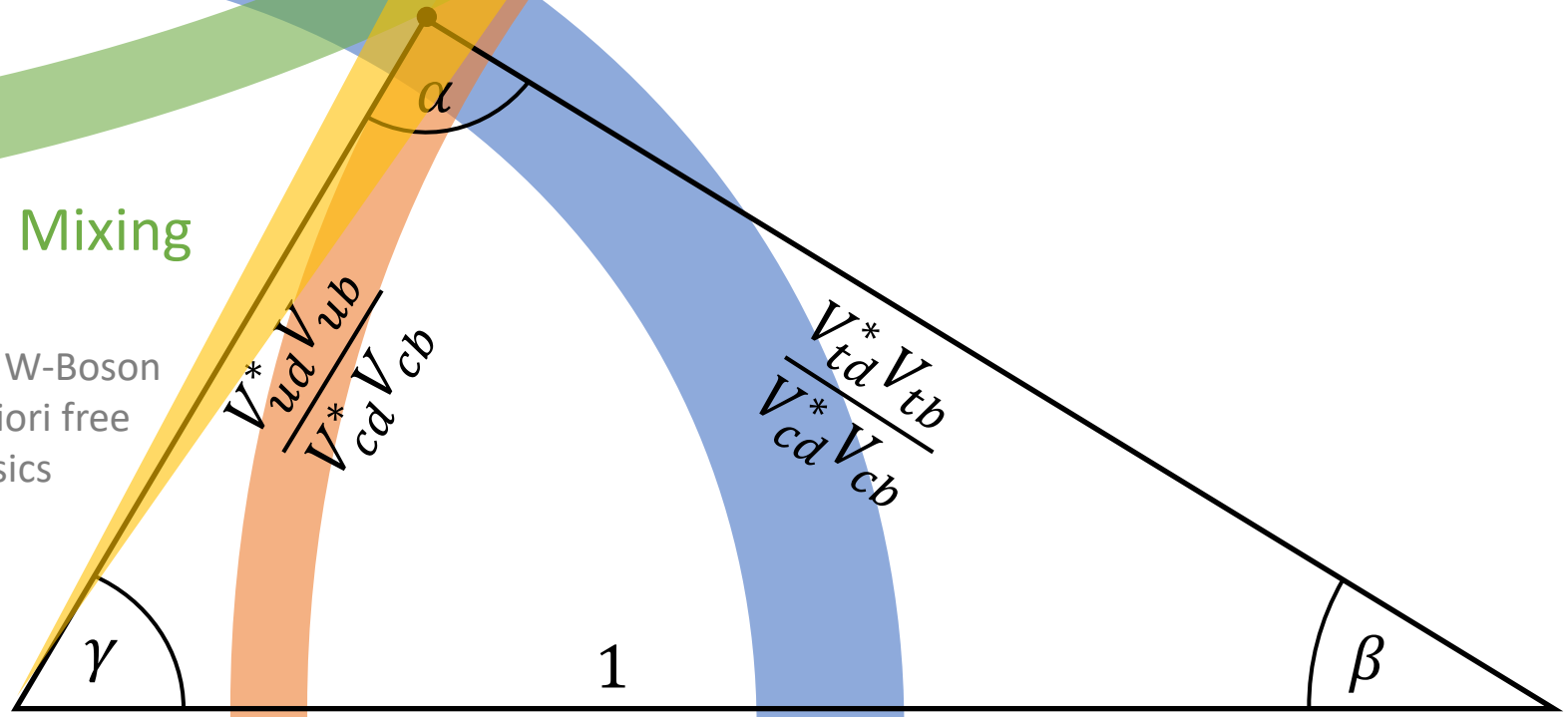
How do we measure the CKM triangle?

B-Meson Mixing

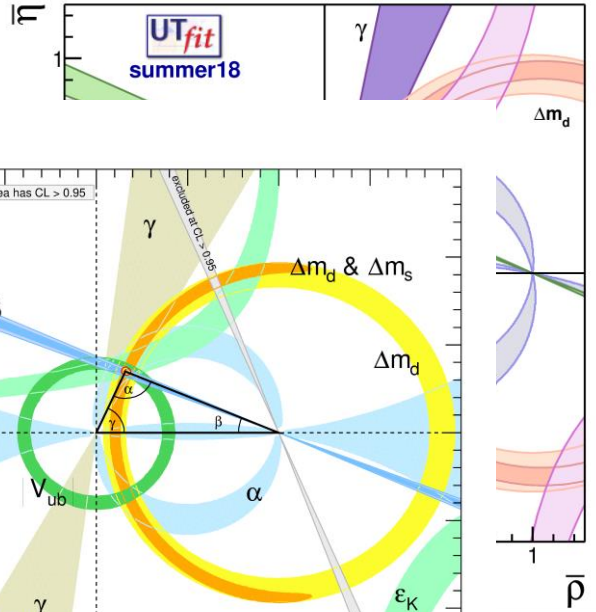
CPV Kaon Mixing

$\frac{|V_{ub}|}{|V_{cb}|}$ Dominated by W-Boson emission, a-priori free from new physics

CKM γ can be measured from tree-level decays



How do we measure the CKM triangle?



B-Meson Mixing

CPV Kaon Mixing

Today

$$\frac{|V_{ub}|}{|V_{cb}|}$$

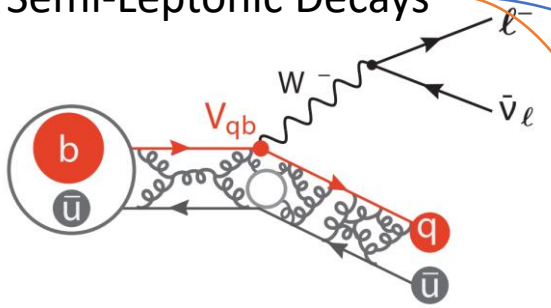
Dominated by W-Boson emission, a-priori free from new physics

CKM γ can be measured from tree-level decays

Measurements of CP violation at Belle II by Pablo Goldenzweig

How can we measure $|V_{ub}|$ and $|V_{cb}|$?

Semi-Leptonic Decays



Inclusive $|V_{ub}|$

$B \to X_u \ell \bar{\nu}_\ell$

Inclusive $|V_{cb}|$

$B \to X_c \ell \bar{\nu}_\ell$

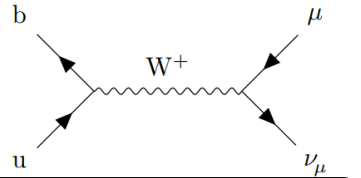
Operator Product Expansion

$$BR \propto |V_{qb}|^2 [1 + \frac{c_5(\mu)\langle O_5 \rangle(\mu)}{m_b^2} + \frac{c_6(\mu)\langle O_6 \rangle(\mu)}{m_b^3} + O(m_b^4)]$$

+ Shape Function / Fermi Motion

Leptonic Decays

Leptonic $|V_{ub}|$



$$BR \propto |V_{ub}|^2 f_B^2 m_l^2$$

f_B : B-Meson decay constant

Exclusive $|V_{ub}|$

$B \to \pi, \rho, \omega \ell \bar{\nu}_\ell$

$\Lambda_b \to p \mu \bar{\nu}_\ell$

$B_s \to K \mu \bar{\nu}_\mu$

Exclusive $|V_{cb}|$

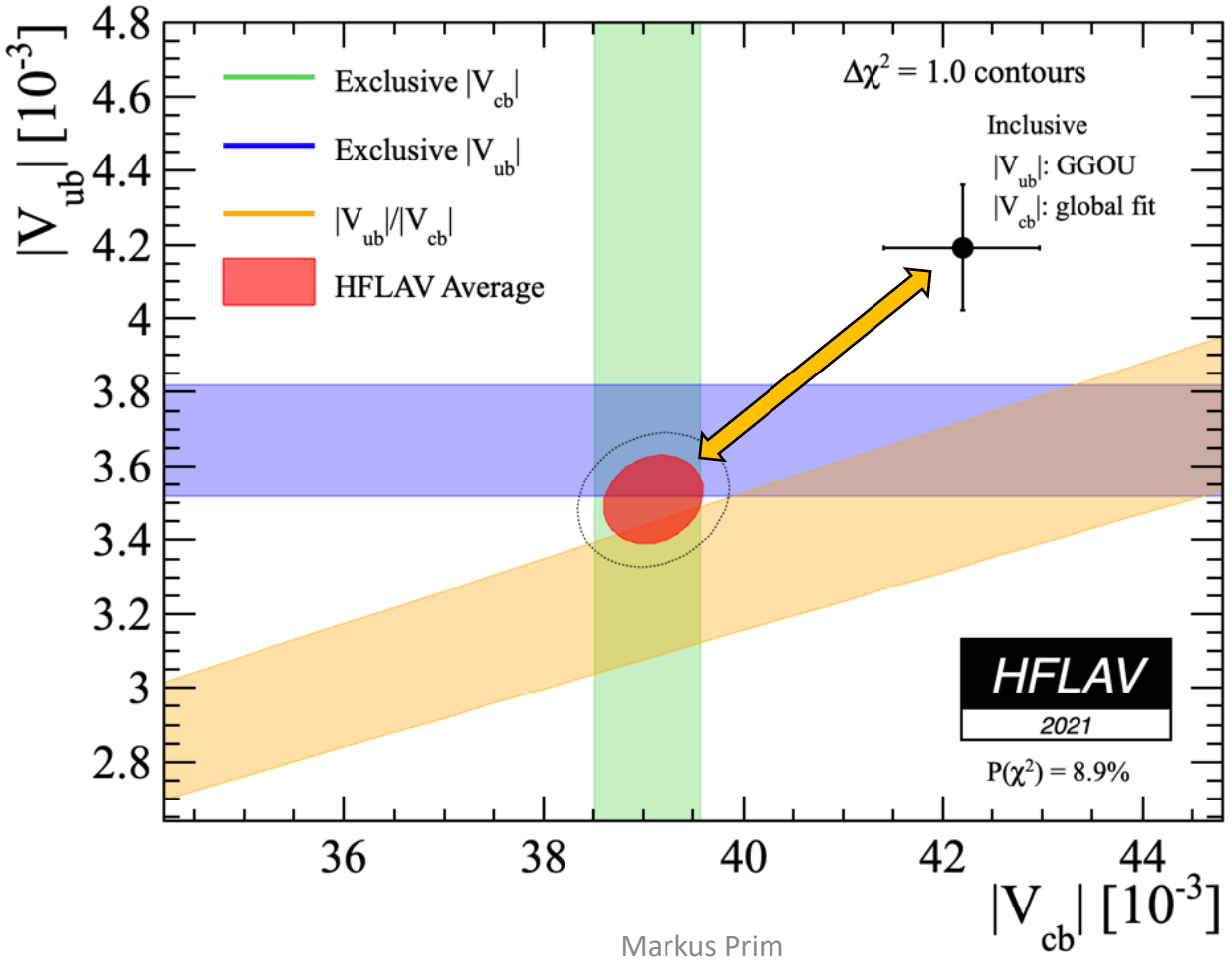
$B_{(s)} \to D_{(s)}^{(*)} \ell \bar{\nu}_\ell$

$$BR \propto |V_{qb}|^2 f^2$$

Form Factors

$\langle B|H_\mu|P\rangle = (p + p')_\mu f_+$

Where do we stand with $|V_{ub}|$ and $|V_{cb}|$?

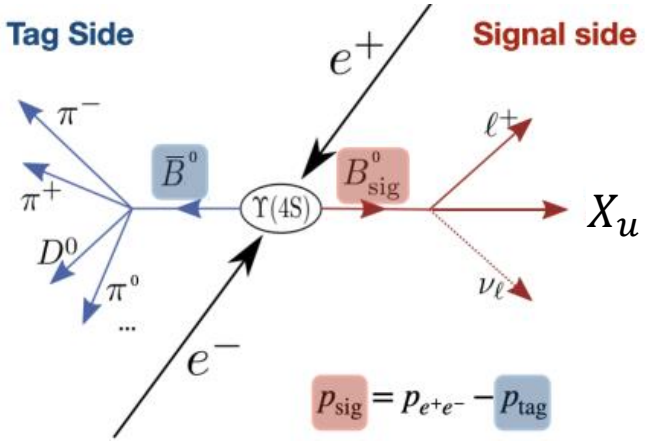


Significant tension between inclusive and exclusive determinations

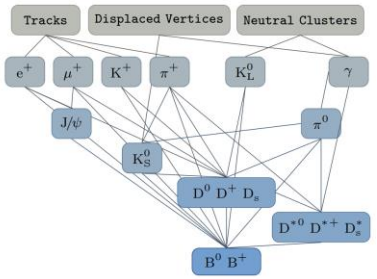
Inclusive $|V_{ub}|$

1. Measurement of partial & differential branching fractions of inclusive $B \to X_u \ell \nu_\ell$ decays with hadronic tagging

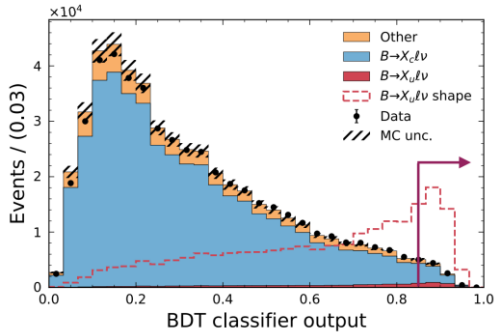
Measurement of partial & differential branching fractions of inclusive $B \to X_u \ell \nu_\ell$ decays



Full Recon.



Biggest challenge:
Suppress X_c background



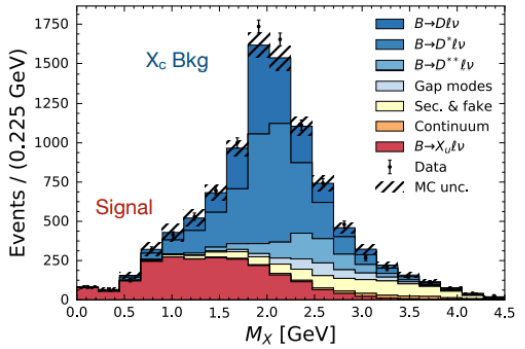
Charged Tracks Neutral Clusters

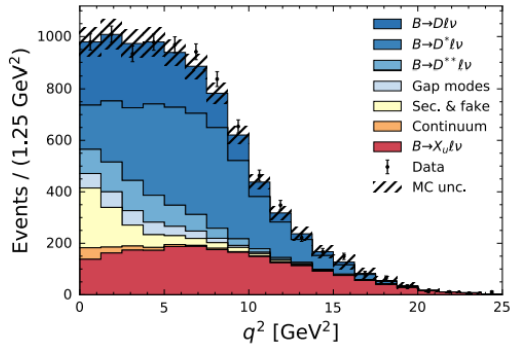
$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

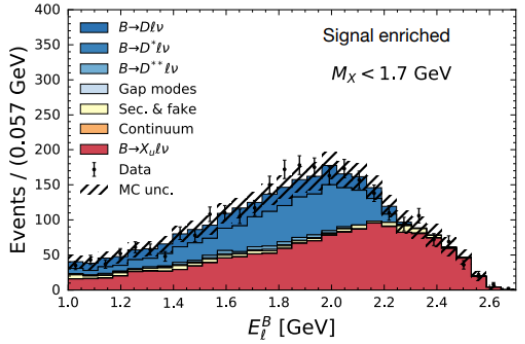
$$q^2 = (p_{sig} - p_X)^2 \qquad M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

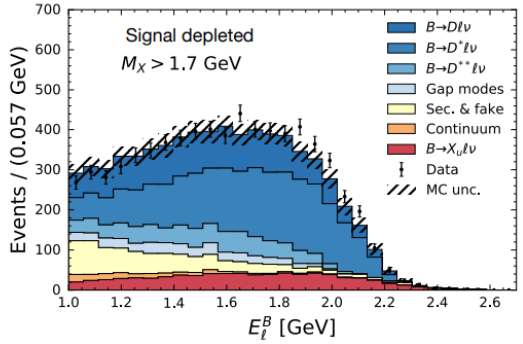
$$m^2_{miss} = \left(p_{sig} - p_X - p_\ell \right)^2 \approx m_\nu^2 = 0\,\text{GeV}^2$$

Partial: [PRD 104, 012008(2021)]
Differential: [Phys. Rev. Lett. 127, 261801 (2021)]

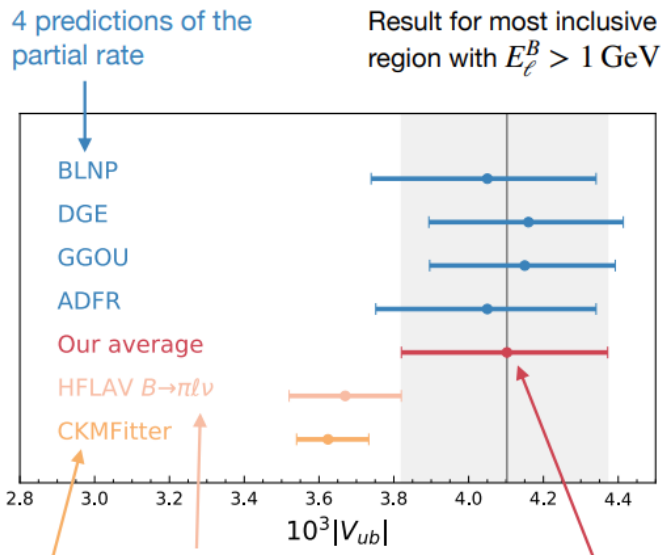






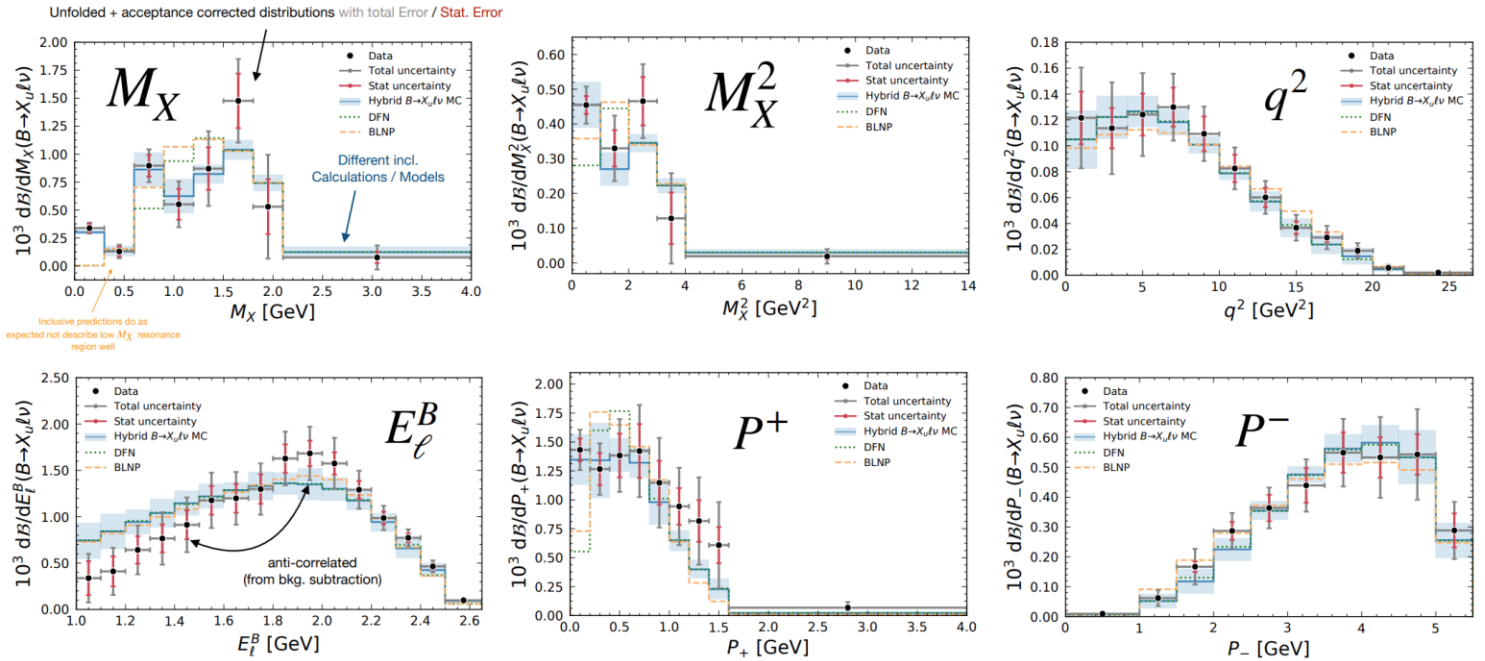


Measurement of partial & differential branching fractions of inclusive $B \to X_u \ell \nu_\ell$ decays



Exclusive Average for $B \to \pi \ell \bar{\nu}_\ell$:
$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = (3.62^{+0.11}_{-0.08}) \times 10^{-3}$



Arithmetic average:
$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$

Can be used for future shape-function independent determination of Vub

NNVub

P. Gambino, K. Healey, C. Mondino, Phys. Rev. D 94, 014031 (2016), [arXiv:1604.07598]

F. Bernlochner, H. Lacker, Z. Ligeti, I. Stewart, F. Tackmann, K. Tackmann Phys. Rev. Lett. 127, 102001 (2021) [arXiv:2007.04320]

Exclusive $|V_{ub}|$

1. First observation of the decay $B_s^0 \to K^- \mu^+ \nu_\mu$ & measurement of $|V_{ub}|/|V_{cb}|$ 

2. First glimpse at $|V_{ub}|$ in $B \to \pi \ell \nu_\ell$ with Belle II data 

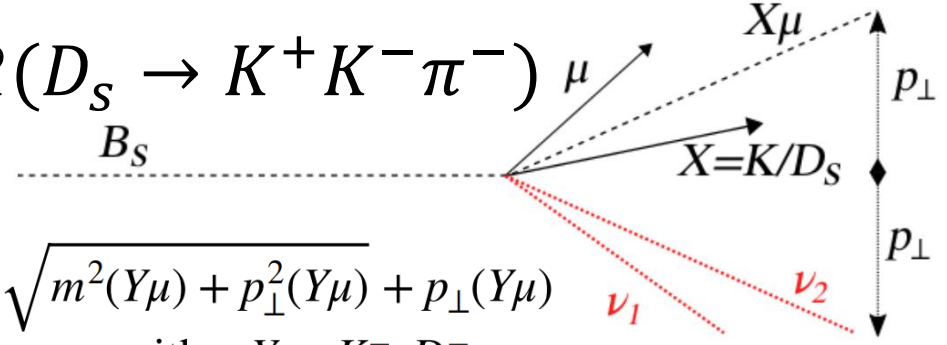
First observation of the decay $B_s^0 \to K^- \mu^+ \nu_\mu$ & measurement of $|V_{ub}|/|V_{cb}|$

[Phys.Rev.Lett. 126 (2021) 8, 081804]

- Directly measure $\frac{|V_{ub}|}{|V_{cb}|}$ via the ratio

$$R = \frac{BR(B_s^0 \to K^- \mu^+ \nu_\mu)}{BR(B_s^0 \to D_s^- \mu^+ \nu_\mu)} = \frac{N_K}{N_{D_s}} \frac{\epsilon_{D_s}}{\epsilon_K} BR(D_s \to K^+ K^- \pi^-)$$

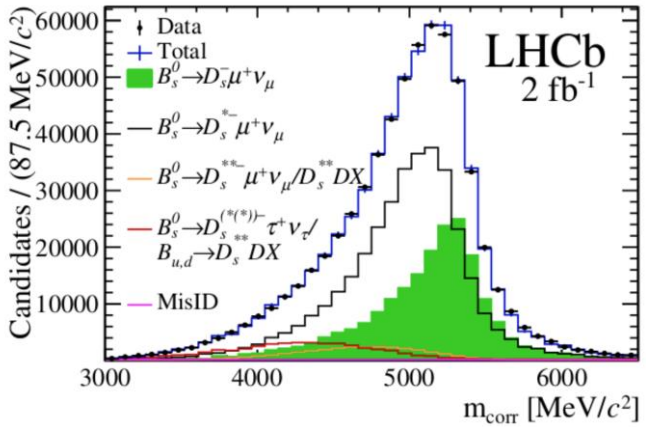
- Separation of decay vertex from primary vertex is utilized to reconstruct B_s flight direction
- Reconstruct *corrected mass* m_{corr}



$$m_{\text{corr}} = \sqrt{m^2(Y\mu) + p_\perp^2(Y\mu)} + p_\perp(Y\mu)$$

with $Y = K^-, D_s^-$

$B_s^0 \to D_s^- \mu^+ \nu_\mu$

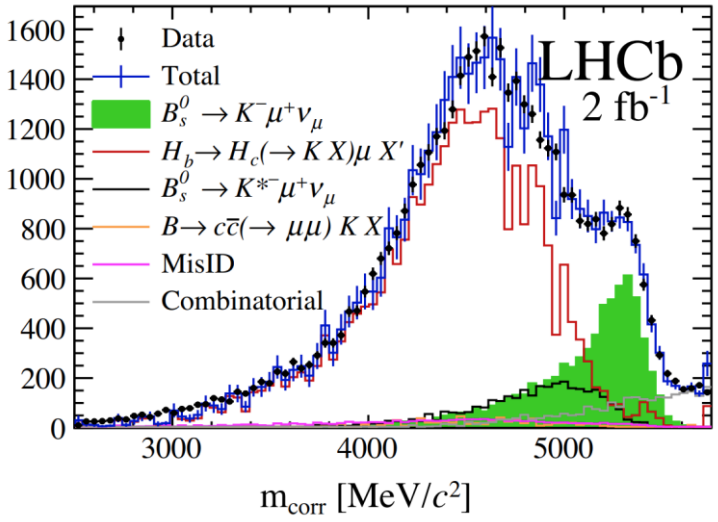


First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ & measurement of $|V_{ub}|/|V_{cb}|$

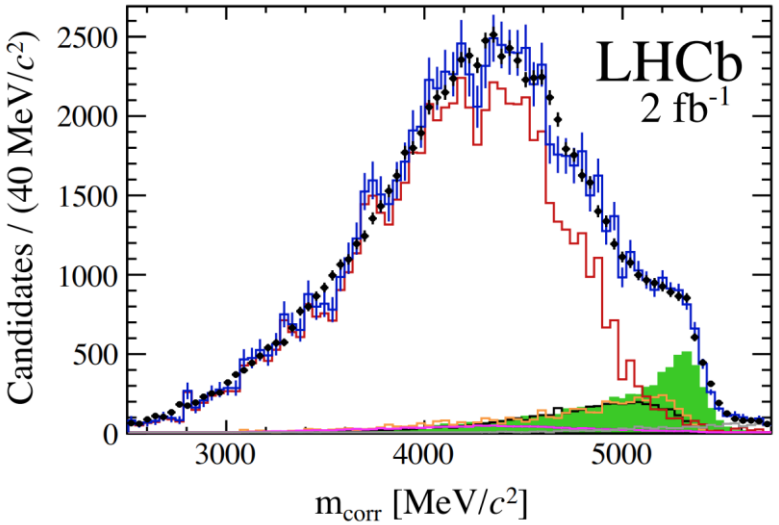
[Phys.Rev.Lett. 126 (2021) 8, 081804]

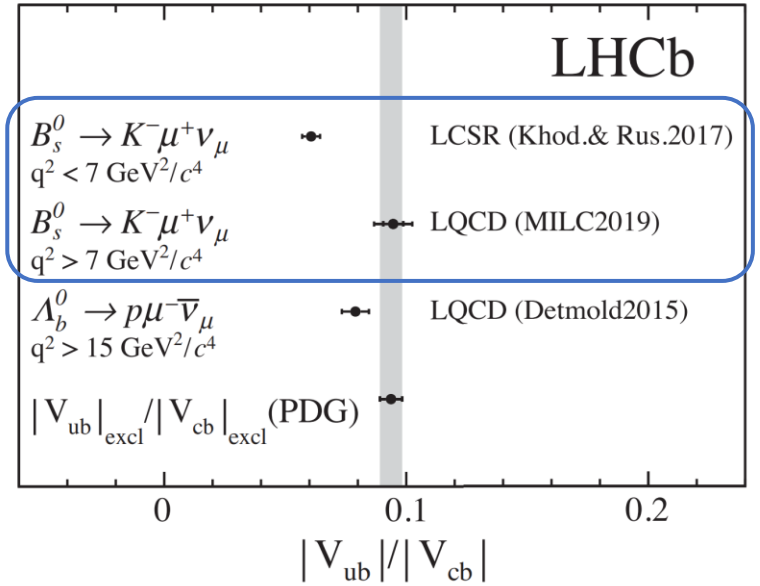
Extraction at low and high $q^2 = (p_B - p_K)^2$

$q^2 < 7\ \mathrm{GeV}^2$



$q^2 > 7\ \mathrm{GeV}^2$

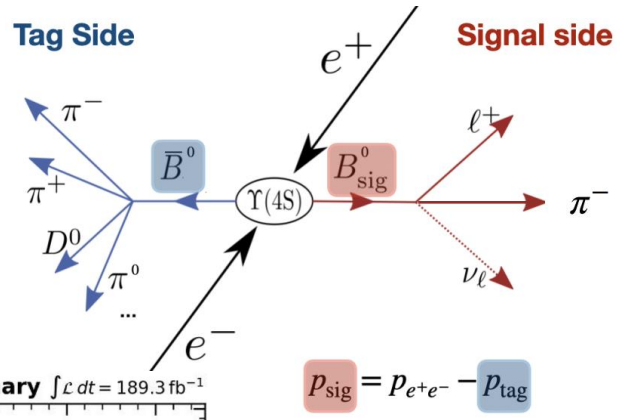


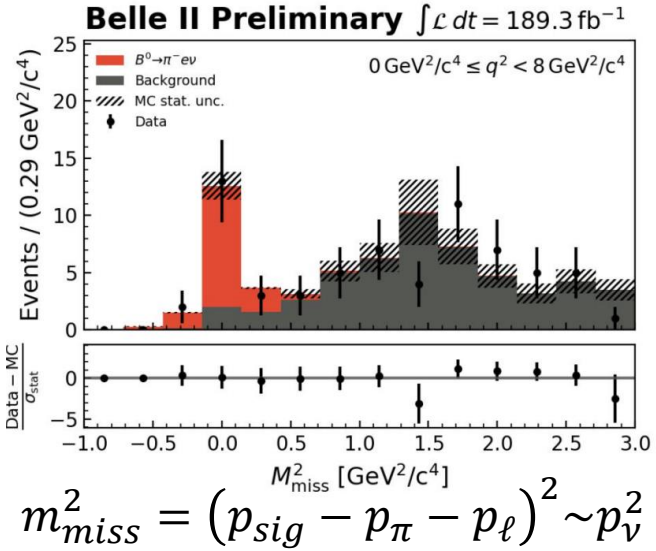


$|V_{ub}|$ in $B \to \pi \ell \nu_\ell$ with Belle II data

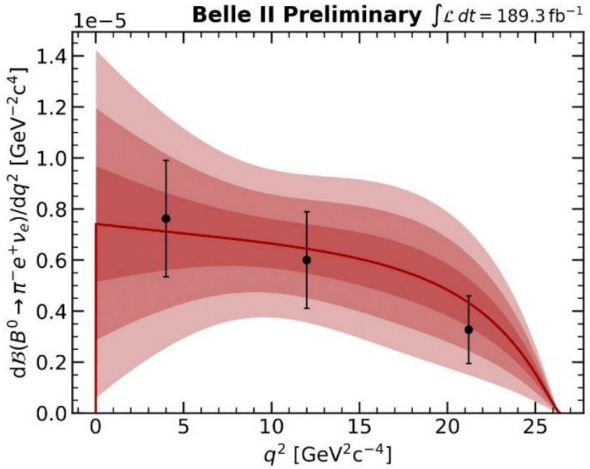
[2206.08102]

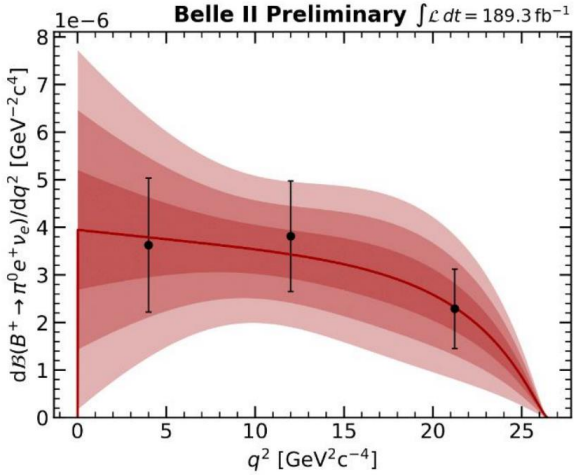
- Hadronic tag-side reconstruction
- Fit in 3 bins of q^2 to subtract background
- Form factor & $|V_{ub}|$ fit





$$m^2_{miss} = \left(p_{sig} - p_\pi - p_\ell\right)^2 \sim p^2_\nu$$





$$|V_{ub}| = (3.88 \pm 0.45) \times 10^{-3}$$

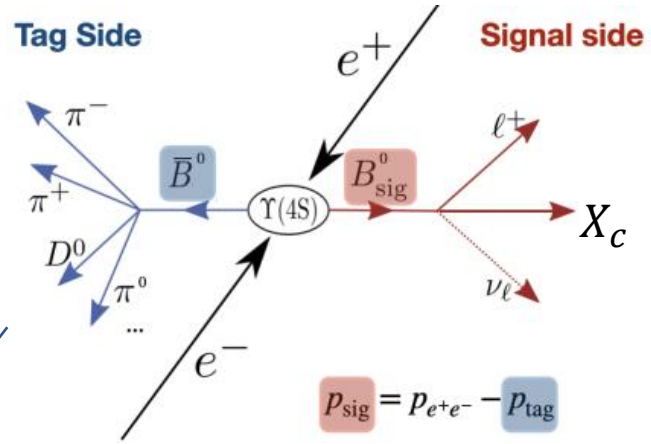
with LQCD data from FNAL/MILC Phys.Rev.D 92 (2015) 1, 014024, [arXiv: 1503.07839]

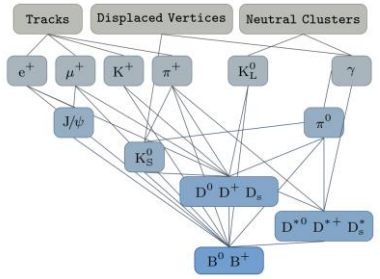
Inclusive $|V_{cb}|$

1. Measurement of q^2 moments of inclusive $B \to X_c \ell \nu_\ell$ decays with hadronic tagging
   Measurement of Lepton mass squared moments in inclusive $B \to X_c \ell \nu_\ell$ decays with the Belle II experiment
2. First determination of $|V_{cb}|$ from q^2 moments
3. Third order correction to the semileptonic $b \to c$ and the muon decays
   Three loop calculations and $|V_{cb}|$

Measurement of q^2 moments of inclusive $B \to X_c \ell \nu_\ell$ decays with hadronic tagging

- Similar analysis strategy for Belle and Belle II
- Hadronic tag-side reconstruction

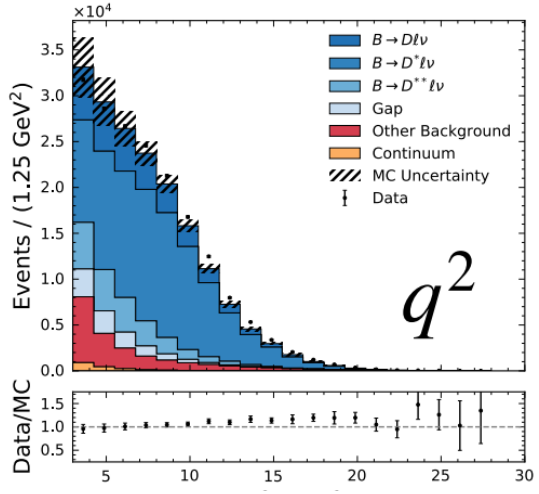


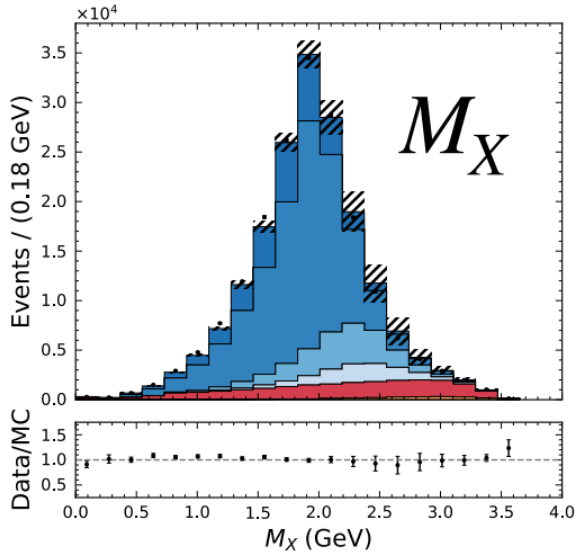


FEI / FR

$$q^2 = (p_{sig} - p_{X_c})^2$$

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$





Belle: [PRD 104, 112011 (2021)]
Belle II: [2205.06372]

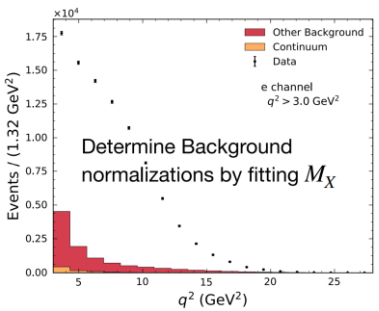
Measurement of q^2 moments of inclusive $B \to X_c \ell \nu_\ell$ decays with hadronic tagging

- Event-wise Master-formula

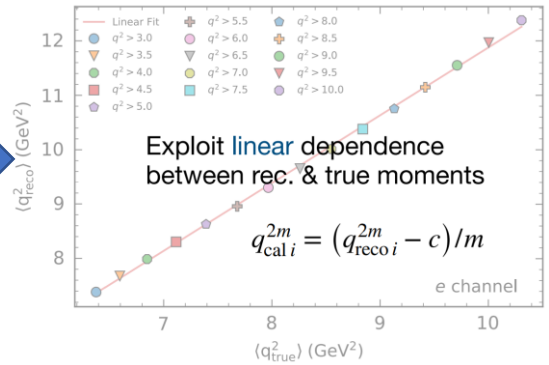
Belle: [PRD 104, 112011 (2021)]
Belle II: [2205.06372]

$$\langle q^{2m} \rangle = \frac{C_{cal} \, C_{acc}}{\sum_i^{events} w(q_i^2)} \times \sum_i^{events} w(q_i^2) \, q_{cal,i}^{2m}$$

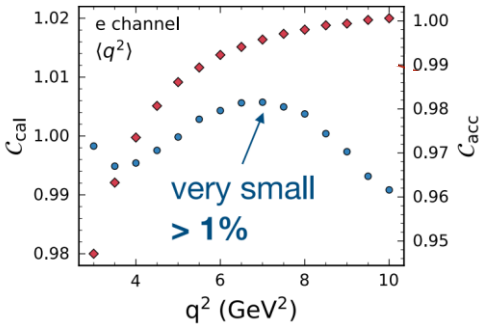
1. Subtract Background

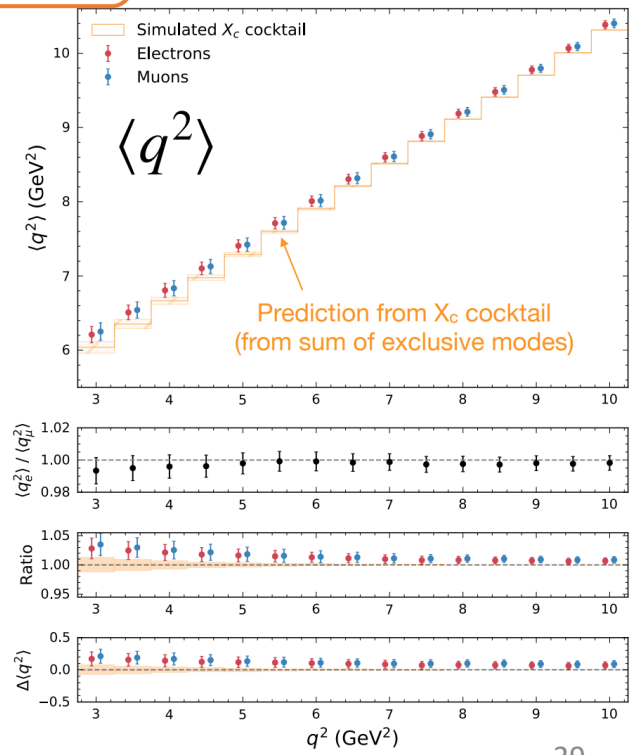


2. Calibrate moments



3. Correct for selection efficiencies

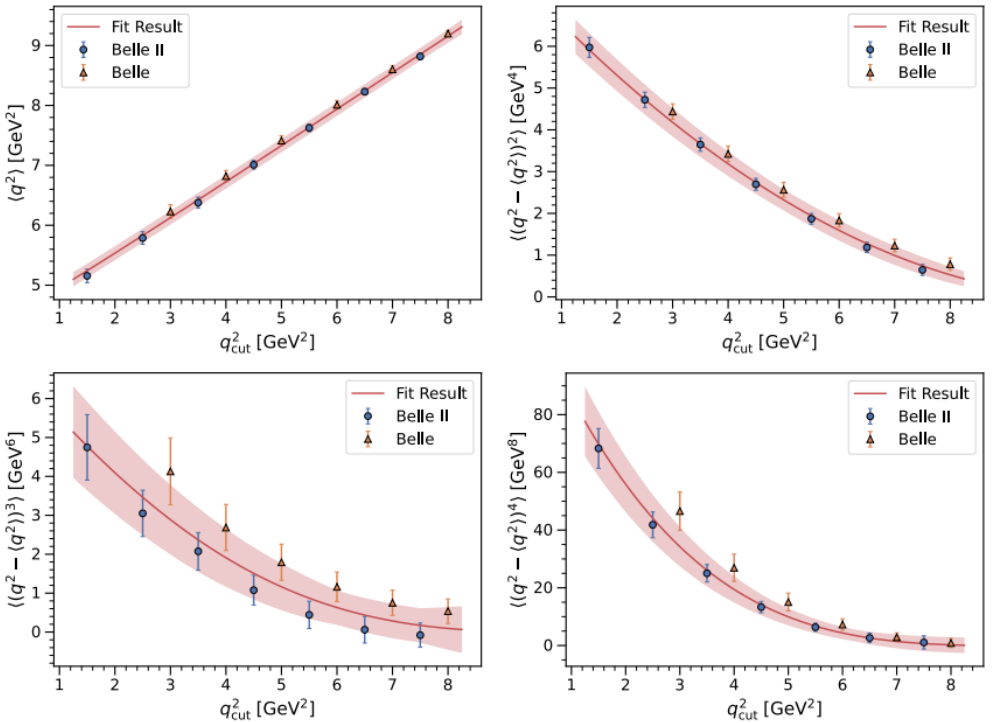


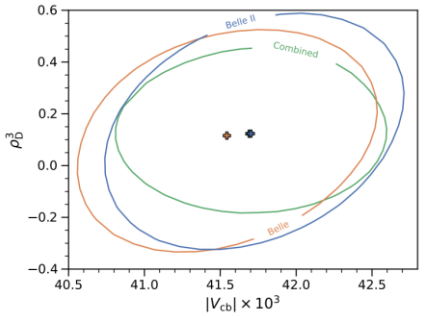


LFU Ratio

First determination of $|V_{cb}|$ from q^2 moments

- Fit to both Belle and Belle II measurements
- $|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$



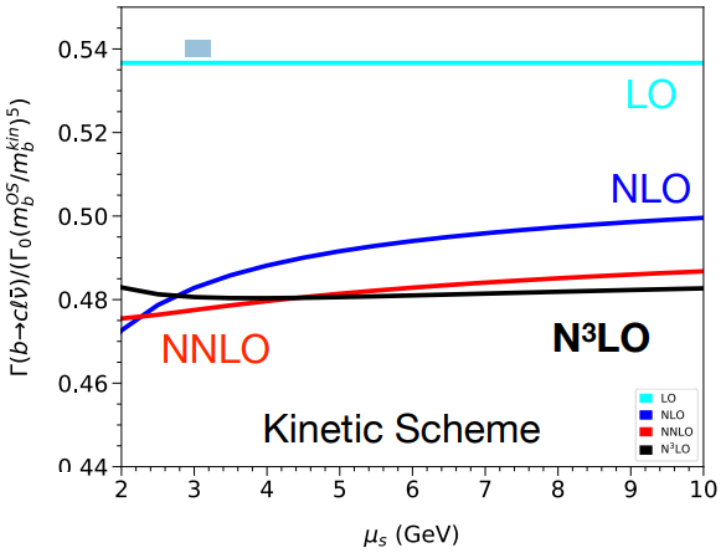


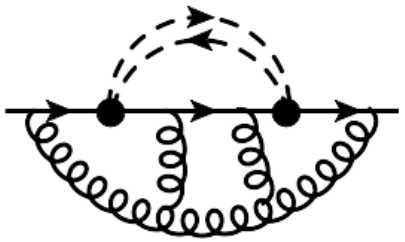
F. Bernlochner, M. Fael, K. Olschwesky, E. Persson, R. Van Tonder, K. Vos, M. Welsch
[JHEP 10 (2022) 068]

- Leverage reparametrization invariance to reduce the set of HQE parameters → only 8 non-perturbative parameters up to order $\frac{1}{m_b^4}$
- Consistent with inclusive $|V_{cb}|$ from lepton energy and hadronic invariant mass moments

Theory Progress

- Semi-leptonic rate at N3LO
  M.Fael, K. Schönwald, M. Steinhauser
  [Phys. Rev. D 104 (2021) 1, 016003]





→Update inclusive fit of lepton energy and hadronic invariant mass moments
M. Bordone, B.Capdevila, P.Gambino
[Phys.Lett.B. 822 (2021) 136679]

$$|V_{cb}| = 41.16(30)_{th}(32)_{exp}(25)_{\Gamma}\ 10^{-3}$$

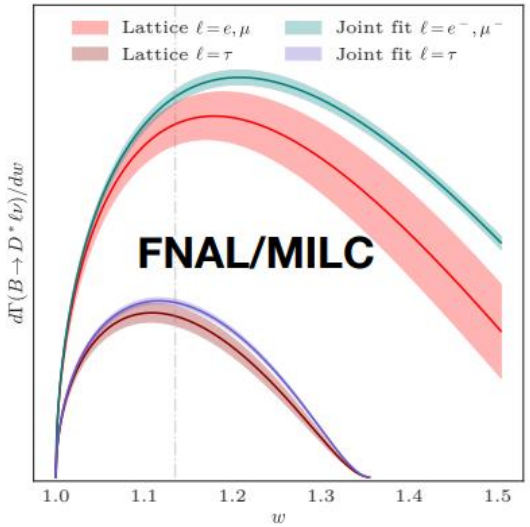
$$\frac{\Delta|V_{cb}|}{|V_{cb}|} = 1.2\%$$

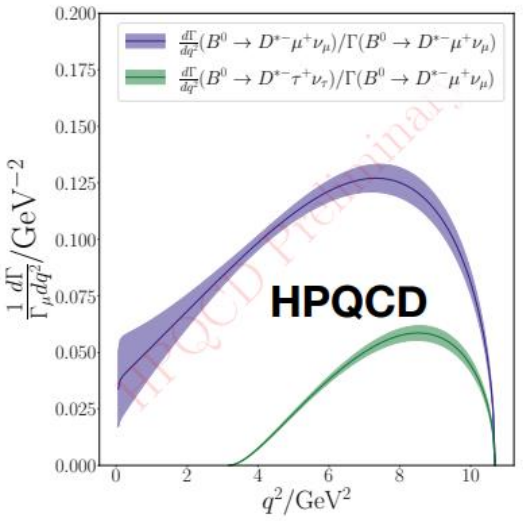
Exclusive $|V_{cb}|$

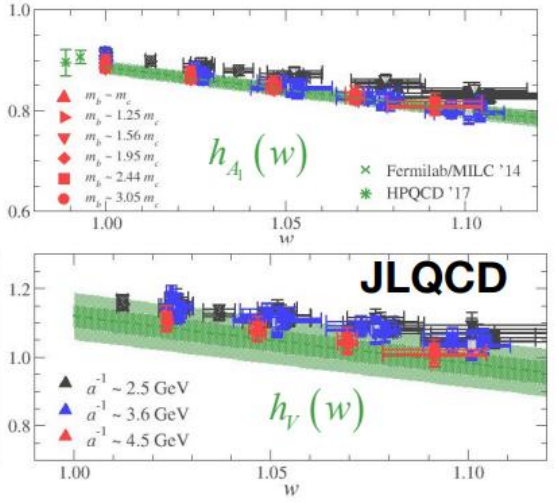
1. Beyond zero-recoil lattice prediction for form factors
2. Measurement of $|\mathrm{V_{cb}}|$ with $\mathrm{B_s} \to \mathrm{D_s^{(*)}} \mu \nu_\mu$ decays
3. $|\mathrm{V_{cb}}|$ in $B \to \mathrm{D} \ell \nu_\ell$ with Belle II data
4. $|\mathrm{V_{cb}}|$ in $\mathrm{B^0} \to \mathrm{D^*} \ell \nu_\ell$ with Belle II data
5. Measurement of Differential Distributions of $\mathrm{B} \to \mathrm{D^*} \ell \nu_\ell$ and Determination of $|\mathrm{V_{cb}}|$

Beyond zero-recoil lattice prediction for form factors

- Theory progresses and delivers beyond zero-recoil predictions for the $B \to D^* \ell \nu_\ell$ form factors for the first time
- FNAL/MILC under review A. Bazarvov et. al [2105.14019]
- HPQCD & JLQCD in preparation



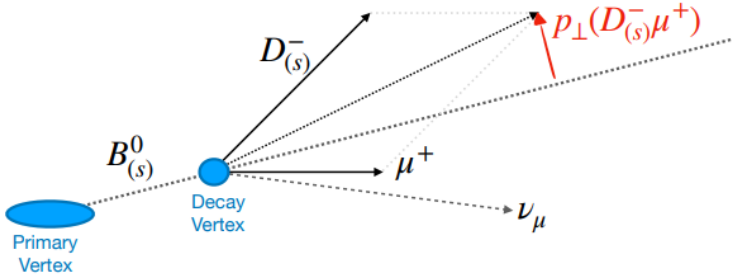




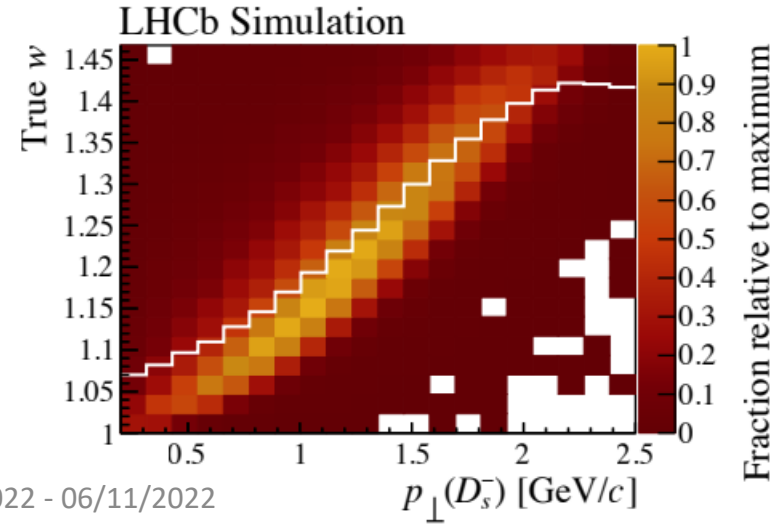
Measurement of $|V_{cb}|$ with $B_s \to D_s^{(*)} \mu \nu_\mu$ decays

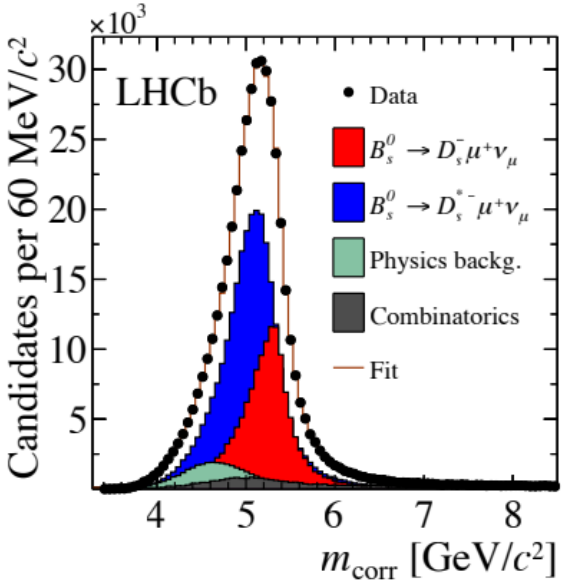
[Phys. Rev. D. 101, 072004]

- Separation of decay vertex from primary vertex is utilized to reconstruct B_s flight direction
- Reconstruct *corrected mass* m_{corr}
- Hadronic recoil w reconstructed via correlation to $p_\perp(D_s)$



$$m_{corr} \equiv \sqrt{m^2(D_s^-\mu^+) + p_\perp^2(D_s^-\mu^+)} + p_\perp(D_s^-\mu^+)$$

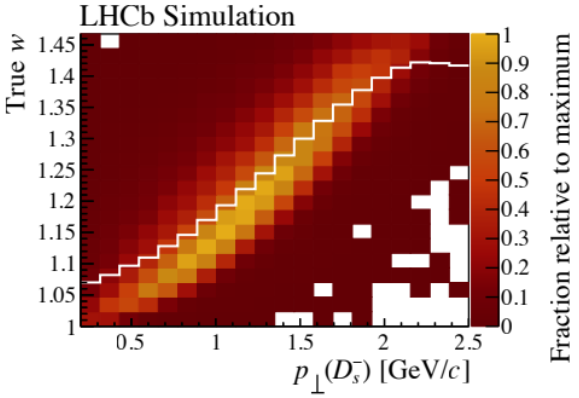


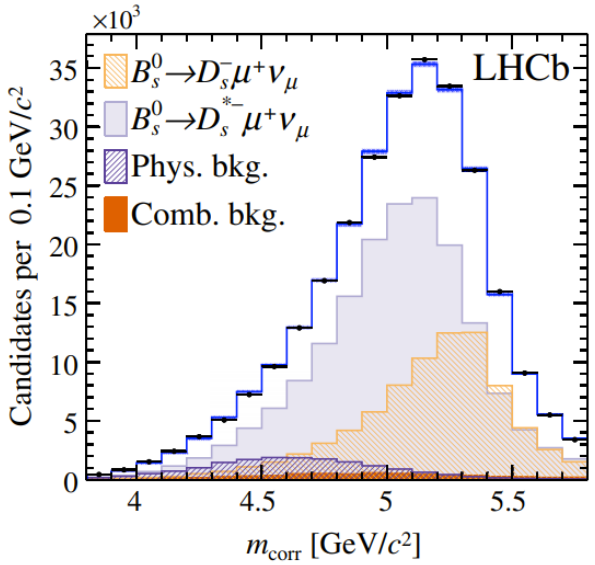


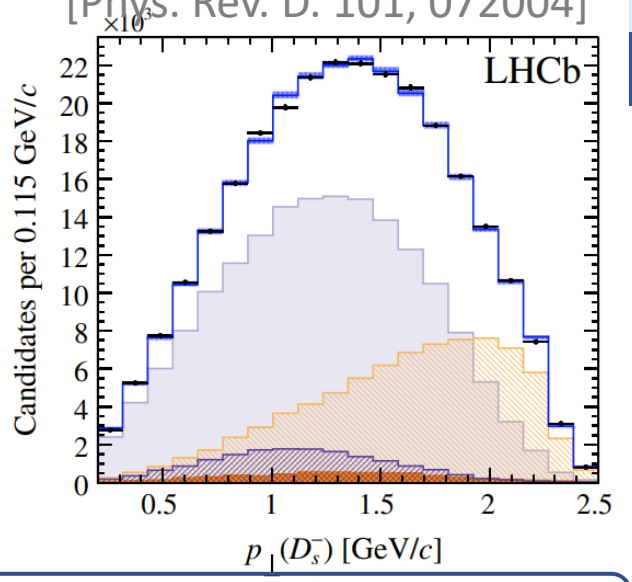
Measurement of $|V_{cb}|$ with $B_s \to D_s^{(*)} \mu \nu_\mu$ decays

[Phys. Rev. D. 101, 072004]

- Fit to m_{corr}- $p_\perp$



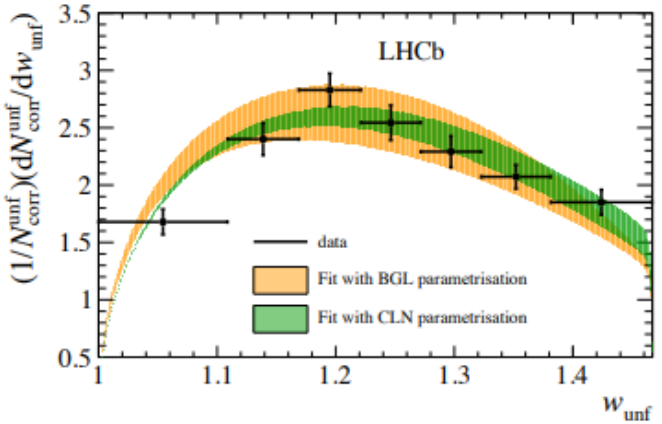




- $|V_{cb}|_{BGL} = \left(41.7 \pm 0.8(stat) \pm 0.9(sys) \pm 1.1(ext)\right) \times 10^{-3}$

[JHEP12(2020)144]

- Unfolded w distributions for $B_s \to D_s^* \mu \nu_\mu$



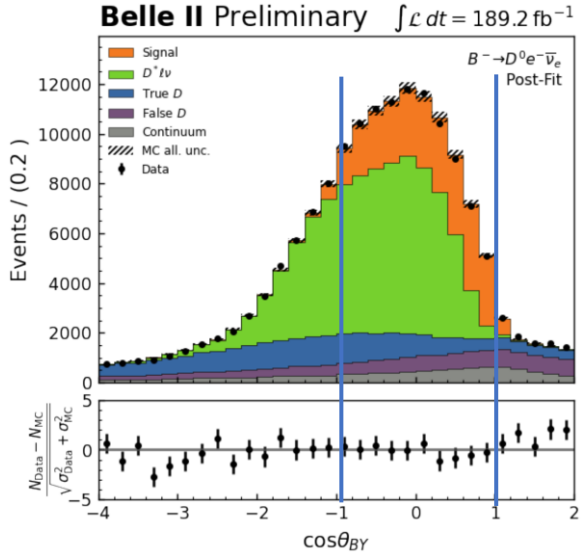
$|V_{cb}|$ in $B \to D\ell\nu_\ell$ with Belle II data

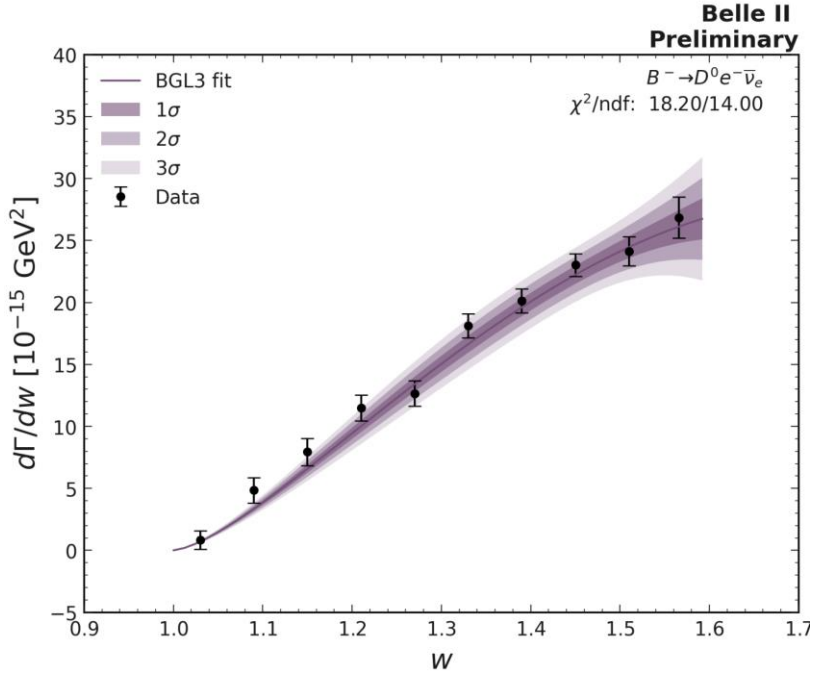
[2210.13143]

- Untagged reconstruction and $189.3 fb^{-1}$ for $B^{\pm,0}, \ell = e, \mu$
- Signal extraction in

$$\cos\theta_{BY} = \frac{2E_B^* E_Y^* - m_B^2 - m_Y^2}{2|p_B^*||p_Y^*|}$$

- Signal peaks [-1, 1]
- Main background: D^*



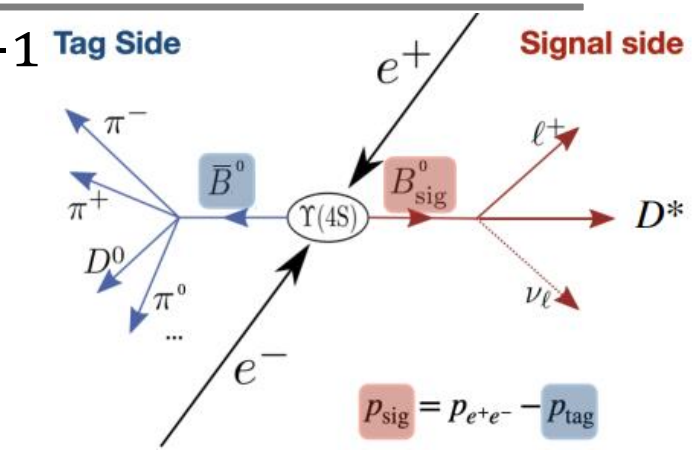


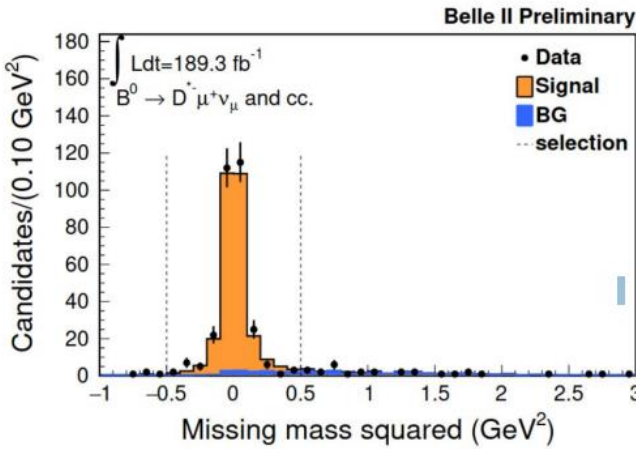
$$\eta_{EW}|V_{cb}|_{BGL} = (38.53 \pm 1.15) \times 10^{-3}$$

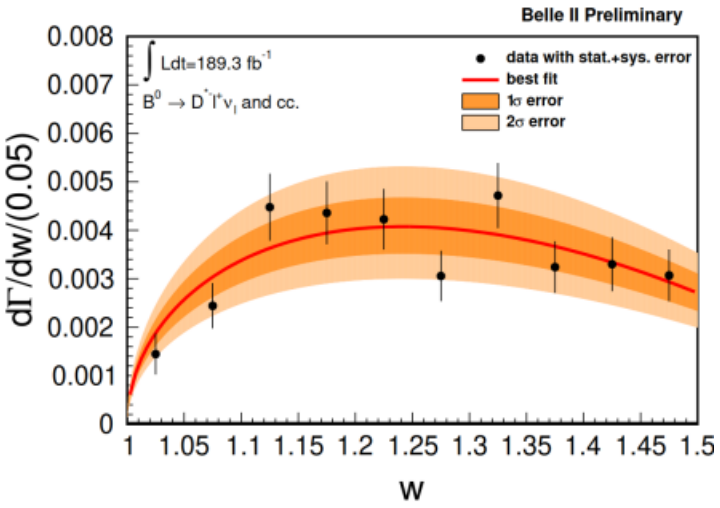
$|V_{cb}|$ in $B^0 \to D^* \ell \nu_\ell$ with Belle II data

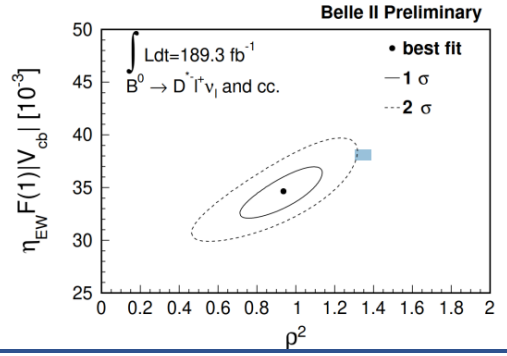
- Reconstructed with hadronic tagging and $189.3 fb^{-1}$
- Background subtraction in

$$m^2_{\text{miss}} = (p_{\text{sig}} - p_{D^*} - p_\ell)^2 \sim p_\nu^2 = 0$$









$$|V_{cb}|_{CLN} = (37.9 \pm 2.7) \times 10^{-3}$$

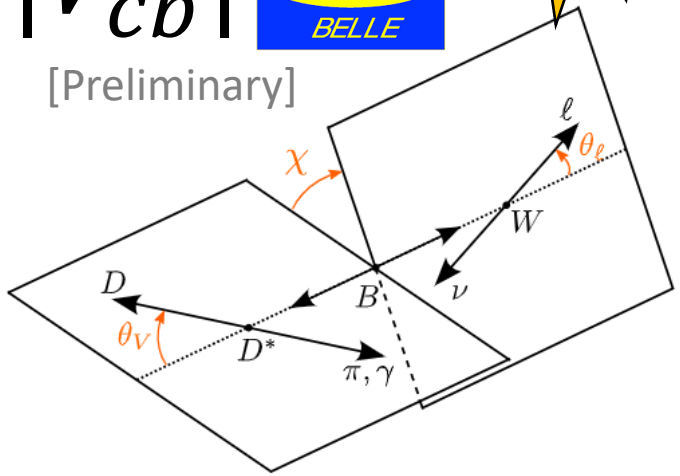
Measurement of Differential Distributions of $B \to D^* \ell \nu_\ell$ and Determination of $|V_{cb}|$

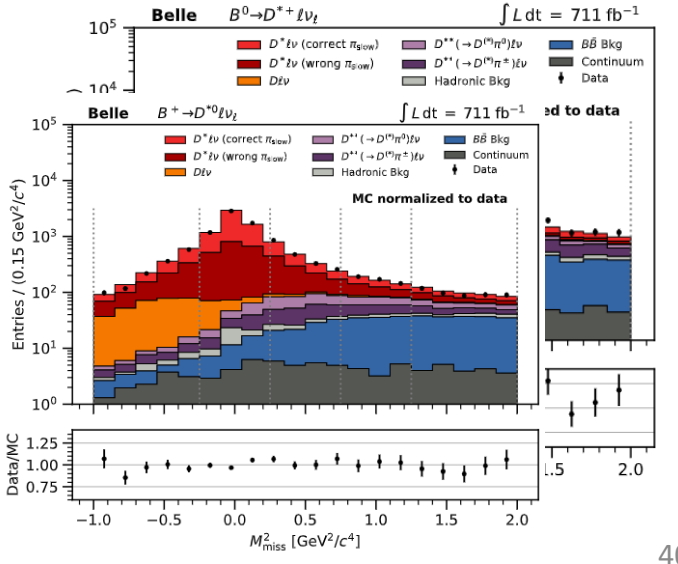
New!

[Preliminary]

- Analysis in 4 separate decay modes: $B^{\pm,0}, \ell = e, \mu$
- Utilize hadronic tagging (Full Event Interpretation)
- Extract form factors from differential shapes, and use world averaged absolute branching ratio
- Extraction with model-independent variable

$$M^2_{\text{miss}} = p^2_{\text{miss}} = \left(p_{e^+e^-} - p_{\text{tag}} - p_{D^*} - p_\ell\right)^2$$

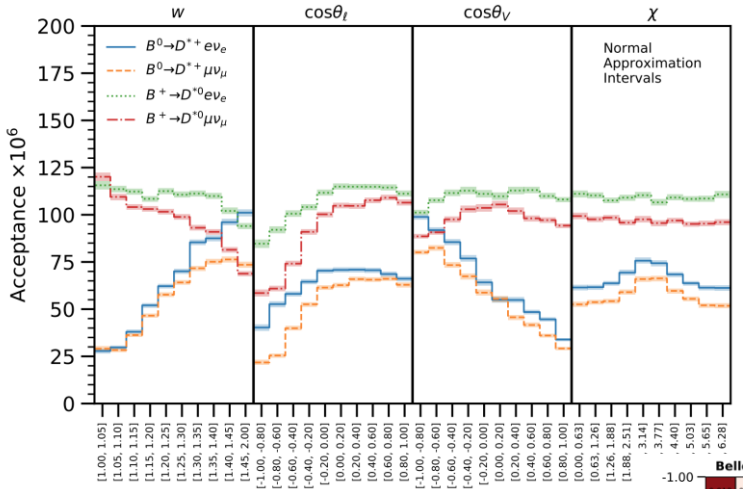


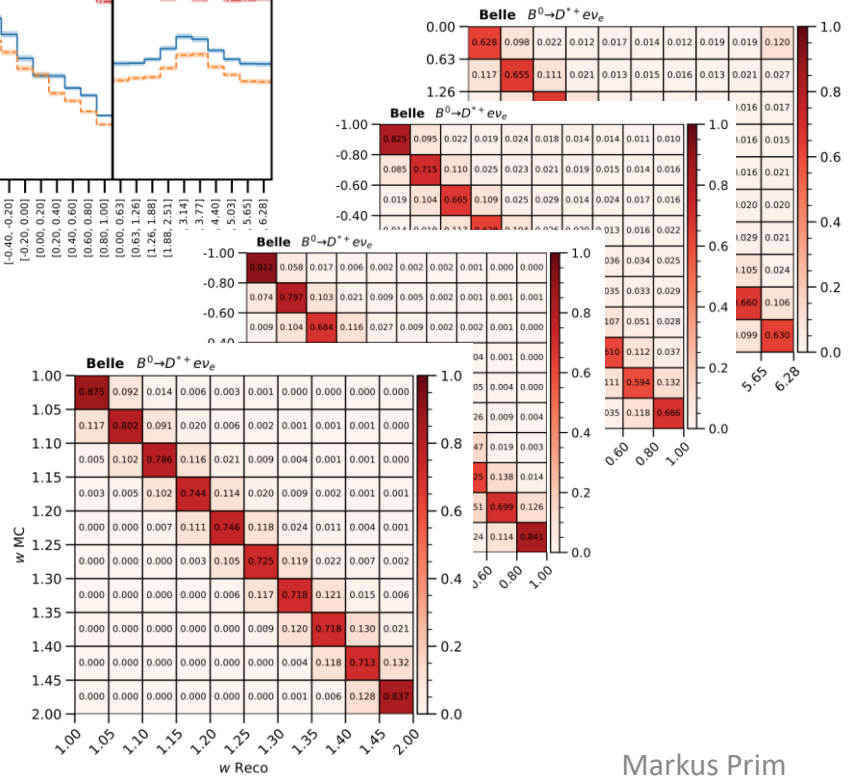


Measurement of Differential Distributions of $B \to D^* \ell \nu_\ell$ and Determination of $|V_{cb}|$

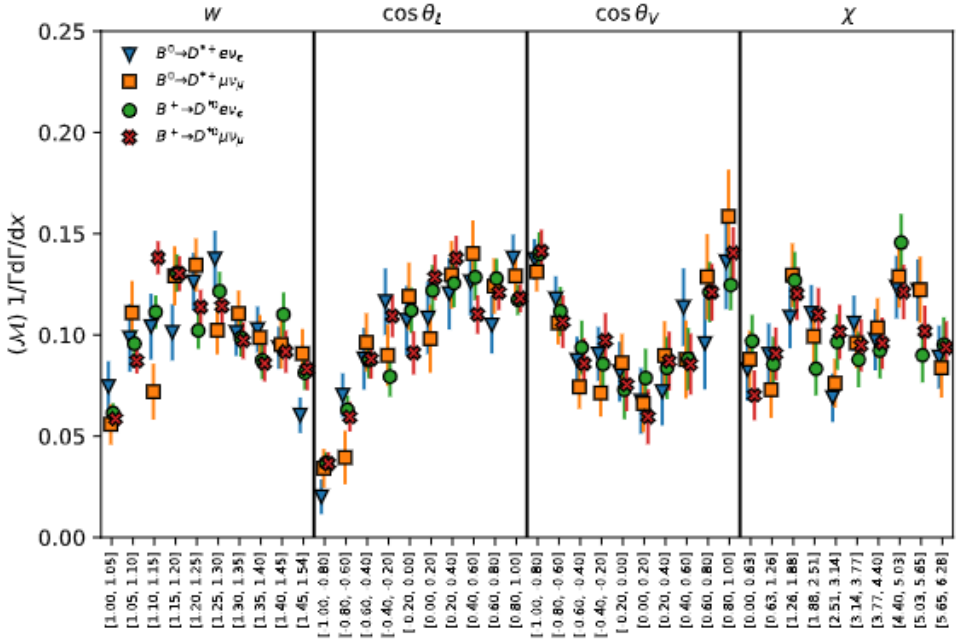
New!

[Preliminary]





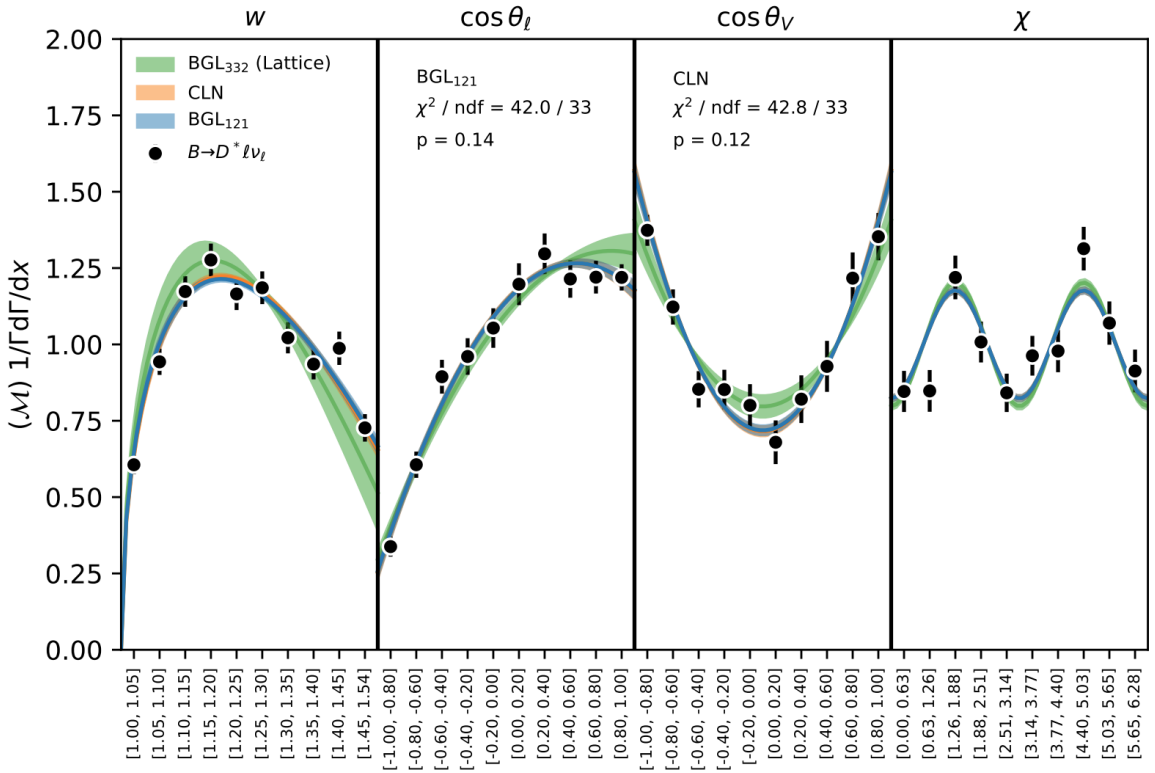
Signal extraction in bins of M_{miss}^2
+ Unfolding + Acceptance Correction



Measurement of Differential Distributions of $B \to D^* \ell \nu_\ell$ and Determination of $|V_{cb}|$

New!

[Preliminary]



Measured Shapes + External Branching Ratio Input

BGL(121)	Value	Correlation				
$a_0 \times 10^3$	24.93 ± 1.41	1.00	0.25	−0.21	0.26	−0.30
$b_0 \times 10^3$	13.11 ± 0.18	0.25	1.00	−0.01	−0.01	−0.62
$b_1 \times 10^3$	-11.93 ± 12.72	−0.21	−0.01	1.00	0.25	−0.48
$c_1 \times 10^3$	-0.87 ± 0.97	0.26	−0.01	0.25	1.00	−0.49
$\lvert V_{cb}\rvert \times 10^3$	40.77 ± 0.92	−0.30	−0.62	−0.48	−0.49	1.00

CLN	Value	Correlation			
ρ^2	1.25 ± 0.09	1.00	0.56	−0.89	0.38
$R_1(1)$	1.32 ± 0.08	0.56	1.00	−0.63	−0.03
$R_2(1)$	0.85 ± 0.07	−0.89	−0.63	1.00	−0.15
$\lvert V_{cb}\rvert \times 10^3$	40.30 ± 0.86	0.38	−0.03	−0.15	1.00

Based on the lattice input at zero-recoil:

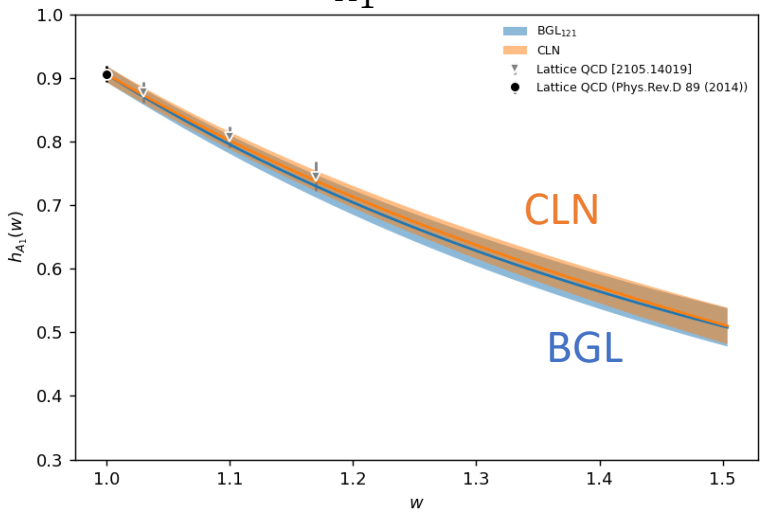
$$h_{A_1}(1) = 0.906 \pm 0.013$$

Measurement of Differential Distributions of $B \to D^* \ell \nu_\ell$ and Determination of $|V_{cb}|$

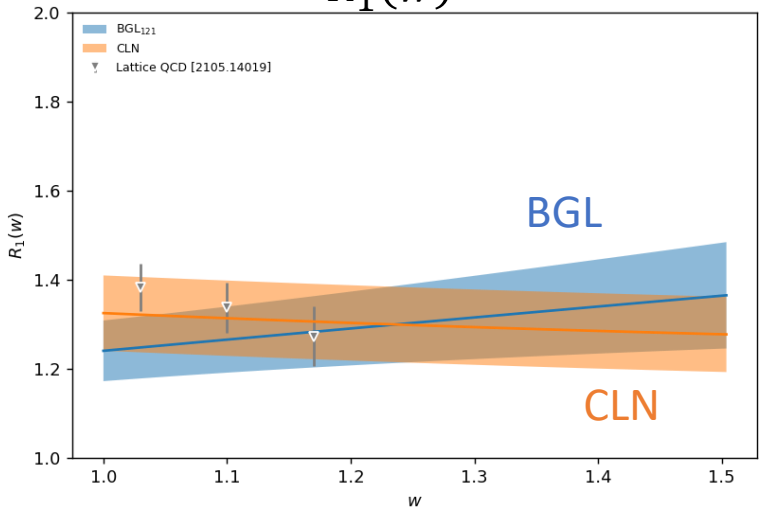
New!

[Preliminary]

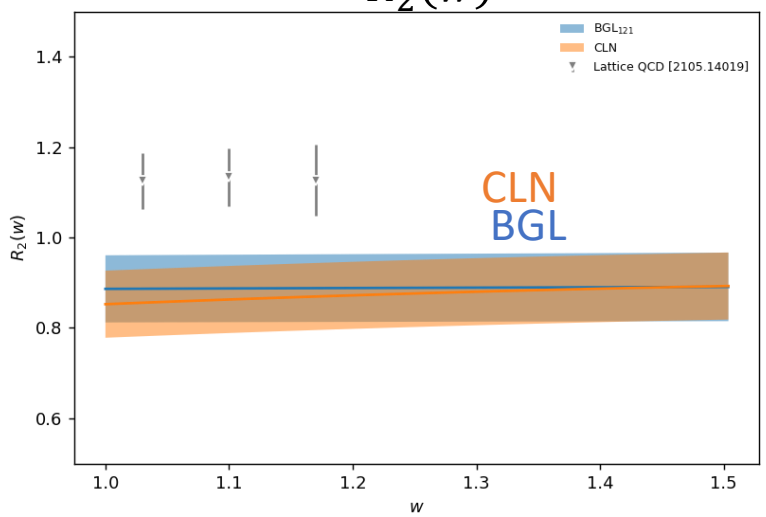
$h_{A_1}(w)$



$R_1(w)$

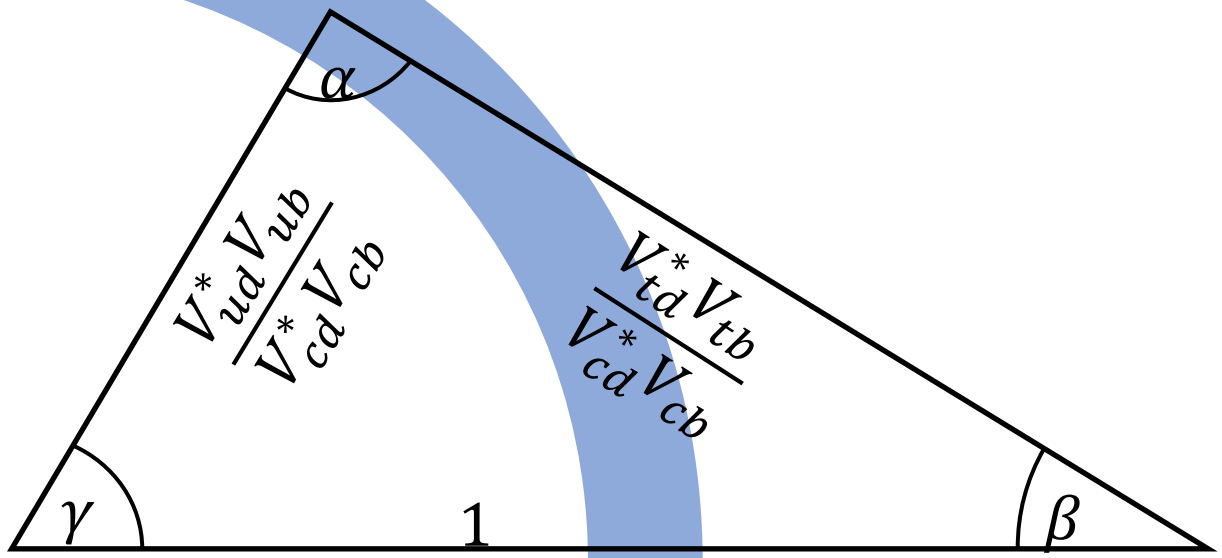


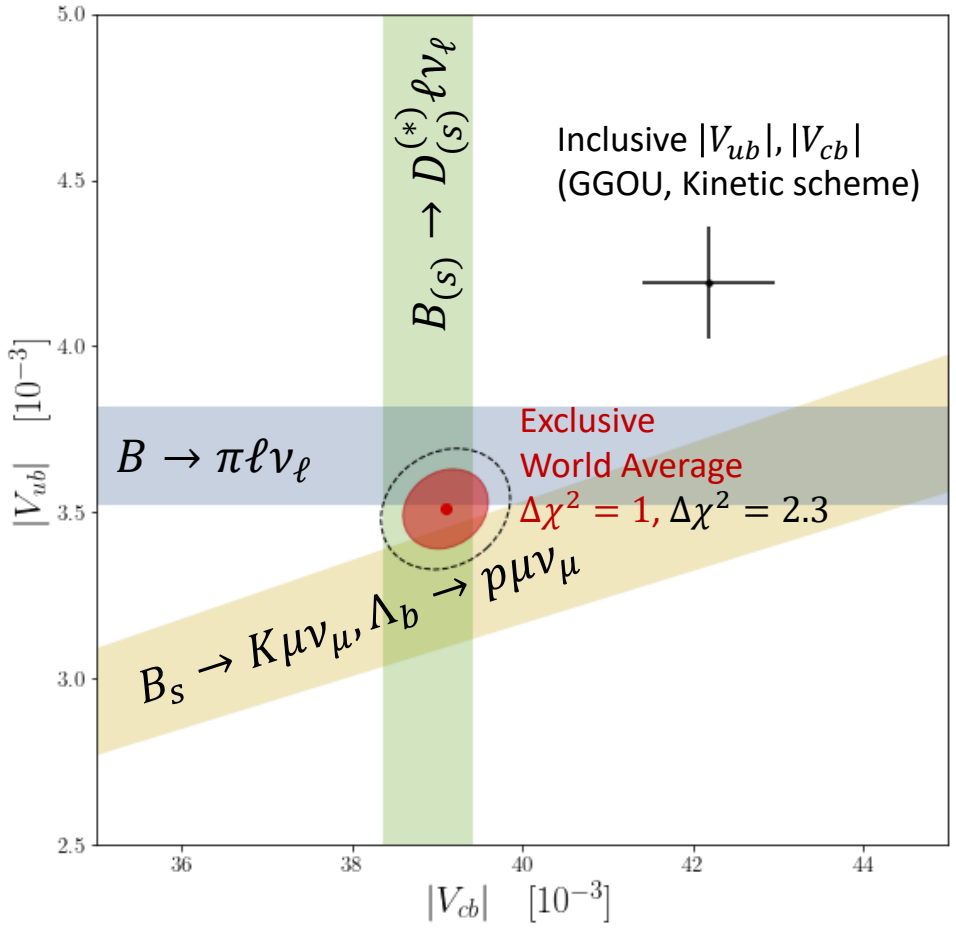
$R_2(w)$



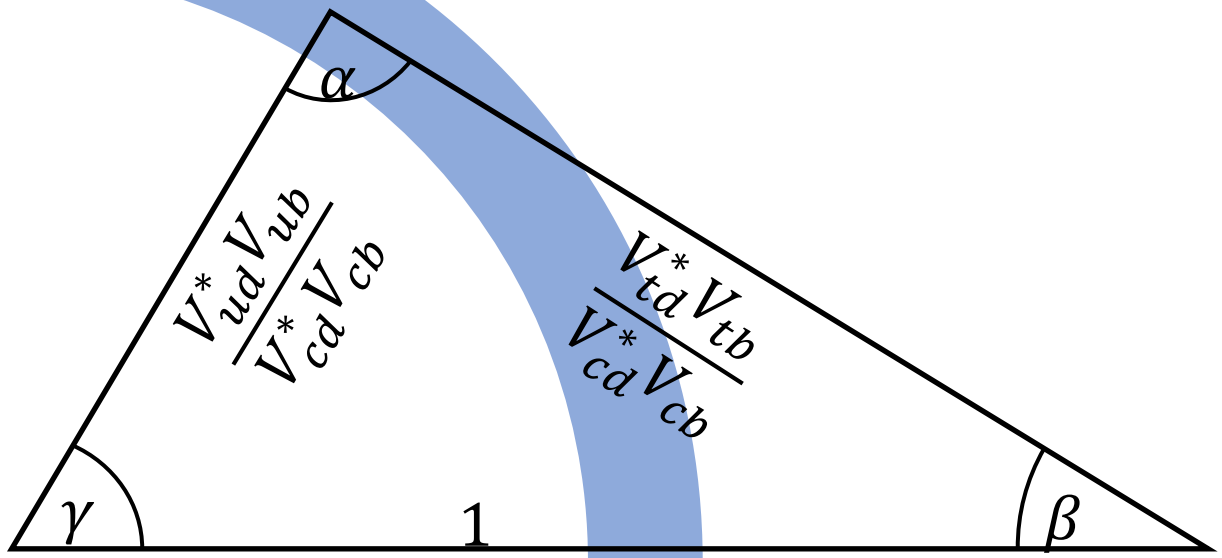
Here: beyond zero-recoil points overlayed (not in fit)

Summary

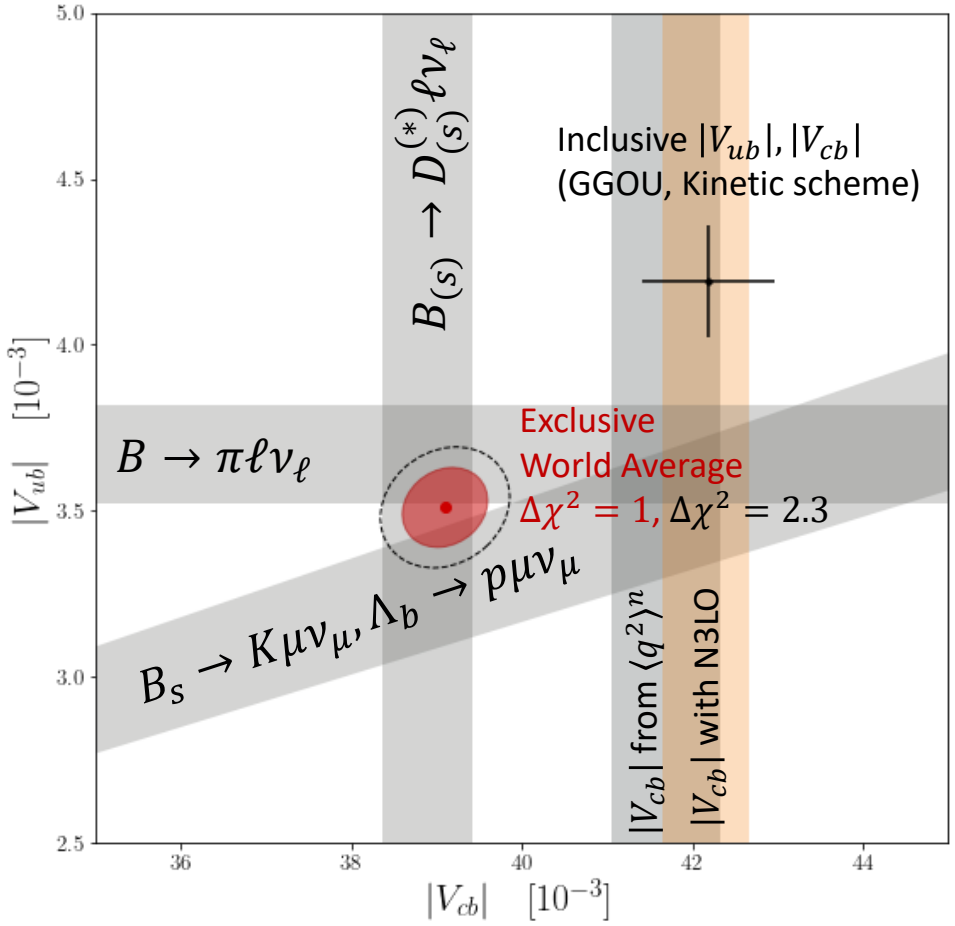

α
$\frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}$
$\frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}$
γ
1
β


Inclusive $|V_{ub}|, |V_{cb}|$
(GGOU, Kinetic scheme)
$B_{(s)} \to D_{(s)}^{(*)} \ell \nu_\ell$
$B \to \pi \ell \nu_\ell$
Exclusive
World Average
$\Delta\chi^2 = 1, \Delta\chi^2 = 2.3$
$B_s \to K \mu \nu_\mu, \Lambda_b \to p \mu \nu_\mu$
$|V_{ub}|$ $[10^{-3}]$
$|V_{cb}|$ $[10^{-3}]$
5.0
4.5
4.0
3.5
3.0
2.5
36
38
40
42
44

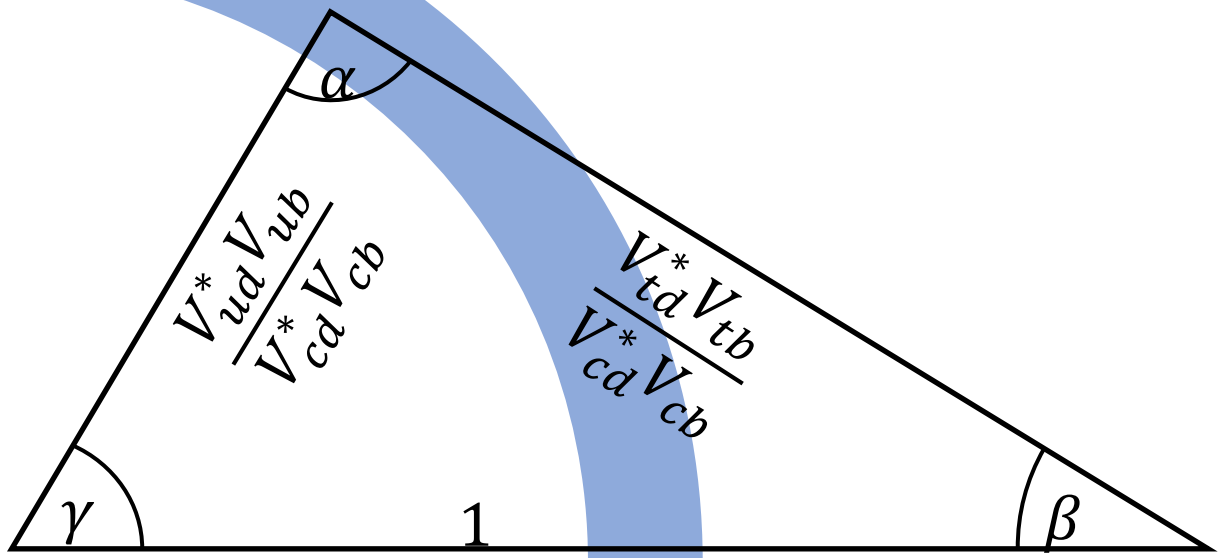
Summary

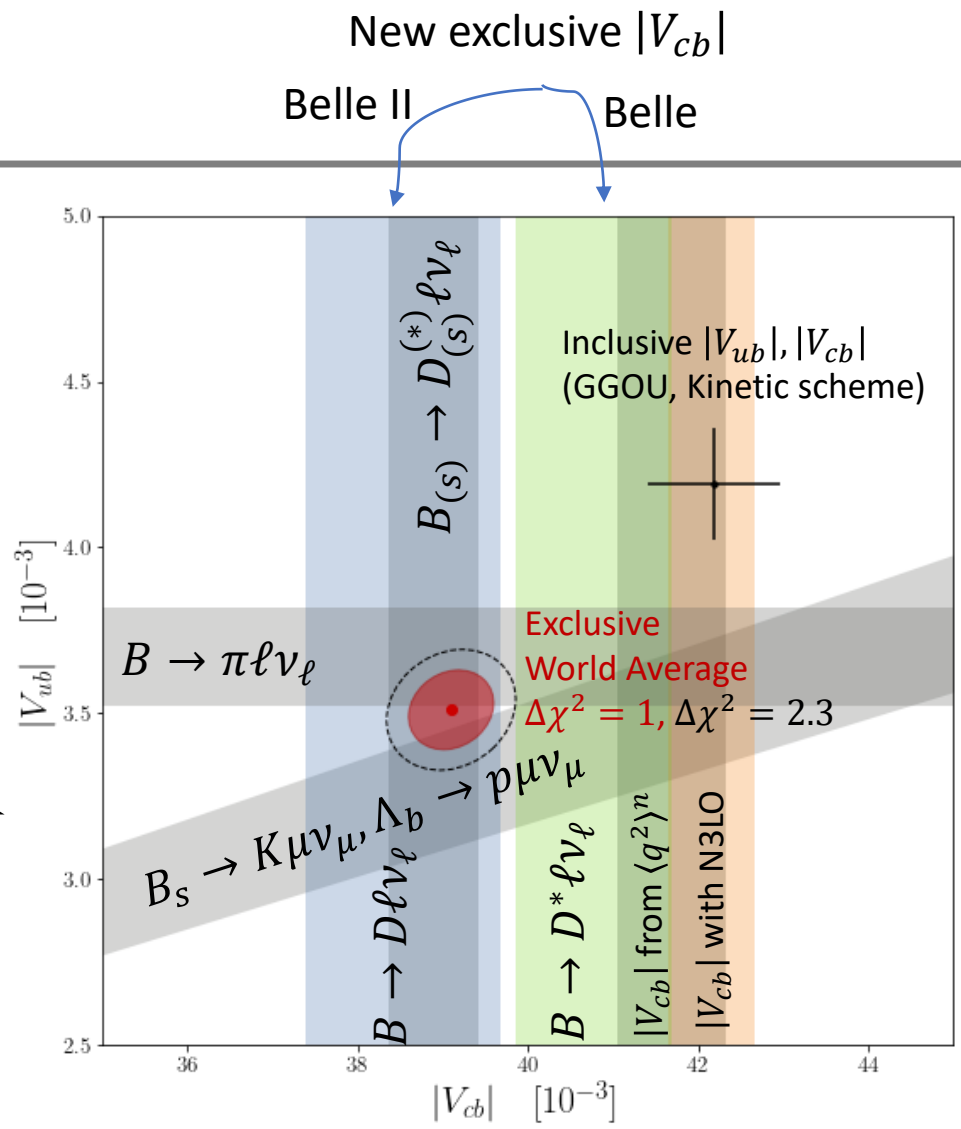


New inclusive $|V_{cb}|$

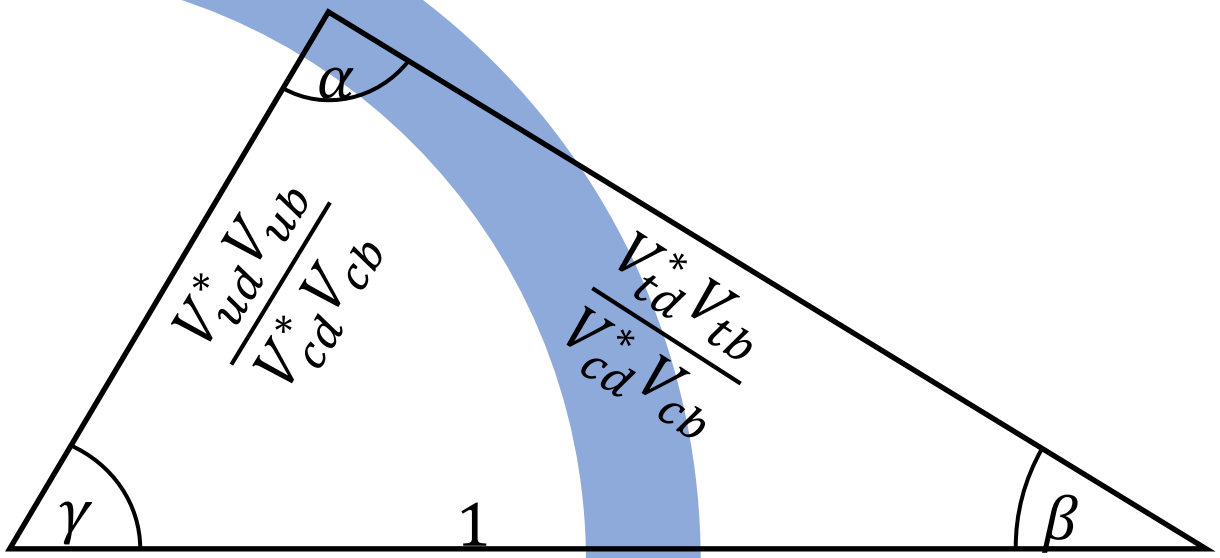


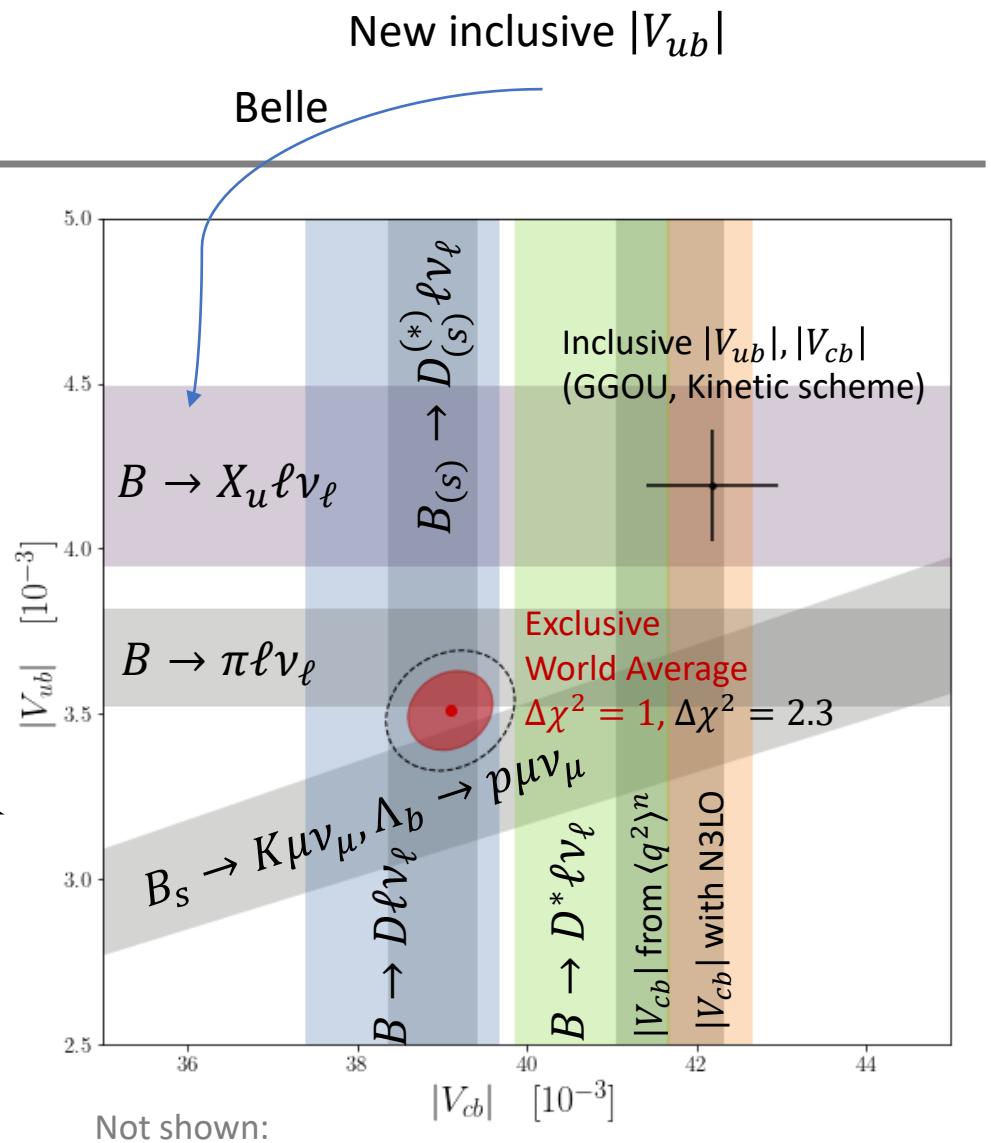
Summary


α
$\frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}$
$\frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}$
γ
1
β


New exclusive $|V_{cb}|$
Belle II
Belle
Inclusive $|V_{ub}|, |V_{cb}|$
(GGOU, Kinetic scheme)
Exclusive
World Average
$\Delta\chi^2 = 1, \Delta\chi^2 = 2.3$
$B \to \pi\ell\nu_\ell$
$B_s \to K\mu\nu_\mu, \Lambda_b \to p\mu\nu_\mu$
$B_{(s)} \to D_{(s)}^{(*)}\ell\nu_\ell$
$B \to D\ell\nu_\ell$
$B \to D^*\ell\nu_\ell$
$|V_{cb}|$ from $\langle q^2 \rangle^n$
$|V_{cb}|$ with N3LO
$|V_{ub}|$ $[10^{-3}]$
$|V_{cb}|$ $[10^{-3}]$
5.0
4.5
4.0
3.5
3.0
2.5
36
38
40
42
44

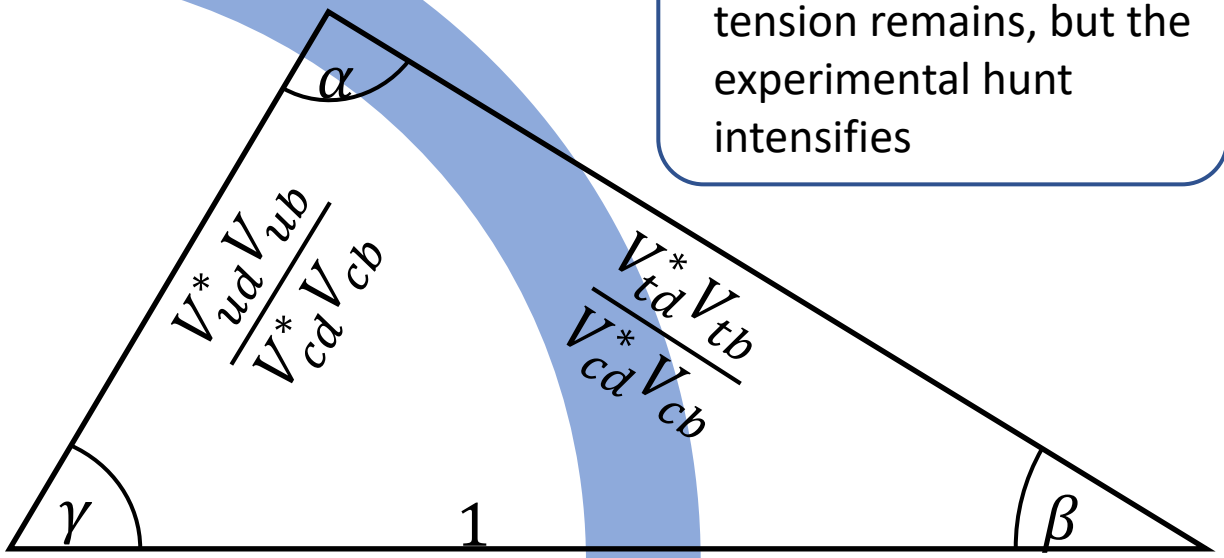
Summary



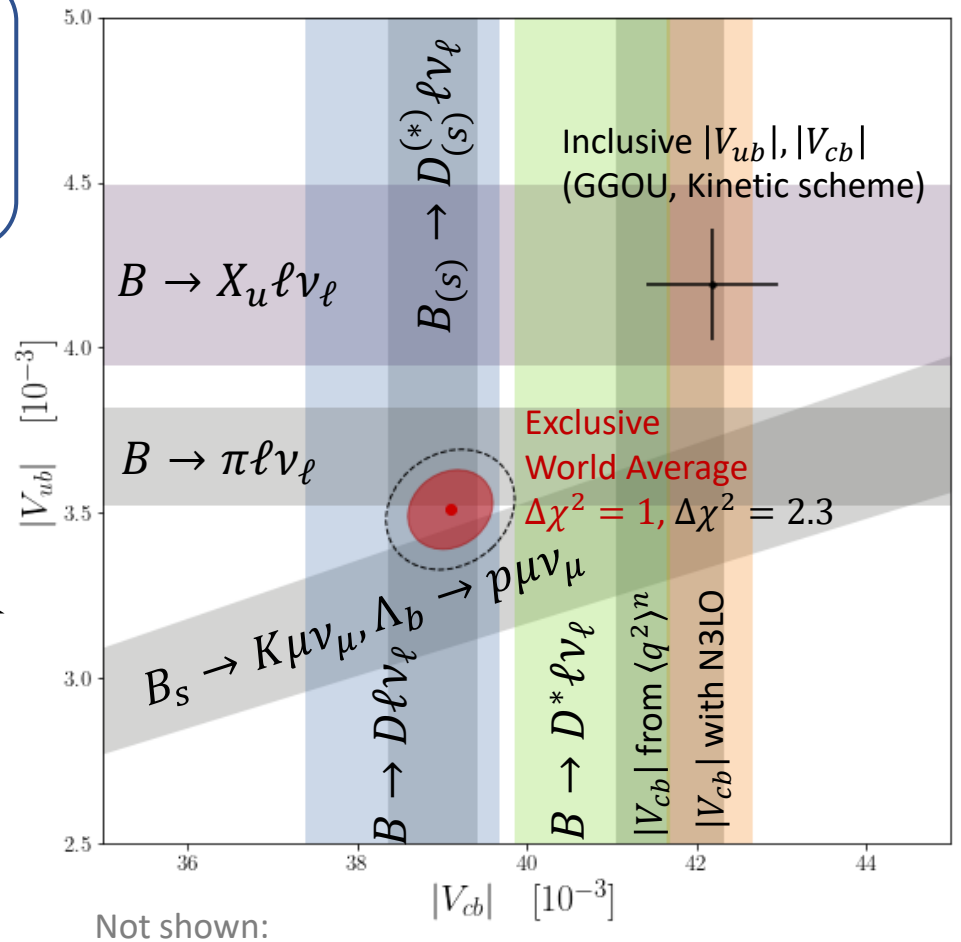


Not shown:
Tagged Belle II $|V_{ub}|, |V_{cb}|$

Summary



The inclusive-exclusive tension remains, but the experimental hunt intensifies



Not shown:
Tagged Belle II $|V_{ub}|, |V_{cb}|$