

# Recent Belle II results on hadronic $B$ decays

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on behalf of the Belle II collaboration**

**FPCP2023 - Parallel session  
May 30, 2023**

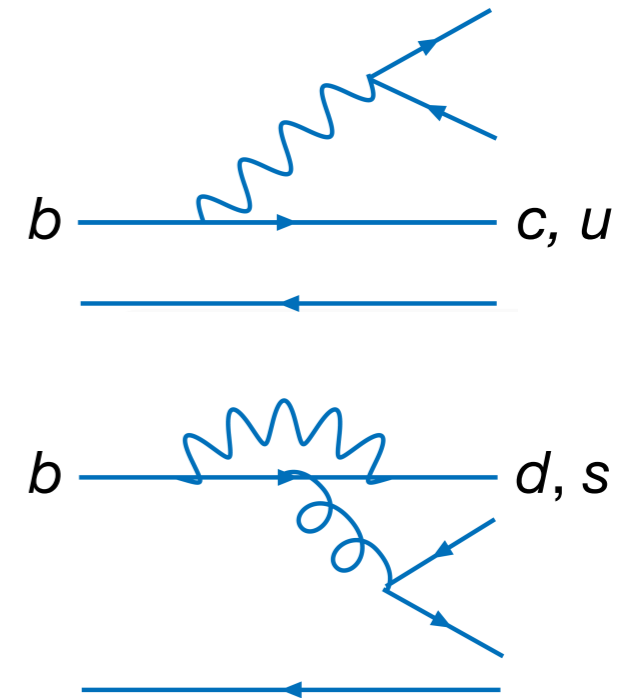


# Hadronic $B$ decays

$\mathbf{b} \rightarrow \mathbf{c}, \mathbf{u}$  trees and  $\mathbf{b} \rightarrow \mathbf{d}, \mathbf{s}$  penguins.

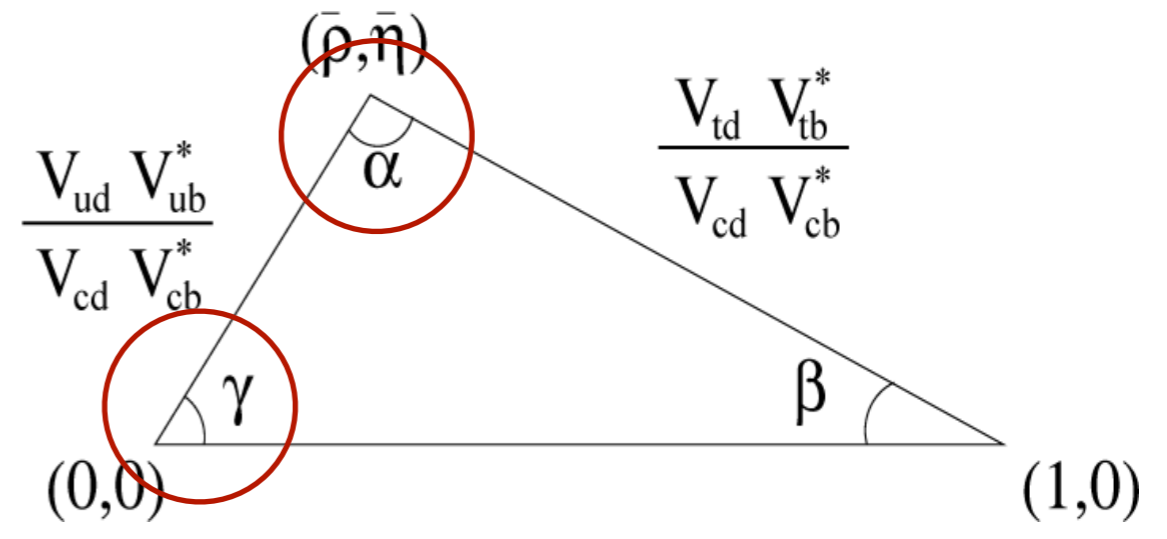
Probe SM dynamics in all three CKM angles

- $\gamma$  with theoretically clean modes  $B \rightarrow DK$ ,
  - $\alpha$  with  $B \rightarrow \rho\rho$ ,  $B \rightarrow \rho\pi$ ,  $B \rightarrow \pi\pi$  isospin analyses,
  - $\beta$  with  $B^0 \rightarrow J/\psi K_S^0$ ,  $B^0 \rightarrow \eta' K_S^0$ ,  $B^0 \rightarrow \phi K_S^0$
- and by testing isospin sum rules, chiral structure, ...



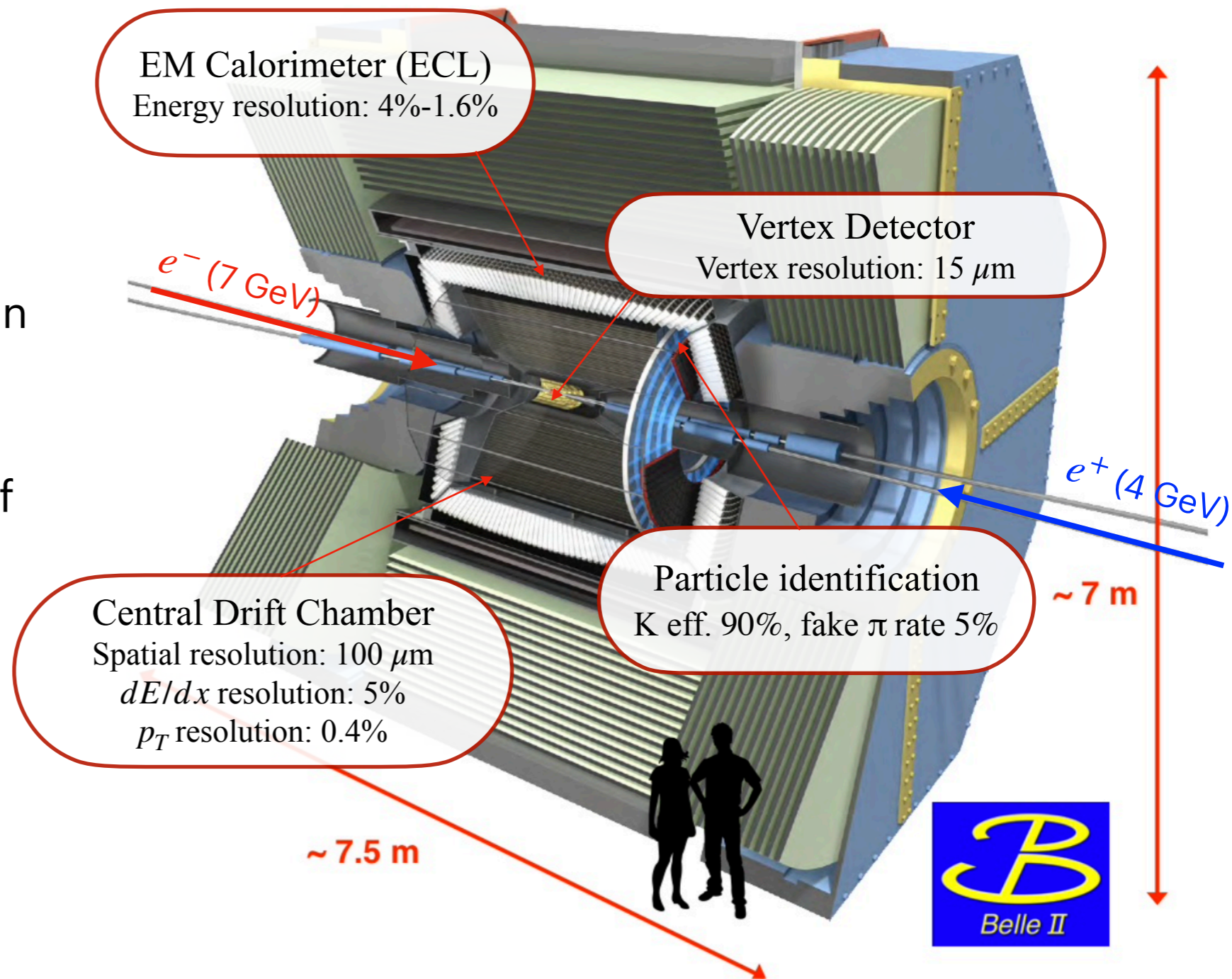
Today:

$\gamma$  determination using two different methods,  
 $B \rightarrow \rho\rho$  and  $B \rightarrow \pi\pi$  towards  $\alpha$ ,  
 $K\pi$  isospin sum rule,  
 observation of new  $B \rightarrow D^{(*)}K^-K_S^0$  decays.



# The Belle II detector

- ▶ SuperKEKB: 7-on-4 GeV  $e^-e^+$  collider at 10.58 GeV;
- ▶ Aim at 700  $B\bar{B}$  pairs/second in low-bkg environment;
- ▶ 424 fb<sup>-1</sup> (400 x10<sup>6</sup>  $B\bar{B}$  pairs) of data collected;
- ▶ Record peak luminosity: 4.7x10<sup>34</sup> cm<sup>-2</sup> s<sup>-1</sup>



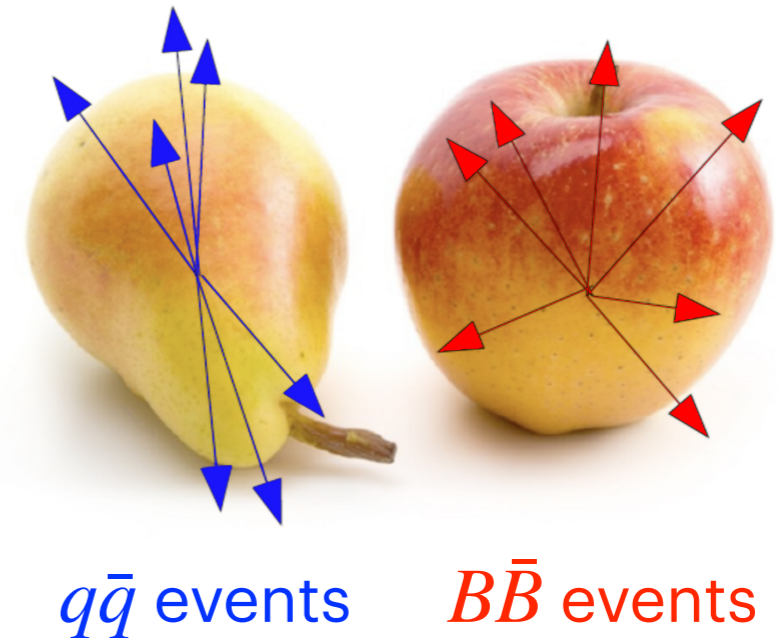
Unique reach on final states with multiple neutrinos and  $\pi^0$ /photons.

# Analysis workflow

~1/5 of hadronic events from  $e^+e^-$  are  $B\bar{B}$ .

**Typical  $B$  hadronic event:** 10 tracks/clusters — easy to trigger on unbiased variables (e.g. number of tracks) — isotropically distributed in space.

**Main backgrounds:**  $e^+e^- \rightarrow q\bar{q}$  (collimated jets, very different event shape), other misidentified  $B$  events.



## Reconstruction

- combine final state particles ( $K, \pi, \dots$ ) in kinematic fits to form the  $B$  decay

## Selection

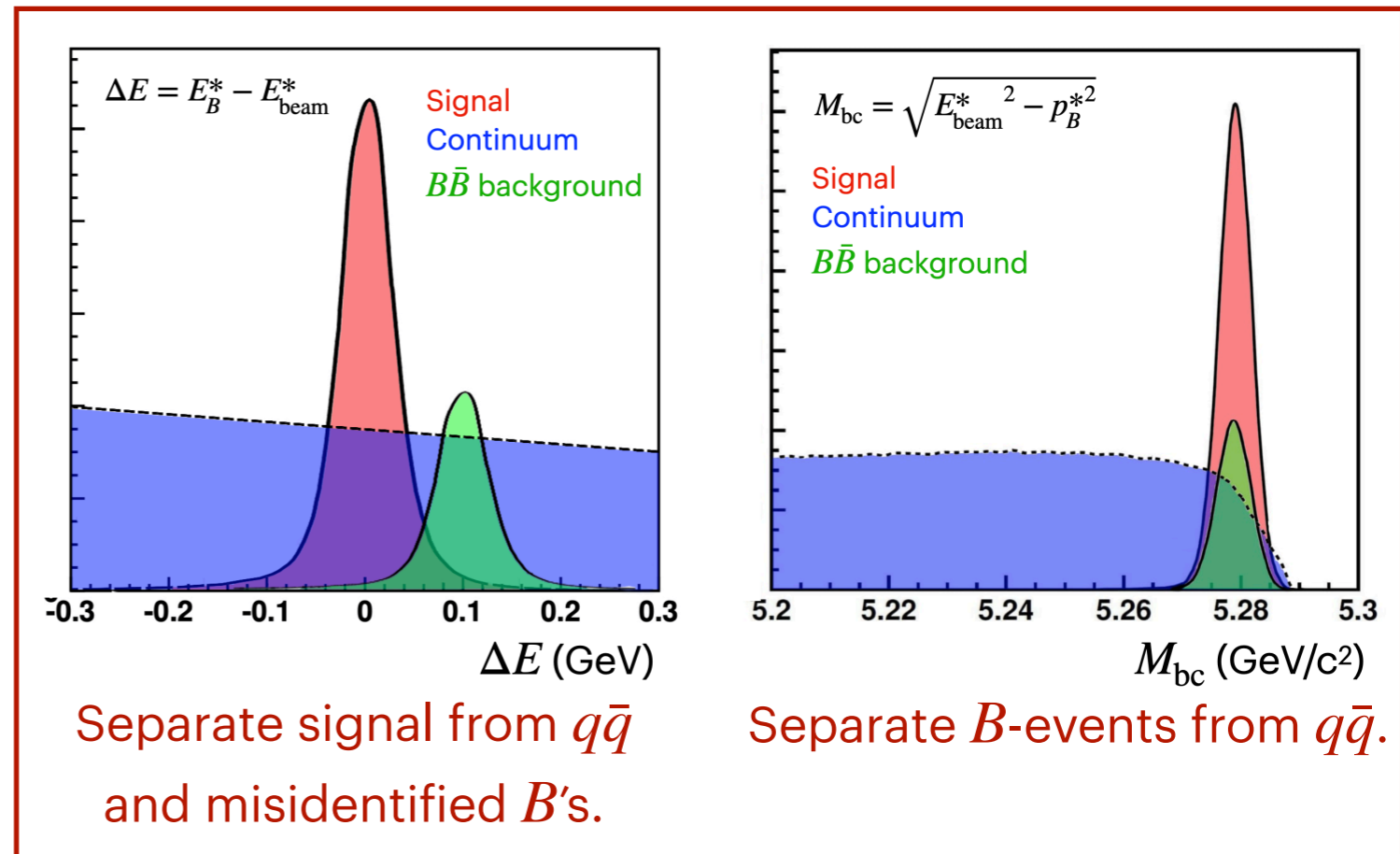
- optimize event-shape multivariate classifier (CS) and particle ID criteria

## Fit

- extract models from simulation (calibrate on data), fit in to data and evaluate physics quantities

## Systematic uncertainties

- with control modes and simulations



# Measurement of $\gamma$

# $\gamma$ from $B \rightarrow DK$ decays

$\gamma$ : phase between  $b \rightarrow u$  and  $b \rightarrow c$  transitions.  
 Accessible via tree-level decays: no direct new physics  $\rightarrow$  strong constraints on SM.

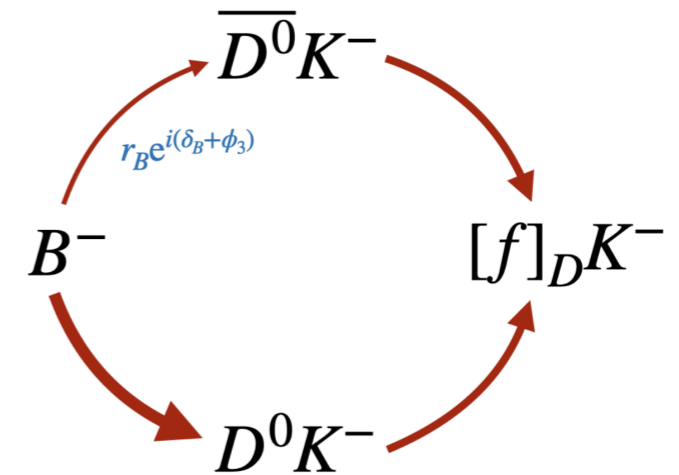
Current WA dominated by LHCb:

$$\gamma[^\circ] = 65.9^{+3.3}_{-3.5} \quad \text{HFLAV}$$

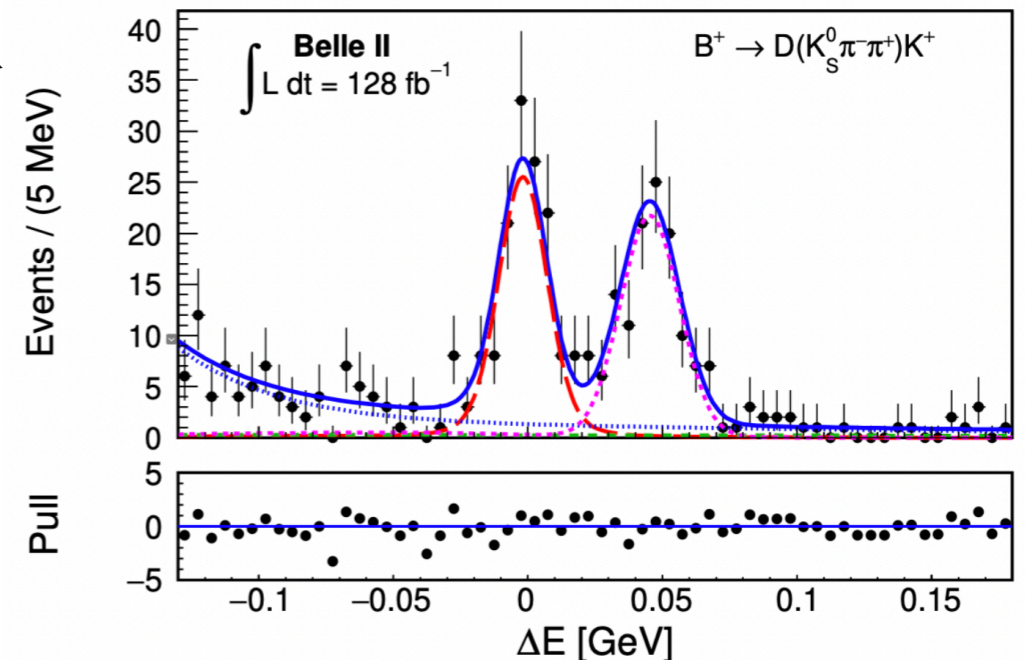
Various approaches — different  $D$  final states:

- Self-conjugate final states  $D \rightarrow K_S^0 h^+ h^-$   
 Belle + Belle II  $\gamma = (78.4 \pm 11.4 \pm 0.5 \pm 1.0)^\circ$   
[https://link.springer.com/article/10.1007/JHEP02\(2022\)063](https://link.springer.com/article/10.1007/JHEP02(2022)063)
- Cabibbo-suppressed decays  $D \rightarrow K_S^0 K^\pm \pi^\mp$
- $CP$  eigenstates  $D \rightarrow K^+ K^-, K_S^0 \pi^0$

Interference between two decays to same final state gives access to phase:



$$\frac{\mathcal{A}^{\text{suppr.}}(B^- \rightarrow \bar{D}^0 K^-)}{\mathcal{A}^{\text{favor.}}(B^- \rightarrow D^0 K^-)} = r_B e^{i(\delta_B + \gamma)}$$



# $\gamma$ using Cabibbo-suppressed decays

$$B^\pm \rightarrow DK^\pm, D\pi^\pm \quad (D \rightarrow K_S^0 K^\pm \pi^\mp)$$

SS: same-sign, OS: opposite sign.

2D fit ( $\Delta E, CS'$ ) of 8 categories:

(+, -)  $\times$  (SS, OS)  $\times$  (DK, D $\pi$ ) in full  $D$  phase space and in interference-enhanced  $D \rightarrow K^*K$  region.

Combination of  $\mathcal{R}$  and  $\mathcal{A}$  constraints  $\gamma$ .

Full  $D$  phase space

$$\mathcal{A}_{SS}^{DK} = -0.089 \pm 0.091 \pm 0.011$$

$$\mathcal{A}_{OS}^{DK} = +0.109 \pm 0.133 \pm 0.013$$

$$\mathcal{A}_{SS}^{D\pi} = +0.018 \pm 0.026 \pm 0.009$$

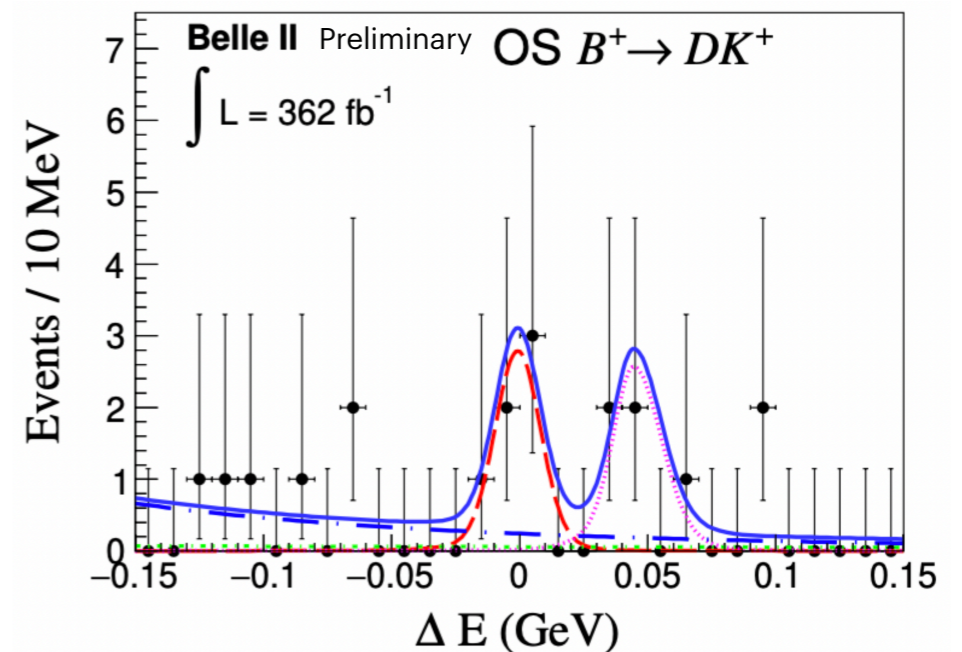
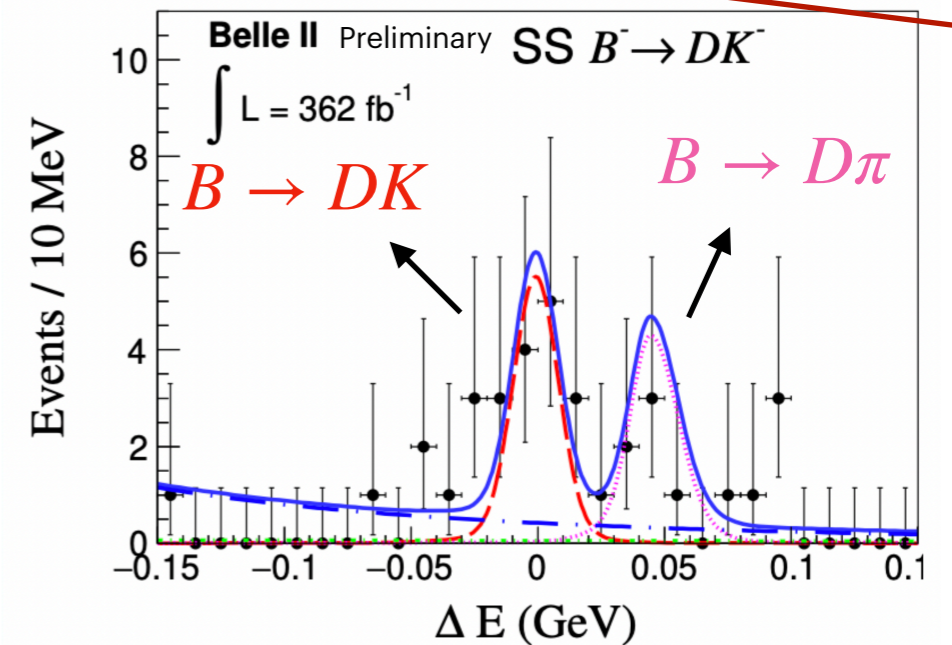
$$\mathcal{A}_{OS}^{D\pi} = -0.028 \pm 0.031 \pm 0.009$$

$$\mathcal{R}_{SS}^{DK/D\pi} = 0.122 \pm 0.012 \pm 0.004$$

$$\mathcal{R}_{OS}^{DK/D\pi} = 0.093 \pm 0.013 \pm 0.003$$

$$\mathcal{R}_{SS/OS}^{D\pi} = 1.428 \pm 0.057 \pm 0.002$$

362 fb<sup>-1</sup> Belle II  
+ 711 fb<sup>-1</sup> Belle



Results consistent with LHCb, but not competitive.

Contribute to constrain  $\gamma$  in combination with other measurements.

# $\gamma$ using $CP$ eigenstates

189 fb<sup>-1</sup> Belle II  
+ 711 fb<sup>-1</sup> Belle

$$B^\pm \rightarrow D_{CP^\pm}, \quad D \rightarrow K^+K^- \text{ (CP even)} \quad D \rightarrow K_S^0\pi^0 \text{ (CP odd)}$$

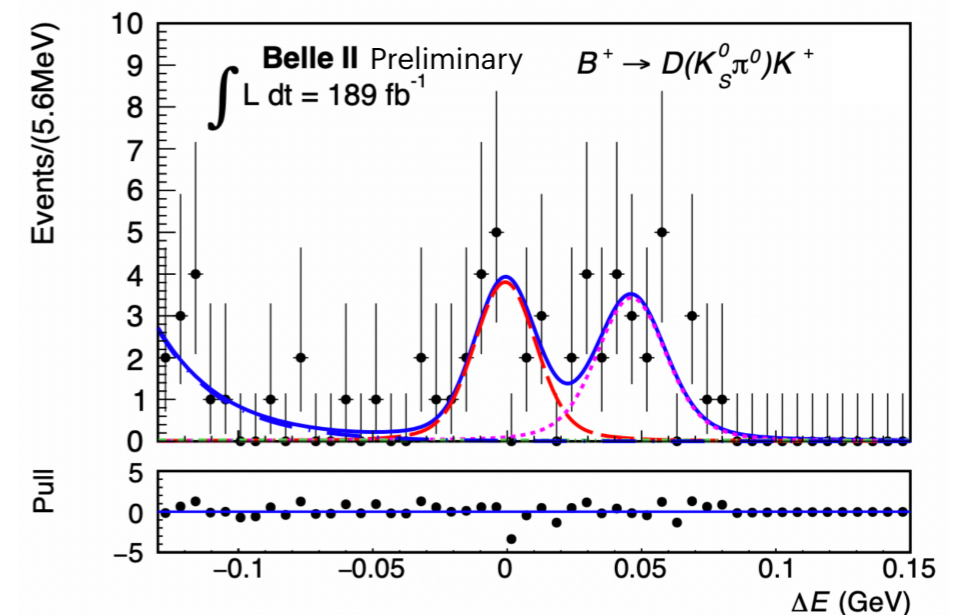
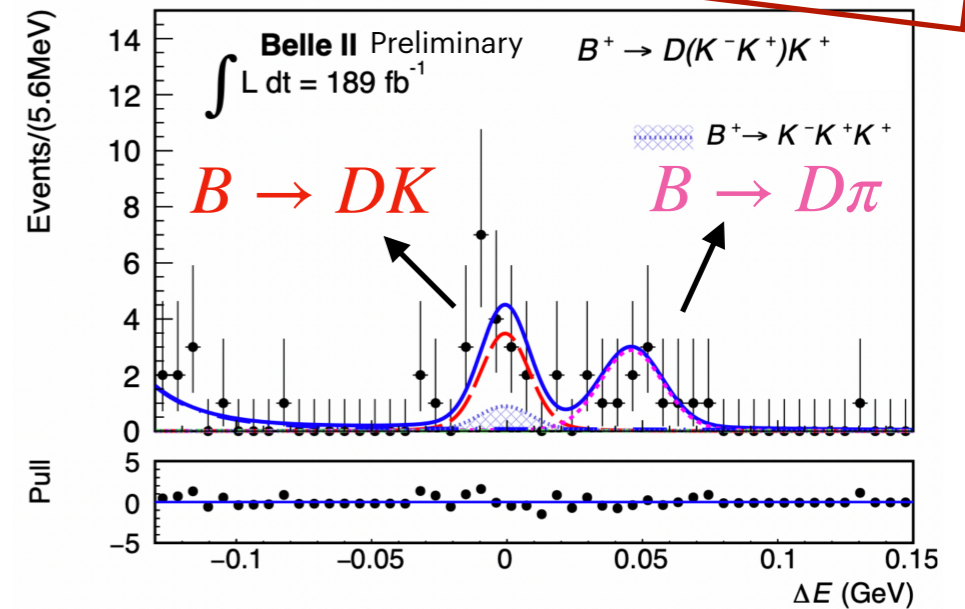
2D fit ( $\Delta E, CS'$ ) of 6 categories:  
( $DK, D\pi$ )  $\times$  ( $K^+K^-, K_S^0\pi^0, K^+\pi^-$ )

Only accessible  
to Belle/Belle II

Combination of  $\mathcal{R}$  and  $\mathcal{A}$  gives access to  $\gamma$ .

$$\begin{aligned} \mathcal{R}_{CP^+} &= 1.164 \pm 0.081 \pm 0.036 \\ \mathcal{R}_{CP^-} &= 1.151 \pm 0.074 \pm 0.019 \\ \mathcal{A}_{CP^+} &= +0.125 \pm 0.058 \pm 0.014 \\ \mathcal{A}_{CP^-} &= -0.167 \pm 0.057 \pm 0.006 \end{aligned}$$

Evidence for difference in  $\mathcal{A}_{CP^\pm}$ .



Results consistent with BaBar and LHCb, but not competitive.  
Contribute to constrain  $\gamma$  in combination with other measurements.



Towards CKM angle  $\alpha$

# Towards CKM angle $\alpha$

$\alpha = \arg \left[ -V_{td}V_{tb}^*/V_{ud}V_{ub}^* \right]$  less precisely known angle, may limit the global testing power of CKM fits.

$$\alpha[^\circ] = 85.2 \begin{matrix} + 4.8 \\ - 4.3 \end{matrix}$$

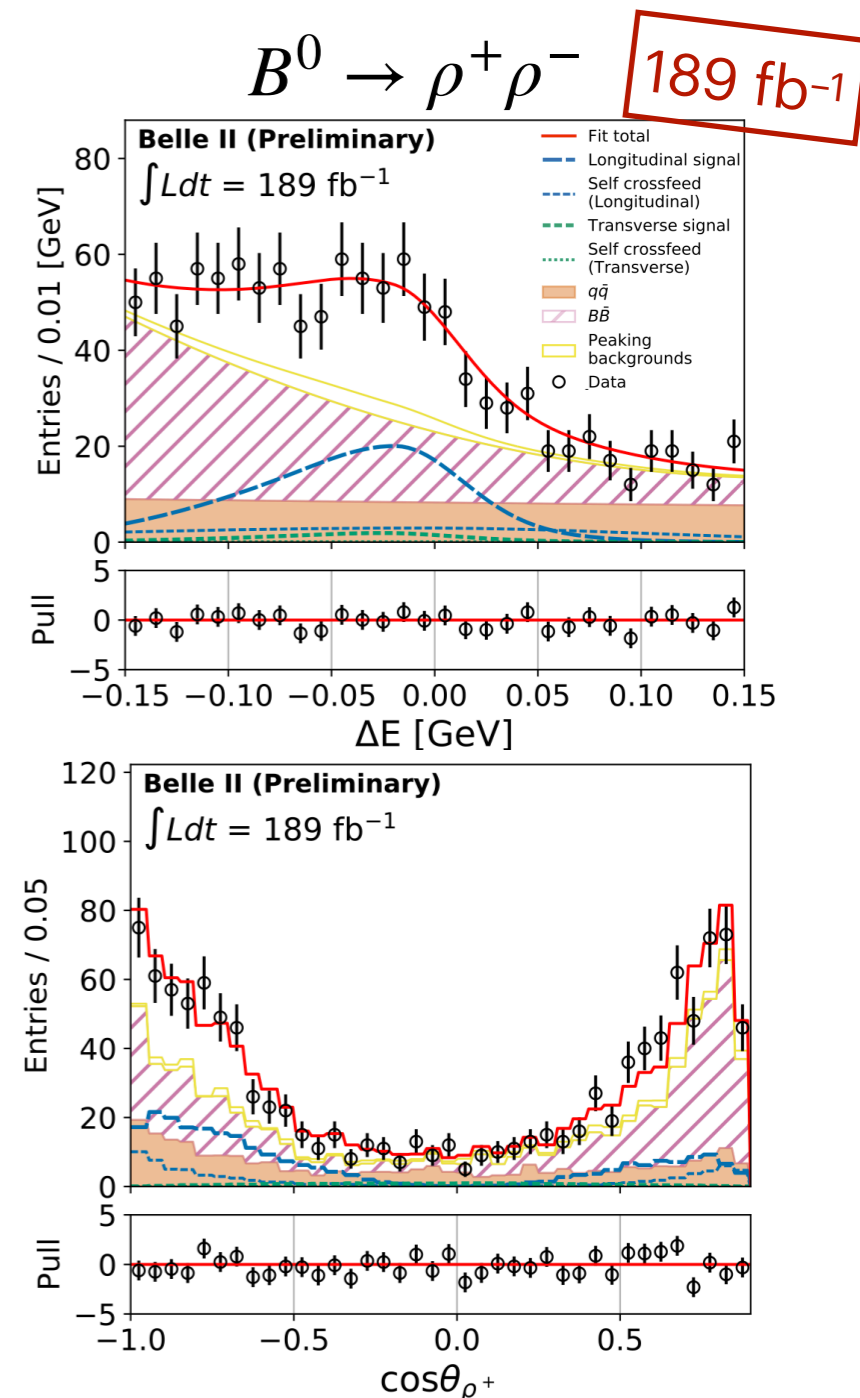
[HFLAV](https://hflav.cern.ch/)

Determined using  $B \rightarrow \rho\rho$  and  $B \rightarrow \pi\pi$  isospin analyses: combine information from  $BF$  and  $A_{CP}$  to reduce impact of hadronic uncertainties – non-perturbative QCD.

Unique Belle II capability to study in consistent way all  $B \rightarrow \rho\rho$  and  $B \rightarrow \pi\pi$  channels.

$B \rightarrow \rho\rho$  measurements require angular analysis:

- Winter 2022  $B^+ \rightarrow \rho^+\rho^0$  result: [arxiv.org/abs/2206.12362](https://arxiv.org/abs/2206.12362);
- result for  $B^0 \rightarrow \rho^+\rho^-$ .



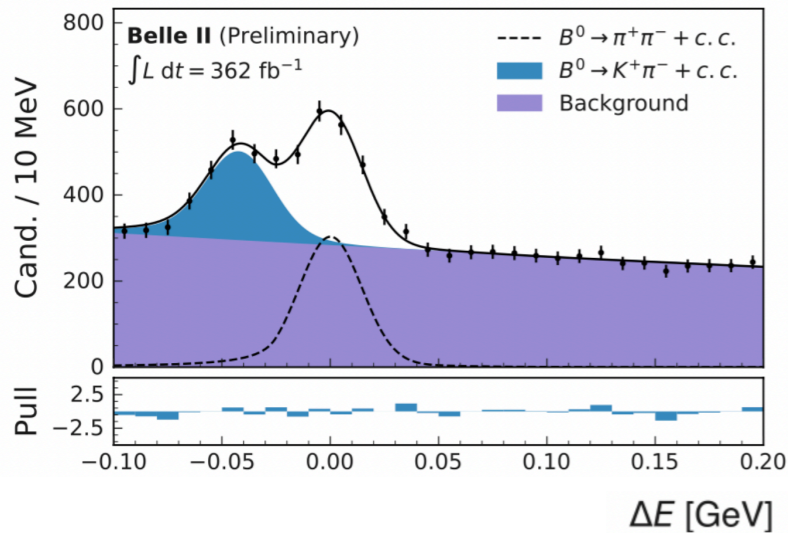
$$\mathcal{B} = (26.7 \pm 2.8 \pm 2.8) \times 10^{-6}$$

$$f_L = 0.956 \pm 0.035 \pm 0.033$$

[arxiv.org/abs/2208.03554](https://arxiv.org/abs/2208.03554)

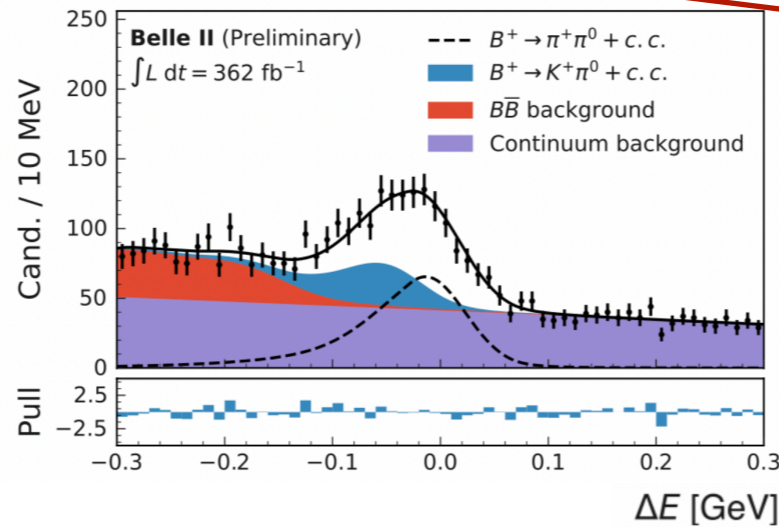
# $B \rightarrow \pi\pi$ results

$B^0 \rightarrow \pi^+\pi^-$



$B^+ \rightarrow \pi^+\pi^0$

362 fb<sup>-1</sup>



$$\mathcal{B}(\pi^+\pi^-) = (5.83 \pm 0.22 \pm 0.17) \times 10^{-6}$$

$$\mathcal{B}(\pi^+\pi^0) = (5.10 \pm 0.29 \pm 0.32) \times 10^{-6}$$

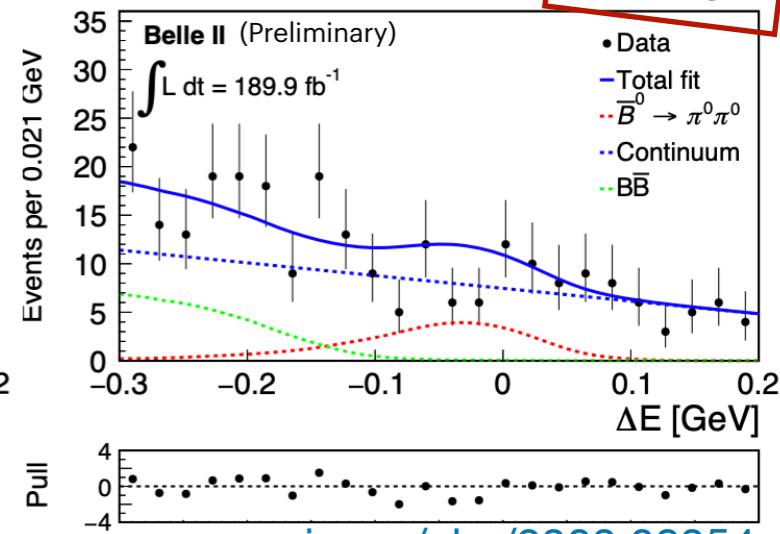
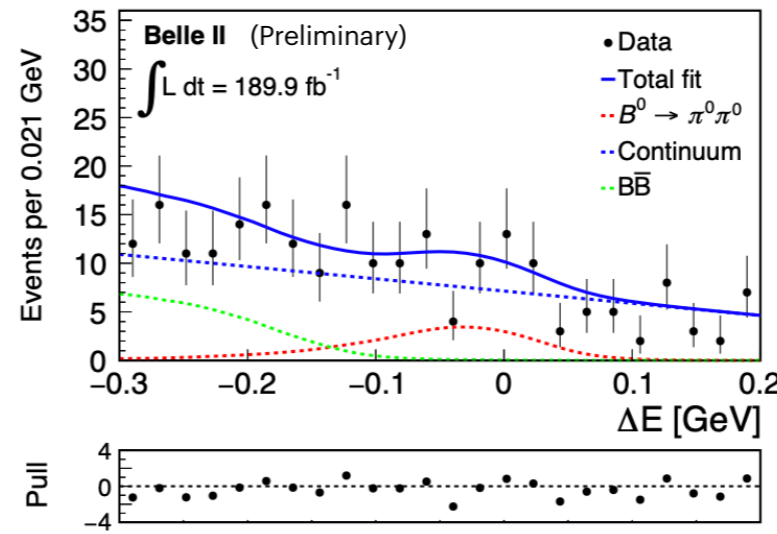
$$\mathcal{A}(\pi^+\pi^0) = -0.081 \pm 0.54 \pm 0.008$$

First  $B^0 \rightarrow \pi^0\pi^0$  measurement at Belle II:

- rare, small  $BF$  ( $10^{-6}$ ),
- only photons in the final state — dominated by signal-like background,
- large theoretical uncertainties.

$B^0 \rightarrow \pi^0\pi^0$

189 fb<sup>-1</sup>



[arxiv.org/abs/2303.08354](https://arxiv.org/abs/2303.08354)

Achieved Belle  $BF$  precision using only 1/3 of data.

$$\mathcal{B}(\pi^0\pi^0) = (1.38 \pm 0.27 \pm 0.22) \times 10^{-6}$$

$$\mathcal{A}(\pi^0\pi^0) = 0.14 \pm 0.46 \pm 0.07$$

Preliminary Belle II results on par with best performance from Belle/Babar.

# Isospin sum rule

# Isospin sum rule

Stringent null test of SM, sensitive to presence of non-SM dynamics. Inconsistency between current measurements: “ $K\pi$  puzzle” (anomalously enhanced amplitudes or new physics):

$$I_{K\pi} = \mathcal{A}_{\text{CP}}^{K^+\pi^-} + \mathcal{A}_{\text{CP}}^{K^0\pi^+} \frac{\mathcal{B}(K^0\pi^+) \tau_{B^0}}{\mathcal{B}(K^+\pi^-) \tau_{B^+}} - 2\mathcal{A}_{\text{CP}}^{K^+\pi^0} \frac{\mathcal{B}(K^+\pi^0) \tau_{B^0}}{\mathcal{B}(K^+\pi^-) \tau_{B^+}} - 2\mathcal{A}_{\text{CP}}^{K^0\pi^0} \frac{\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)} \approx 0$$

[Gronau \(Phys. Lett. B 627 \(2005\) no.1, 82-88\)](#)

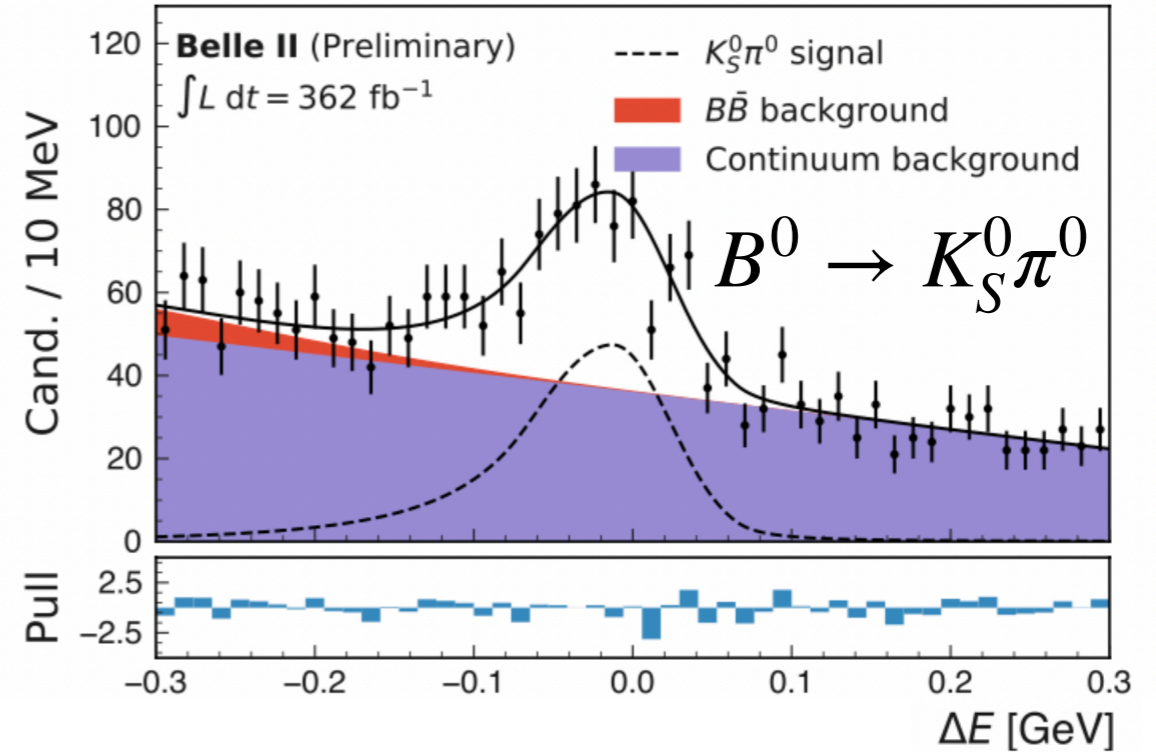
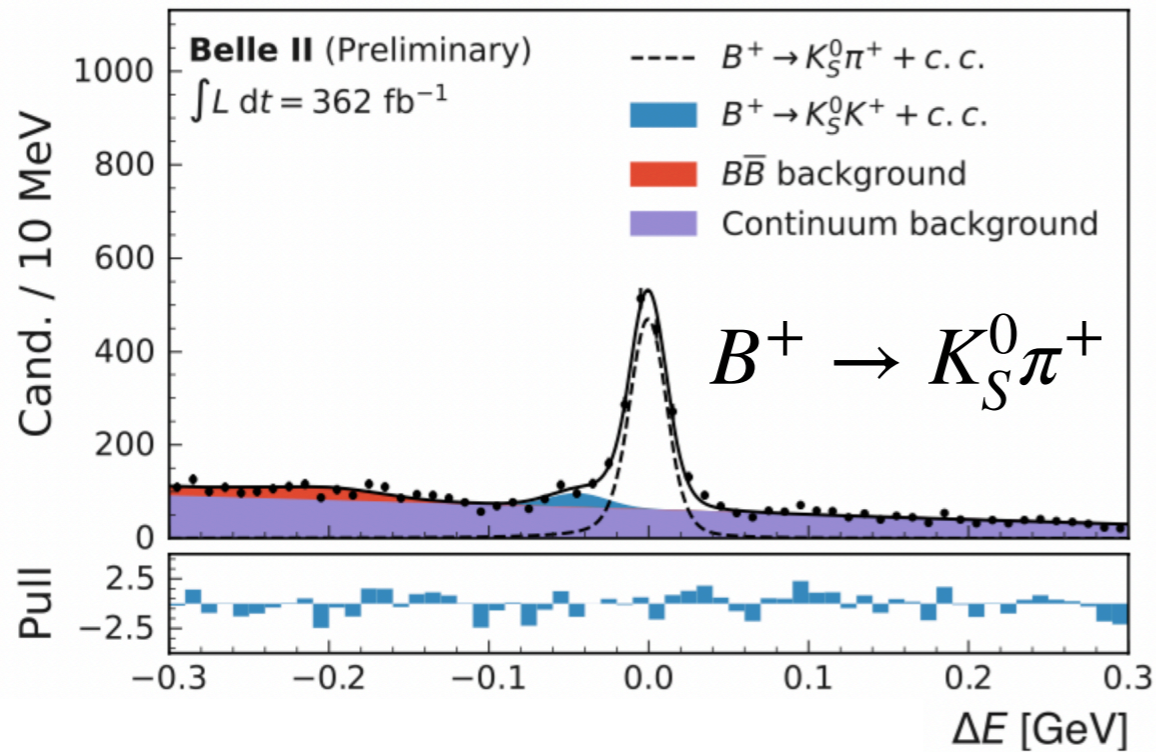
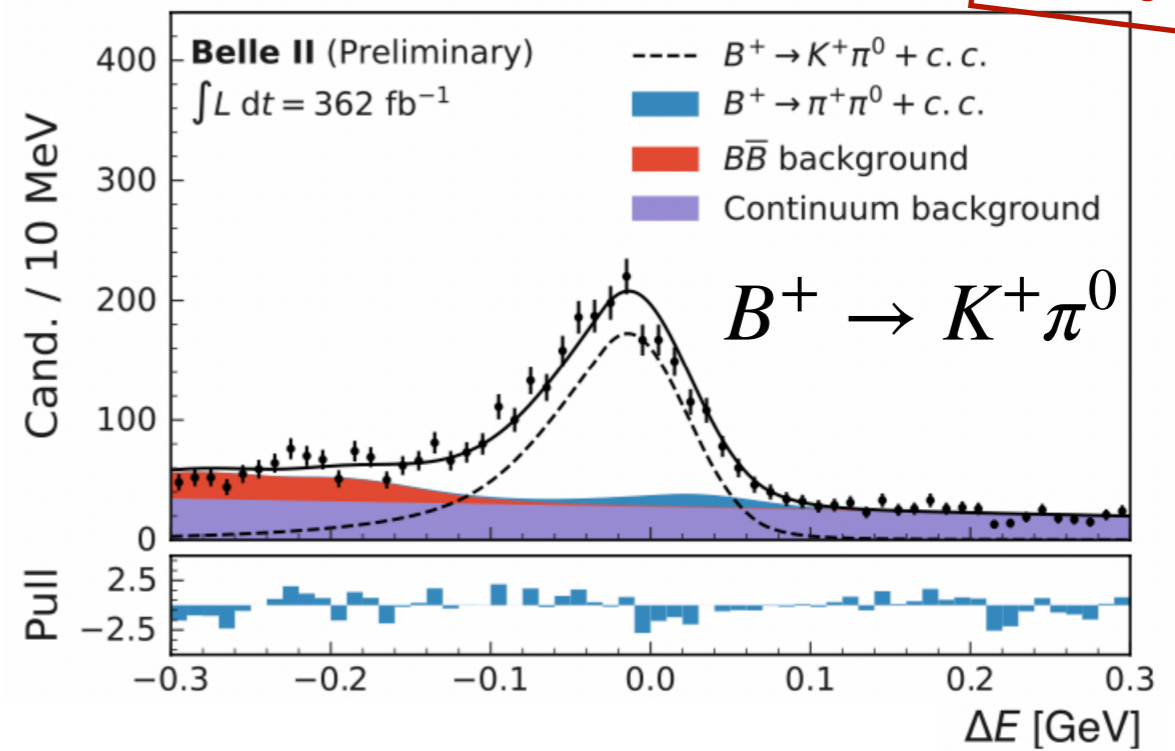
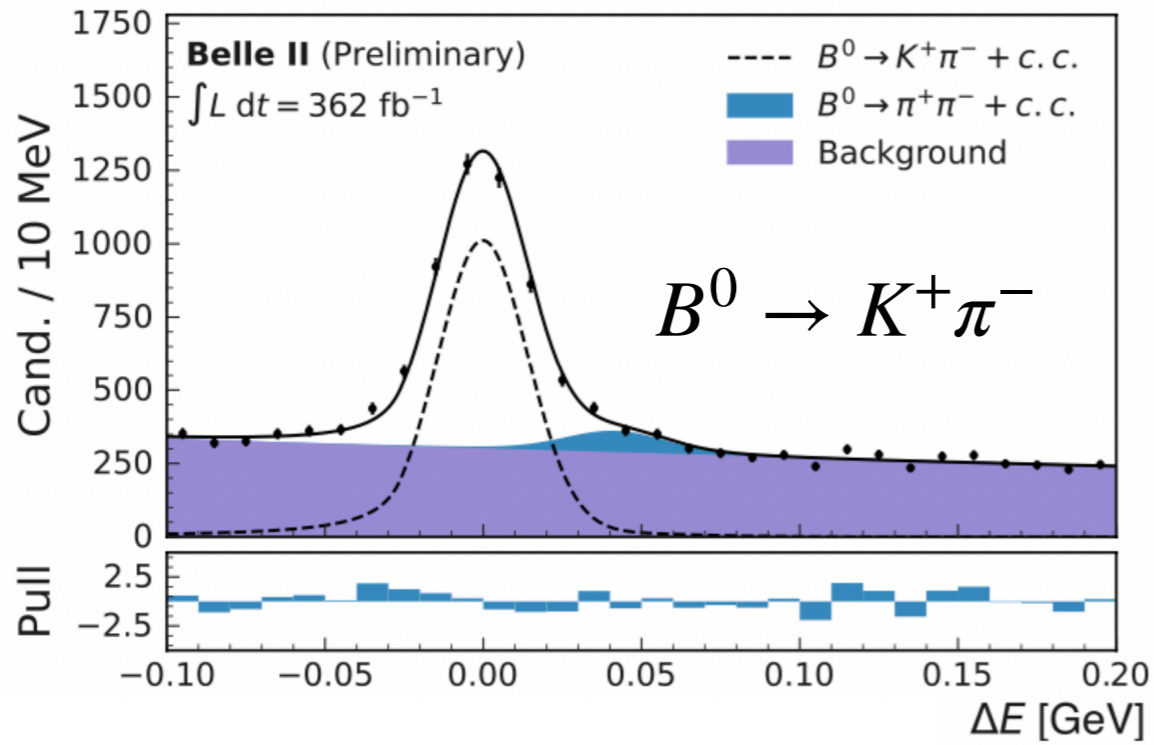
Belle II: measure all final states, with unique access to  $B^0 \rightarrow K^0\pi^0$  (major limitation in  $I_{K\pi}$ ).

Similar strategy for all the modes:

- common selection for final-state particles,
- continuum suppression,
- 2D fit ( $\Delta E, CS'$ ) for branching fractions and time-integrated  $\mathcal{A}_{\text{CP}}$ .

# Isospin sum rule results

362 fb<sup>-1</sup>



# Isospin sum rule results

$$B^0 \rightarrow K^+ \pi^-$$

$$\mathcal{B}(K^+ \pi^-) = (20.67 \pm 0.37 \pm 0.62) \times 10^{-6}$$
$$\mathcal{A}_{CP}(K^+ \pi^-) = -0.072 \pm 0.019 \pm 0.007$$

$$B^+ \rightarrow K^+ \pi^0$$

$$\mathcal{B}(K^+ \pi^0) = (13.93 \pm 0.38 \pm 0.84) \times 10^{-6}$$
$$\mathcal{A}_{CP}(K^+ \pi^0) = +0.013 \pm 0.027 \pm 0.005$$

$$B^+ \rightarrow K_S^0 \pi^+$$

$$\mathcal{B}(K_S^0 \pi^+) = (24.40 \pm 0.71 \pm 0.86) \times 10^{-6}$$
$$\mathcal{A}_{CP}(K_S^0 \pi^+) = +0.046 \pm 0.029 \pm 0.007$$

$$B^0 \rightarrow K_S^0 \pi^0$$

$$\mathcal{B}(K_S^0 \pi^0) = (10.16 \pm 0.65 \pm 0.67) \times 10^{-6}$$
$$\mathcal{A}_{CP}(K_S^0 \pi^0) = -0.006 \pm 0.15 \pm 0.05$$

$\mathcal{B}$  and  $\mathcal{A}_{CP}$  agree and are competitive with world's best,  $\mathcal{B}$  systematically limited.

$B^0 \rightarrow K_S^0 \pi^0$  result combined with time-dependent analysis ([arxiv.org/abs/2206.07453](https://arxiv.org/abs/2206.07453)), obtaining world's best:

$$A_{CP}(K_S^0 \pi^0) = -0.10 \pm 0.12 \pm 0.05$$

More detail in [talk by Jake Bennett](#)

$$I_{K\pi} = -0.03 \pm 0.13 \pm 0.05 \text{ (world average } 0.13 \pm 0.11)$$

⇒ Competitive precision to world's best already with this data size.

$B \rightarrow D^{(*)} K^- K_S^0$  decays



# $B \rightarrow D^{(*)}K^-K_S^0$ decays

$B \rightarrow D^{(*)}KK$  makes up a few % of hadronic decay, but only a small fraction is known.

Improve simulation and tagging techniques: need to know well  $BF$ 's and possible intermediate states.

Fit  $\Delta E$ , subtract background, and look at  $m(K^-K_S^0)$  and Dalitz distributions.

Structures observed in low mass region.

$$\mathcal{B}(B^- \rightarrow D^0K^-K_S^0) = (1.89 \pm 0.16 \pm 0.10) \times 10^{-4}$$

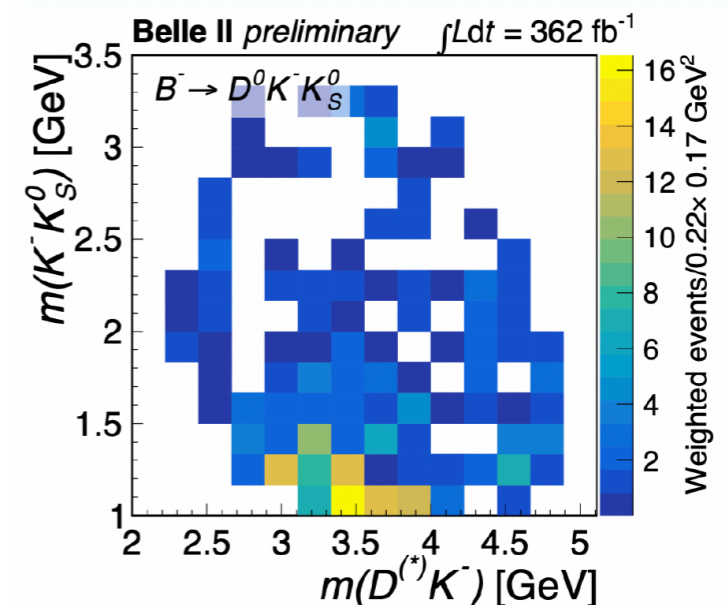
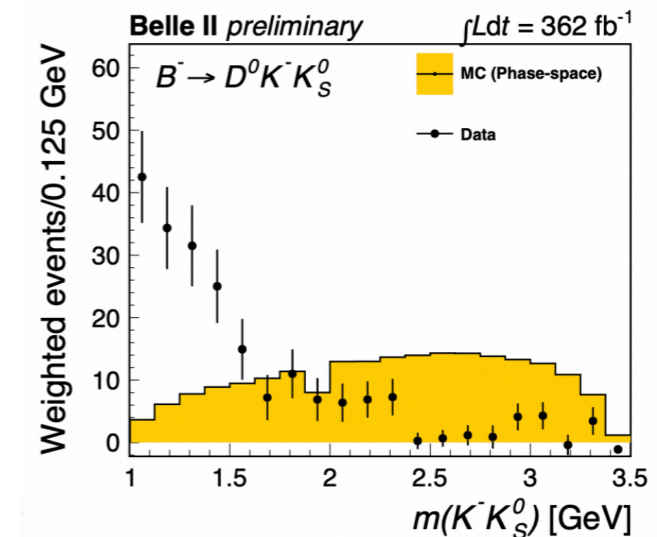
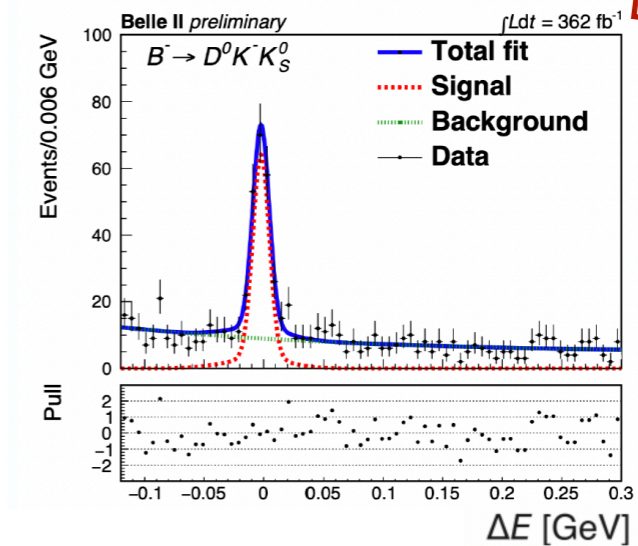
$$\mathcal{B}(\bar{B}^0 \rightarrow D^+K^-K_S^0) = (0.85 \pm 0.11 \pm 0.05) \times 10^{-4}$$

$$\mathcal{B}(B^- \rightarrow D^{*0}K^-K_S^0) = (1.57 \pm 0.27 \pm 0.12) \times 10^{-4}$$

$$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}K^-K_S^0) = (0.96 \pm 0.18 \pm 0.06) \times 10^{-4}$$

First observation of three new decay channels.

362 fb<sup>-1</sup>



# Summary

Hadronic decays important element in Belle II  $B$  physics program. First analyses using the full data sample (362 fb<sup>-1</sup>).

- $B \rightarrow DK$  decay measurements, with  $D$  decaying in Cabibbo-suppressed or  $CP$  eigenstates final states contribute in Belle + Belle II combined  $\gamma$  program.
- Measurements of  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  contribute in Belle II program for angle  $\alpha$ .
- $B^0 \rightarrow K_S^0\pi^0$  asymmetry achieves world's best precision, competitive  $I_{K\pi}$  sensitivity.
- Three new decay channels observed in  $B \rightarrow DKK$ , with structures observed in  $m(K^-K_S^0)$  and Dalitz distributions.

Backup

# $\gamma$ using GLS method

## Parameters physics meanings

- 2  $\mathcal{A}_{CP}$  for  $DK (D\pi)$ :

$$\mathcal{A}_{SS}^{DK} \equiv \frac{N_{SS}^- - N_{SS}^+}{N_{SS}^- + N_{SS}^+}$$

$$\mathcal{A}_{OS}^{DK} \equiv \frac{N_{OS}^- - N_{OS}^+}{N_{OS}^- + N_{OS}^+}$$

Physics meanings

$$\mathcal{A}_{SS}^{DK} = \frac{2r_B r_D \kappa \sin(\delta_B - \delta_D) \sin \phi_3}{1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D) \cos \phi_3}$$

$$\mathcal{A}_{OS}^{DK} = \frac{2r_B r_D \kappa \sin(\delta_B + \delta_D) \sin \phi_3}{1 + r_B^2 + r_D^2 + 2r_B r_D \kappa \cos(\delta_B + \delta_D) \cos \phi_3}$$

- 3 ratios:

$$\mathcal{R}_{SS}^{DK/D\pi} \equiv \frac{N_{SS}^- + N_{SS}^+}{N'_{SS}^- + N'_{SS}^+}$$

$$\mathcal{R}_{OS}^{DK/D\pi} \equiv \frac{N_{OS}^- + N_{OS}^+}{N'_{OS}^- + N'_{OS}^+}$$

$$\mathcal{R}_{SS/OS}^{D\pi} \equiv \frac{N_{SS}^- + N_{SS}^+}{N'_{OS}^- + N'_{OS}^+}$$

Physics meanings

$$\mathcal{R}_{SS}^{DK/D\pi} = R \frac{1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D) \cos \phi_3}{1 + r_B'^2 r_D^2 + 2r_B' r_D \kappa \cos(\delta_B' - \delta_D) \cos \phi_3}$$

$$\mathcal{R}_{OS}^{DK/D\pi} = R \frac{r_B^2 + r_D^2 + 2r_B r_D \kappa \cos(\delta_B + \delta_D) \cos \phi_3}{r_B'^2 + r_D^2 + 2r_B' r_D \kappa \cos(\delta_B' + \delta_D) \cos \phi_3}$$

$$\mathcal{R}_{SS}^{D\pi/OS} = \frac{1 + r_B'^2 r_D^2 + 2r_B' r_D \kappa \cos(\delta_B' - \delta_D) \cos \phi_3}{r_B'^2 + r_D^2 + 2r_B' r_D \kappa \cos(\delta_B' + \delta_D) \cos \phi_3}$$

# $\gamma$ using GLS method

Full  $D$  phase space

$$\mathcal{A}_{SS}^{DK} = -0.089 \pm 0.091 \pm 0.011$$

$$\mathcal{A}_{OS}^{DK} = +0.109 \pm 0.133 \pm 0.013$$

$$\mathcal{A}_{SS}^{D\pi} = +0.018 \pm 0.026 \pm 0.009$$

$$\mathcal{A}_{OS}^{D\pi} = -0.028 \pm 0.031 \pm 0.009$$

$$\mathcal{R}_{SS}^{DK/D\pi} = 0.122 \pm 0.012 \pm 0.004$$

$$\mathcal{R}_{OS}^{DK/D\pi} = 0.093 \pm 0.013 \pm 0.003$$

$$\mathcal{R}_{SS/OS}^{D\pi} = 1.428 \pm 0.057 \pm 0.002$$

$K^*$  region

$$\mathcal{A}_{SS}^{DK} = +0.055 \pm 0.119 \pm 0.020$$

$$\mathcal{A}_{OS}^{DK} = +0.231 \pm 0.184 \pm 0.014$$

$$\mathcal{A}_{SS}^{D\pi} = +0.046 \pm 0.029 \pm 0.016$$

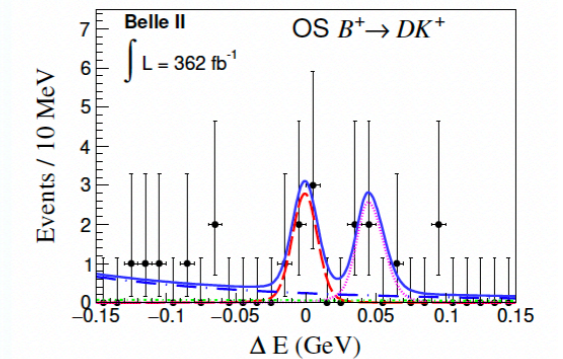
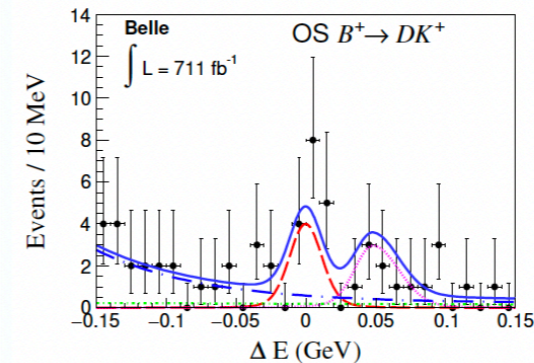
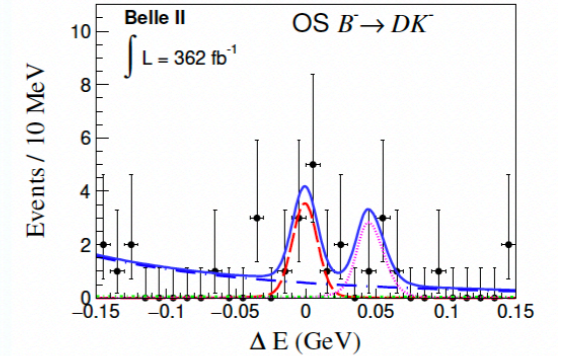
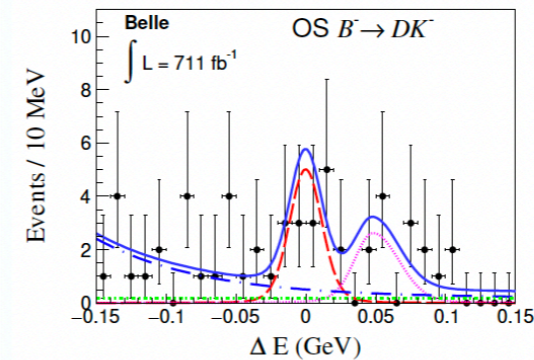
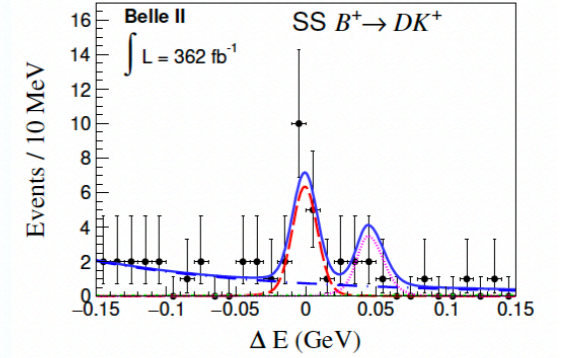
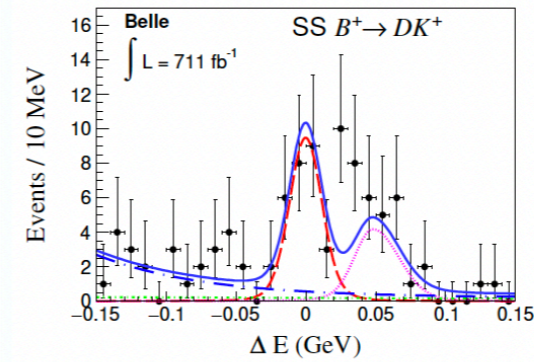
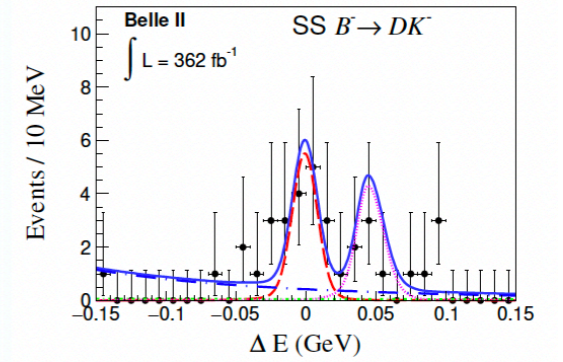
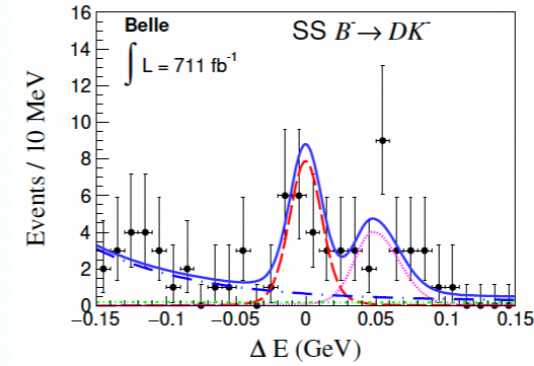
$$\mathcal{A}_{OS}^{D\pi} = +0.009 \pm 0.046 \pm 0.009$$

$$\mathcal{R}_{SS}^{DK/D\pi} = 0.093 \pm 0.012 \pm 0.005$$

$$\mathcal{R}_{OS}^{DK/D\pi} = 0.103 \pm 0.020 \pm 0.006$$

$$\mathcal{R}_{SS/OS}^{D\pi} = 2.412 \pm 0.132 \pm 0.019$$

Full  $D$  phase space



# $\gamma$ using GLS method

## Systematic uncertainties (absolute)

	$A_{SS}^{DK}$	$A_{OS}^{DK}$	$A_{SS}^{D\pi}$	$A_{OS}^{D\pi}$	$R_{SS}^{DK/D\pi}$	$R_{OS}^{DK/D\pi}$	$R_{SS/OS}^{D\pi}$
Full $D$ phase space							
$\epsilon_{K^\pm}, \epsilon_{\pi^\pm}$	0.38	0.56	0.19	0.14	0.05	0.06	0.09
$\delta$	—	0.03	—	—	0.04	0.03	0.02
Model	0.62	0.78	0.02	0.02	0.30	0.22	0.07
$\epsilon_{K_S^0 K^- \pi^+} / \epsilon_{K_S^0 K^+ \pi^-}$	0.82	0.83	0.82	0.83	0.01	0.01	0.02
Total syst. unc.	1.1	1.3	0.9	0.9	0.4	0.3	0.2
Stat. unc.	9.1	13.3	2.6	3.1	1.2	1.3	5.7
$K^*(892)^\pm$ region							
$\epsilon_{K^\pm}, \epsilon_{\pi^\pm}$	0.37	0.61	0.17	0.15	0.03	0.08	0.13
$\delta$	0.02	0.02	0.01	0.01	0.03	0.04	0.04
Model	1.04	0.97	0.20	0.03	0.46	0.49	0.61
$\epsilon_{K_S^0 K^- \pi^+} / \epsilon_{K_S^0 K^+ \pi^-}$	1.6	0.8	1.6	0.8	0.1	0.1	1.7
Total syst. unc.	2.0	1.4	1.6	0.9	0.5	0.6	1.9
Stat. unc.	11.9	18.4	2.9	4.6	1.2	2.0	13.2

# $\gamma$ using GLW method

Physics meanings

$$\mathcal{A}_{CP^\pm} = \frac{\Gamma(B^- \rightarrow D_{CP^\pm} K^-) - \Gamma(B^+ \rightarrow D_{CP^\pm} K^+)}{\Gamma(B^- \rightarrow D_{CP^\pm} K^-) + \Gamma(B^+ \rightarrow D_{CP^\pm} K^+)} = \pm \frac{r_B \sin \delta_B \sin \phi_2}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \phi_3},$$

$$\mathcal{R}_{CP^\pm} = \frac{\mathcal{B}(B^- \rightarrow D_{CP^\pm} K^-) + \mathcal{B}(B^+ \rightarrow D_{CP^\pm} K^+)}{\mathcal{B}(B^- \rightarrow D_{flav} K^-) + \mathcal{B}(B^+ \rightarrow D_{flav} K^+)} \approx \frac{R_{CP^\pm}}{R_{flav}}, \text{ with}$$

$$R_X \equiv \frac{\mathcal{B}(B^- \rightarrow D_X K^-) + \mathcal{B}(B^+ \rightarrow D_X K^+)}{\mathcal{B}(B^- \rightarrow D_X \pi^-) + \mathcal{B}(B^+ \rightarrow D_X \pi^+)}.$$

$$\Rightarrow \begin{cases} \mathcal{R}_{CP^\pm} = 1 + r_B^2 \pm 2 \cos \delta_B \cos \phi_3 \\ \mathcal{A}_{CP^\pm} = \pm 2r_B \sin \phi_3 / \mathcal{R}_{CP^\pm} \end{cases}, \text{ assuming } CP \text{ conservation in } B^\pm \rightarrow D\pi^\pm$$

• Channels:

- Signal:  $B \rightarrow D(\rightarrow KK, K_S^0 \pi^0)K$
- $R_{flav}$  control channel:  $B \rightarrow D(\rightarrow K\pi)K$
- $R_X$  control channel:  $B \rightarrow D\pi$

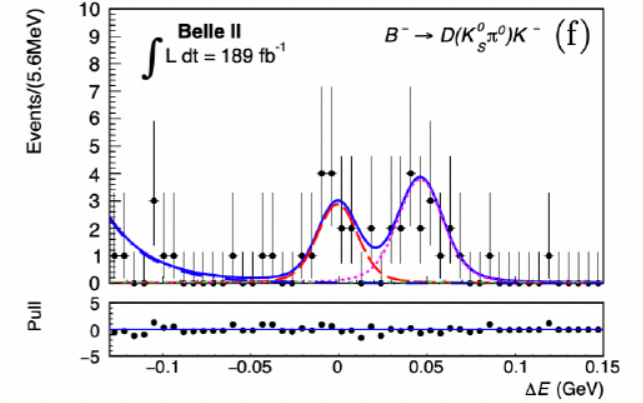
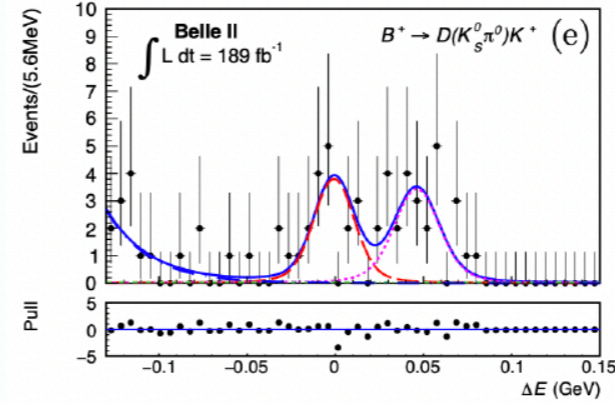
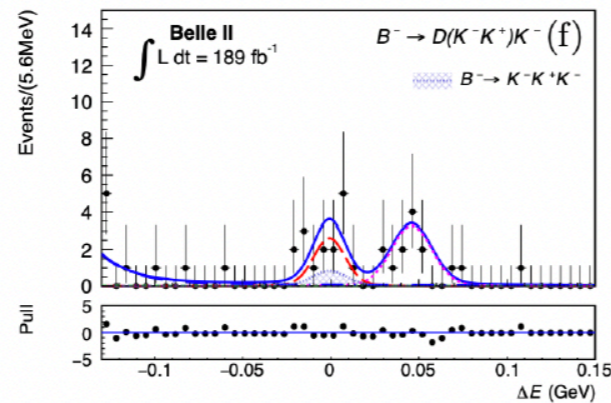
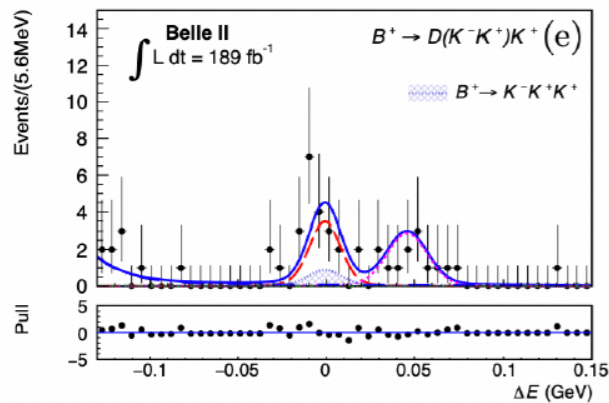
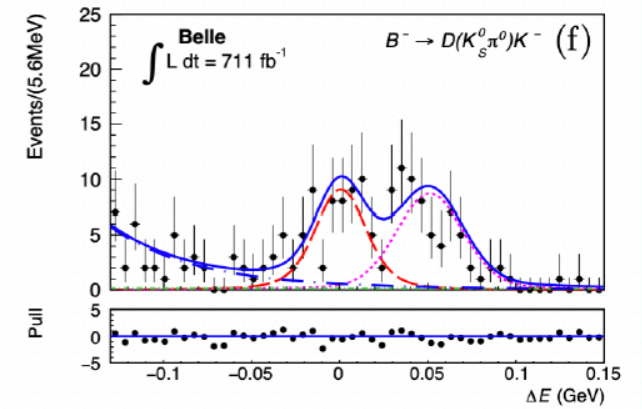
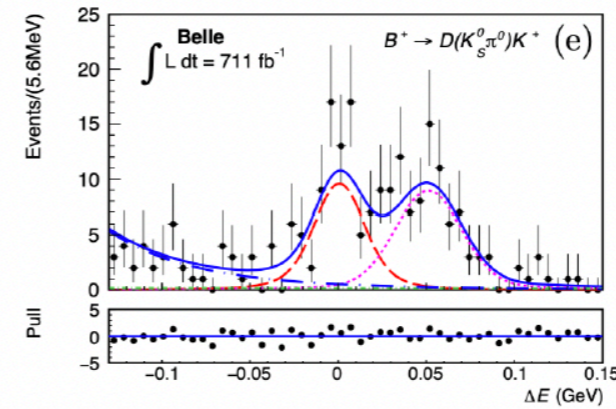
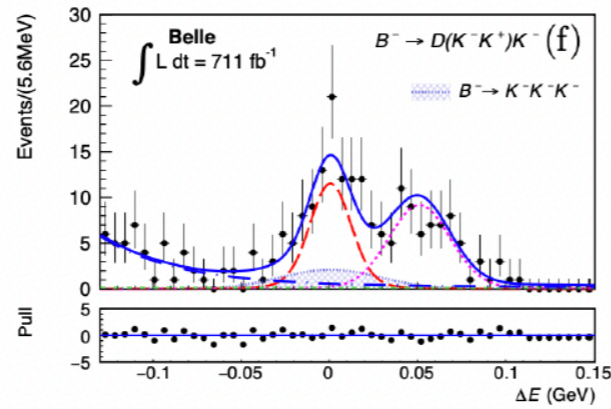
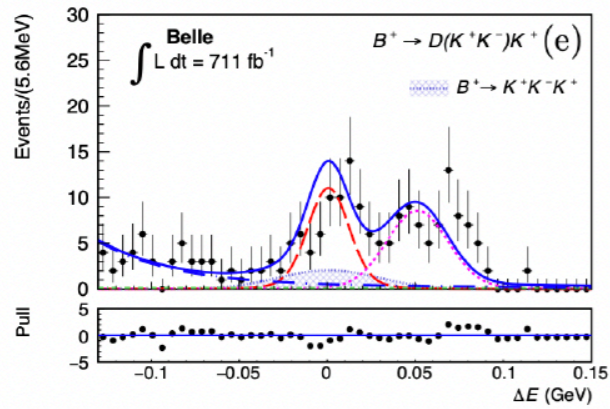
# $\gamma$ using GLW method

## Results

	68.3% CL	95.4% CL
$\phi_3$ ( $^\circ$ )	[8.5, 16.5]	[5.0, 22.0]
	[84.5, 95.5]	[80.0, 100.0]
	[163.3, 171.5]	[157.5, 175.0]
$r_B$	[0.321, 0.465]	[0.241, 0.522]

## Yields

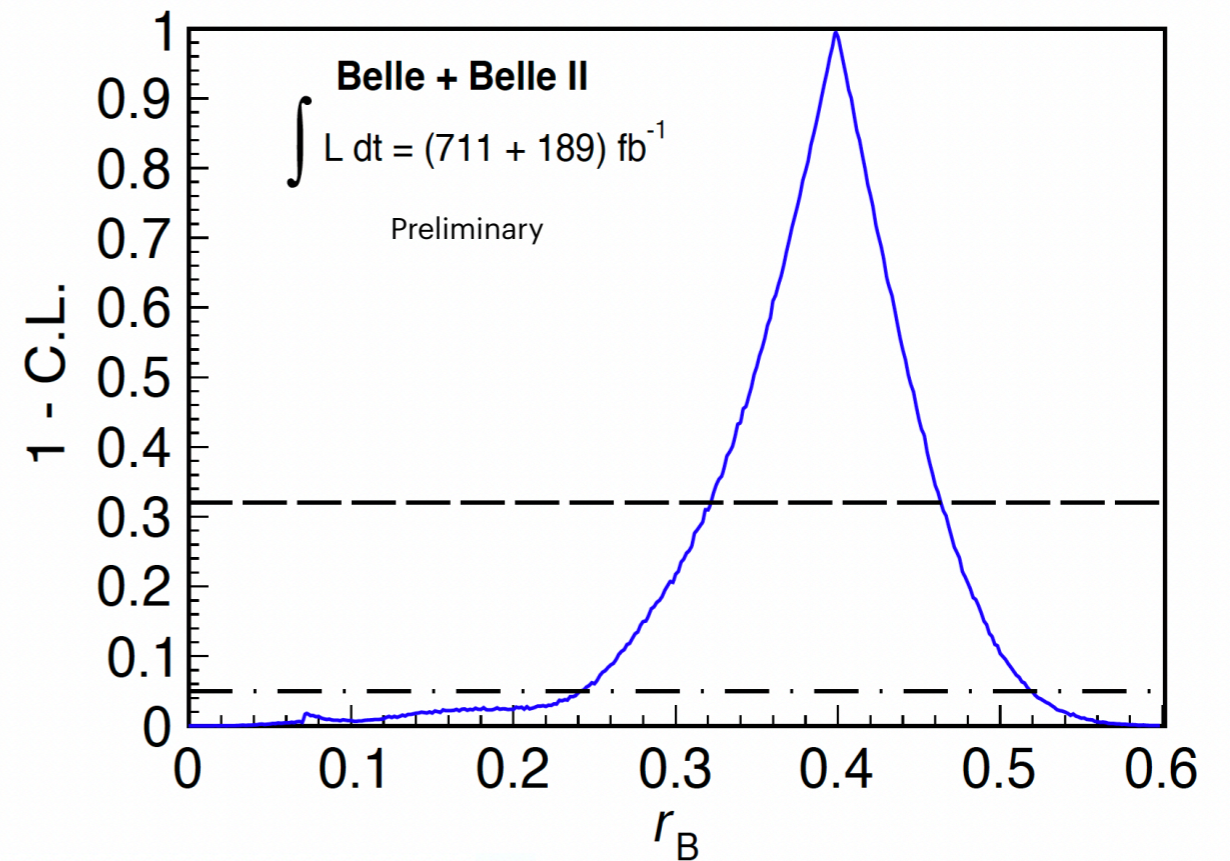
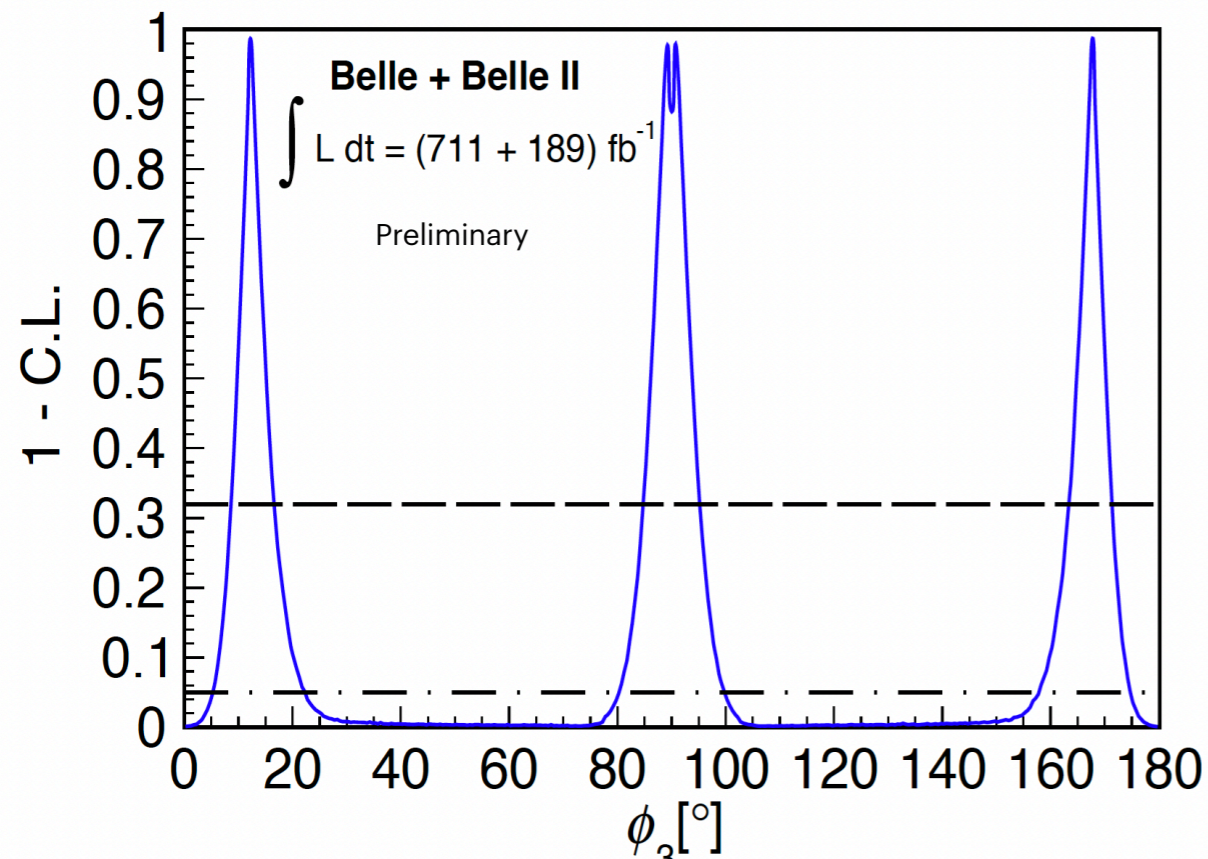
$D_X$ mode		$N(B \rightarrow D_X K)$	$N(B \rightarrow D_X \pi)$
$D \rightarrow K^\pm \pi^\mp$	Belle	4238(94)	59 481(267)
	Belle II	1084(44)	14 229(126)
$D \rightarrow K^+ K^-$	Belle	476(36)	5559(85)
	Belle II	107(15)	1336(40)
$D \rightarrow K_S^0 \pi^0$	Belle	541(42)	6484(95)
	Belle II	145(16)	1763(46)





# $\gamma$ using GLW method

## $\gamma$ estimation



	68.3% CL	95.4% CL
$\phi_3$ ( $^\circ$ )	[8.5, 16.5]	[5.0, 22.0]
	[84.5, 95.5]	[80.0, 100.0]
	[163.3, 171.5]	[157.5, 175.0]
$r_B$	[0.321, 0.465]	[0.241, 0.522]

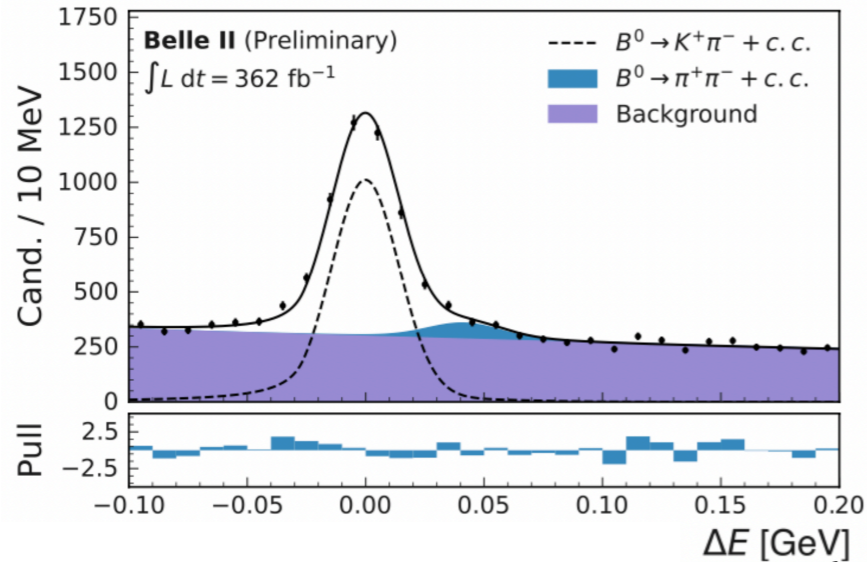
# $\gamma$ using GLW method

## Systematic uncertainties (absolute)

	$\mathcal{R}_{CP+}$	$\mathcal{R}_{CP-}$	$\mathcal{A}_{CP+}$	$\mathcal{A}_{CP-}$
PDF parameters	0.012	0.014	0.002	0.002
PID parameters	0.009	0.010	0.003	0.005
peaking background yields	0.033	0.002	0.013	—
Efficiency ratio	0.001	0.001	<0.001	<0.001
commonality of $\Delta E$ modes	-0.005	-0.006	<0.001	<0.001
Total systematic uncertainty	0.036	0.019	0.014	0.006
Statistical uncertainty	0.081	0.074	0.058	0.057

# Isospin sum rule

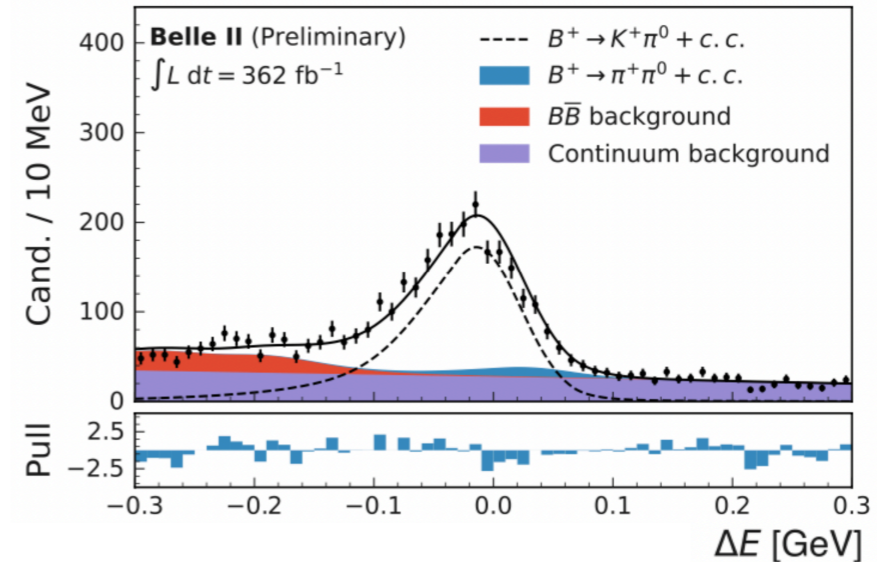
$$B^0 \rightarrow K^+ \pi^-$$



$$\mathcal{B} = (20.67 \pm 0.37 \pm 0.62) \times 10^{-6}$$

$$\mathcal{A}_{CP} = -0.072 \pm 0.019 \pm 0.007$$

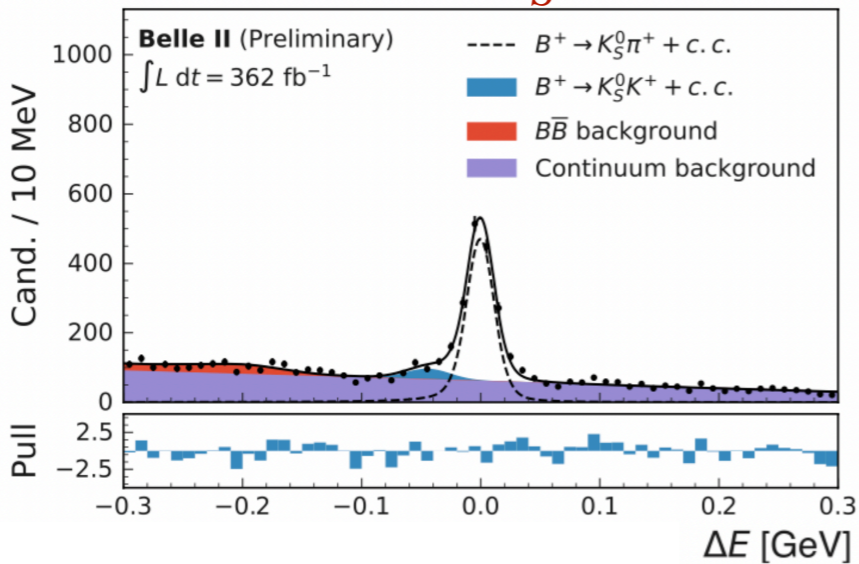
$$B^+ \rightarrow K^+ \pi^0$$



$$\mathcal{B}(K^+ \pi^0) = (14.21 \pm 0.38 \pm 0.85) \times 10^{-6}$$

$$\mathcal{A}_{CP}(K^+ \pi^0) = +0.013 \pm 0.027 \pm 0.005$$

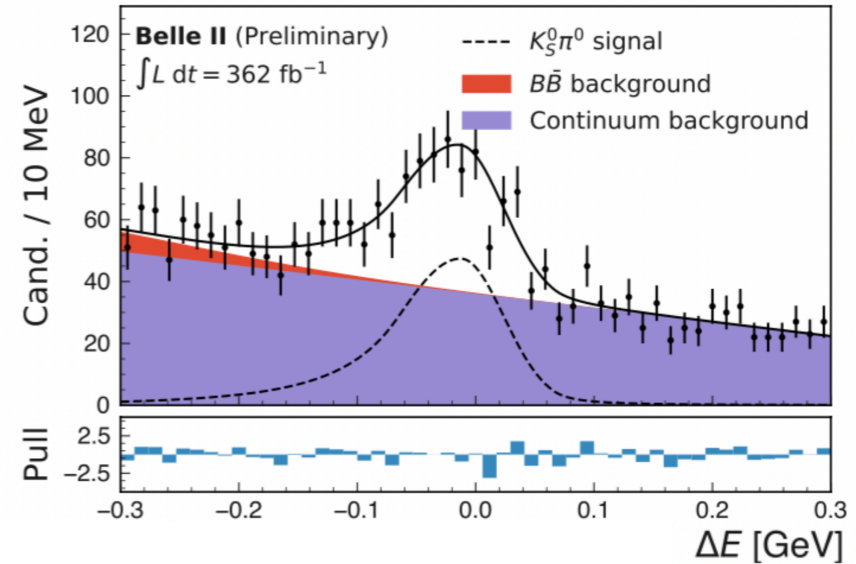
$$B^+ \rightarrow K_S^0 \pi^+$$



$$\mathcal{B}(K_S^0 \pi^+) = (24.40 \pm 0.71 \pm 0.86) \times 10^{-6}$$

$$\mathcal{A}_{CP}(K_S^0 \pi^+) = +0.046 \pm 0.029 \pm 0.007$$

$$B^0 \rightarrow K_S^0 \pi^0$$



$$\mathcal{B}(K_S^0 \pi^0) = (10.16 \pm 0.65 \pm 0.67) \times 10^{-6}$$

$$\mathcal{A}_{CP}(K_S^0 \pi^0) = -0.006 \pm 0.15 \pm 0.05$$

# Isospin sum rule

## Systematic uncertainties

TABLE II. Summary of the relative systematic uncertainties (%) on the branching ratios.

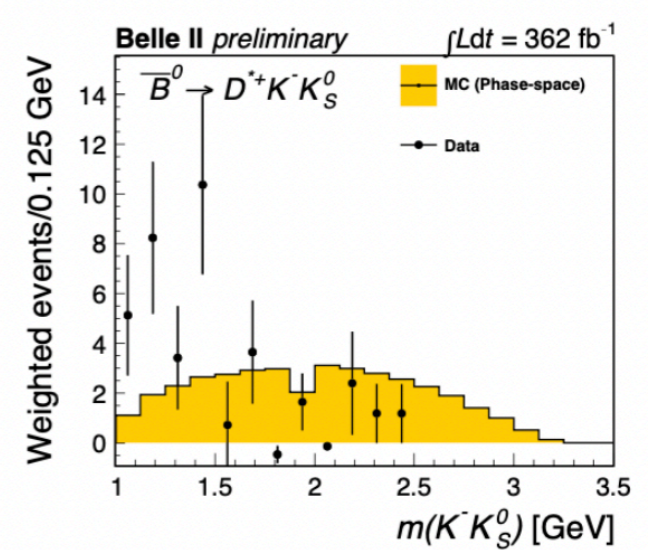
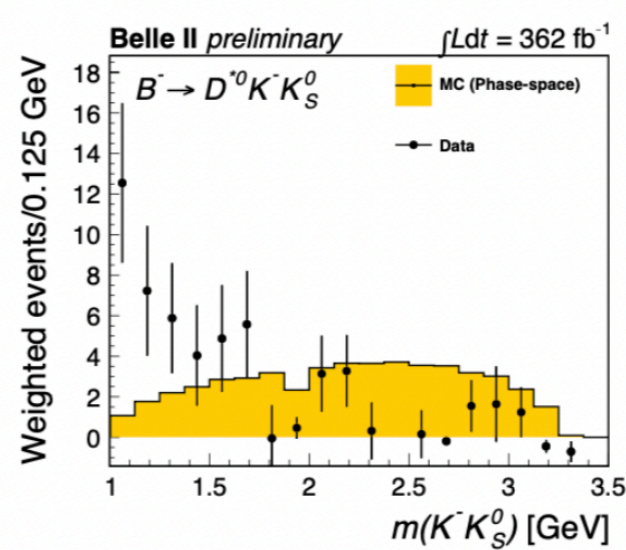
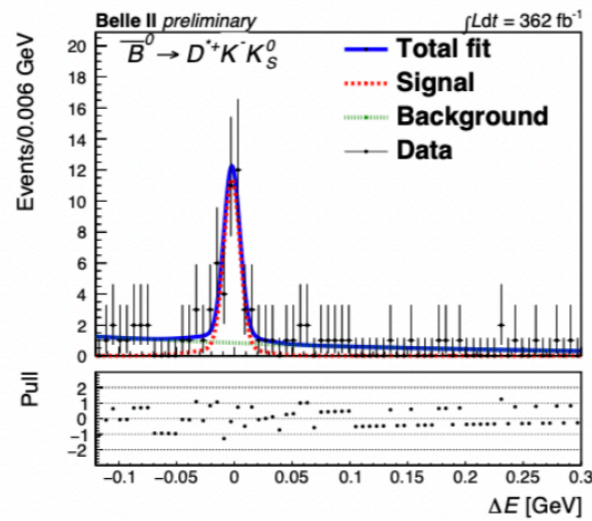
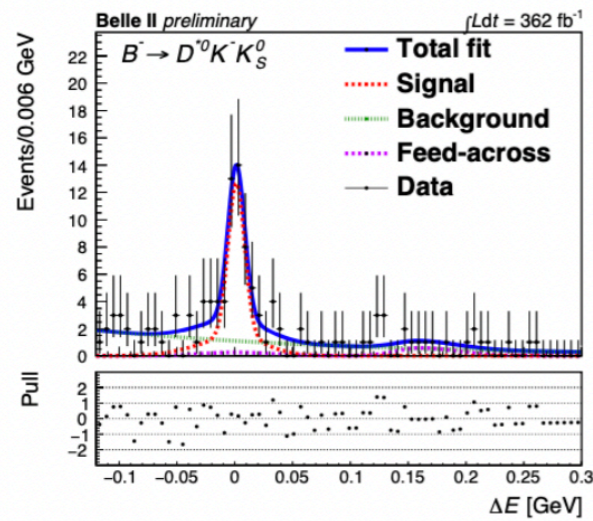
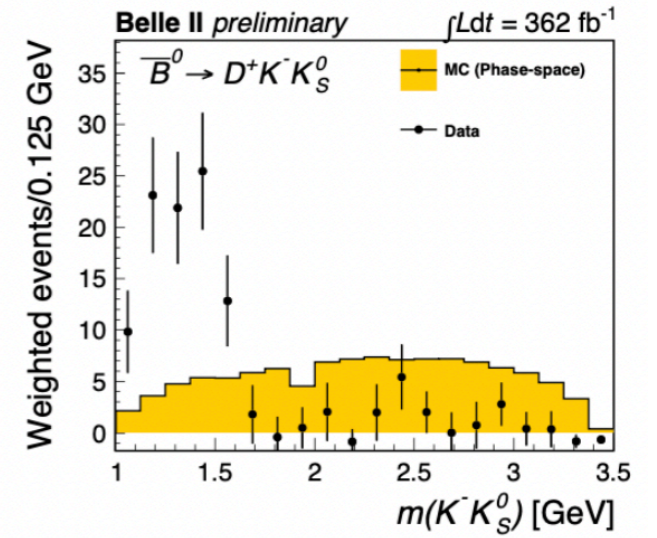
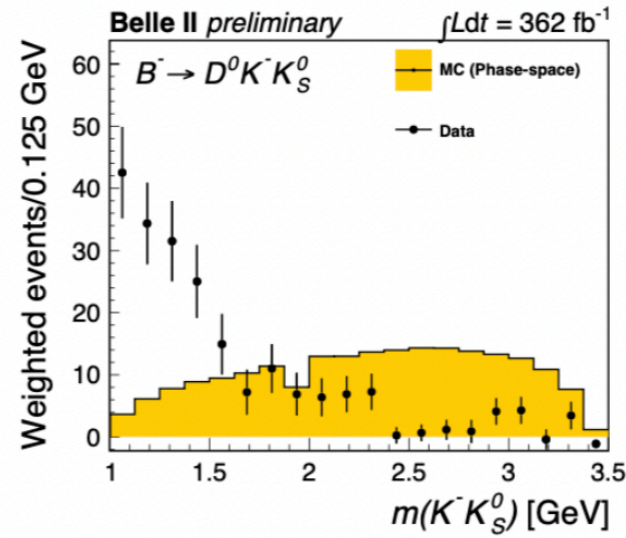
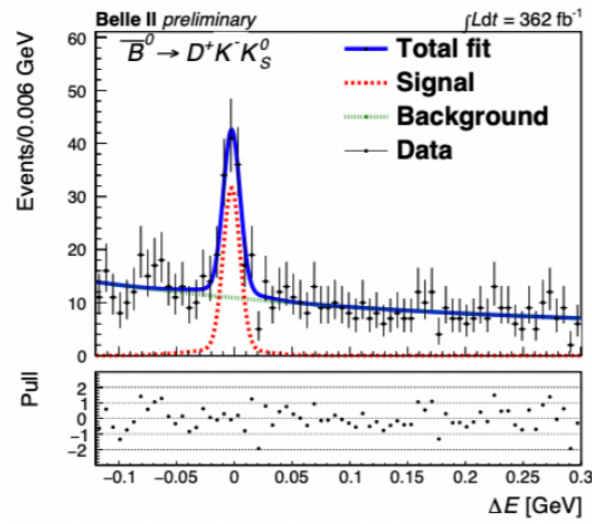
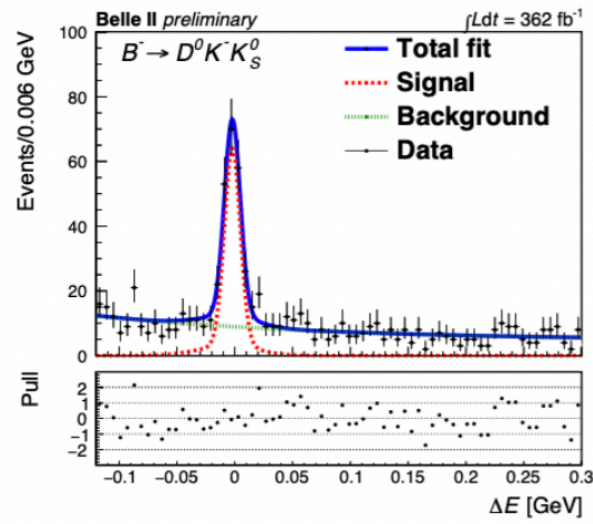
Source	$B^0 \rightarrow K^+\pi^-$	$B^0 \rightarrow \pi^+\pi^-$	$B^+ \rightarrow K^+\pi^0$	$B^+ \rightarrow \pi^+\pi^0$	$B^+ \rightarrow K_S^0\pi^+$	$B^0 \rightarrow K_S^0\pi^0$
Tracking	0.5	0.5	0.2	0.2	0.7	0.5
$N_{B\bar{B}}$	1.5	1.5	1.5	1.5	1.5	1.5
$f^{+-/00}$	2.5	2.5	2.4	2.4	2.4	2.5
$\pi^0$ efficiency	-	-	5.0	5.0	-	5.0
$K_S^0$ efficiency	-	-	-	-	2.0	2.0
CS efficiency	0.2	0.2	0.7	0.7	0.5	1.7
PID correction	0.1	0.1	0.1	0.2	-	-
$\Delta E$ shift and scale	0.1	0.2	1.2	2.0	0.3	1.7
$K\pi$ signal model	0.1	0.2	0.1	<0.1	<0.1	0.1
$\pi\pi$ signal model	<0.1	0.1	<0.1	<0.1	-	-
$K\pi$ CF model	<0.1	0.1	<0.1	0.1	-	-
$\pi\pi$ CF model	0.1	0.2	<0.1	0.1	-	-
$K_S^0K^+$ model	-	-	-	-	0.1	-
$B\bar{B}$ model	-	-	0.3	0.5	<0.1	0.3
Multiple candidates	<0.1	<0.1	1.0	0.3	0.1	0.3
Total	3.0	3.0	6.0	6.2	3.6	6.6

TABLE III. Summary of the absolute systematic uncertainties on the  $CP$  asymmetries.

Source	$B^+ \rightarrow K^+\pi^-$	$B^+ \rightarrow K^+\pi^0$	$B^+ \rightarrow \pi^+\pi^0$	$B^+ \rightarrow K_S^0\pi^+$	$B^0 \rightarrow K_S^0\pi^0$
$\Delta E$ shift and scale	<0.001	0.001	0.002	0.001	0.003
$K_S^0K^+$ model	-	-	-	0.001	-
$B\bar{B}$ background asymmetry	-	-	-	-	0.046
$q\bar{q}$ background asymmetry	-	-	-	-	0.024
Fitting bias	-	-	0.007	0.006	-
Instrumental asymmetry	0.007	0.005	0.004	0.004	-
Total	0.007	0.005	0.008	0.007	0.052

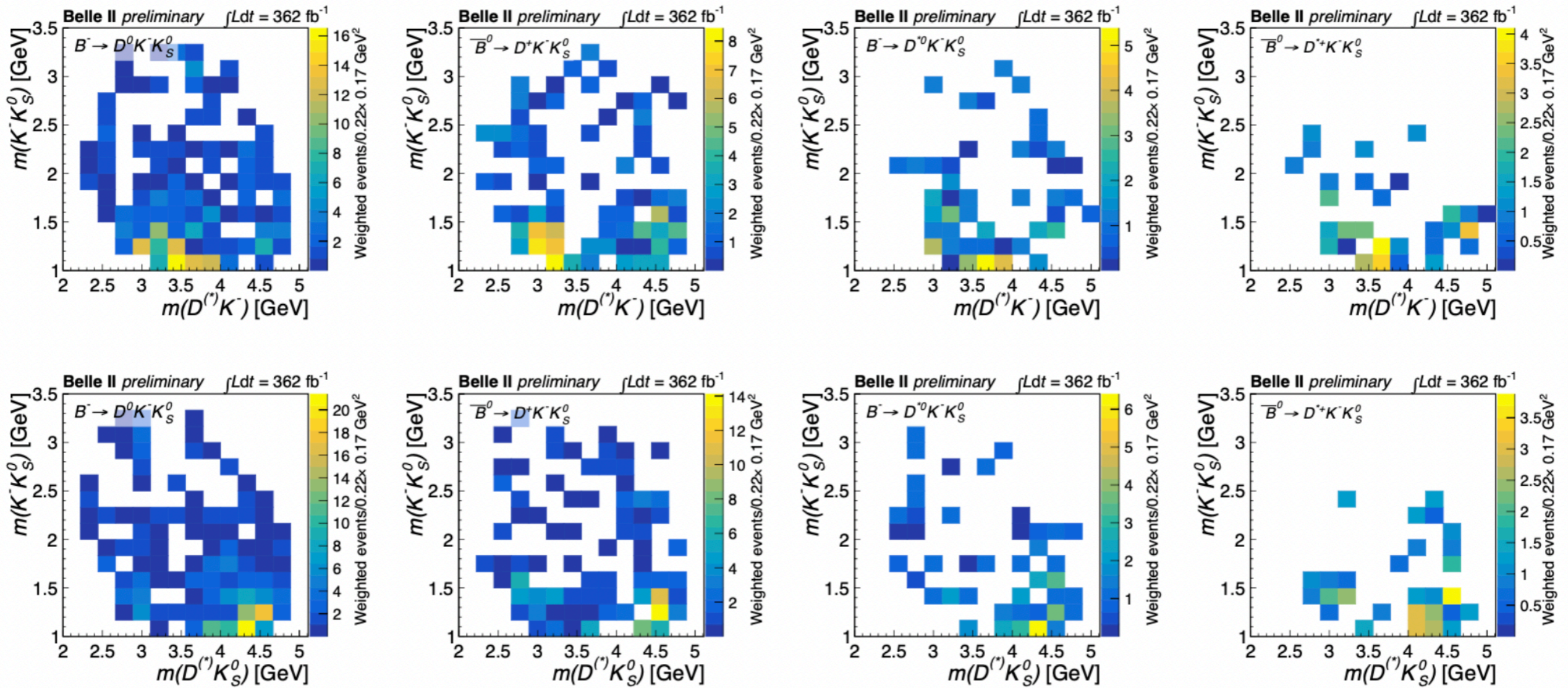
# $B \rightarrow D^{(*)}K^-K_S^0$ decays

## $\Delta E$ fit and $m(K^-K_S^0)$ distributions



# $B \rightarrow D^{(*)}K^-K_S^0$ decays

## Dalitz distributions



# $B \rightarrow D^{(*)}K^-K_S^0$ decays

## Systematic uncertainties (relative)

Source	$B^- \rightarrow D^0K^-K_S^0$	$\bar{B}^0 \rightarrow D^+K^-K_S^0$	$B^- \rightarrow D^{*0}K^-K_S^0$	$\bar{B}^0 \rightarrow D^{*+}K^-K_S^0$
Eff. - MC sample size	0.6	0.9	1.0	0.8
Eff. - tracking	0.7	1.0	0.7	1.0
Eff. - $\pi^+$ from $D^{*+}$	-	-	-	2.7
Eff. - $K_S^0$	3.4	3.4	3.4	3.3
Eff. - PID	1.3	1.4	0.5	0.6
Eff. - $\pi^0$	-	-	5.1	-
Signal model	1.9	3.3	2.7	3.1
Bkg model	1.1	0.8	0.1	0.1
Self-cross-feed	-	-	2.7	-
$D^{*0}$ peaking bkg	-	-	0.9	-
$N_{B\bar{B}}, f_{+-,00}$	2.7	2.8	2.7	2.8
Intermediate $\mathcal{B}$ s	0.7	1.7	1.6	1.1
Total systematic	5.2	6.1	7.6	6.2
Statistical	8.3	13.5	17.1	19.0