

Recent tau-lepton results at Belle and Belle II

Kenta Uno (KEK)

on behalf of the Belle/Belle II collaboration

EPS-HEP2023 conference

21 August 2023



2023/8/21



Unravelling the mysteries of
matter, life and the universe.

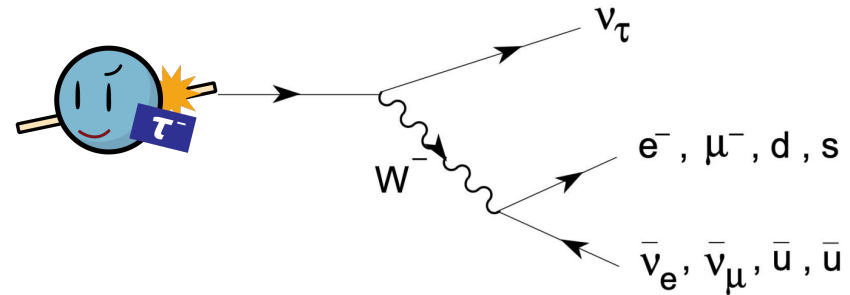


Introduction: τ physics

Leptons	0.511 MeV/c ² -1 1/2 e electron	105.7 MeV/c ² -1 1/2 μ muon	1.777 GeV/c ² -1 1/2 τ tau
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τ lepton: heaviest lepton in the Standard Model (SM)

- Leptonic and Hadronic decays: > 200 decay channels!
- Sensitive to new physics



Search for forbidden decays of tau

- Lepton Flavor Violation: $\tau \rightarrow \ell\gamma, \tau \rightarrow \ell\ell\ell, \tau \rightarrow \ell V^0$
 - Observation → Clear signs of new physics

Precision measurements of the tau properties

- Lepton Flavor Universality
 - Deviation from the SM → Indirect signs of new physics

Belle, Belle II experiments

Belle experiment (1999 – 2010)

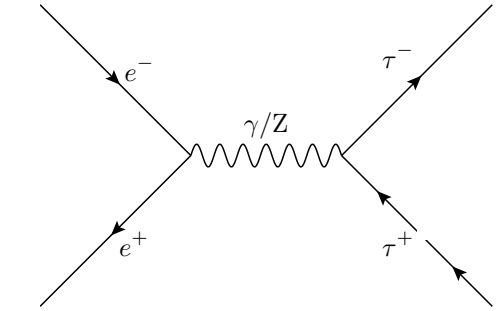
- 8 GeV e^- and 3.5 GeV e^+
 - Recorded $\sim 1000 \text{ fb}^{-1}$ data

Belle II experiment (2018 -)

- 7 GeV e^- and 4 GeV e^+
 - Recorded 424 fb^{-1} data



$e^+e^- \rightarrow \tau^+\tau^-$ at Belle (II)



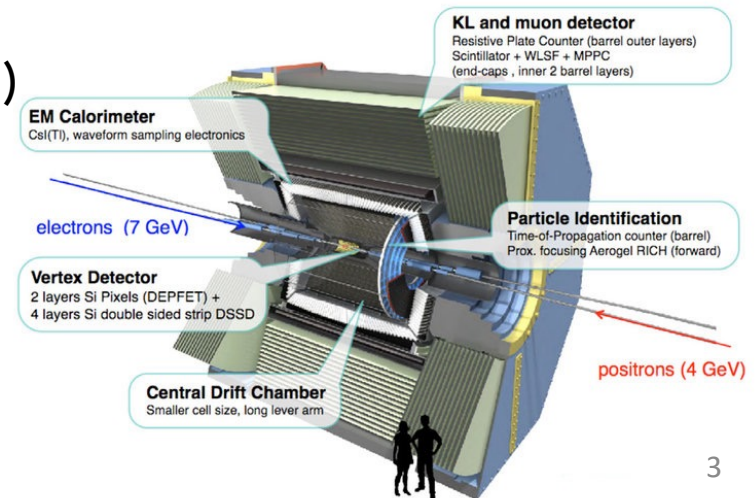
$\sigma(ee \rightarrow bb) \sim 1.1 \text{ nb}$, $\sigma(ee \rightarrow \tau\tau) \sim 0.9 \text{ nb}$

τ factory ($N_{\tau\tau} \sim 4.0 \times 10^8$ Belle II)

- Enables high precision studies
- Searches for rare processes

Belle (II) detectors

- Good efficiency of neutral particles (π^0, η)
- Good reconstruction of missing energy
- Specific low-multi triggers at Belle II
 - Eg. single track trigger



How to reconstruct τ at Belle (II)

Not fully reconstruct τ in the SM due to neutrinos

→ Identify τ -pair events using **thrust axis, \vec{n}_{th}**

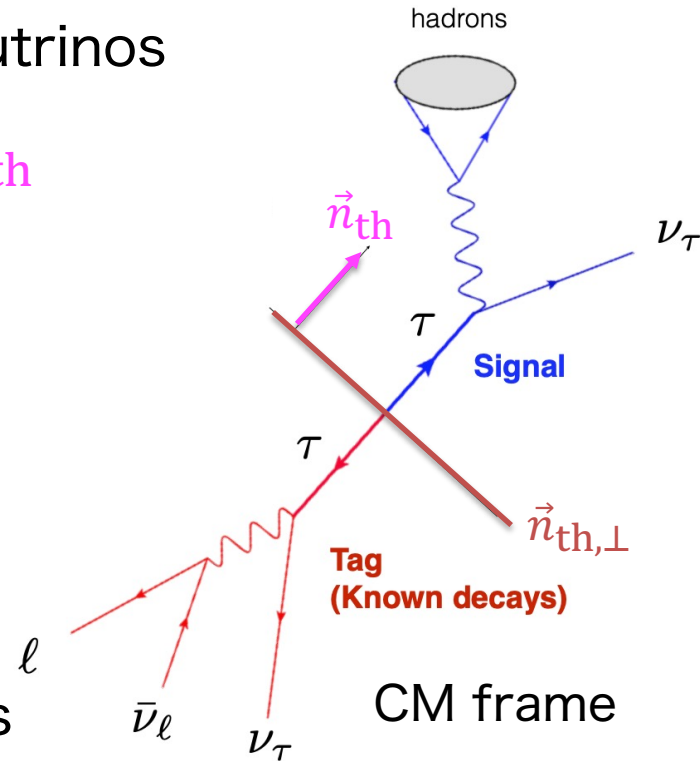
\vec{n}_{th} is defined so that V_{th} is maximized

$$V_{\text{th}} = \frac{\sum_i |\vec{p}_i^{\text{CM}} \cdot \vec{n}_{\text{th}}|}{\sum_i |\vec{p}_i^{\text{CM}}|}$$

all the reconstructed particles

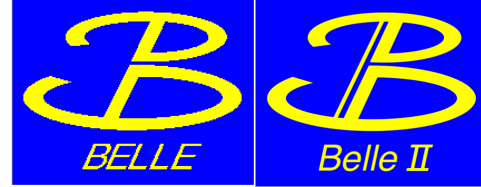
τ -pair event is divided into two hemispheres

- **Signal-side:** Target τ decay, Eg. $\tau \rightarrow \ell\gamma, \ell\ell\ell, \ell V^0$ in LFV search
- **Tag-side:** 1-prong or 3-prong τ decay in the SM



This method is used in Belle (II) τ analysis to identify an τ event.

LFV decay: $\tau^\pm \rightarrow \ell^\pm V^0$

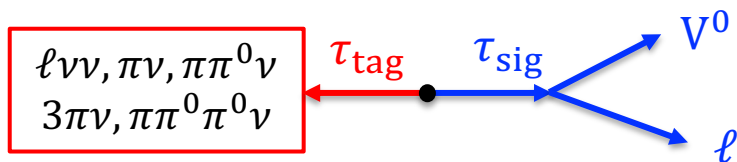


Charged Lepton Flavor Violation

- Forbidden in the SM but possible in several new physics scenarios
 - $\tau \rightarrow \ell V^0$: sensitive to leptoquark model
- Exotic decay \rightarrow High signal detection efficiency is crucial!

Belle analysis ($V^0 = \rho^0, \phi, \omega, K^*$)

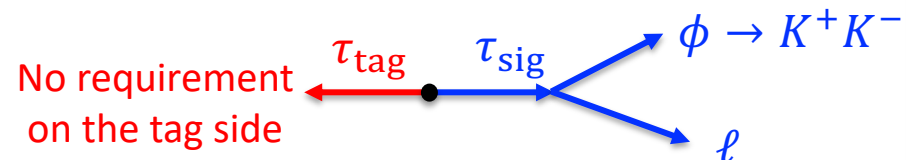
- Use full data (980 fb^{-1})
- Consider 3-prong decay in τ_{tag}
- Event selection by BDT



[JHEP06\(2023\)118](#)

Belle II analysis ($V^0 = \phi$)

- Use 190 fb^{-1} data
- Untagged reconstruction
- Event selection by BDT



Signal detection efficiency $\sim 2 \times$ Belle

[arXiv:2305.04759](#)

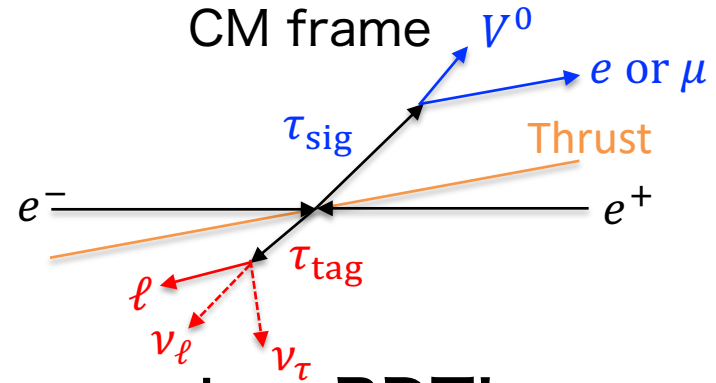
Improved the sensitivities by untagged reconstruction and BDT

Belle: Analysis approach

Signal-side: Reconstruct ℓ and V^0

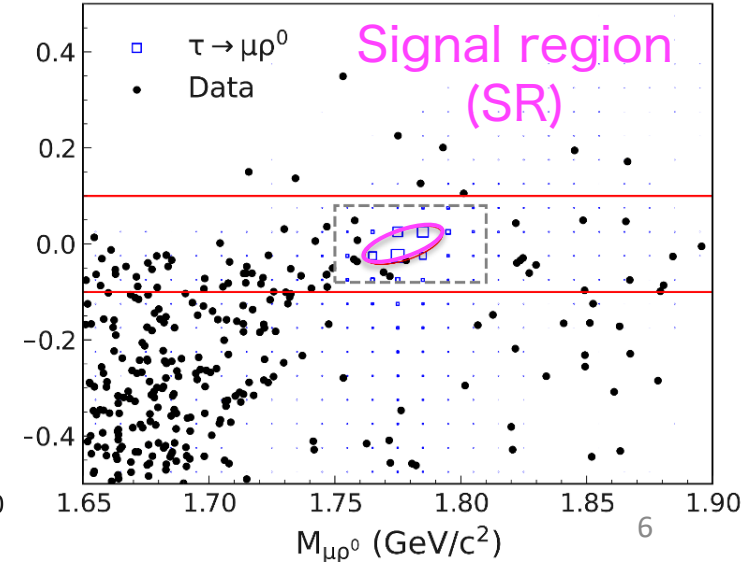
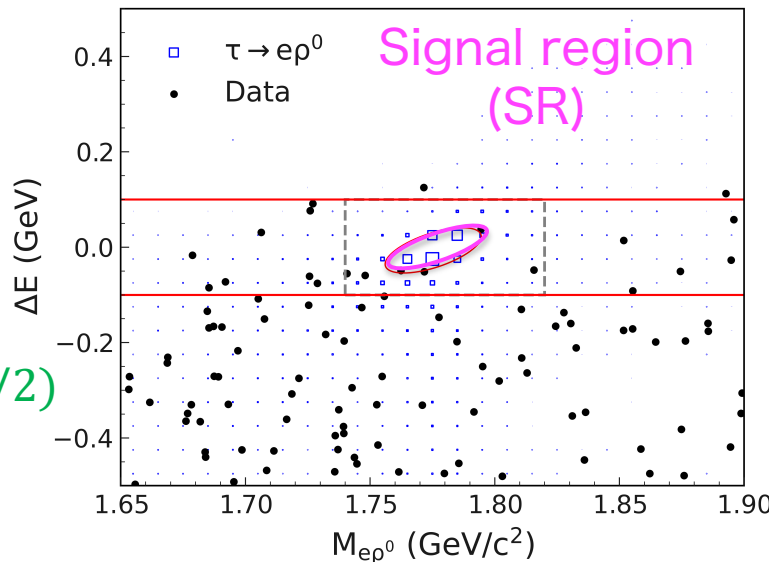
- V^0 : $\rho^0, \phi, \omega, K^*(\bar{K}^{*0})$

Tag-side: Require 1, 3-prong τ



Background (eg. $\tau \rightarrow 3\pi\nu, ee \rightarrow q\bar{q}$) suppression: **BDT!**

- Prepare BDT classifier for each ℓV^0 mode
- Training: 11 input variables for $\ell\omega$, 9 input variables for others



$$\Delta E = (E_{\ell V^0}^{\text{CM}} - \sqrt{s}/2)$$

Belle: $\tau^\pm \rightarrow \ell^\pm V^0$ results

No significant excess in all ℓV^0 modes

World leading results

Mode	ϵ (%)	N_{BG}	σ_{syst} (%)	N_{obs}	$\mathcal{B}_{\text{obs}} (\times 10^{-8})$
$\tau^\pm \rightarrow \mu^\pm \rho^0$	7.78	$0.95 \pm 0.20(\text{stat.}) \pm 0.15(\text{syst.})$	4.6	0	< 1.7
$\tau^\pm \rightarrow e^\pm \rho^0$	8.49	$0.80 \pm 0.27(\text{stat.}) \pm 0.04(\text{syst.})$	4.4	1	< 2.2
$\tau^\pm \rightarrow \mu^\pm \phi$	5.59	$0.47 \pm 0.15(\text{stat.}) \pm 0.05(\text{syst.})$	4.8	0	< 2.3 *
$\tau^\pm \rightarrow e^\pm \phi$	6.45	$0.38 \pm 0.21(\text{stat.}) \pm 0.00(\text{syst.})$	4.5	0	< 2.0 *
$\tau^\pm \rightarrow \mu^\pm \omega$	3.27	$0.32 \pm 0.23(\text{stat.}) \pm 0.19(\text{syst.})$	4.8	0	< 3.9 *
$\tau^\pm \rightarrow e^\pm \omega$	5.41	$0.74 \pm 0.43(\text{stat.}) \pm 0.06(\text{syst.})$	4.5	0	< 2.4 *
$\tau^\pm \rightarrow \mu^\pm K^{*0}$	4.52	$0.84 \pm 0.25(\text{stat.}) \pm 0.31(\text{syst.})$	4.3	0	< 2.9 *
$\tau^\pm \rightarrow e^\pm K^{*0}$	6.94	$0.54 \pm 0.21(\text{stat.}) \pm 0.16(\text{syst.})$	4.1	0	< 1.9 *
$\tau^\pm \rightarrow \mu^\pm \bar{K}^{*0}$	4.58	$0.58 \pm 0.17(\text{stat.}) \pm 0.12(\text{syst.})$	4.3	1	< 4.3 *
$\tau^\pm \rightarrow e^\pm \bar{K}^{*0}$	7.45	$0.25 \pm 0.11(\text{stat.}) \pm 0.02(\text{syst.})$	4.1	0	< 1.7 *

Set ULs at 90% CL by counting approach

$$B(\tau \rightarrow e V^0) < (1.7 - 2.4) \times 10^{-8}$$

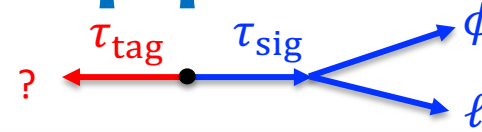
$$B(\tau \rightarrow \mu V^0) < (1.7 - 4.3) \times 10^{-8}$$

The ULs are improved by ~30% from the previous results

Belle II: Analysis approach

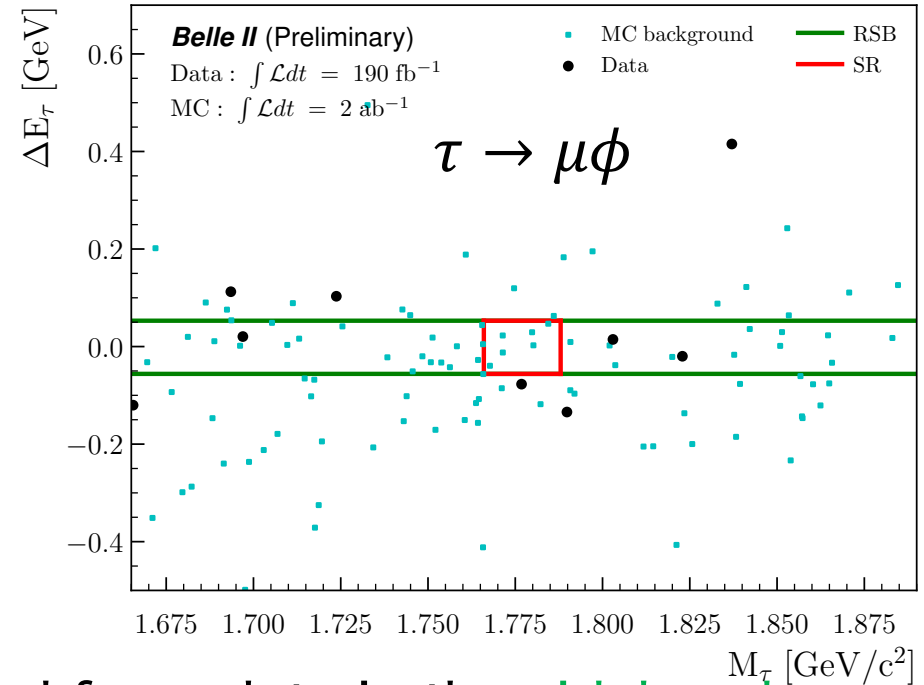
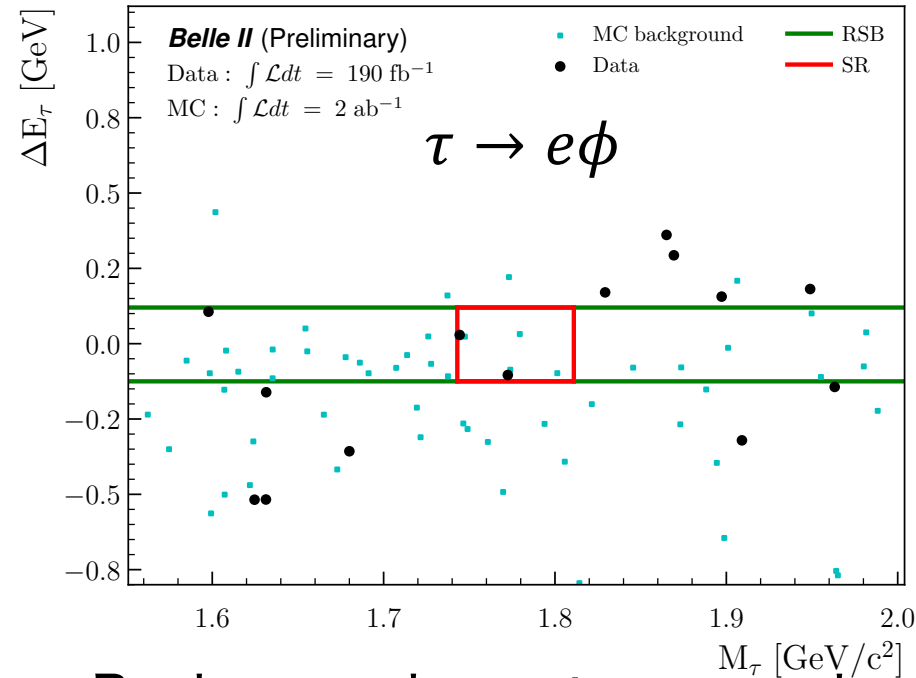


Untagged approach



- Reconstruct $\tau_{\text{sig}} \rightarrow \ell\phi$
- SR: $M_{\ell\phi}$ and $\Delta E_\tau = (E_{\ell\phi}^{\text{CM}} - \sqrt{s}/2)$

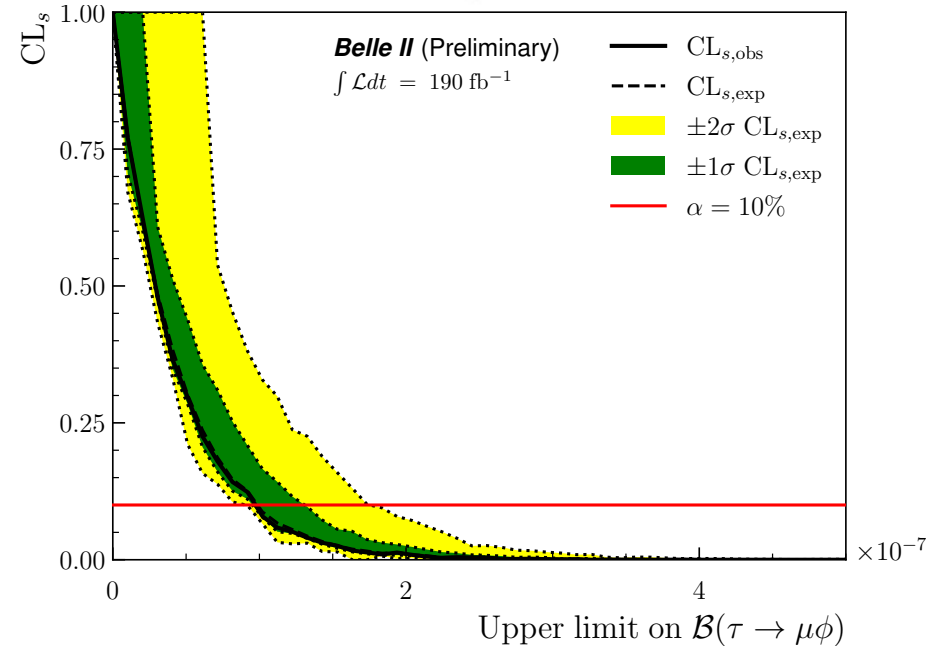
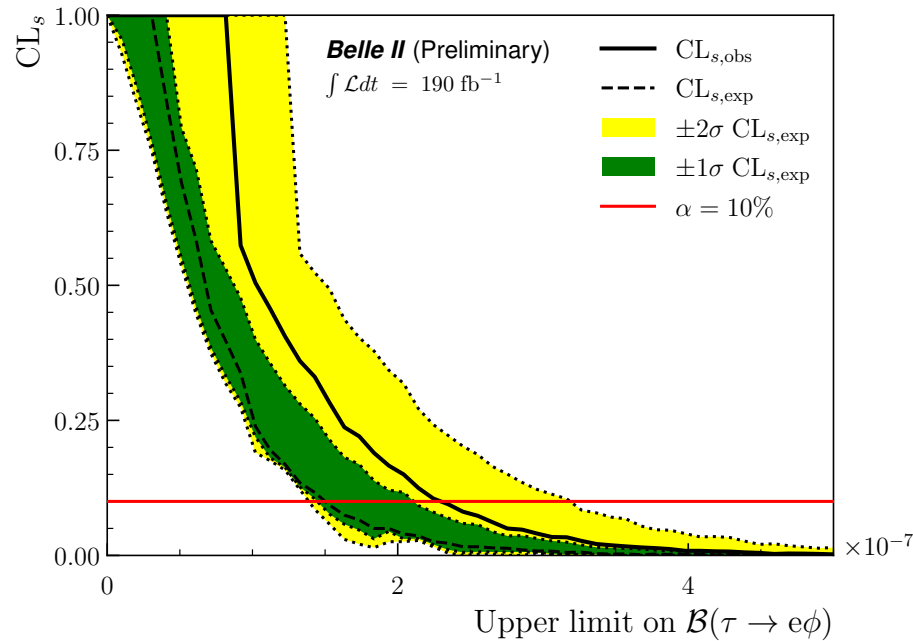
Result	Region	Mode	
		$e\phi$	$\mu\phi$
N_{exp}	SR	$0.23^{+0.55}_{-0.21}$ stat	$0.36^{+0.39}_{-0.23}$ stat
N_{obs}	SR	$2.0^{+2.6}_{-1.3}$ stat	$0.0^{+1.8}_{-0.0}$ stat



- Background events are evaluated from data in the **sidebands**
- Perform Poisson counting experiment approach in **SR**

Belle II: $\tau^\pm \rightarrow \ell^\pm \phi$ results

Set ULs at 90% CL $B_{UL}(\tau \rightarrow \ell\phi) = \frac{s}{L \times 2\sigma_{\tau\tau} \times \epsilon_{\ell\phi}},$



Obs. $B_{UL}(\tau \rightarrow e\phi) = 23 \times 10^{-8}$

Exp. $B_{UL}(\tau \rightarrow e\phi) = 15 \times 10^{-8}$

$B_{UL}(\tau \rightarrow e\phi) = 2.0 \times 10^{-8}$ at Belle

Obs. $B_{UL}(\tau \rightarrow \mu\phi) = 9.7 \times 10^{-8}$

Exp. $B_{UL}(\tau \rightarrow \mu\phi) = 9.9 \times 10^{-8}$

$B_{UL}(\tau \rightarrow \mu\phi) = 2.3 \times 10^{-8}$ at Belle

**Successful first application of untagged approach
in τ analysis at Belle II**

$\tau^\pm \rightarrow \ell^\pm \alpha$ search at Belle II



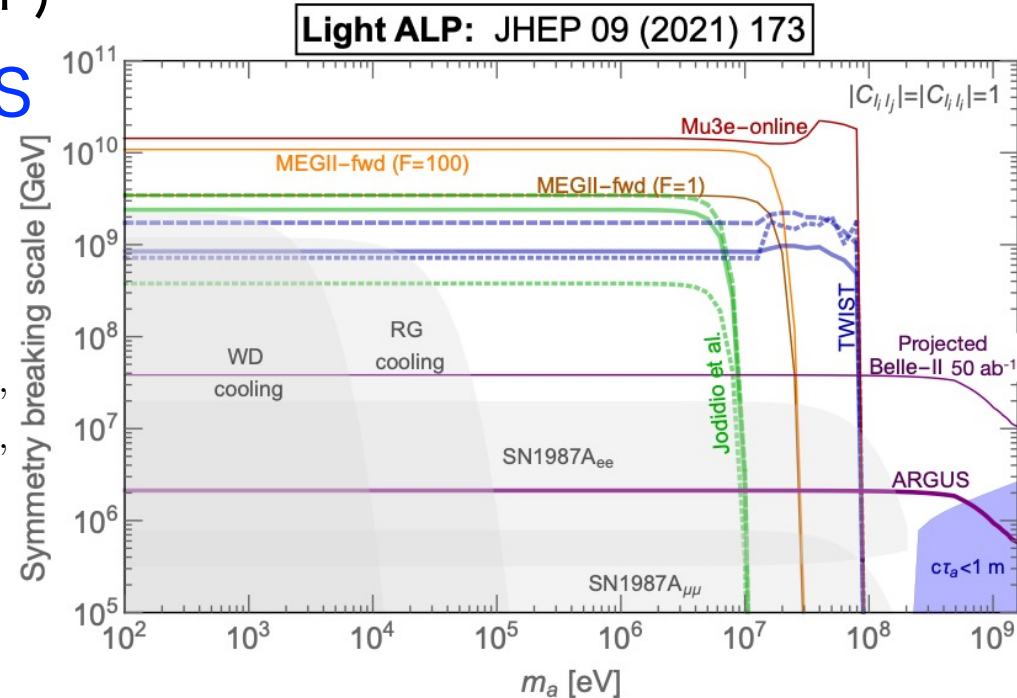
Phys.Rev.Lett.130,181803

- Search for $\tau \rightarrow \ell + \alpha$ (invisible)
 - eg. Axion-like particle (ALP)
- Upper Limit from ARGUS
 - 476 pb⁻¹ data (1995)

UL at 95% CL

$$\mathcal{B}(\tau^\pm \rightarrow e^\pm \alpha) / \mathcal{B}(\tau^\pm \rightarrow e^\pm \nu \bar{\nu}) < (0.6 - 3.4) \times 10^{-2},$$
$$\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \alpha) / \mathcal{B}(\tau^\pm \rightarrow \mu^\pm \nu \bar{\nu}) < (0.3 - 3.6) \times 10^{-2},$$

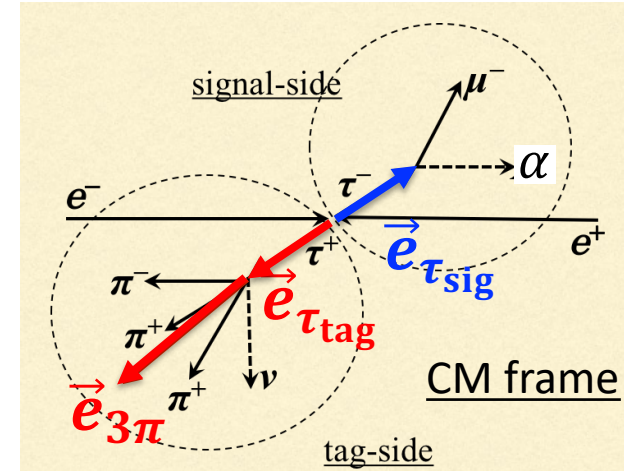
$$0.0 < m_\alpha < 1.6 \text{ GeV}$$



Belle (II) can set more stringent limits on the invisible boson

$\tau^\pm \rightarrow \ell^\pm \alpha$: Analysis approach

- Split event in two hemispheres based on the thrust axis
 - **Signal side:** 1 lepton track ($\ell = e, \mu$)
 - **Tag side:** 3-pion ($\tau \rightarrow 3\pi\nu$ decay)
 - Veto neutrals to suppress hadronic bkg
- Exploit the shape differences
 - Signals: $\tau \rightarrow \ell\alpha$ two-body decays
 - Backgrounds: $\tau \rightarrow \ell\nu\nu$ three-body decays

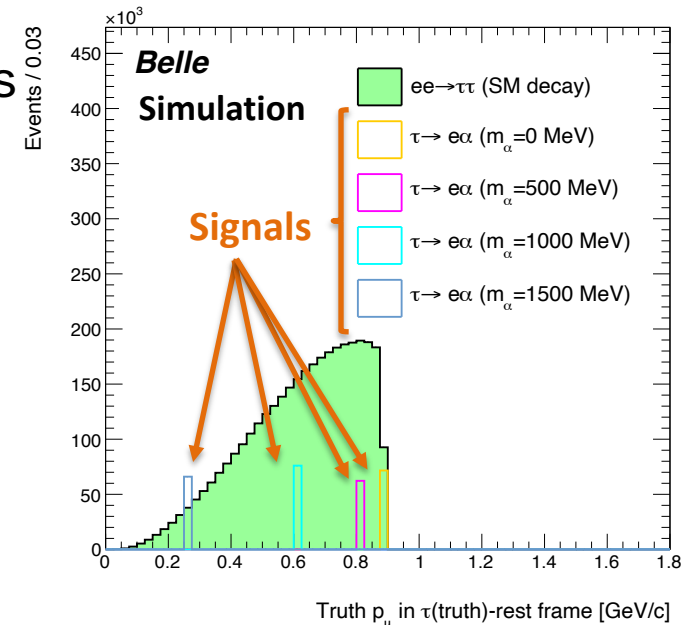


$\rightarrow p_\ell, E_\ell$ in tau rest frame: monochromatic

How to obtain tau direction?

- Require $\tau \rightarrow a_1(\rightarrow 3\pi)\nu$ in tag side

$$\rightarrow \vec{e}_{\tau_{\text{tag}}} \approx \vec{e}_{3\pi} \quad \begin{aligned} m_\tau &= 1.78 \text{ GeV} \\ m_{a_1} &= 1.26 \text{ GeV} \end{aligned}$$



Energy in τ pseudo-rest frame



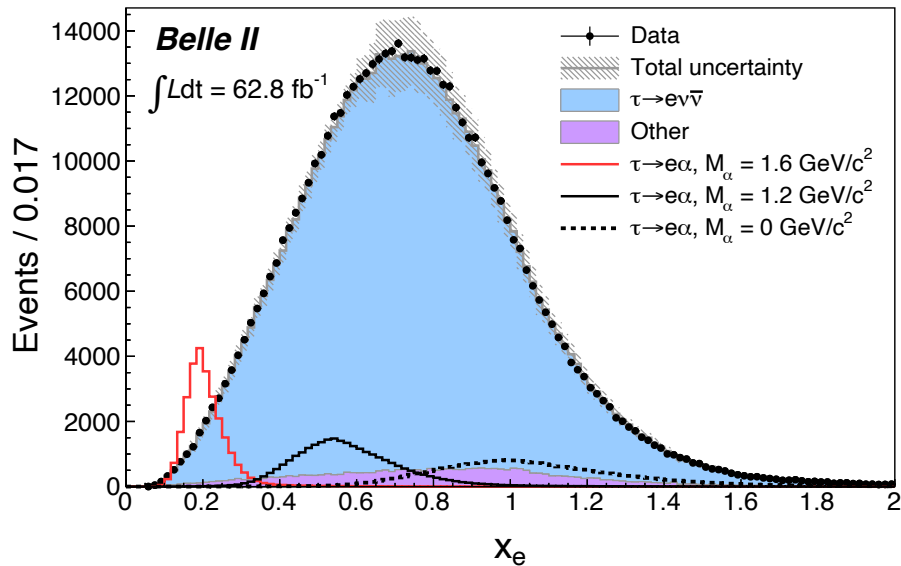
Discriminating variable: normalized lepton energy

$$x_\ell \equiv \frac{E_\ell^*}{m_\tau c^2 / 2},$$

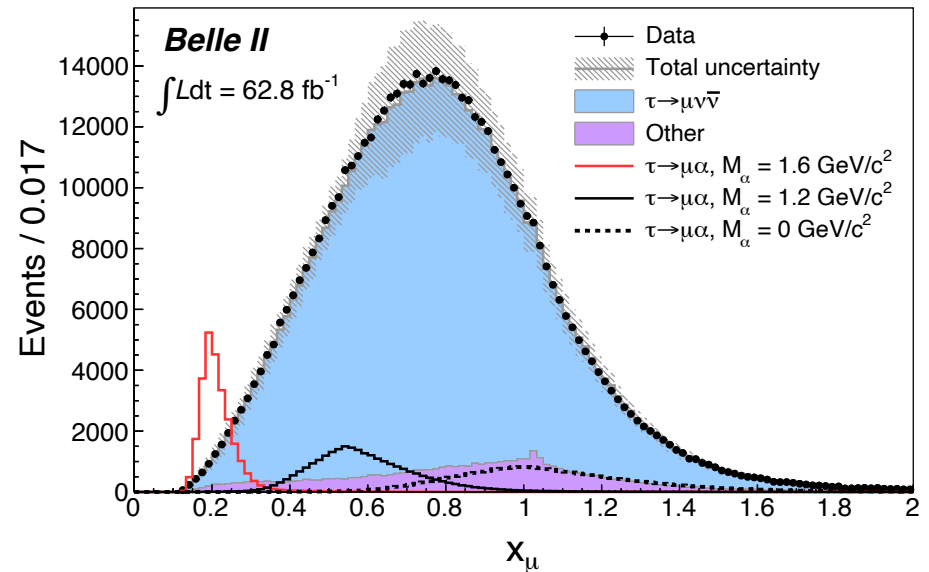
Lepton energy in the τ rest frame

- Bump hunt above broad distribution from $\tau \rightarrow \ell \nu \nu$

$\tau \rightarrow e \alpha$ search



$\tau \rightarrow \mu \alpha$ search



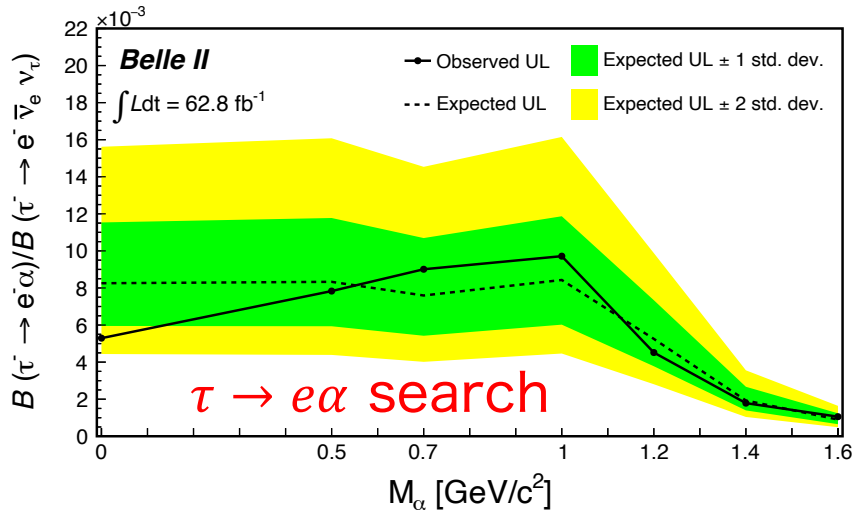
No significant excess over the background predictions..

$\tau^\pm \rightarrow \ell^\pm \alpha$: Upper limits

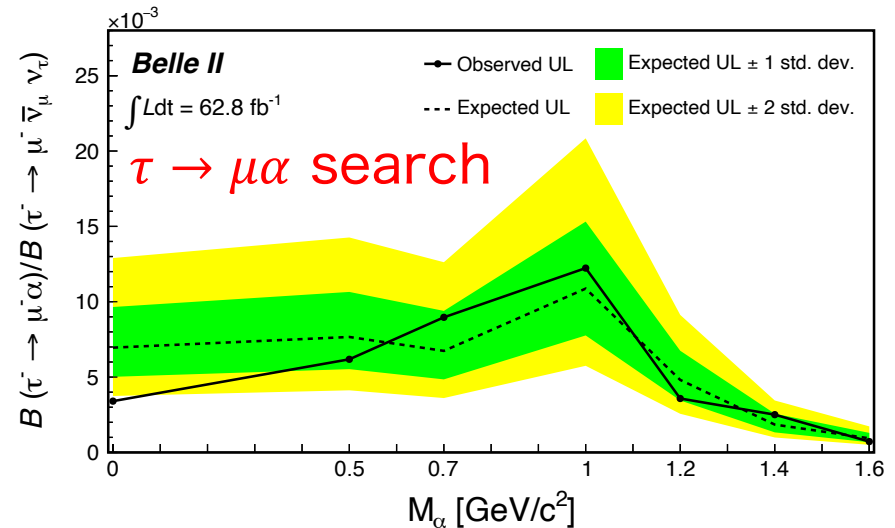
Perform the maximum likelihood fit

- Use x_ℓ distributions as signal/bkg PDF (MC simulation)

Set upper limits on branching fractions



Dominant syst. unc.: Lepton ID efficiency



2 - 14 times more stringent limits than ARGUS

Heavy neutrino search in τ decay

Submitted to PRL ([arXiv:2212.10095](https://arxiv.org/abs/2212.10095))



Neutrino mass, $m_\nu \neq 0 \rightarrow$ Need a mechanism to establish it

- One approach is to include right-handed neutrino

$$\tau^\pm \rightarrow \pi^\pm \nu_h \quad (\nu_h \rightarrow \pi^\pm \ell^\mp) \quad \begin{array}{l} \nu_h: \text{Majorana neutrino (long-lived)} \\ \ell = e, \mu \end{array}$$

Signal-side: Require $\pi\pi\ell$ ($\ell = e, \mu$)

- $\nu_h \rightarrow \pi^\pm \ell^\mp$: π and ℓ with a common vertex

Tag-side: Require 1, 3-prong τ

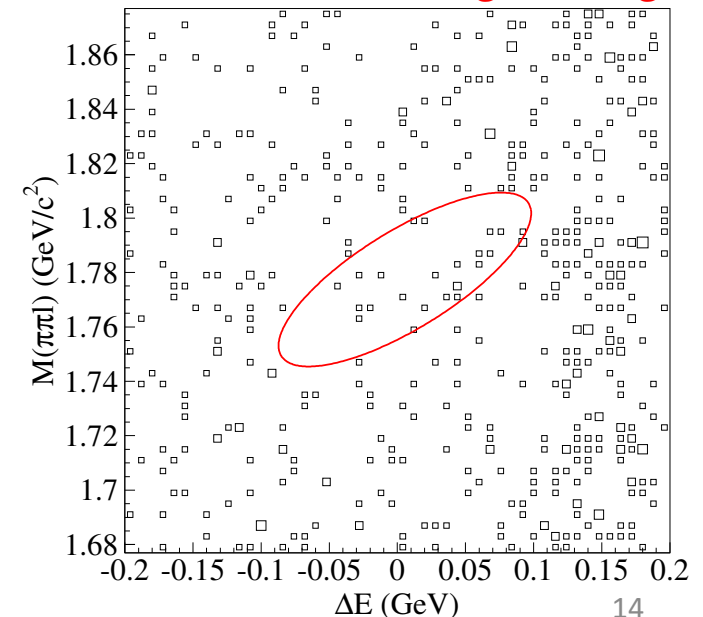
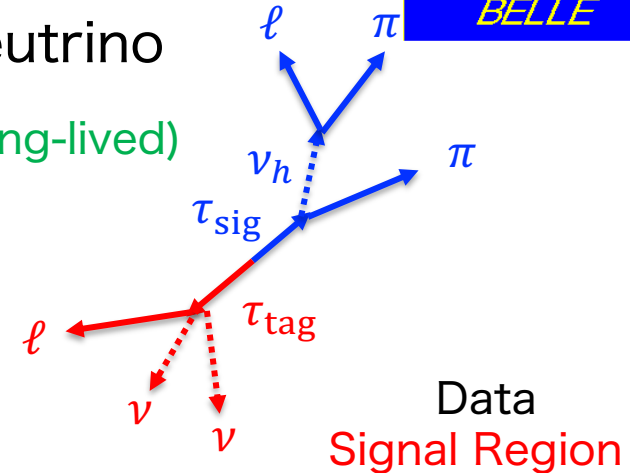
Background: $ee \rightarrow q\bar{q}, \tau\tau, \ell\ell, e\ell\ell$

\rightarrow Suppress the bkg by $M(\pi\pi\ell), \Delta E$ cuts.

$$\Delta E = (E_{\pi\pi\ell}^{\text{CM}} - \sqrt{s}/2)$$



Peak search in $m_{\pi\ell}$ distribution

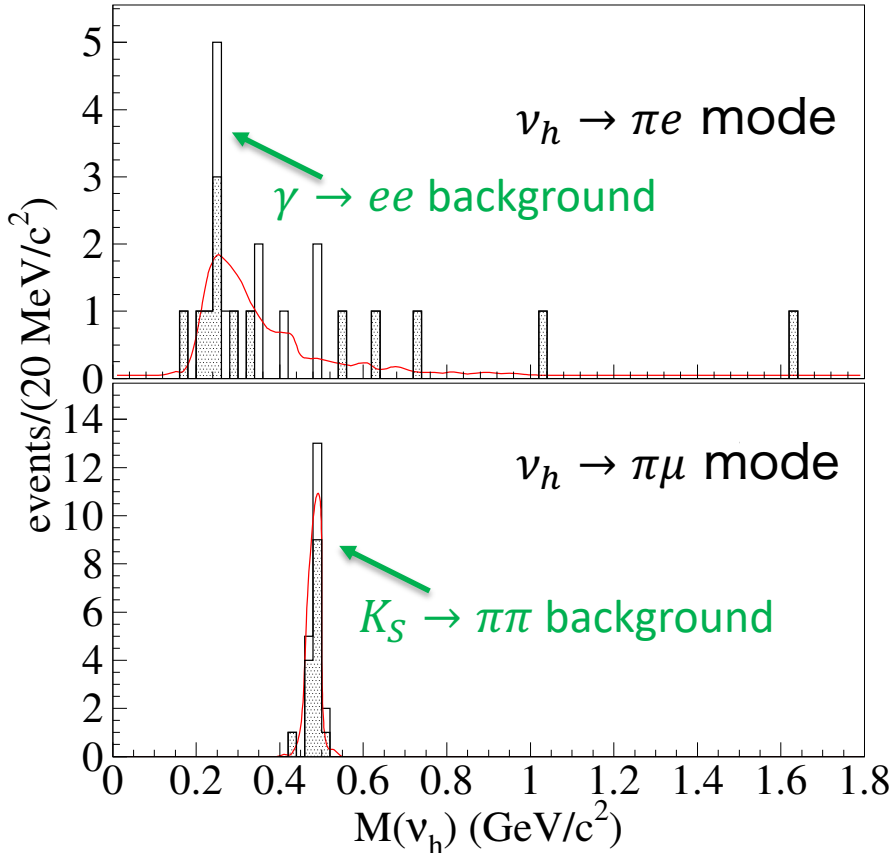


Result: $\nu_h \rightarrow \pi \ell$

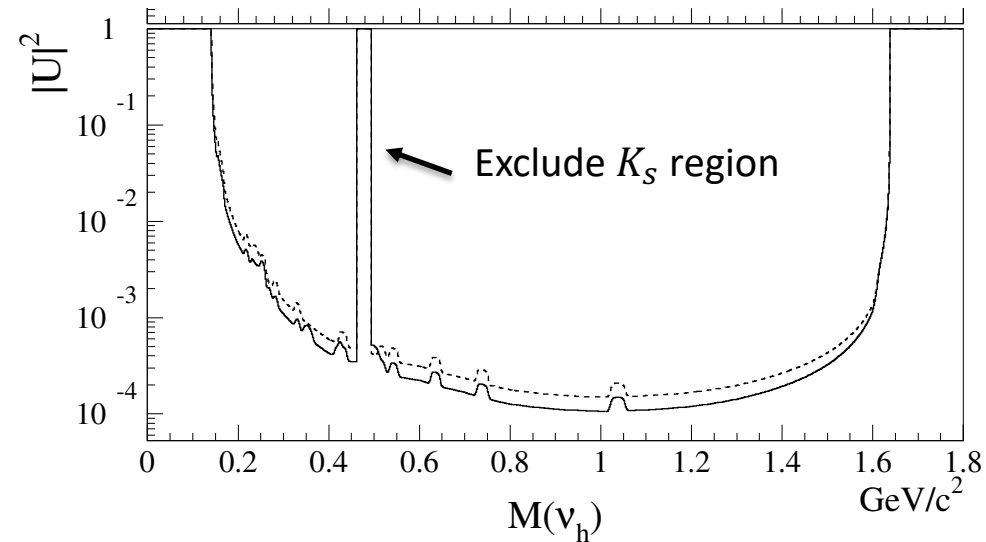
No narrow (signal-like) peak in $M(\nu_h \rightarrow \pi \ell)$ distribution

Data, **Background only fit**

Set ULs at 95% CL on $|U|^2$



$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \quad \alpha = e, \mu, \tau, \dots, i = 1, 2, 3, 4, \dots$$



ULs on the mixing of heavy neutral leptons to the active neutrinos are set for $0.2 < M(\nu_h) < 1.6$ GeV/c²

Belle II: m_τ measurement

Phys.Rev.D108,032006

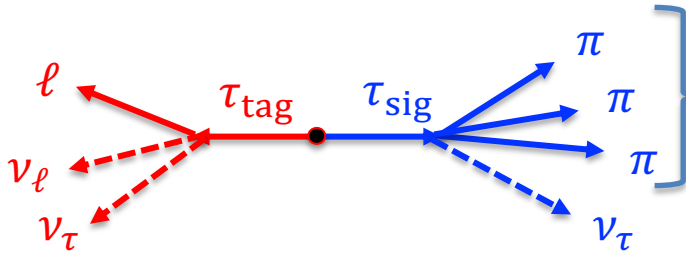


m_τ : one of the fundamental parameters of the SM

- Precise measurement is important for tests of LFU (τ vs e/μ)

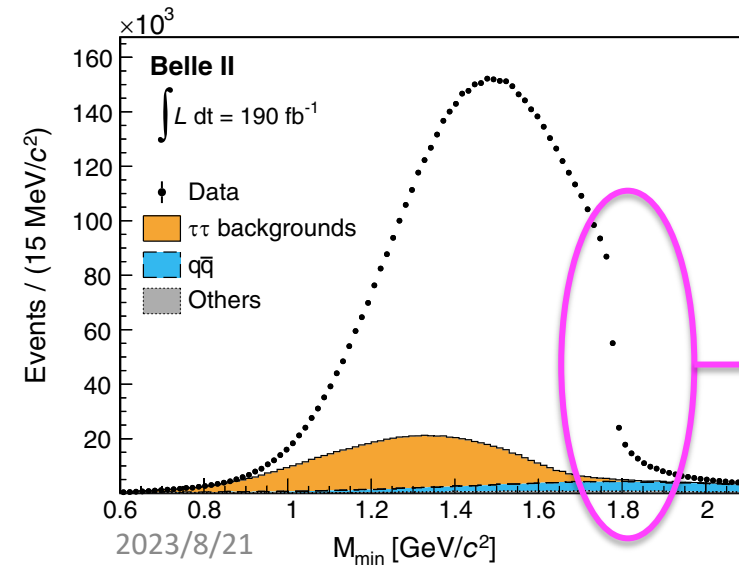
Belle II: Use Pseudomass endpoint (M_{min}) method

→ Use kinematics edge of M_{min} distribution in $\tau \rightarrow 3\pi\nu$ decays

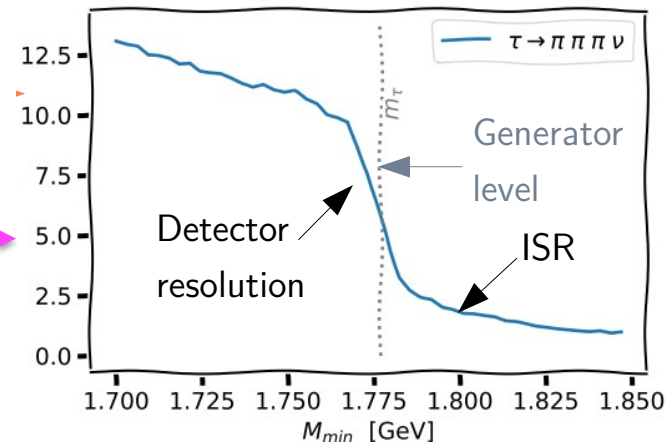


$$M_{min} = \sqrt{m_{3\pi}^2 + 2(\sqrt{s}/2 - E_{3\pi})(E_{3\pi} - |\vec{p}_{3\pi}|)} \leq m_\tau$$

Fit to the endpoint with empirical function



Smearing edge due to detector resolution/ISR

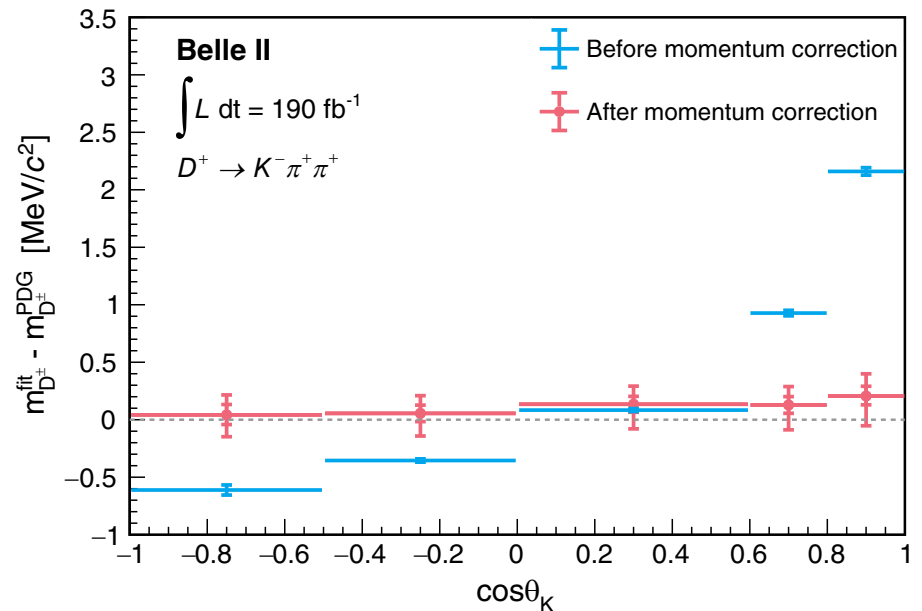
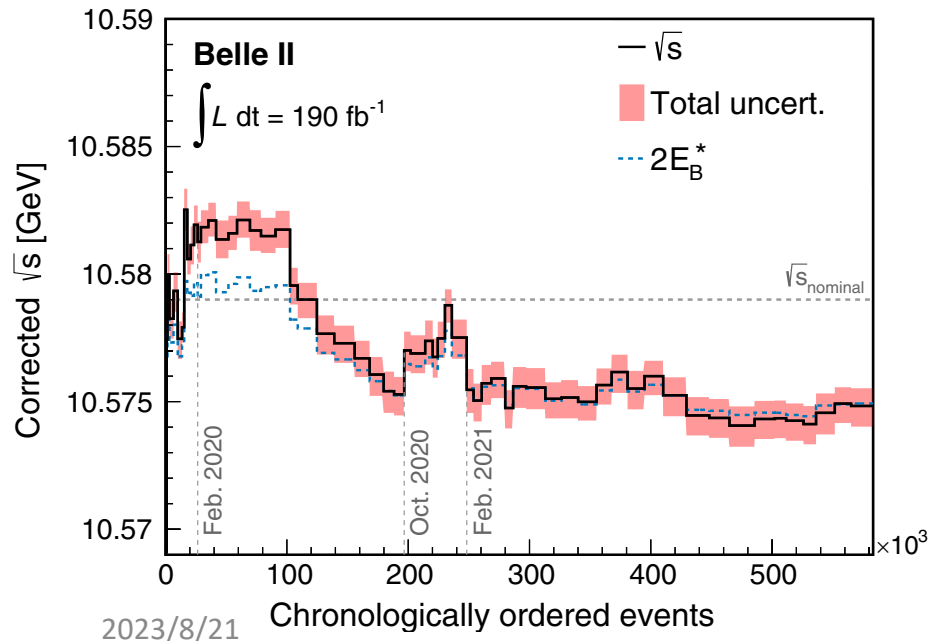


Corrections factors

$$M_{\min} = \sqrt{m_{3\pi}^2 + 2(\sqrt{s} - E_{3\pi})(E_{3\pi} - |\vec{p}_{3\pi}|)}$$

Beam energy calibration and momentum correction are crucial

- Beam energy is corrected using B-meson hadronic decays
- Momentum scale : extract scale factors for K/π using $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$
 - \vec{p} due to imperfect \vec{B} , mismodeling in material \rightarrow bias mass extraction



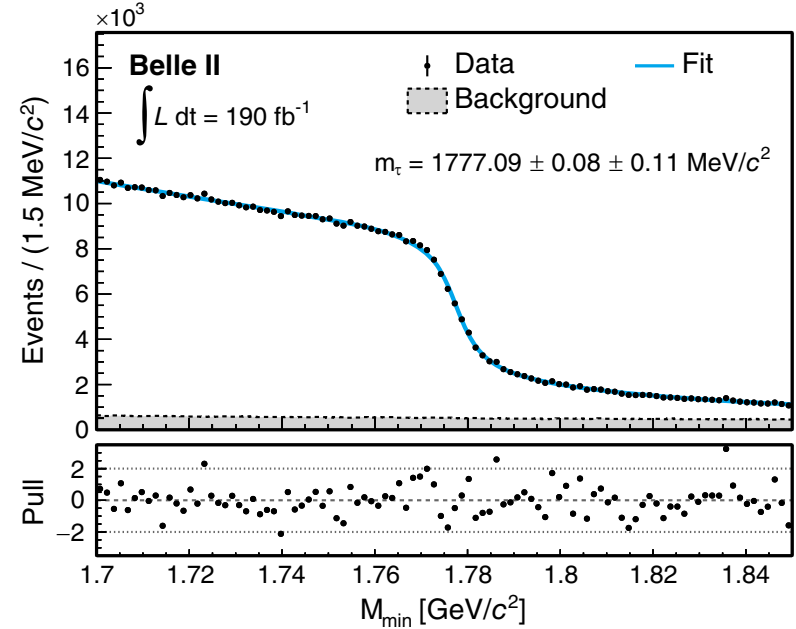
m_τ measurement: Result

Perform an unbinned maximum likelihood fit

Fit function

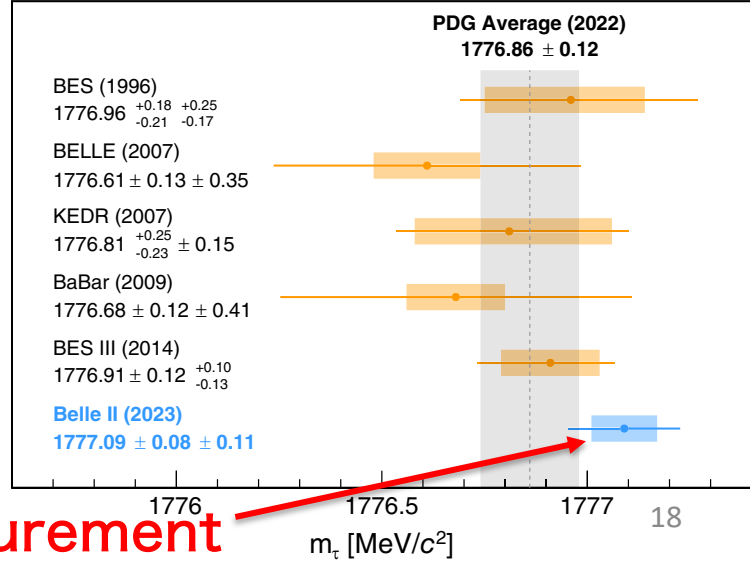
$$F(M_{\min}) = 1 - P_3 \cdot \arctan\left(\frac{M_{\min} - P_1}{P_2}\right) + P_4(M_{\min} - P_1) + P_5(M_{\min} - P_1)^2.$$

$$m_\tau = 1777.09 \pm 0.08 \pm 0.11 \text{ MeV}/c^2$$



Systematic uncertainties

Source	Uncertainty (MeV/c ²)
Knowledge of the colliding beams:	
Beam-energy correction	0.07
Boost vector	< 0.01
Reconstruction of charged particles:	
Charged-particle momentum correction	0.06
Detector misalignment	0.03
Fit model:	
Estimator bias	0.03
Choice of the fit function	0.02
Mass dependence of the bias	< 0.01
Imperfections of the simulation:	
Detector material density	0.03
Modeling of ISR, FSR and τ decay	0.02
Neutral particle reconstruction efficiency	≤ 0.01
Momentum resolution	< 0.01
Tracking efficiency correction	< 0.01
Trigger efficiency	< 0.01
Background processes	< 0.01
Total	0.11



Conclusions and outlook

Belle (II) has an excellent sensitivity for τ physics

- LFV decays, $\tau \rightarrow \ell\alpha, \ell V^0$: most stringent BF limit
- World's most precise measurement of τ mass

Statistical uncertainty is still dominant for τ decay searches, eg. LFV

- Now, 424 fb⁻¹ at Belle II.
 - Expect more results on larger statistics → Stay tuned!

Backup

Recent tau physics result

Summary of Journal/Conference papers in 2022-2023

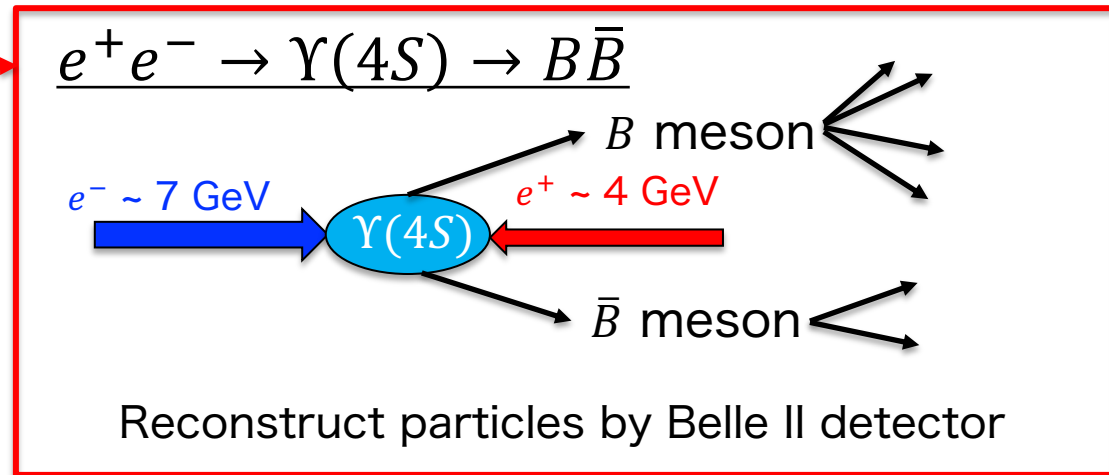
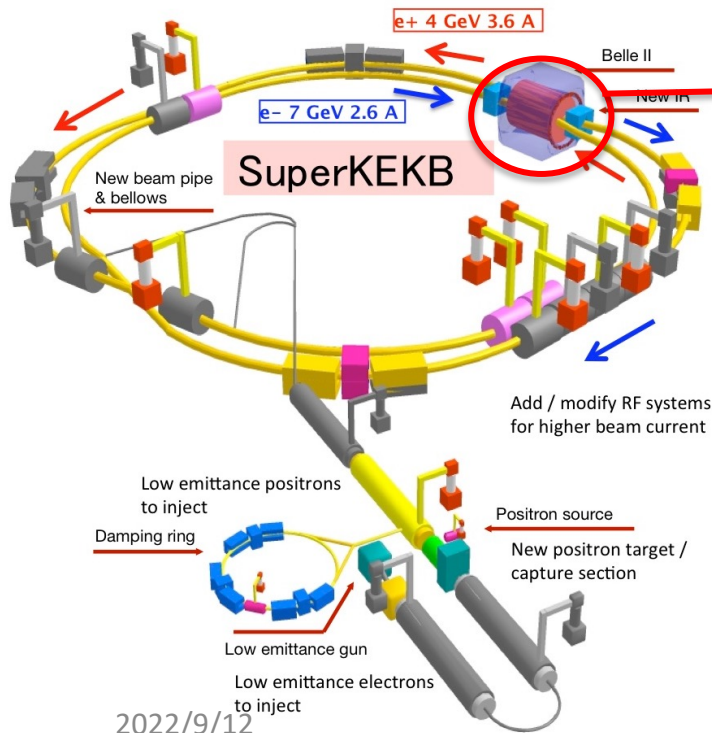
- Belle: 5 papers [[Link](#)]
- Belle II: 3 papers [[Link](#)]

	Short Title	Luminosity	Journal
Belle	Leptophilic search, $ee \rightarrow \tau\tau\phi_L$	626 fb ⁻¹	Submitted to PRL arXiv:2207.07476
	Search for LFV, $\tau \rightarrow \ell V_0$	980 fb ⁻¹	JHEP06(2023)118
	Heavy neutrino search in tau decays	980 fb ⁻¹	Submitted to PRL arXiv:2212.10095
	Michel parameter measurement in $\tau \rightarrow \mu\nu\nu$	980 fb ⁻¹	Phys.Rev.Lett.131,021801
	Search for τ EDM	833 fb ⁻¹	JHEP04(2022)110
Belle II	τ -lepton mass measurement	190 fb ⁻¹	Phys.Rev.D108,032006
	Search for LFV, $\tau \rightarrow \ell\alpha$	62.8 fb ⁻¹	Phys.Rev.Lett.130,181803
	Search for LFV, $\tau \rightarrow \ell\phi$	190 fb ⁻¹	Conference Paper (arXiv:2305.04759)

Belle II experiment

Flavor physics experiment to search for new physics

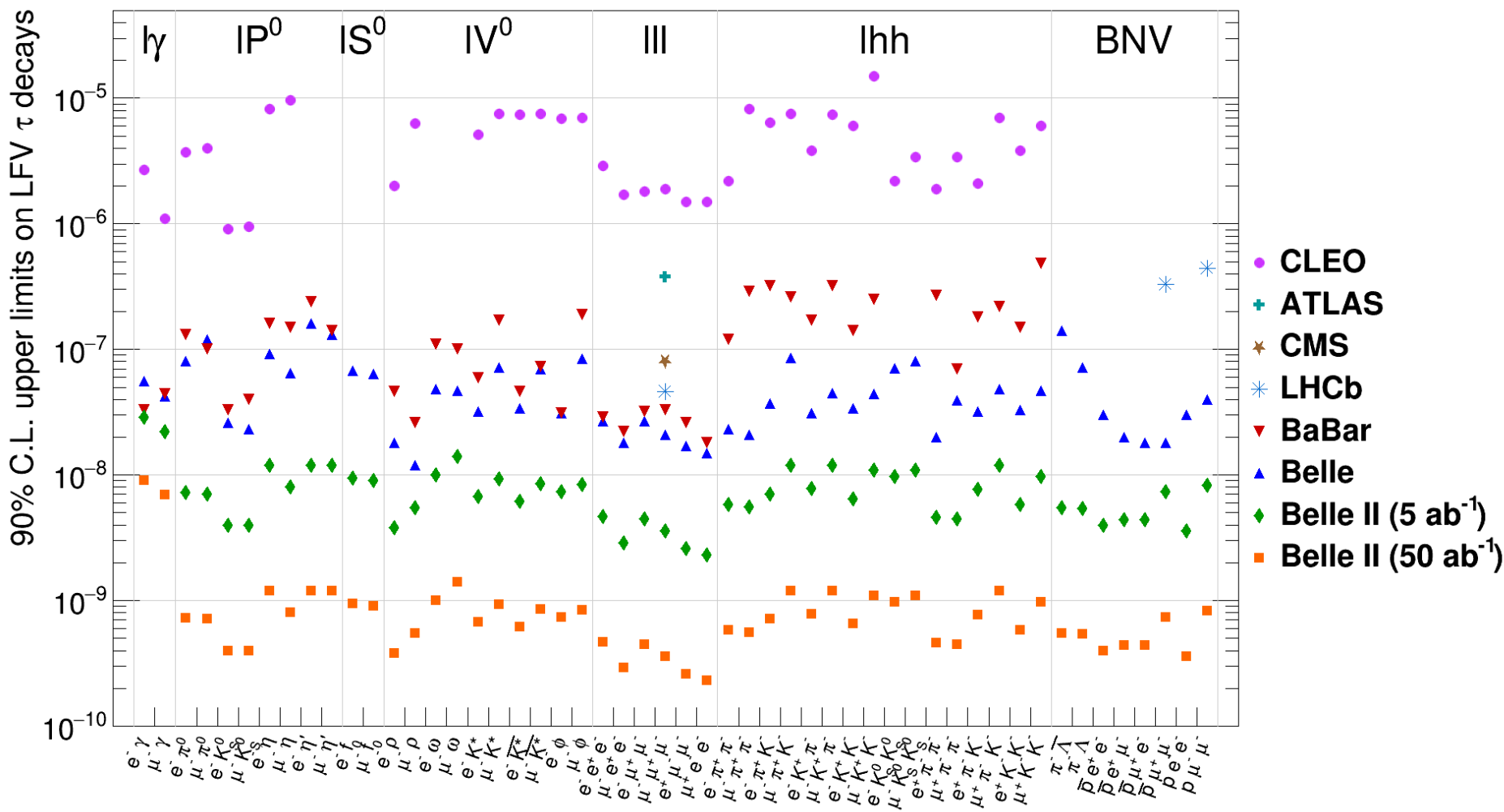
- Asymmetric e^+e^- collider mainly at $\sqrt{s} = 10.58$ GeV
 - Produce B, D, τ , etc..
- Goal: **50 ab^{-1} data** in ~ 10 years
 - $50 \times$ Belle data: $N_{B\bar{B}} \sim 50 \times 10^9$



Belle II operation started in 2019

Summary of LFV in tau decay

[arXiv.2203.14919](https://arxiv.org/abs/2203.14919)

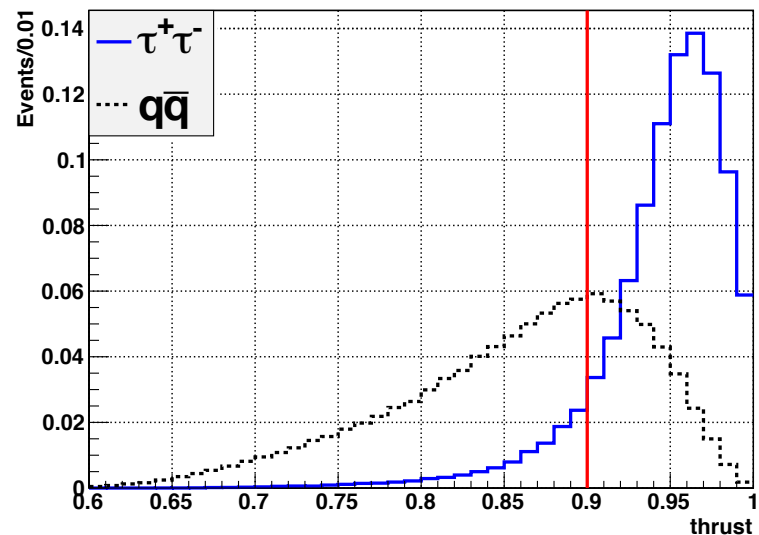
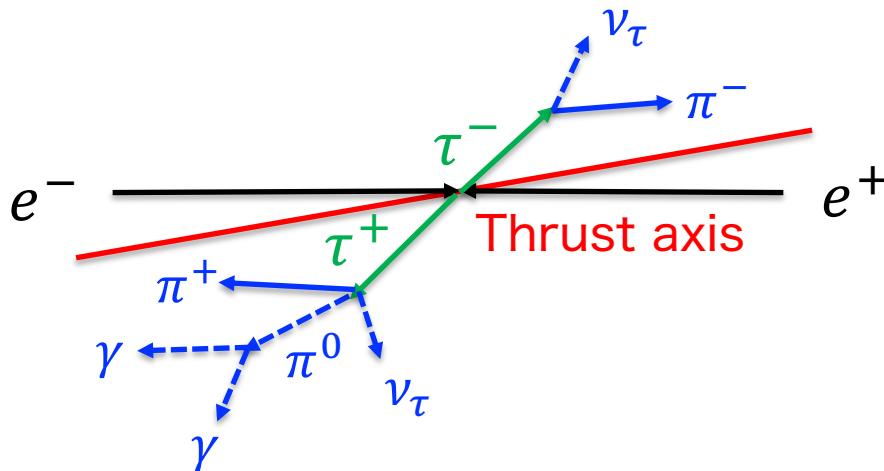


Thrust

- V_{th} is the magnitude of thrust in the event. The thrust axis, \hat{n}_{th} , is defined so that the value V_{th} ,

$$V_{\text{th}} = \sum \frac{|\vec{p}_i^{\text{CM}} \cdot \hat{n}_{\text{th}}|}{\sum \vec{p}_i^{\text{CM}}} \quad (4.3)$$

is maximized. Here, \vec{p}_i^{CM} is the three-momentum of each particle in the CM frame.



$\tau \rightarrow \ell V^0$: Belle

V^0 meson reco.

V^0 meson	ρ^0	ϕ	ω	$K^{*0}(\bar{K}^{*0})$
Decay particles	$\pi^+\pi^-$	K^+K^-	$\pi^+\pi^-\pi^0$	$K^\pm\pi^\mp$
Mass window [GeV/c ²]	0.445 – 1.08	1.00 – 1.04	0.7 – 0.9	0.7 – 1.1

- M_{V^0} , M_ν^2 , $P_\nu^{\text{c.m.}}$, T , P_ℓ^{sig} , $E_{\text{tag}}^{\text{hemi}}$, $\cos\theta_{\text{miss-tag}}^{\text{c.m.}}$
- (categorical variables) τ_{tag} decay mode, collision energy
- (additional for the $\ell\omega$ modes) $P_{\pi^0}^{\text{sig}}$, E_γ^{low} ,

BDT: LightGBM library

Systematic uncertainties

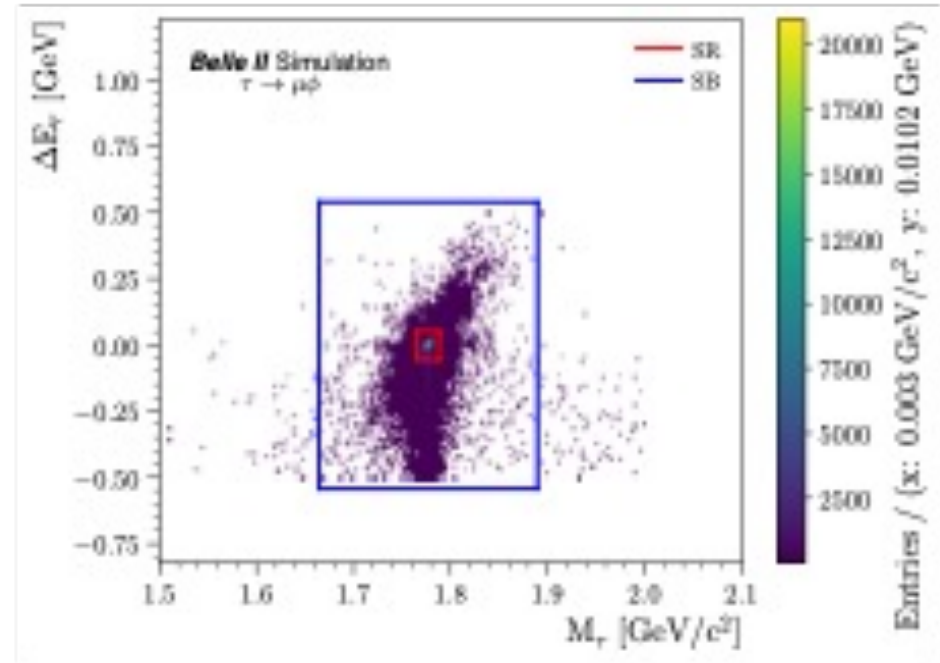
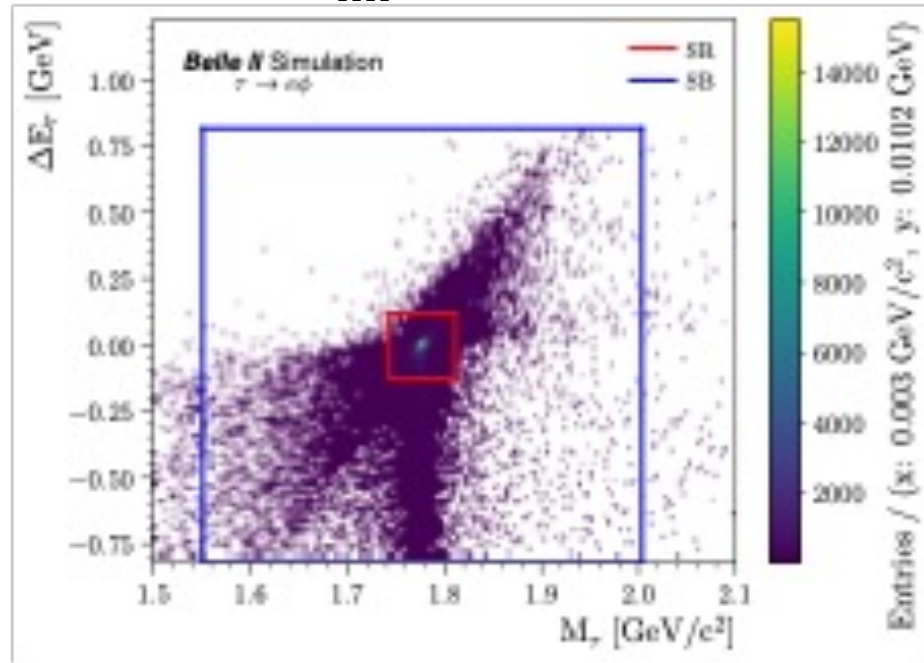
Source	σ_{syst} (%)
Integrated luminosity	1.4
$ee \rightarrow \tau\tau(\gamma)$ cross section [48]	0.3
$\mathcal{B}(\phi \rightarrow K^+K^-)$ and $\mathcal{B}(\omega \rightarrow \pi^+\pi^-\pi^0)$	1.2 and 0.7
Trigger efficiency	0.2–0.9
Tracking efficiency	$0.35 \times N_{\text{track}}$
Electron identification efficiency	$1.7 \times N_{\text{electron}}$
Muon identification efficiency	$1.8 \times N_{\text{muon}}$
K^\pm and π^\pm identification efficiency	1.6 (ρ^0), 1.8 (ϕ) and 1.1 (K^{*0} and \bar{K}^{*0})
π^0 efficiency	$2.2 \times N_{\pi^0}$
Electron veto for hadrons	0.4–1.2
MC statistics	0.3–0.5
Track energy resolution	0.3–1.3
Photon energy resolution	0.0–0.4

$\tau \rightarrow \ell V^0$: Belle II

Signal: $\tau \rightarrow \ell \phi (\rightarrow KK)$

- $1.014 < m_{KK} < 1.024 \text{ GeV}/c^2$

Bkg: $ee \rightarrow q\bar{q}, \tau \rightarrow 3\pi\nu$



BDT: XGBoost library

- Event-shape
- Kinematic properties of τ_{sig}, ϕ
- Variables related to the ROE

Affected quantity	Source	Mode	
		$e\phi$	$\mu\phi$
$\epsilon_{\ell\phi}$	Particle identification	0.8%	0.3%
	Tracking efficiency		0.9%
	Trigger efficiency	0.4%	0.9%
	Signal variable mismodeling	15.2%	8.5%
N_{exp}	Momentum scale	0.6%	0.4%
L	Luminosity	0.6%	
$\sigma_{\tau\tau}$	Tau-pair cross section	0.3%	

Heavy neutrino

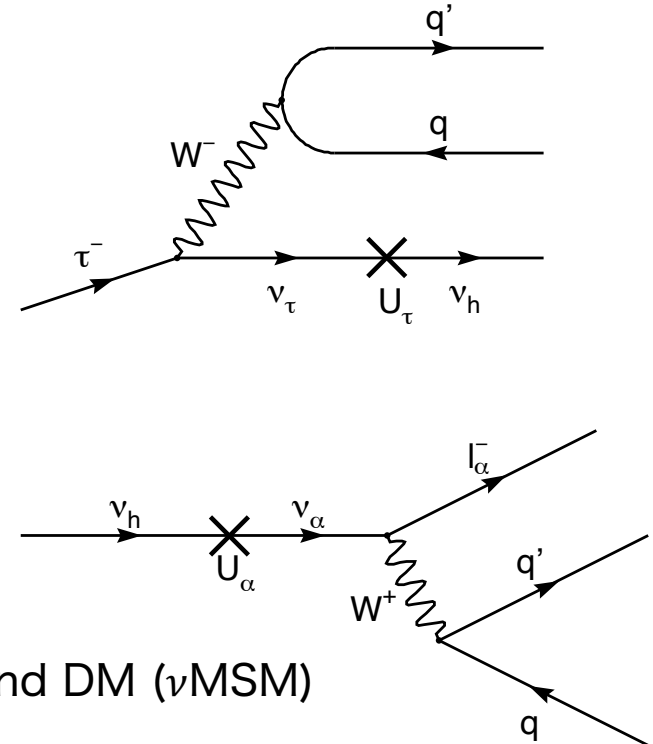
Right handed neutrino (eg. Heavy Neutral Lepton, HNL)

- No strong interaction (it is lepton)
- No weak interaction (it is right-handed)
- No electromagnetic interaction (it is neutral)

→ The only way to interact is to mix with left-handed neutrino:

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \quad \alpha = e, \mu, \tau, \dots, \quad i = 1, 2, 3, 4, \dots$$

α : flavor eigenstates, i : mass eigenstates



May also contribute to explanation of baryogenesis and DM (ν MSM)

#neutrinos in this method

Particle with a mass m and width Γ has a momentum p , the probability that it travels distance ℓ or greater is

$$P(l) = \exp\left(-\frac{m\Gamma l}{p}\right), \quad \longrightarrow \quad dP(l) = \frac{m\Gamma}{p} \exp\left(-\frac{m\Gamma l}{p}\right) dl.$$

Estimate $c\tau \sim |U|^{-2} m(\nu_h)^{-5}$

The number of neutrinos detected in the Belle detector is

$$\begin{aligned} n(\nu_h) &= \underline{N_0} \int \varepsilon(m, l) dP(l) \\ &= \underline{2N_{\tau\tau} \mathcal{B}(\tau \rightarrow \pi\nu_h) \mathcal{B}(\nu_h \rightarrow \pi\ell)} \frac{m\Gamma}{p} \int \exp\left(-\frac{m\Gamma l}{p}\right) \varepsilon(m, l) dl \\ &= |U_\tau|^2 |U_\ell|^2 2N_{\tau\tau} f_1(m) f_2(m) \frac{m}{p} \int \exp\left(-\frac{m\Gamma l}{p}\right) \varepsilon(m, l) dl, \end{aligned}$$

$|U_\tau|^2, |U_\ell|^2$ coupling come from $B(\tau \rightarrow \pi\nu_h), B(\nu_h \rightarrow \pi\ell)\Gamma$

To factor out the $|U_\ell|^2$ dependence, we define functions $f_{1,2}(m)$ as

$$|U_\tau|^2 f_1(m) = B(\tau \rightarrow \pi\nu_h) \text{ and } |U_\ell|^2 f_2(m) = \Gamma(\nu_h \rightarrow \pi\ell) = \Gamma B(\tau \rightarrow \pi\nu_h)$$

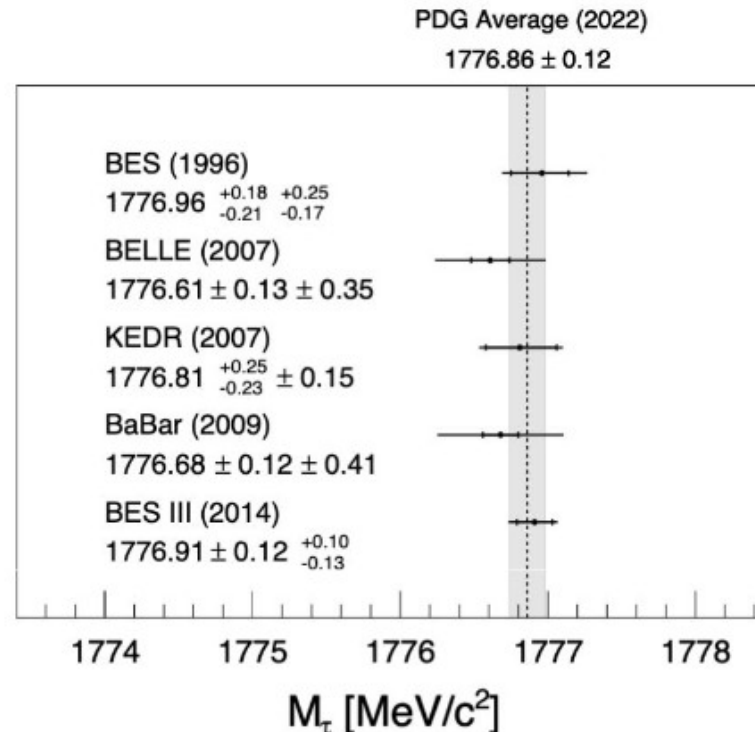
Relative mixing coefficients are different b.t.w normal and inverted hierarchy

$$x_\alpha = |U_\alpha|^2 / |U|^2 \quad (\alpha = e, \mu, \tau) \rightarrow x_e = 0.06, x_\mu = 0.48, x_\tau = 0.46 \text{ from oscillation data}$$

τ mass measurement

Two possible methods to measure the mass

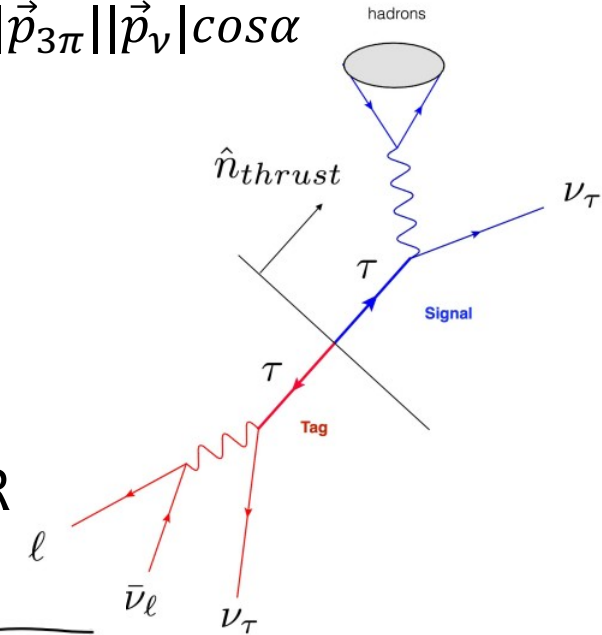
- Pair-production cross section scan used by BESIII
 - Analyze the $ee \rightarrow \tau\tau$ cross-section near the $\tau\tau$ production threshold
- Pseudomass method at Belle, BaBar
 - Exploit the kinematics of the three pions in $\tau \rightarrow 3\pi\nu$ decays



τ mass: Pseudomass method

Reconstruct $\tau_{\text{sig}} \rightarrow 3\pi\nu$ and $\tau_{\text{tag}} \rightarrow \ell\nu\nu$

- $m_\tau^2 = (p_{3\pi} + p_\nu)^2 = m_{3\pi}^2 + m_\nu^2 + 2E_{3\pi}(E_\tau - E_{3\pi}) - 2|\vec{p}_{3\pi}||\vec{p}_\nu|\cos\alpha$
- $= m_{3\pi}^2 + 2(\sqrt{s}/2 - E_{3\pi})(E_{3\pi} - |\vec{p}_{3\pi}|\cos\alpha)$
- $m_{\text{min}} = \sqrt{m_{3\pi}^2 + 2(\sqrt{s}/2 - E_{3\pi})(E_{3\pi} - |\vec{p}_{3\pi}|)} \leq m_\tau$



Fit to the end point with an empirical function

- Smearing edge due to detector resolution/ISR

