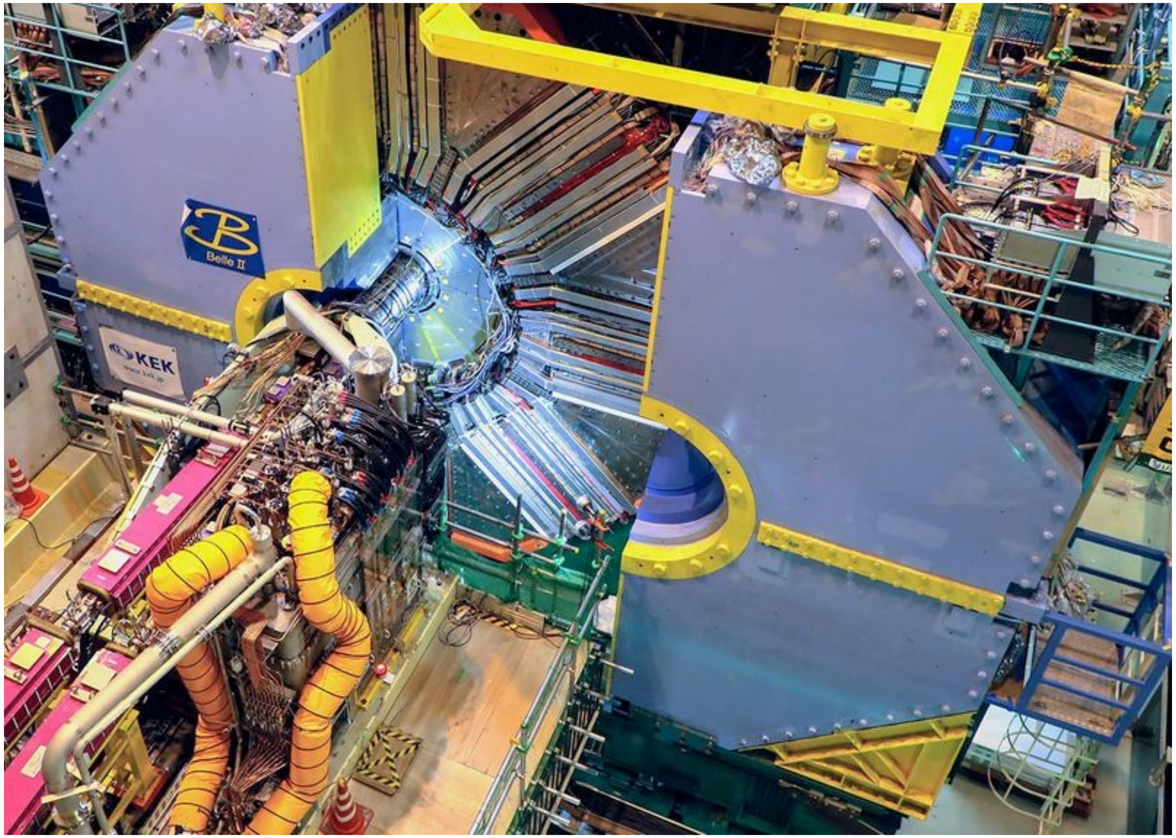
Charm lifetimes and prospects for semileptonic decays at Belle II

Alan Schwartz
University of Cincinnati, USA
(on behalf of Belle/Belle II)

12th International Workshop on the CKM Unitarity Triangle
Santiago de Compostela, Spain
19 September 2023



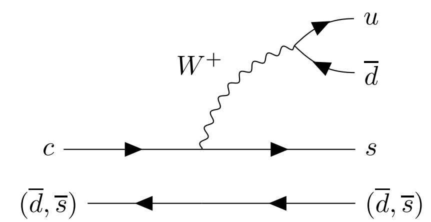
- why measure charm lifetimes?
- measurements
  - mesons: D^0, D^+, D_s^+
  - baryons: Λ_c^+, Ω_c^0
- comparison with theory
- why measure leptonic/semileptonic decays?
- prospects for Belle II

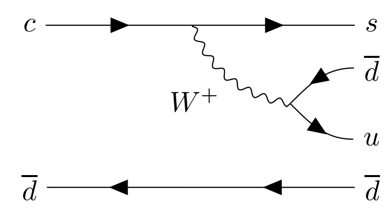
Why measure charm lifetimes?

Lenz, IJMP A30 (2015)
Lenz et al., JHEP 12 (2020) 199
King, Lenz et al., JHEP 08 (2022) 241
Gratrex et al., JHEP 07 (2022) 058

Theory:

- ***qualitatively*** *understood in terms of simple diagrams, e.g., $c \to s\, e^+ \nu$ partial width gives $G_F^2 m_c^5\, |V_{cs}|^2/(192\pi^3)$ dependence. Long D^+ lifetime can be understood as arising from destructive interference between spectator and color-suppressed amplitudes. But this doesn't include QCD…*





- ***to include QCD:*** *calculate using the Heavy Quark Expansion*

$$\Gamma(D) = \frac{1}{2m_D}\sum_X \int_{PS} (2\pi)^4 \delta^{(4)}(p_D - p_X)\ |\langle X(p_X)|\mathcal{H}_{\rm eff}|D(p_D)\rangle|^2,$$

ΣX is sum over final states

$$\to \frac{1}{2m_D}{\rm Im}\langle D|\mathcal{T}|D\rangle \quad \text{where} \quad \mathcal{T} = i\int d^4x\, T\left\{\mathcal{H}_{\rm eff}(x)\,,\mathcal{H}_{\rm eff}(0)\right\}$$

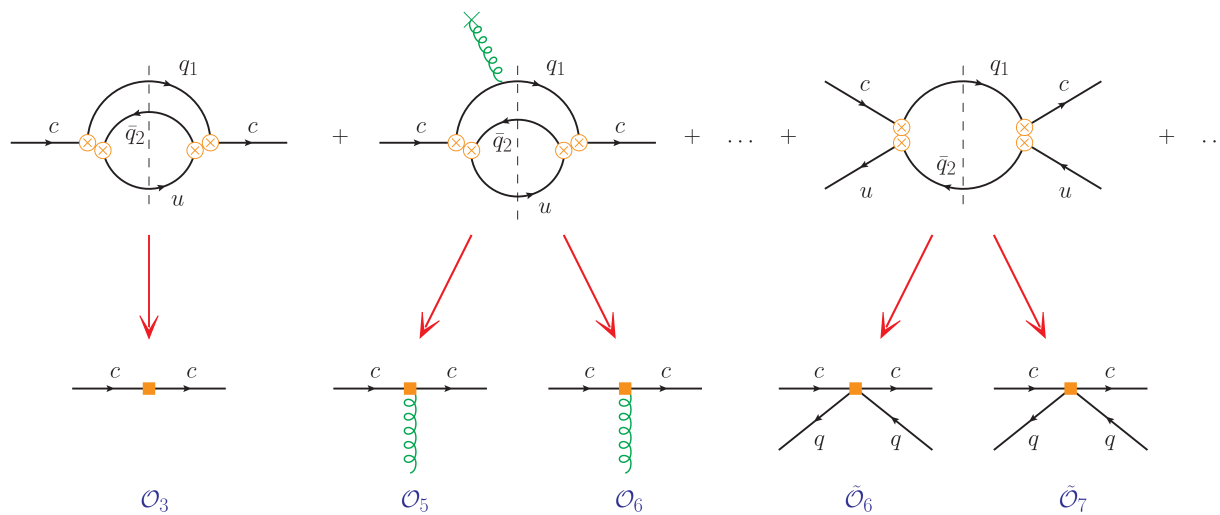
via optical theorem

$$\to \Gamma_3 + \Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_c^2} + \Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_c^3} + \ldots + 16\pi^2\left(\tilde{\Gamma}_6\frac{\langle\tilde{\mathcal{O}}_6\rangle}{m_c^3} + \tilde{\Gamma}_7\frac{\langle\tilde{\mathcal{O}}_7\rangle}{m_c^4} + \ldots\right)$$

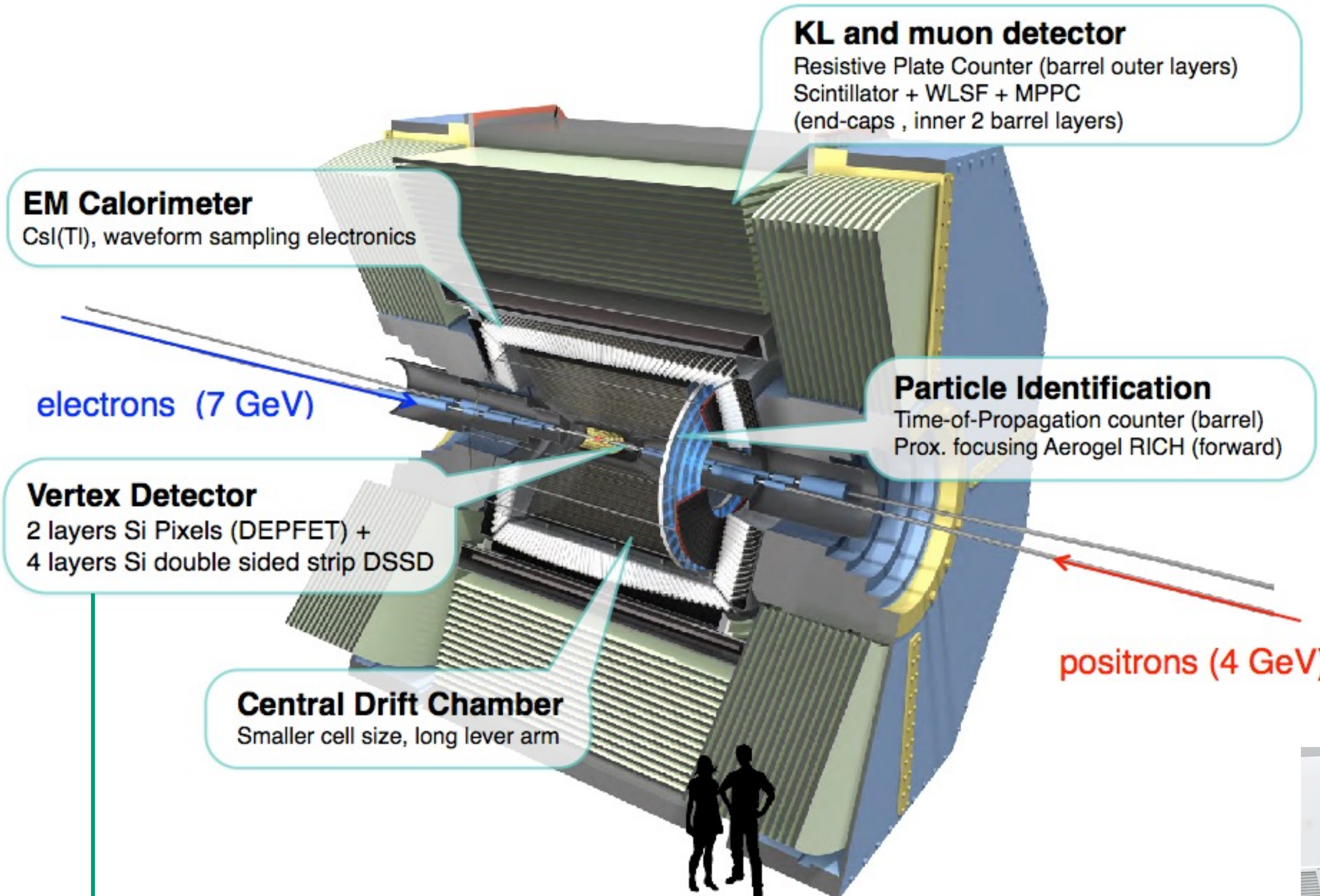
via Heavy Quark Expansion

Wilson coefficients Γ_i are expanded in powers of α_s and calculated perturbatively

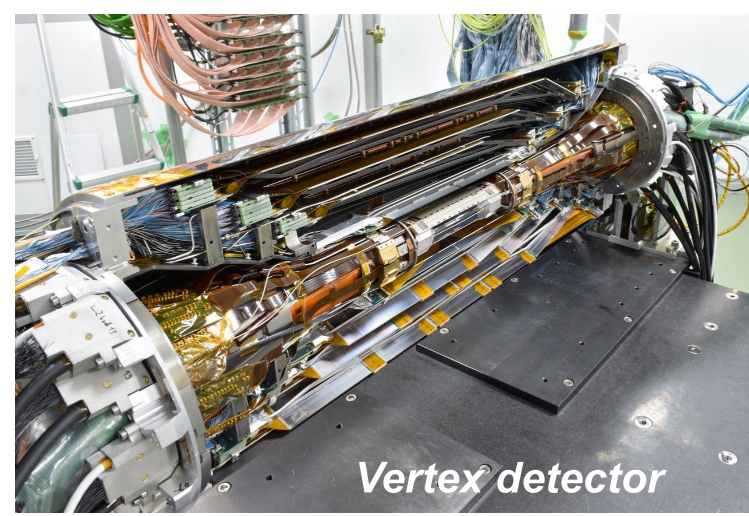
⇒ comparing lifetime calculations with measurements tests/improves our understanding of QCD



The Belle II Experiment



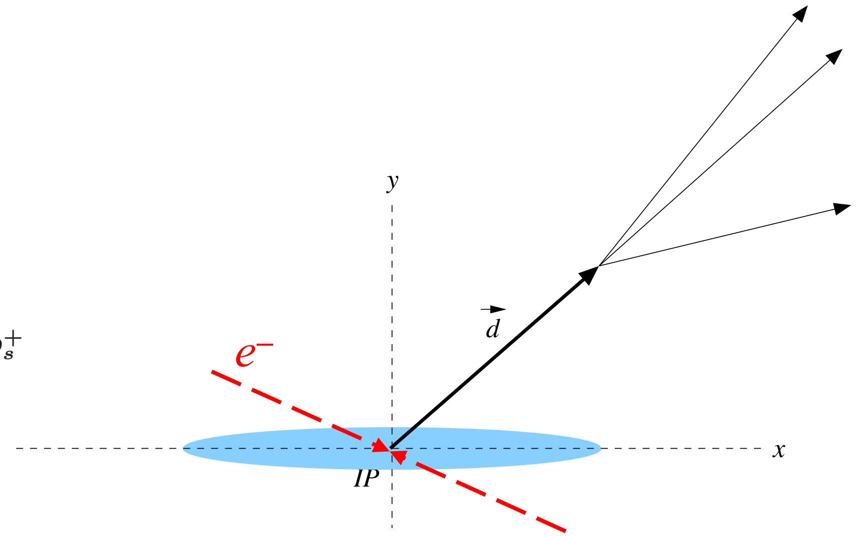
DEPFET pixels:	*Layer 1*	*r = 14 mm*
	Layer 2	*r = 22 mm (1/6 installed)*
double-sided strips:	*Layer 3*	*r = 39 mm*
	Layer 4	*r = 80 mm*
	Layer 5	*r = 104 mm*
	Layer 6	*r = 135 mm*

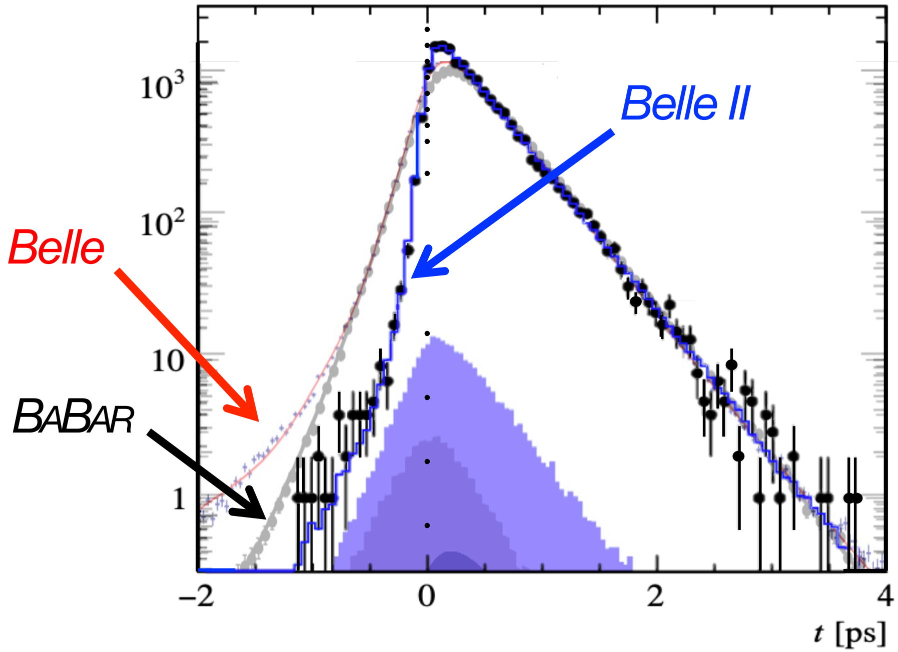


Charm lifetimes: measurement @ Belle II

Determine lifetime by measuring vertex displacement and momentum:

$$t = \left(\frac{\vec{d}\cdot\vec{p}}{p^2}\right) m_{D_s^+}$$



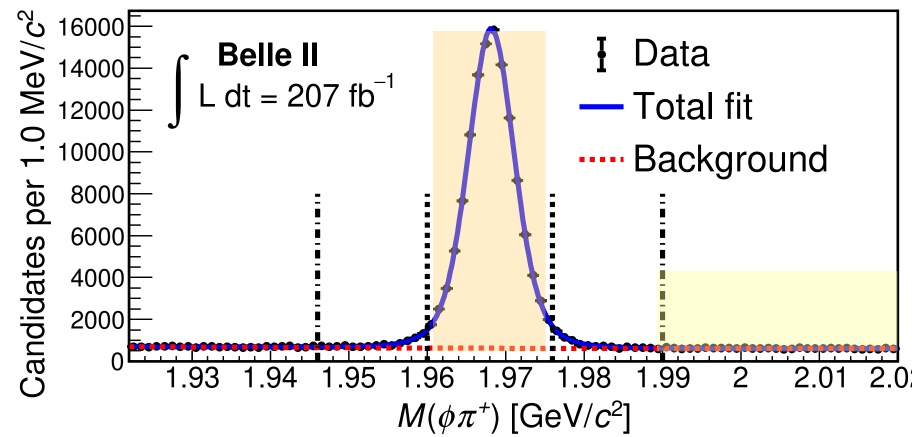


- IP is measured every 30 minutes using $e^+e^- \to \mu^+\mu^-$ events
- Uncertainty on t (σ_t) is calculated event-by-event by propagating uncertainties δd_x, δd_y, δd_z, δp_x, δp_y, δp_z and their correlations.
- The uncertainty σ_t is used as the width of a Gaussian resolution function used to fit the t distribution
- decay time resolution is > 2 times better than Belle/Babar: 80-90 fs vs. 200 fs

D_s^+ lifetime (207 fb^{-1})

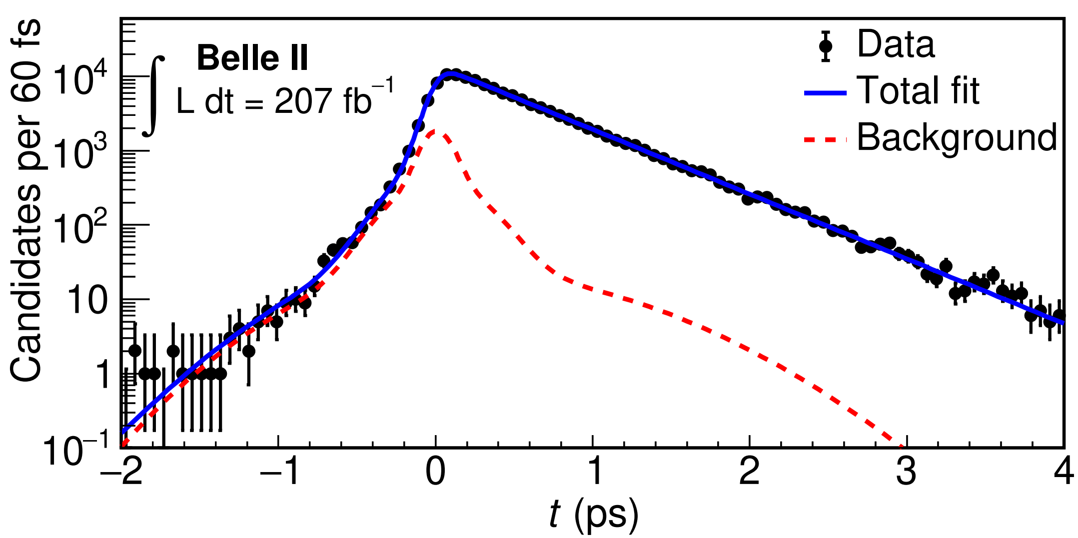
arXiv:2306.00365, submitted to PRL

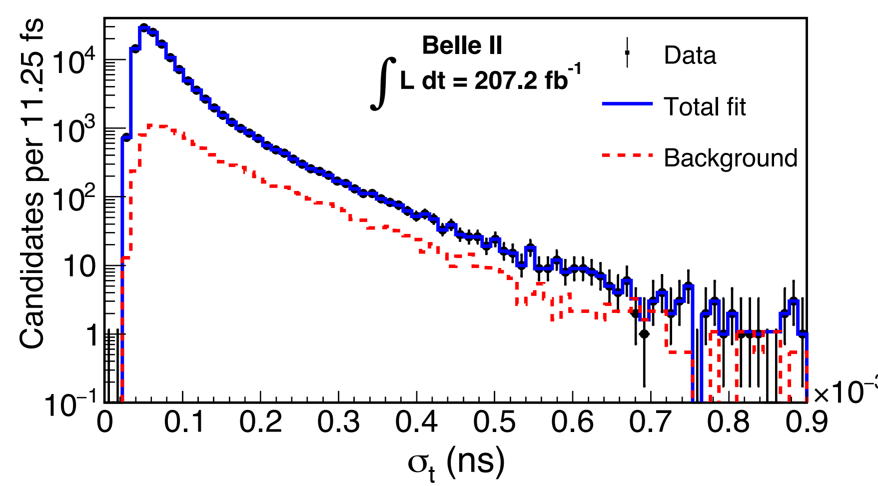
- *Select $D_s^+ \to \phi\,\pi^+$ ($\phi \to K^+K^-$) (low background)*
- *$p_{CM}(D_s^+)$ > 2.5 GeV/c to eliminate $\bar{B} \to D_s^+X$ decays (preserves 2/3 of $e^+e^- \to cc$ events)*
- *require $M(\phi\pi^+) \in [1.960, 1.976]$ GeV/c^2; unbinned ML fit give 116k signal, 92% purity. Background from random combinations of ϕ and π^+*
- *lifetime determined from unbinned ML fit to t. Likelihood function for event i:*



$$\mathcal{L}(\tau|t^i,\sigma_t^i) = f_{\rm sig}\, P_{\rm sig}(t^i|\tau,\sigma_t^i)\, P_{\rm sig}(\sigma_t^i) + (1-f_{\rm sig})\, P_{\rm bkg}(t^i|\tau,\sigma_t^i)\, P_{\rm bkg}(\sigma_t^i)$$

(to avoid bias: Punzi, arXiv:physics/0401045)





D_s^+ lifetime (207 fb^{-1})

arXiv:2306.00365, to appear in PRL

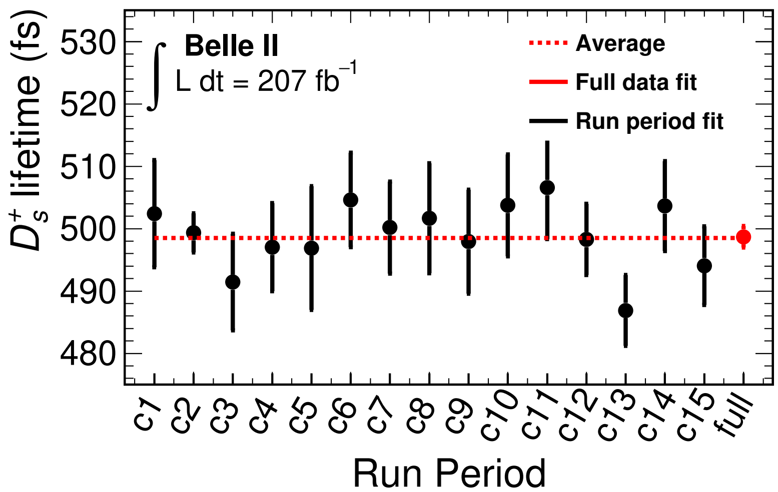
- PDF for signal D_s^+ decays:

$$P_{\rm sig}(t^i|\tau,\sigma_t^i) \;=\; \frac{1}{\tau}\int e^{-t'/\tau}\,R(t^i - t';\mu,s,\sigma_t^i)\,dt'$$

- resolution function R is a single Gaussian with mean μ and per-candidate standard deviation $s \times \sigma_t^i$; μ and scaling parameter s are floated

- PDF for background is taken from fitting $M(\phi\pi^+)$ upper sideband [1.990,2.020] GeV/c^2

- Result: $\tau_{D_s^+} \;=\; (499.5 \pm 1.7 \pm 0.9)$ fs



- Systematic uncertainties:

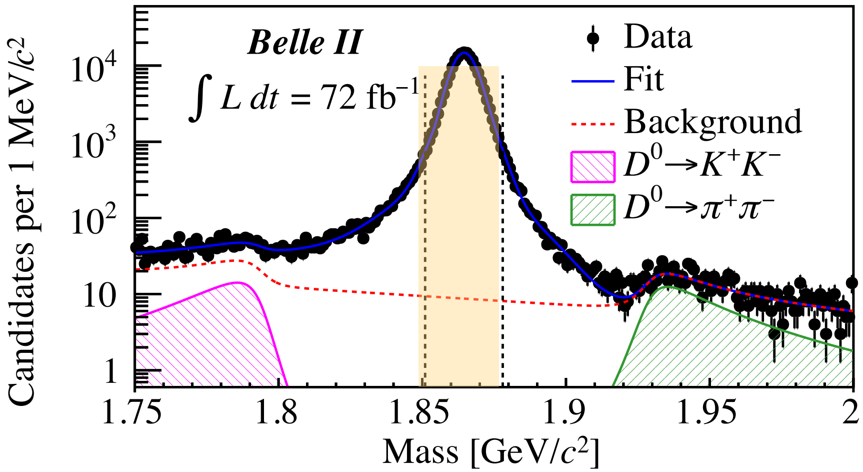
Source	Uncertainty (fs)
Resolution function	$\pm$0.42
Background (t, σ_t) distribution	$\pm$0.40
Binning of σ_t histogram PDF	$\pm$0.10
Imperfect detector alignment	$\pm$0.56
Sample purity	$\pm$0.09
Momentum scale factor	$\pm$0.28
D_s^+ mass	$\pm$0.02
Total	$\pm$0.87

D^0 and D^+ lifetimes (72 fb^{-1})

Abudinen et al., PRL 127, 211801 (2021)
[arXiv:2108.03216]

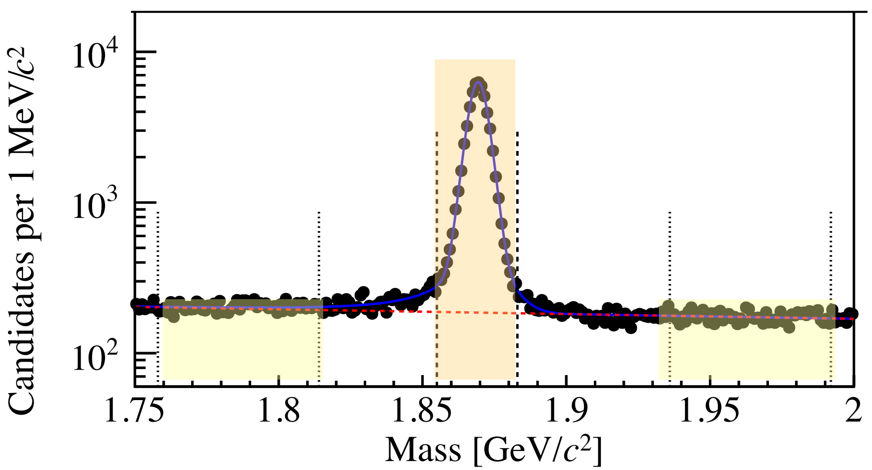
- Select $D^{*+} \to D^0 \pi_s^+$ ($D^0 \to K^- \pi^+$) decays (~no background)
- $p_{CM}(D^{*+})$ > 2.5 GeV/c to eliminate $B \to D^{*+}X$ decays
- require $M(K^-\pi^+) \in [1.851, 1.878]$ GeV/c^2 and $M(K^-\pi^+\pi_s^+) - M(K^-\pi^+) \in [144.94, 145.90]$ MeV/c^2; binned χ^2 fit give 171k signal, 99.8% purity

171k $D^0 \to K^- \pi^+$



- Select $D^{*+} \to D^+ \pi^0$ ($D^+ \to K^- \pi^+ \pi^+$) decays (low background), where $\pi^0 \to \gamma\gamma$ and $m(\gamma\gamma) \in [120, 145]$ MeV/c^2
- $p_{CM}(D^{*+})$ > 2.6 GeV/c to eliminate $B \to D^{*+}X$ decays
- require $M(K^-\pi^+) \in [1.855, 1.883]$ GeV/c^2 and $\Delta M \in [138, 143]$ MeV/c^2; binned χ^2 fit give 59k signal, 91% purity

59k $D^+ \to K^- \pi^+ \pi^+$



D^0 and D^+ lifetimes ($72\ fb^{-1}$)

Abudinen et al., PRL 127, 211801 (2021)
[arXiv:2108.03216]

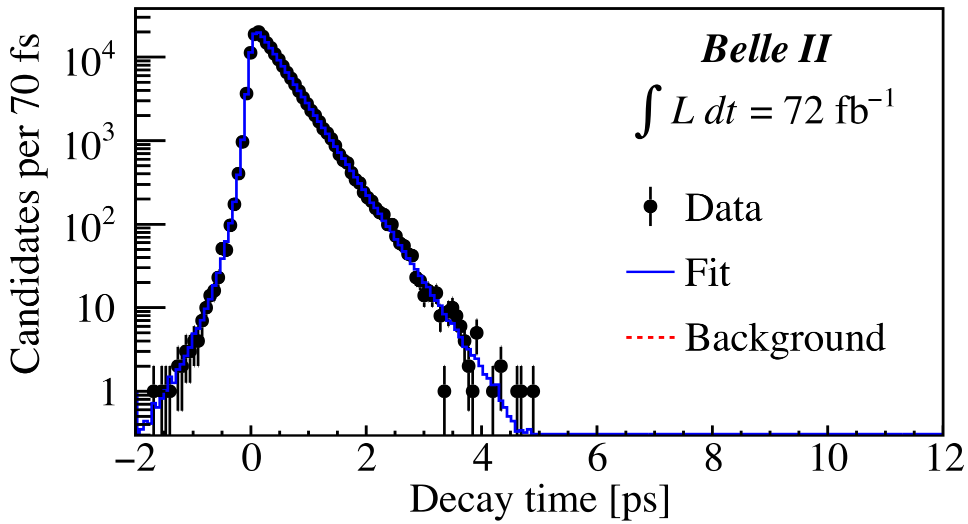
- lifetime determined from unbinned ML fit to (t, σ_t)
- resolution function R is a double Gaussian for D^0 (single Gaussian for D^+) with mean μ and per-candidate standard deviation s x σ_t^i ; μ and scaling parameter s are floated
- PDF for D^+ background is taken from fitting $M(K^-\pi^+\pi^+)$ sidebands [1.758,1.814] and [1.936,1.992] GeV/c^2 . D^0 background is neglected, with a systematic included
- Results:

$$\tau_{D^0} = (410.5 \pm 1.1 \pm 0.8)\ \text{fs}$$
$$\tau_{D^+} = (1030.4 \pm 4.7 \pm 3.1)\ \text{fs}$$

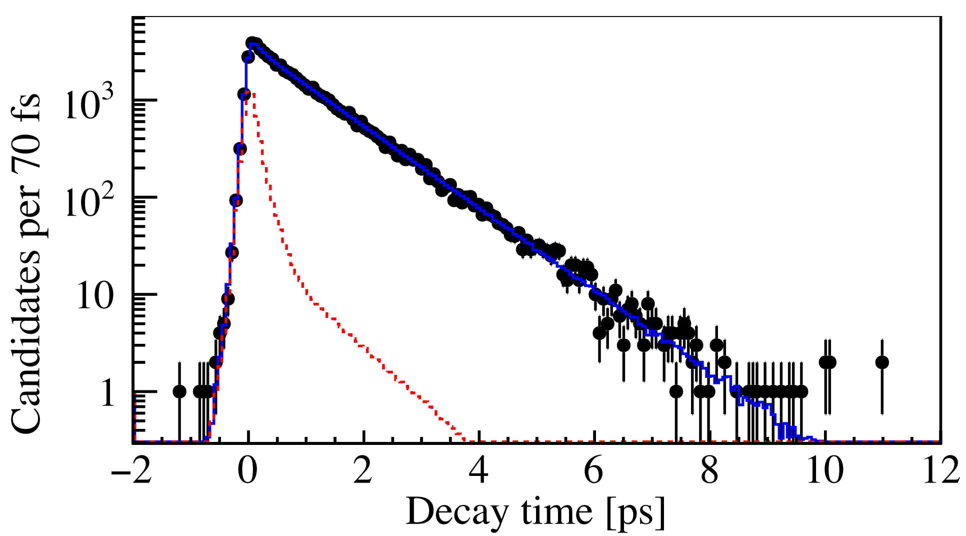
- Systematic uncertainties:

Source	$\tau(D^0)$ (fs)	$\tau(D^+)$ (fs)
Resolution model	0.16	0.39
Backgrounds	0.24	2.52
Detector alignment	0.72	1.70
Momentum scale	0.19	0.48
Total	0.80	3.10

$D^0 \to K^-\pi^+$



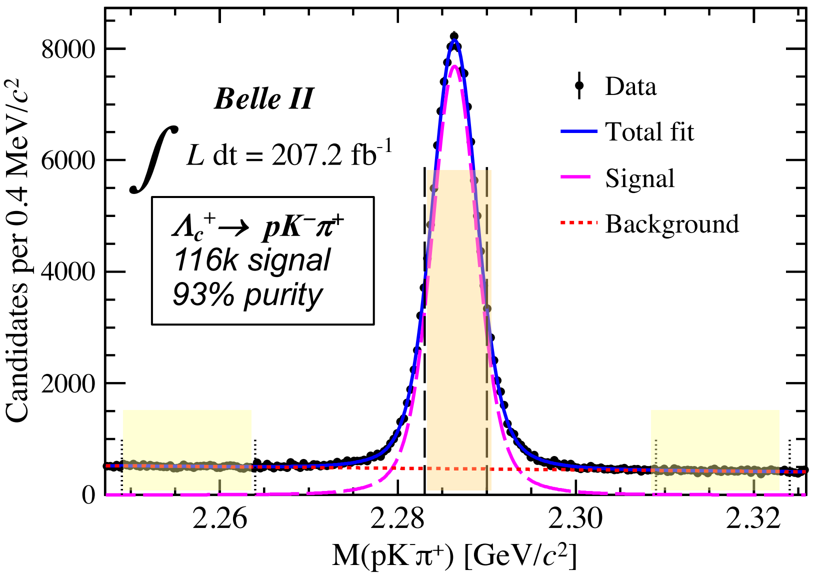
$D^+ \to K^-\pi^+\pi^+$



Λ_c^+ lifetime (207 fb^{-1})

Abudinen et al., PRL 130, 071802 (2023) [arXiv:2206.15227]

- problematic background from $\Xi_c^0 \to \Lambda_c^+\pi^-$, $\Xi_c^+ \to \Lambda_c^+\pi^0$ decays: $\tau(\Xi_c^0)$ = 153 fs, $\tau(\Xi_c^+)$ = 456 fs.
  - Ξ contamination in Λ_c^+ sample is estimated by fitting distribution of Λ_c^+ vertex displacement in plane transverse to the beam. Result: 374 events (0.003% of Λ_c^+ candidates).
  - To reduce, impose vetos:
    $M(pK^-\pi^+\pi^-) - M(pK^-\pi^+) \notin [183.4, 186.4]$ MeV/c^2
    $M(pK^-\pi^+\pi^0) - M(pK^-\pi^+) \notin [175.3, 187.3]$ MeV/c^2
    This reduces Ξ decays by 40%.
  - Effect of remaining decays is estimated via MC simulation; bias of 0.34 fs is subtracted from fitted $\tau(\Lambda_c^+)$

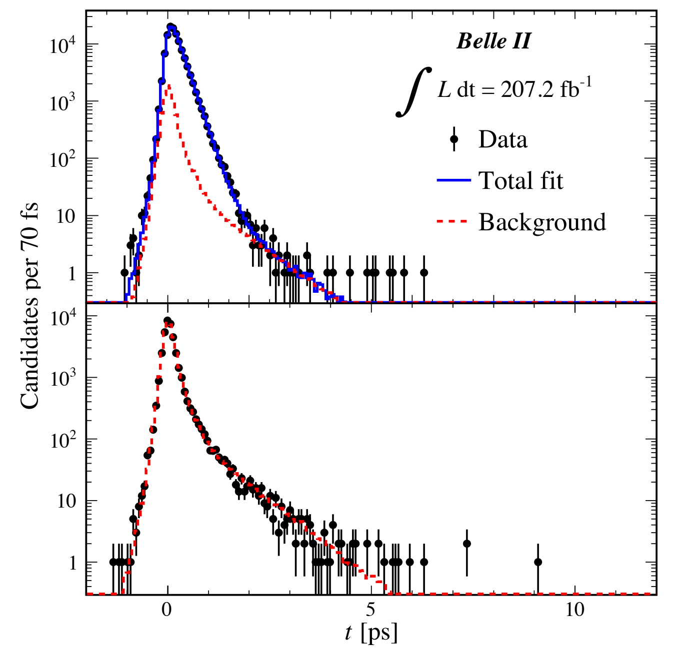


- Result:

$$\tau_{\Lambda_c^+} = (203.20 \pm 0.89 \pm 0.77)\ \text{fs}$$

- Systematic uncertainties:

Source	Uncertainty [fs]
Ξ_c contamination	0.34
Resolution model	0.46
Non-Ξ_c backgrounds	0.20
Detector alignment	0.46
Momentum scale	0.09
Total	0.77



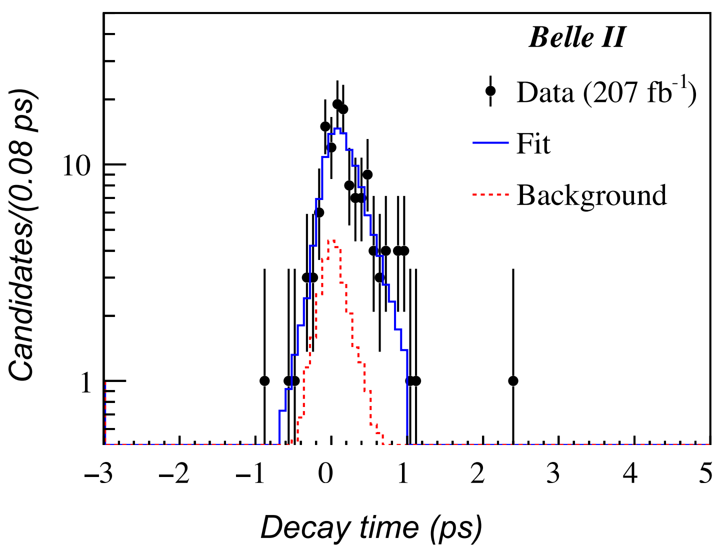
Ω_c^0 lifetime (207 fb^{-1})

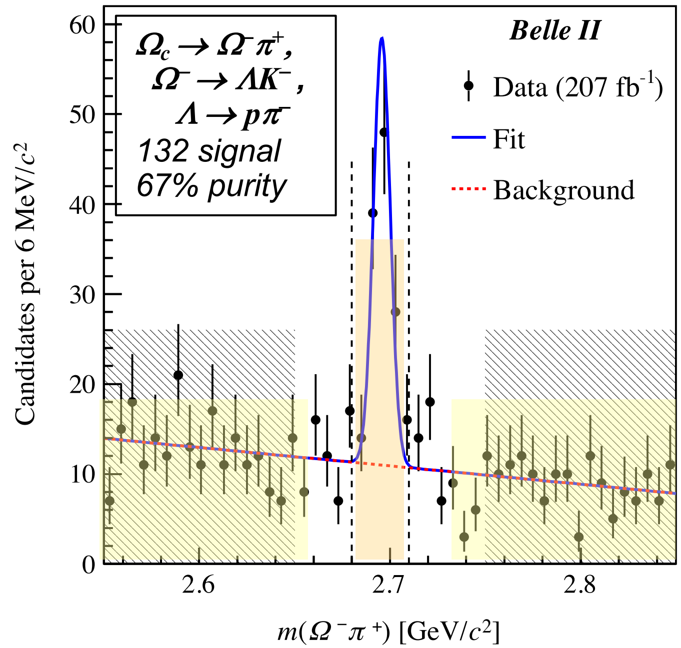
Abudinen et al., PRD 107, L031103 (2023) [arXiv:2208.08573]

Theory expectation: (& E687, WA89)	$\tau(\Omega_c) < \tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Xi_c^+)$	⇒ Belle II can confirm this (useful to have another experiment confirm)
LHCb 2018, 2022:	$\tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Omega_c) < \tau(\Xi_c^+)$	

- $p_{CM}(\Omega_c)/p_{max} > 0.6$ to eliminate $B \to \Omega_c X$ decays, where $p_{max} = \sqrt{[(E^{CM}_{beam})^2 - m(\Omega\pi)^2]}$

- *Result:* $\tau_{\Omega_c^0} = (243 \pm 48 \pm 11)$ fs





Source	Uncertainty (fs)
Fit bias	3.4
Resolution model	6.2
Background model	8.3
Detector alignment	1.6
Momentum scale	0.2
Input Ω_c^0 mass	0.2
Total	11.0

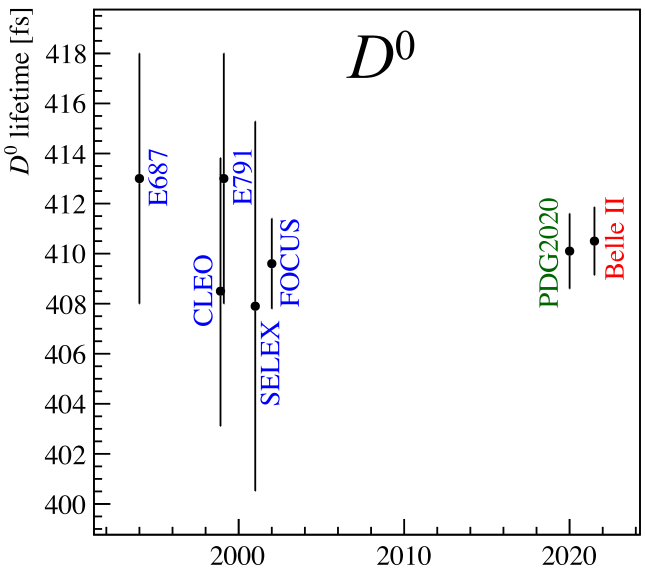
Comparing to theory

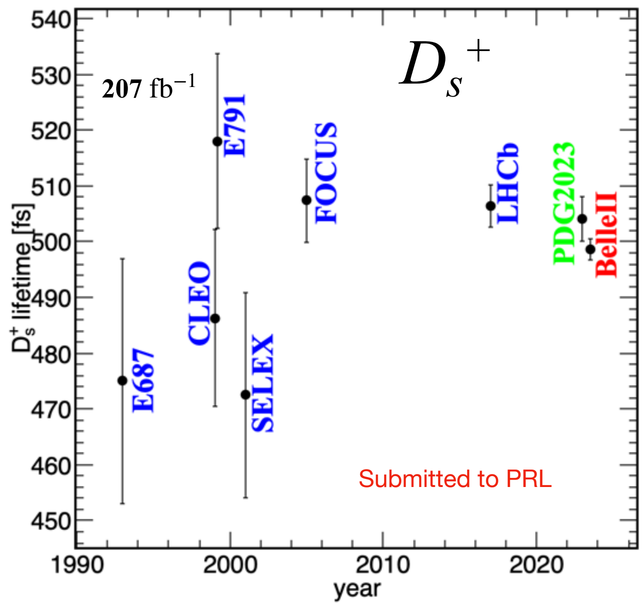
Quantity	Belle II	King et al. JHEP 08 (2022) 241 (Table 15)	Gratrex et al. JHEP 07 (2022) 058 (Tables 10, 14, MSR)
$\tau(D^0)$	410.5 ± 1.1 ± 0.8	$629\ ^{+296}_{-167}$	$595\ ^{+344}_{-166}$
$\tau(D^+)$	1030.4 ± 4.7 ± 3.1	> 897 (90% CL)	> 1260 (90% CL)
$\tau(D^+_s)$	499.5 ± 1.7 ± 0.9	$637\ ^{+381}_{-190}$	$599\ ^{+459}_{-180}$
$\tau(D^+)/\tau(D^0)$	2.510	2.80 ± 0.90	2.89 ± 0.82
$\tau(D^+_s)^*/\tau(D^0)$	1.215	1.01 ± 0.15	1.00 ± 0.22
$\tau(\Lambda_c^+)$	203.20 ± 0.89 ± 0.77		$312\ ^{+128}_{-96}$
$\tau(\Omega_c^0)$	243 ± 48 ± 11		$237\ ^{+111}_{-75}$
$\tau(\Omega_c^0)/\tau(\Lambda_c^+)$	1.20 ± 0.24		$0.83\ ^{+0.30}_{-0.18}$

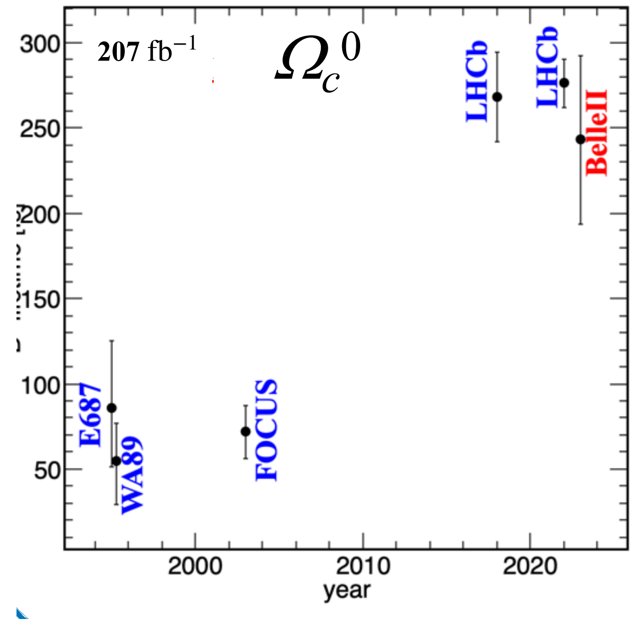
(*subtracting $B(D_s^+ \to \tau^+ \nu)$ = 5.32%)

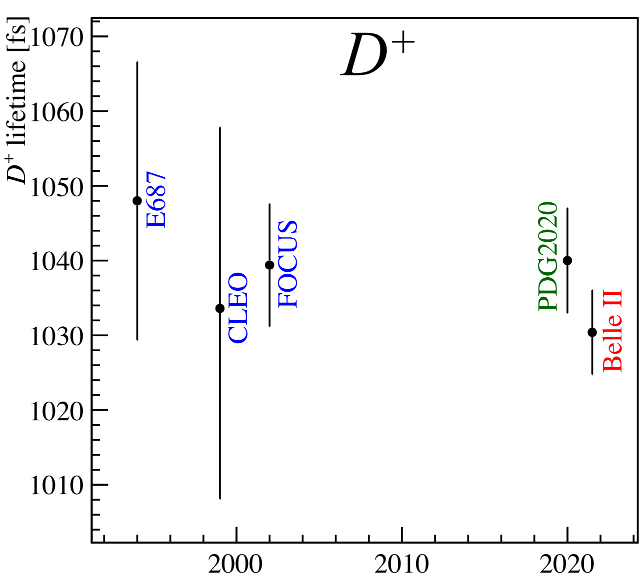
- Experimental precision is much greater than theory precision (large theory uncertainties)
- Even with large theory uncertainties, a few predictions differ from experiment by > 1σ (but less than 2σ). In the future when theory errors are reduced, such differences could become interesting – stay tuned.

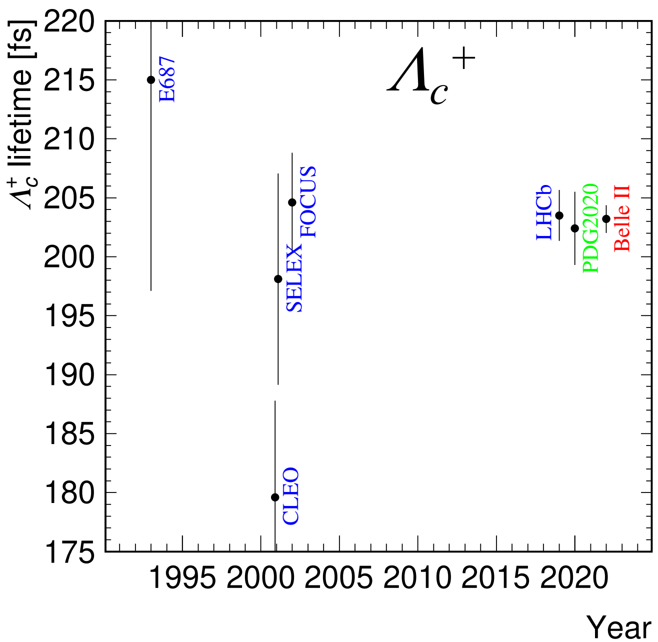
Lifetime summary





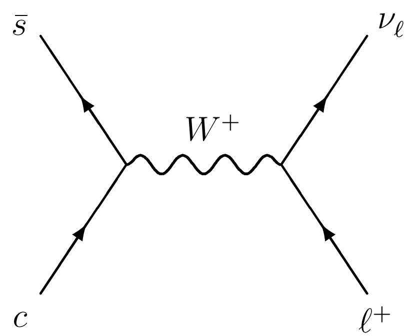






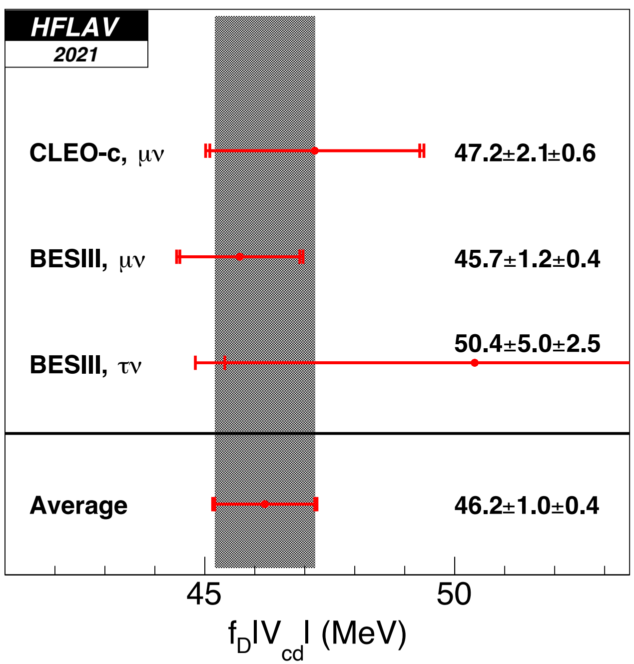
- *In all cases except for Ω_c^0, Belle II has made the world's highest precision measurement (in some cases after 20 years)*
- *For Ω_c^0, the Belle II measurement confirms the longer lifetime measured by LHCb*

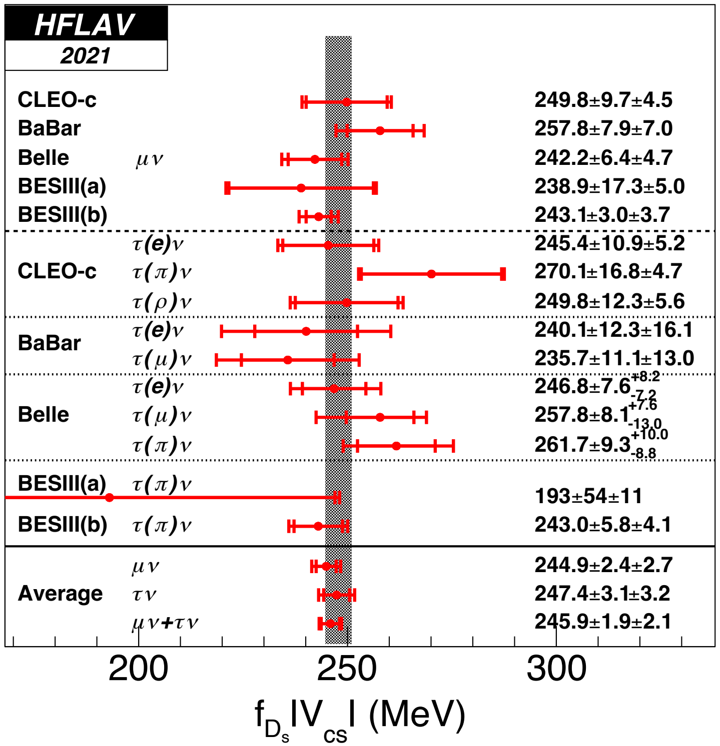
Leptonic Decays $D_{(s)}^+ \to \ell^+ \nu$



$$\mathcal{B}(D_{(s)}^+ \to \ell^+\nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}}^2 |V_{cs,cd}|^2 \tau_D m_D m_\ell^2 \left(1 - \frac{m_\ell^2}{m_D^2}\right)^2$$

1) *Measure $\mathcal{B}$, calculate f_D on lattice, extract $|V_{cs,cd}|$ (compare to unitarity)*
2) *Measure $\mathcal{B}$, take $|V_{cs,cd}|$ from other measurements + unitarity, extract f_D (compare to lattice)*





Using recent LQCD results (FLAG 2022, arXiv:2111.09849):

f_{Ds} = 249.9 ± 0.5
f_{D+} = 212.0 ± 0.7

⇒

$|V_{cs}|$ = 0.9840 ± 0.0113 (exp) ± 0.0020 (LQCD)
$|V_{cd}|$ = 0.218 ± 0.005 (exp) ± 0.001 (LQCD)

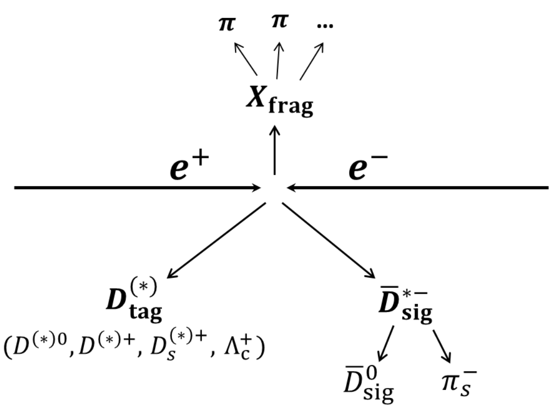
Leptonic Decay $D_s^+ \to \mu^+ \nu$

Zupanc et al., JHEP 09 (2013) 139

Method: *use energy/momentum conservation to search for rare $D^+ \to \ell^+ \nu$, $D^+ \to \nu\nu$, etc.*

$e^+e^- \to D_{\rm tag}\, X_{\rm frag}\, D_{\rm signal}$

$\searrow X\,K$ *(anti-p)*



Tag side:	D^0	D^+	Λ_c^+
1 **Decay mode:**	$K^-\pi^+$ $K^-\pi^+\pi^0$ $K^-\pi^+\pi^+\pi^-$ $K^-\pi^+\pi^+\pi^-\pi^0$ $K_S^0\pi^+\pi^-$ $K_S^0\pi^+\pi^-\pi^0$	$K^-\pi^+\pi^+$ $K^-\pi^+\pi^+\pi^0$ $K_S^0\pi^+$ $K_S^0\pi^+\pi^0$ $K_S^0\pi^+\pi^+\pi^-$ $K^+K^-\pi^+$	$pK^-\pi^+$ $pK^-\pi^+\pi^0$ pK_S^0 $\Lambda\pi^+$ $\Lambda\pi^+\pi^0$ $\Lambda\pi^+\pi^+\pi^-$
3 $X_{\rm frag}$:	$K_S^0\pi^+$ $K_S^0\pi^+\pi^0$ $K_S^0\pi^+\pi^+\pi^-$ K^+ $K^+\pi^0$ $K^+\pi^+\pi^-$ $K^+\pi^+\pi^-\pi^0$	K_S^0 $K_S^0\pi^0$ $K_S^0\pi^+\pi^-$ $K_S^0\pi^+\pi^-\pi^0$ $K^+\pi^-$ $K^+\pi^-\pi^0$ $K^+\pi^-\pi^+\pi^-$	same as for D^+ tag $+\ \bar{p}$

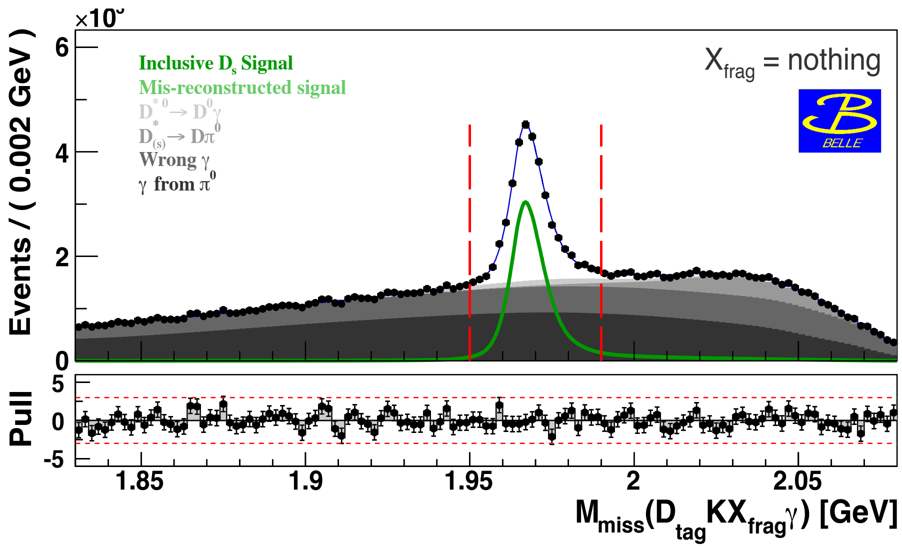
2 *For $D_{\rm signal}$ require 1 lepton track ($D_s^+ \to \ell^+ \nu$)*

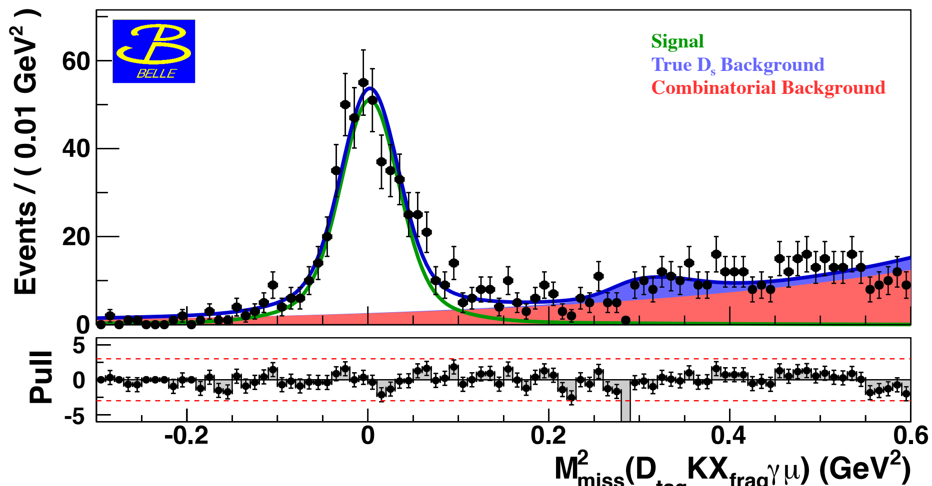
Leptonic Decay $D_s^+ \to \mu^+ \nu$

Zupanc et al., JHEP 09 (2013) 139

$e^+e^- \to D_{tag}\, X_{frag}\, K\, D_s^{*+} \to D_s^+ \gamma$, with $D_s^+ \to \mu^+ \nu$

- $P_{miss} = P_{e+} + P_{e-} - P_{Dtag} - P_K - P_X - P_\gamma - P_\mu$
- $(M_{miss})^2 = (P_{miss})^2$
- Fit to $(M_{miss})^2$





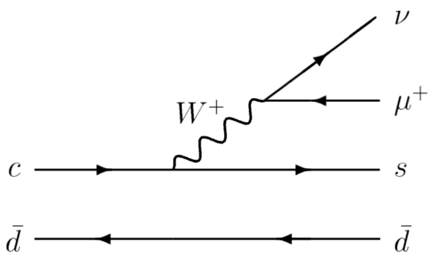
Belle yield (913 fb^{-1}): 94360 inclusive 492 ± 26 exclusive $D_s^+ \to \mu^+ \nu$

⇒ Belle II yield (20 ab^{-1}): 2.07×10^6 inclusive 10800 exclusive $D_s^+ \to \mu^+ \nu$

⇒ $\delta|V_{cs}|$ = 0.56% (stat), not far from the LQCD error on f_{Ds} of 0.20% (FLAG 2022, arXiv:2111.09849)

BESIII with 20 fb^{-1}
(scaling from arXiv:2307.14585): $D_s^+ \to \mu^+ \nu$ 6900

Semileptonic Decays



$D \to (K,\pi)\,\ell^+\nu$:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\, p_h^3}{24\pi^3}\,|V_{cs,cd}|^2\,|f_+(q^2)|^2$$

- *Take $f_+(q^2)$ form factor from theory, determine $|V_{cs}|$ or $|V_{cd}|$*

Simple pole:

$$f_+(q^2) = \frac{f_+(0)}{(1-q^2/m^2_{\rm pole})}$$

Modified pole model:

$$f_+(q^2) = \frac{f_+(0)}{(1-q^2/m^2_{\rm pole})(1-\alpha_p q^2/m^2_{\rm pole})}$$

z expansion:

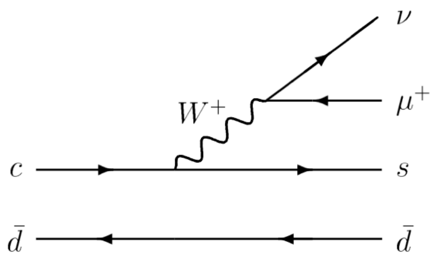
$$t_\pm = (m_D \pm m_P)^2 \qquad t_0 = t_+\left(1-\sqrt{1-t_-/t_+}\right)$$

$$z(q^2,t_0) = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$$

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2,t_0)}\sum_{k=0}^{\infty} a_k z^k$$

$$a_1/a_0 \equiv r_1 \qquad a_2/a_0 \equiv r_2$$

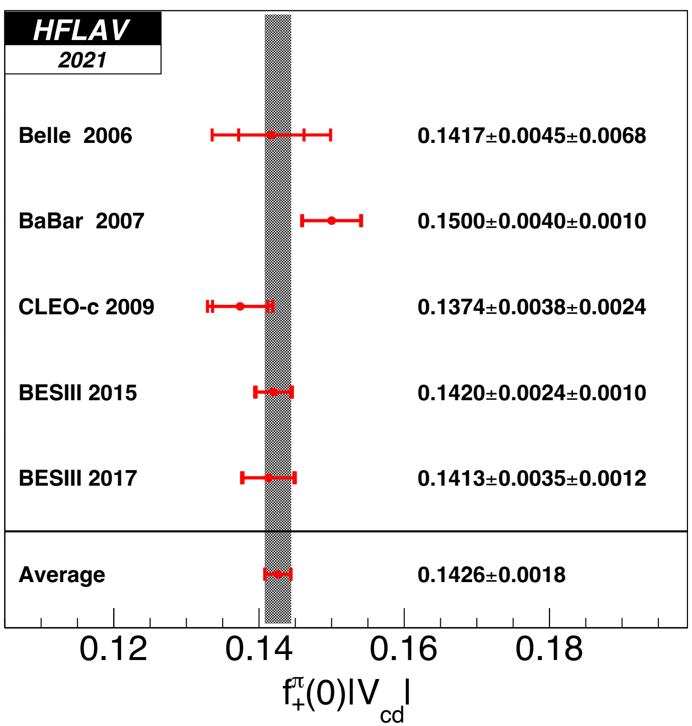
Semileptonic Decays

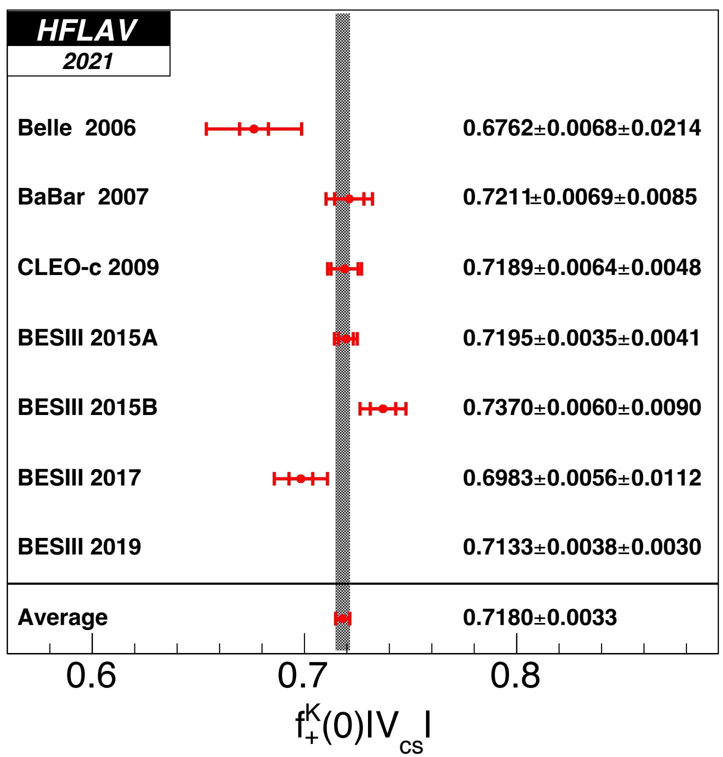


$D \to (K,\pi)\, \ell^+ \nu$:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\, p_h^3}{24\pi^3} |V_{cs,cd}|^2 \left|f_+(q^2)\right|^2$$

- *Take $f_+(q^2)$ form factor from theory, determine $|V_{cs}|$ or $|V_{cd}|$*





Using new LQCD results
[FNAL-MILC, PRD 107, 094516 (2023)]:

$f^K_+(0)$ = *0.7452 ± 0.0031*
$f^\pi_+(0)$ = *0.6300 ± 0.0051*

⇒

$|V_{cs}|$ = *0.9635 ± 0.0044 (exp) ± 0.0040 (LQCD)*
$|V_{cd}|$ = *0.2263 ± 0.0029 (exp) ± 0.0018 (LQCD)*

Semileptonic Decays (Belle II MC)

"The Belle II Physics Book"
PTEP 2019, 123C01 (2019)
[arXiv:1808.10567]

$D \to (K,\pi)\,\ell^+\nu$:

- Fully reconstruct a D^+, D^0 on tag side
- Define $P_{D^*} = P_{e+} + P_{e-} - P_{Dtag} - P_X$
- require $(P_{D^*})^2 = (M_{D^*})^2$
- Identify (K or π) and (μ or e)
- calculate $M_{miss}^2 =$
  $P_{miss}^2 = (P_{D^*} - P_{\pi\,slow} - P_{(K,\pi)} - P_{(\mu,e)})^2$

Tag side:	D^0	D^+
Final state:	$K^-\pi^+$	$K^-\pi^+\pi^+$
	$K^-\pi^+\pi^0$	$K^-\pi^+\pi^+\pi^0$
	$K^-\pi^+\pi^+\pi^-$	$K^0_S\,\pi^+$
	$K^-\pi^+\pi^+\pi^-\pi^0$	$K^0_S\,\pi^+\pi^0$
	$K^0_S\,\pi^+\pi^-$	$K^0_S\,\pi^+\pi^+\pi^-$
	$K^0_S\,\pi^+\pi^-\pi^0$	$K^+K^-\pi^+$
$X_{\rm frag}$:	π^+	none
	$\pi^+\pi^0$	π^0
	$\pi^+\pi^+\pi^-$	$\pi^+\pi^-$
		$\pi^+\pi^-\pi^0$

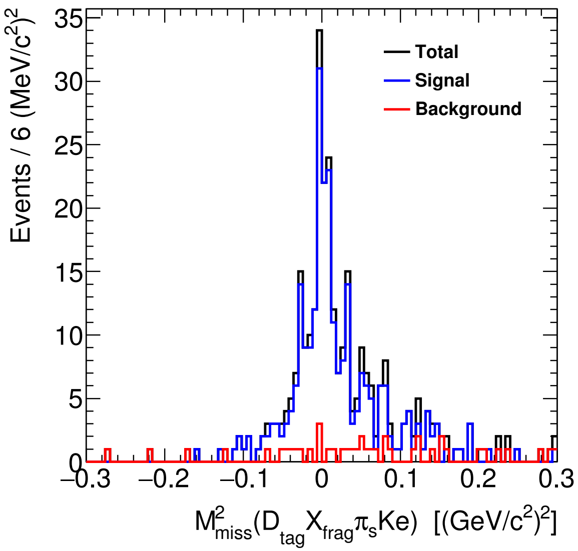
Belle yields
(282 fb^{-1}, 79% purity):
$D^0 \to K^-\mu^+\nu$: 1249
$D^0 \to K^-e^+\nu$: 1318
$D^0 \to \pi^-\mu^+\nu$: 106
$D^0 \to \pi^-e^+\nu$: 126

BaBar yields
(380 fb^{-1}, 53% purity):
$D^0 \to \pi^-e^+\nu$: 5303

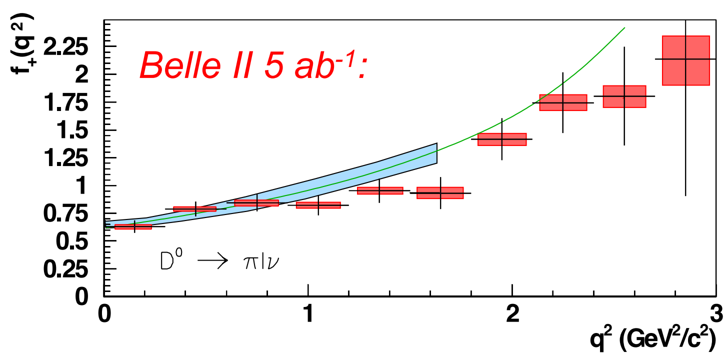
Belle II yields (20 ab^{-1}):
$D^0 \to K^-\ell^+\nu$: 182k
$D^0 \to \pi^-\ell^+\nu$: 16.5k
53% purity:
$D^0 \to \pi^-e^+\nu$: 279k

BESIII with 20 fb^{-1}
(scaling from arXiv:1508.07560):
$D^0 \to K^-e^+\nu$: 484k
$D^0 \to \pi^-e^+\nu$: 43k

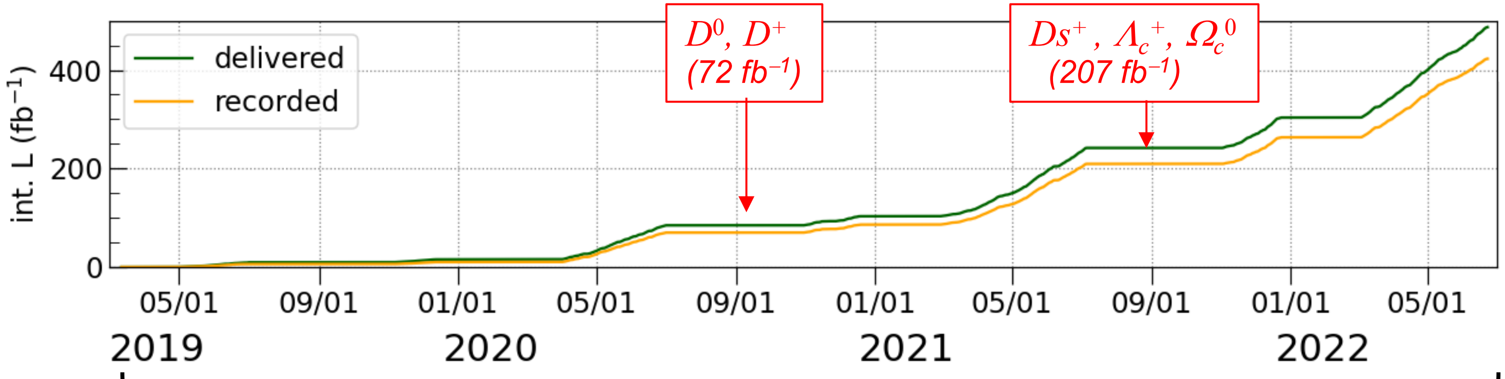
Belle II 1.0 ab^{-1}:



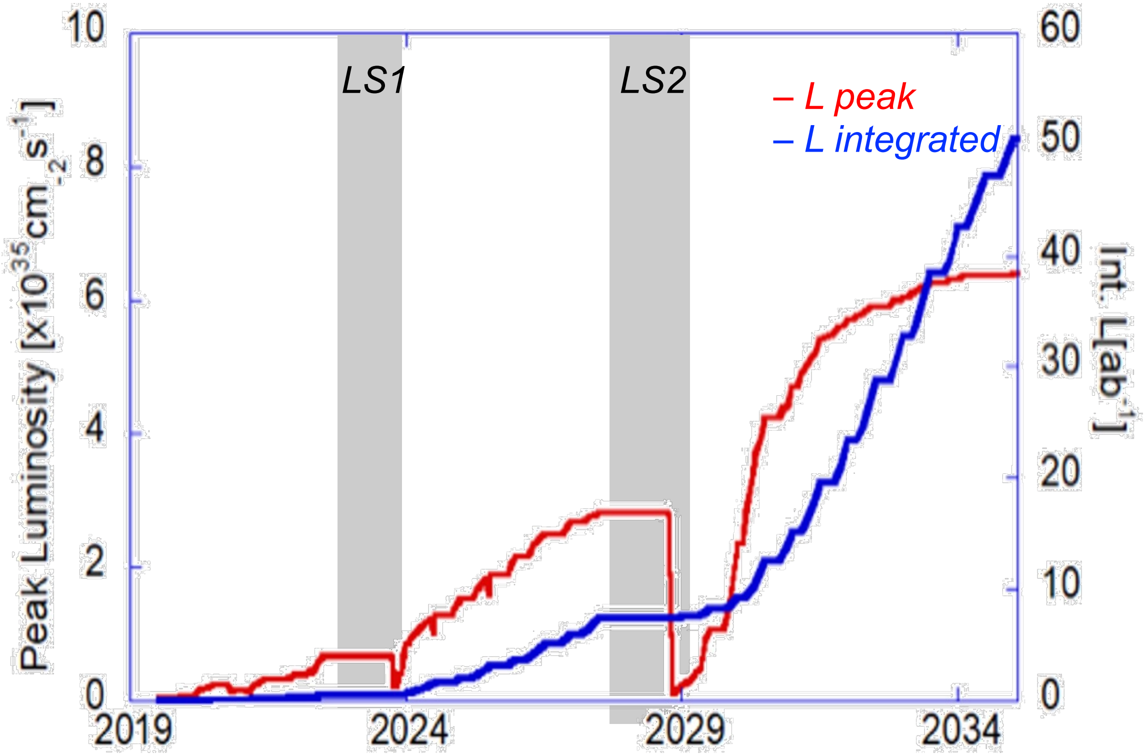
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\,p_h^3}{24\pi^3}\,|V_{cs,cd}|^2\,|f_+(q^2)|^2$$



The future



now: 428 fb^{-1}



- Goal is to ultimately accumulate 50 ab^{-1}
- However: a huge amount of physics will be done with ~5-10 ab^{-1}, possibly uncovering new physics (the amount of physics done with < 0.5 ab^{-1} is surprising: better decay time resolution, expanded D flavor tagging...)

Summary

- *With a very small data set, Belle II has made the world's most precise measurements of the D^0, D^+, D_s^+, and Λ_c lifetimes. Belle has made a relevant measurement of the Ω_c lifetime.*
- *With 20 ab^{-1} of data, Belle II should have competitive samples of D_s^+ leptonic and D^0 semileptonic decays. These should yield among the world's most precise measurements of V_{cd} and V_{cs} .*
- *Belle II is behind in accumulating data. However, as compared to Belle/Babar there are substantial improvements to the detector and reconstruction software. The SuperKEKB accelerator has set world records for instantaneous luminosity and daily/weekly integrated luminosity, and during LS1 there have been substantial improvements to the accelerator. Thus, despite the modest data sample so far, the experiment is expected to have a large physics impact and significant discovery potential.*

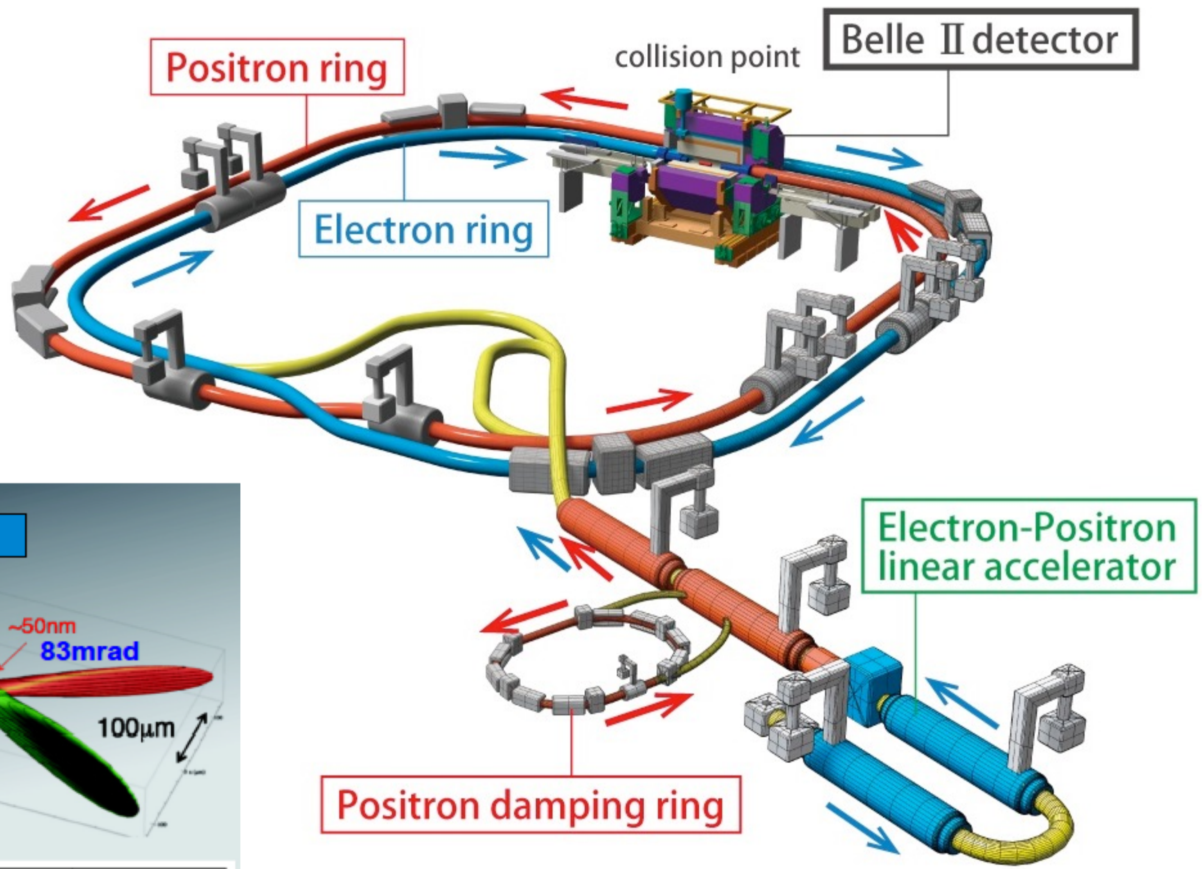
Extra

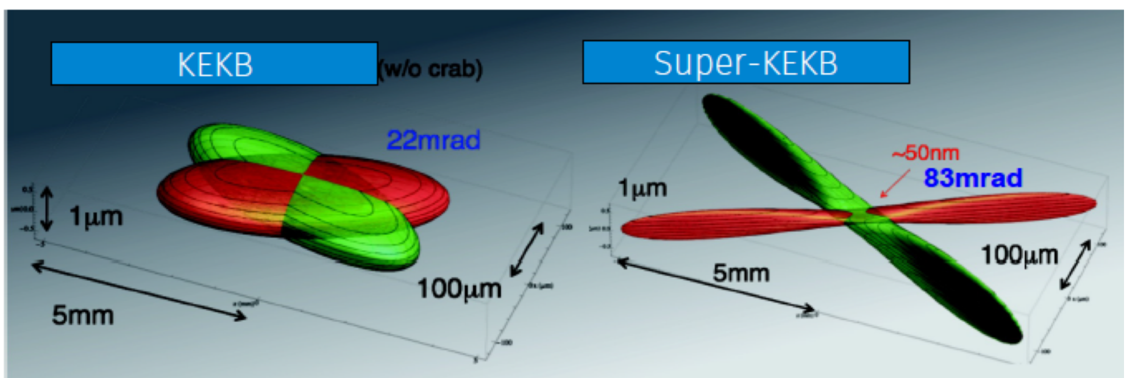
Major accelerator upgrade (KEKB → SuperKEKB)

e^+e^- collider running at the Upsilon(4S) [and Upsilon (5S)] resonances with 7 GeV (e^-) on 4 GeV(e^+) beams.
New e^+ damping ring, new e^+ storage ring, new IR optics, Superconducting FF, new RF

beam size:
100 μm(H) x 2 μm(V)
→ 10 μm(H) x 59 nm(V)

Belle-II Goal:
30 x Belle = ~6 x 10^{35}





	E (GeV) LER/HER	β*y (mm) LER/HER	β*x (cm) LER/HER	φ (mrad)	I (A) LER/HER	L (cm^{-2}s^{-1})
KEKB	3.5/8.0	5.9/5.9	120/120	11	1.6/1.2	2.1 x 10^{34}
SuperKEKB	4.0/7.0	0.27/0.30	3.2/2.5	41.5	3.6/2.6	80 x 10^{34}

factor 20   factor 2-3