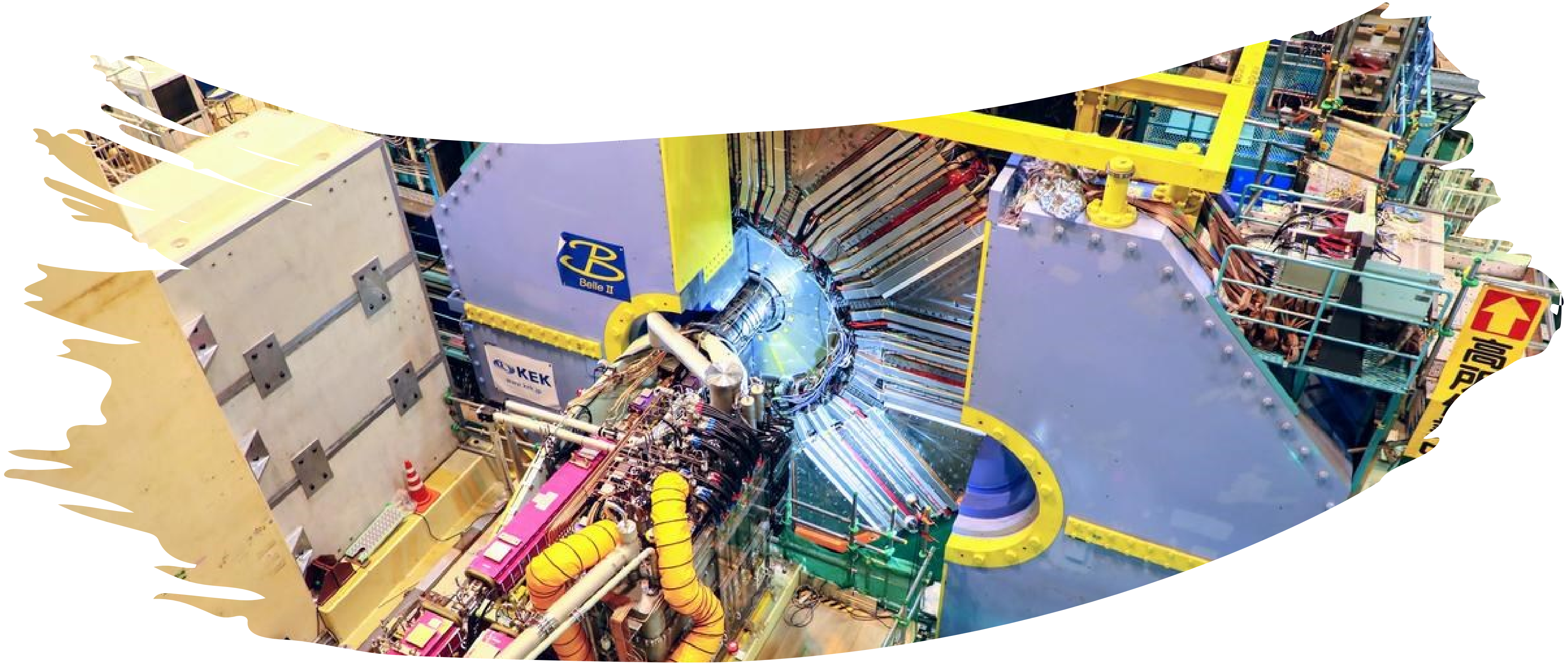

Recent measurements of inclusive SL decays at the beauty and charm factories

Markus Prim for the Belle II Collaboration
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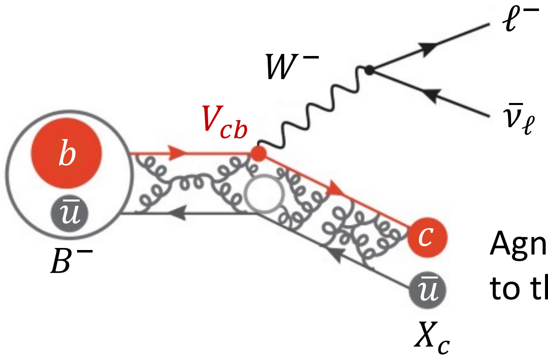
Bundesministerium
für Bildung
und Forschung

UNIVERSITÄT BONN

Inclusive $B \to X_c \ell \bar{\nu}_\ell$

Why? & How?

Inclusive $B \to X_c \ell \bar{\nu}_\ell$



Agnostic with respect to the hadronic system

The theoretical framework is Operator Product Expansion (OPE) and Heavy Quark Expansion (HQE)

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

$d\Gamma$ are calculated perturbatively

→ Available at $\mathcal{O}(\alpha_s^3)$
Fael, Schönwald, Steinhauser
Phys. Rev. D 104, 016003 (2021)

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ encapsulate non-perturbative dynamics

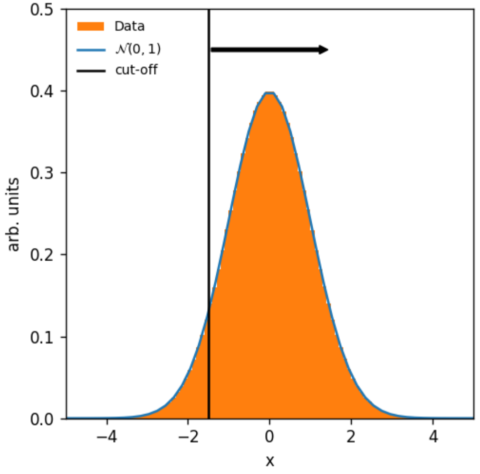
→ HQE parameters must be extracted from data

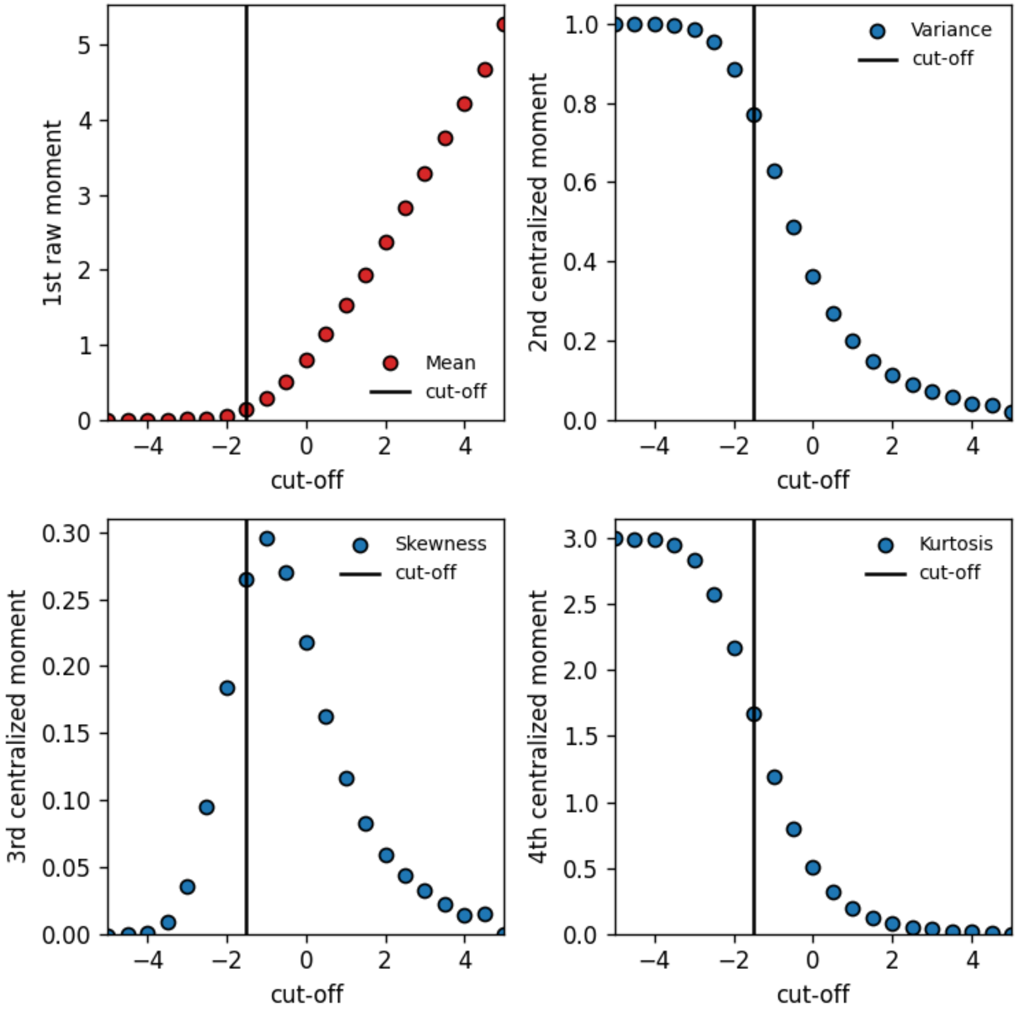
→ requires the spectral moments of $B \to X_c \ell \nu$

Challenge: Proliferation of HQE parameters at higher order

Talk by Keri Vos
"HQE in inclusive SL decays"

Spectral Moments of a Distribution





- The moments are measured with cut-offs in the distribution
- Data points are highly-correlated

$$\mu_n = \int_{-\infty}^{-\infty} (x-c)^n f(x) dx$$

Raw moment: $c = 0$

Central moment: $c = \text{Mean}$

First raw moment: Mean

Measures the location

Second central moment: Variance

Measures the spread

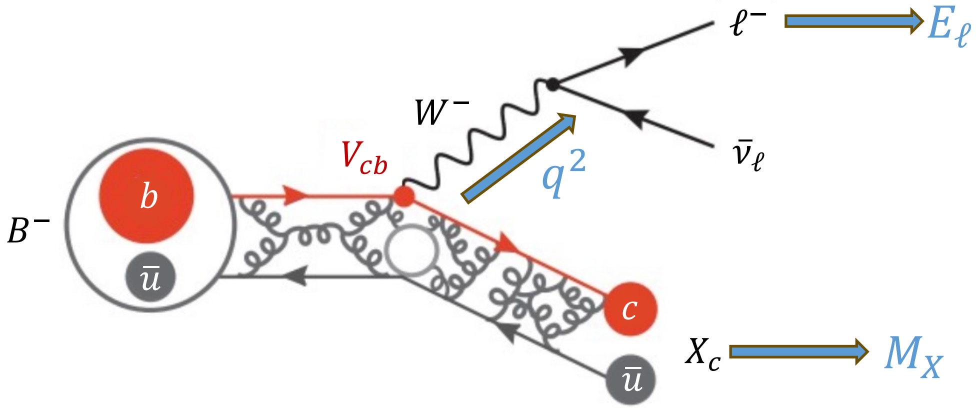
Third central moment: Skewness

Measures asymmetry

Fourth central moment: Kurtosis

Measures "tailedness"

Inclusive $B \to X_c \ell \bar{\nu}_\ell$ - Existing Measurements



$\langle E_\ell \rangle$ and $\langle M_X \rangle$

- DELPHI
  Eur.Phys.J.C45:35-59,2006
- CLEO
  Phys.Rev.D70:032002,2004
  Phys.Rev.D70:032003,2004
- CDF
  Phys.Rev.D71:051103,2005
- Babar
  Phys.Rev.D69:111104,2004
  Phys.Rev.D81:032003,2010
- Belle
  Phys.Rev.D75:032005,2007
  Phys.Rev.D75:032001,2007

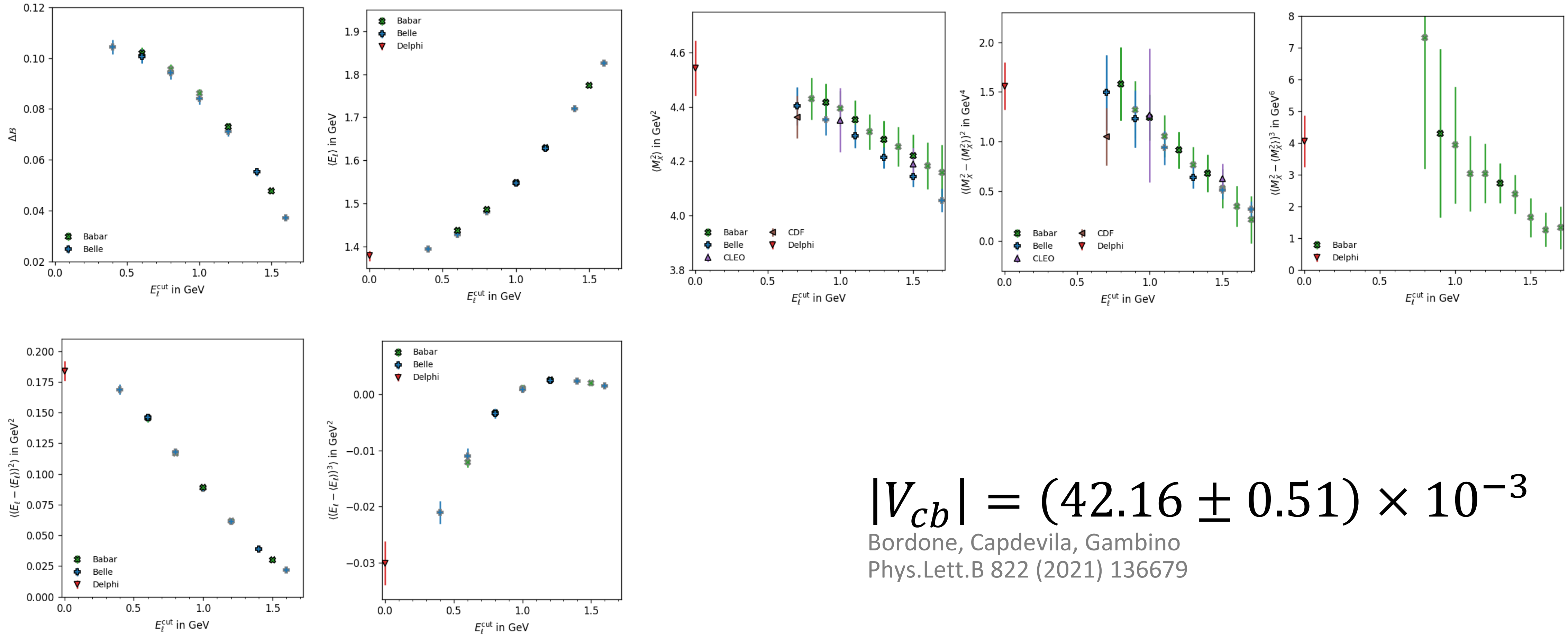
11 years →

$\langle q^2 \rangle$

- Belle
  Phys.Rev.D 104 (2021) 11, 112011
- Belle II
  Phys.Rev.D 107 (2023) 7, 072002

Both analyses are conceptually identical

Inclusive $B \to X_c \ell \bar{\nu}_\ell$ - $\langle E_\ell \rangle, \langle M_X \rangle$ Moments



$$|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3}$$

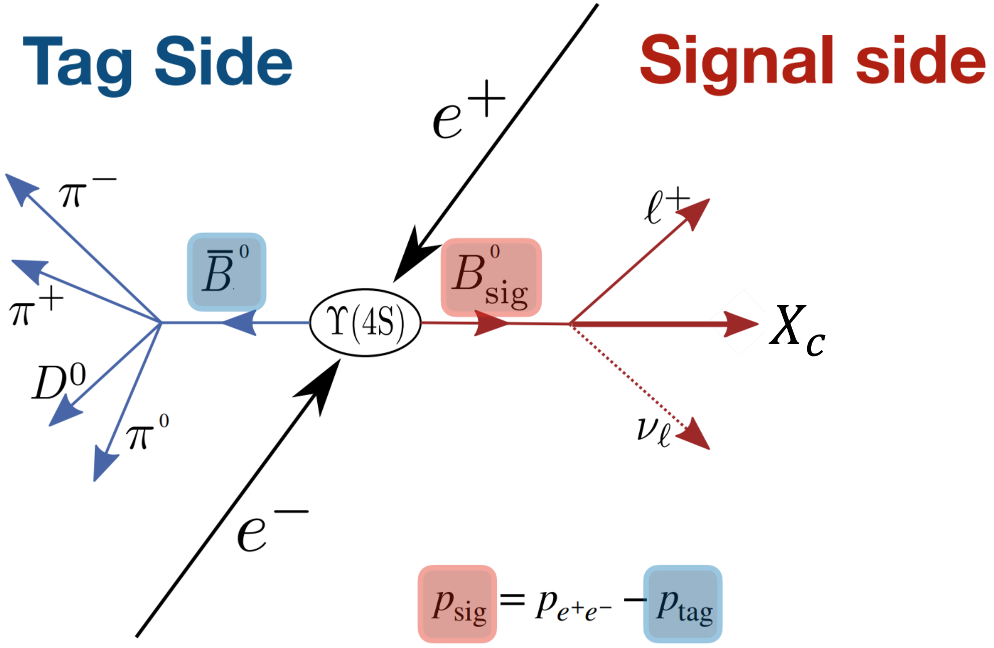
Bordone, Capdevila, Gambino
Phys.Lett.B 822 (2021) 136679

$\langle q^2 \rangle$ Moments

by Belle II

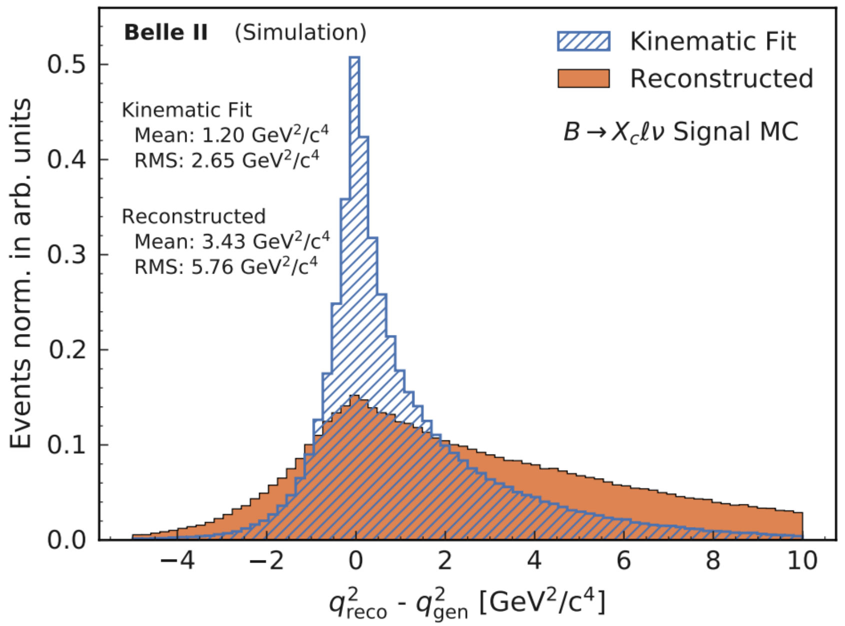
$\langle q^2 \rangle$ Moments – Measurement Strategy

Key-techniques: Hadronic tagging and kinematic fitting exploit the known initial state kinematics



$$q^2 = (p_{sig} - p_{X_c})^2$$

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$



Kinematic constraints:

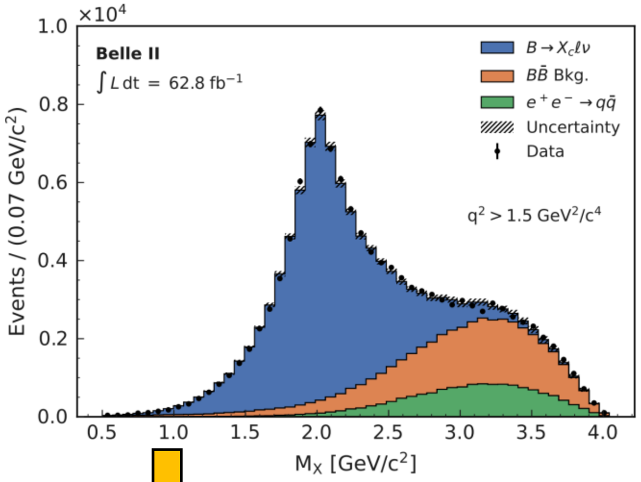
$$\hat{p}_X^2 > 0, \ \hat{p}_{B_{tag}}^2 = m_B^2$$

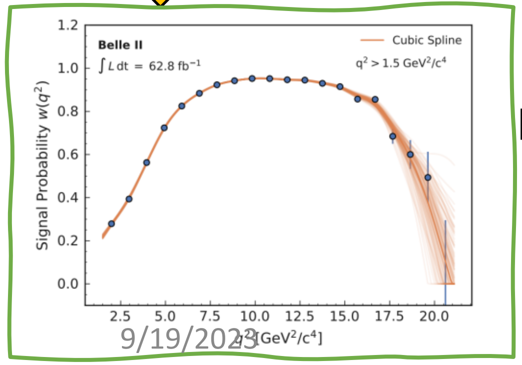
$$(\hat{p}_\ell + \hat{p}_X + \hat{p}_\nu)^2 = m_B^2$$

$$\left(\hat{p}_{e^+e^-} - \hat{p}_{B_{tag}} - \hat{p}_\ell - \hat{p}_X - \hat{p}_\nu\right) = 0$$

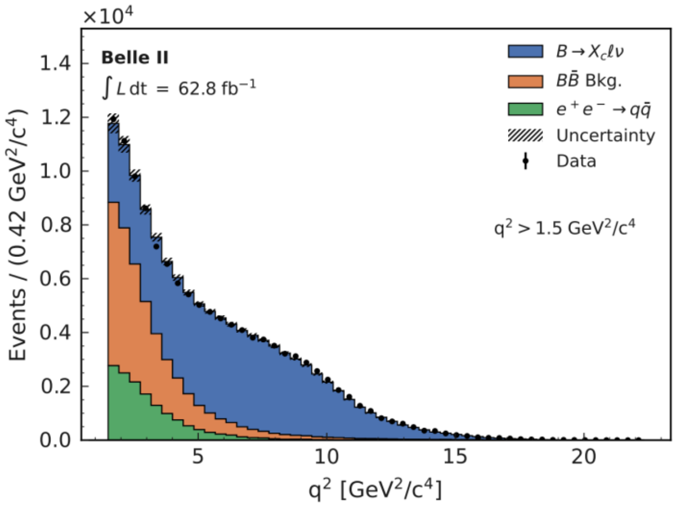
$\langle q^2 \rangle$ Moments – Background Subtraction

Determine background normalization in q^2 through fits to M_X





Calculate event-wise signal probability



Event-wise master formula

$$\langle q^{2n} \rangle = \frac{\Sigma_{i=0}^{N_{data}} w(q_i^2) \times q_{calib,i}^{2n}}{\Sigma_{j=0}^{N_{data}} w(q_j^2)} \times \mathcal{C}_{calib} \times \mathcal{C}_{gen}$$

- Linear calibration function
  $$q_{calib}^{2n} = (q_{reco}^{2n} - c_n)/m_n$$
- Bias from assumed linearity
  $$\mathcal{C}_{calib} = \langle q_{gen,sel}^{2n} \rangle / \langle q_{calib}^{2n} \rangle$$
- Reconstruction effects & final state radiation
  $$\mathcal{C}_{gen} = \langle q_{gen}^{2n} \rangle / \langle q_{gen,sel}^{2n} \rangle$$

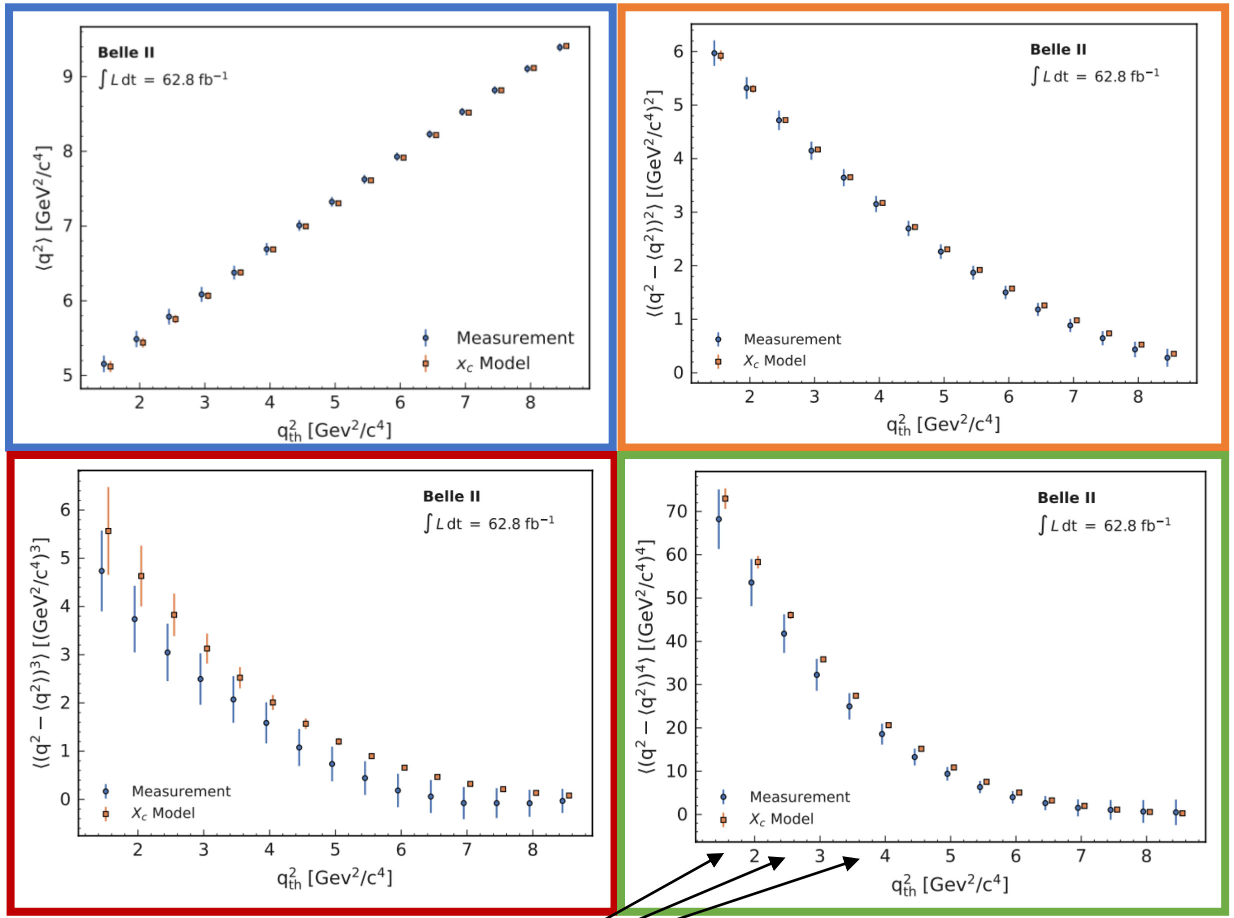
$\langle q^2 \rangle$ Moments – Result

$$\mu_n = \int_{-\infty}^{-\infty} (x-c)^n f(x) dx$$

Raw moment: $c = 0$

Central moment: $c = \text{Mean}$

$\langle q^2 \rangle$ Moments



cut-offs

First raw moment: Mean

Measures the location

Second central moment: Variance

Measures the spread

Third central moment: Skewness

Measures asymmetry

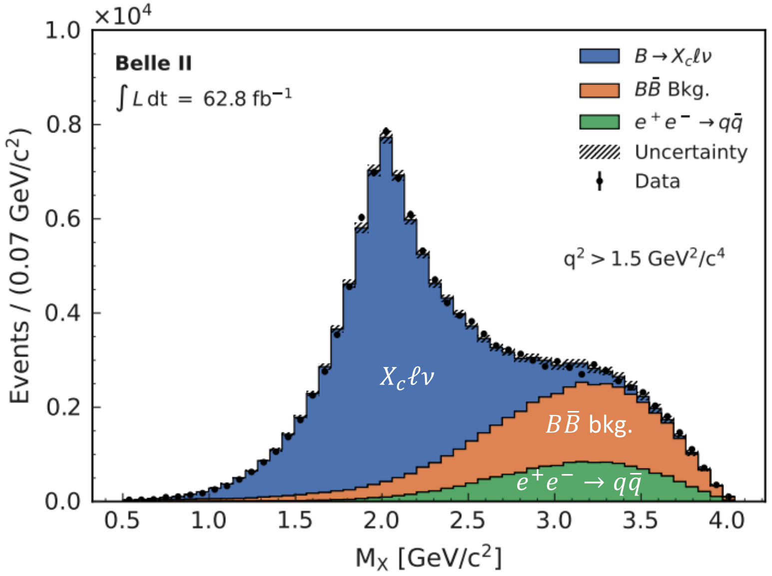
Fourth central moment: Kurtosis

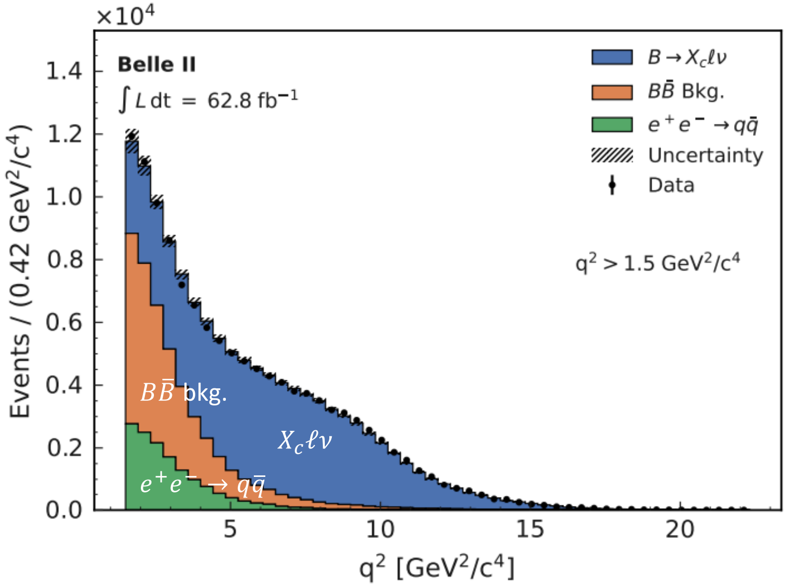
Measures "tailedness"

Systematics

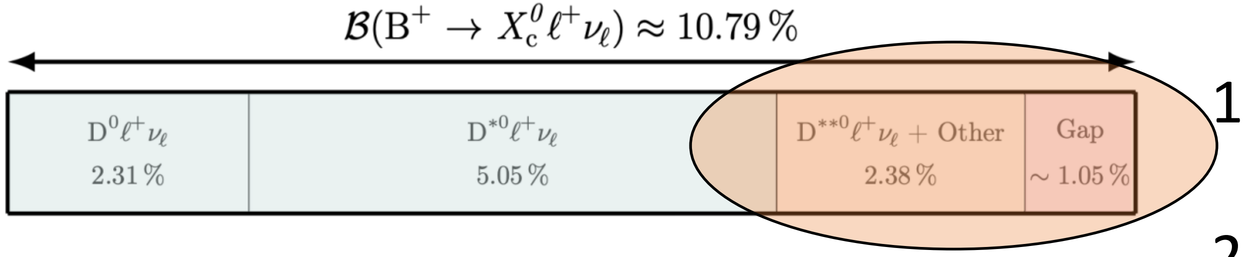
- Background yields and shape
- Composition of the X_c system
- Simulated detector resolution

$\langle q^2 \rangle$ Moments – Systematics





Sizeable uncertainty from the $B \to X_c \ell \bar{\nu}_\ell$ modelling

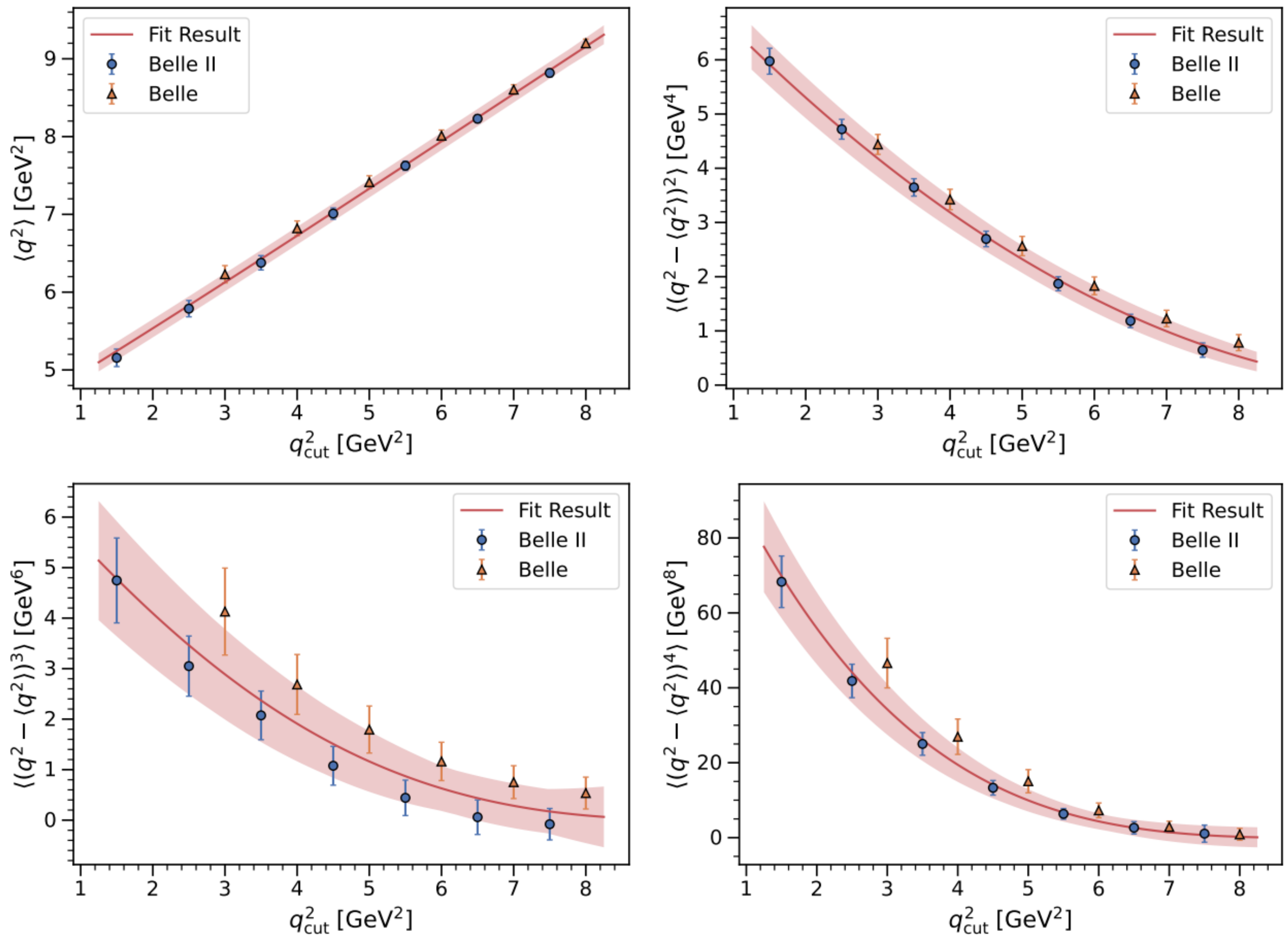


1. Better understanding of $B \to D^{**} \ell \bar{\nu}_\ell$
   → Differential measurements
2. Better understanding of D^{**} states
   → Spectroscopy

$|V_{cb}|$ from $\langle q^2 \rangle$ Moments

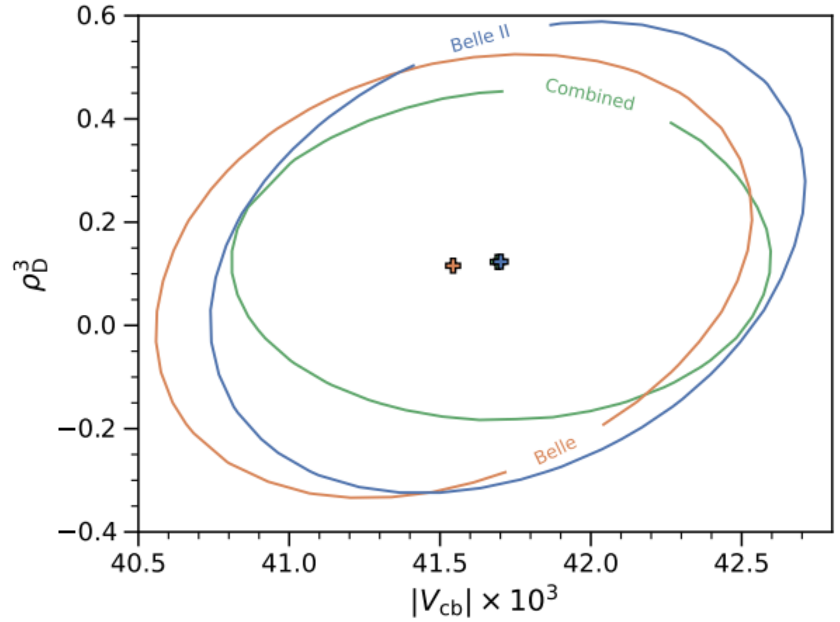
a brief excursion

Combined fit to Belle & Belle II $\langle q^2 \rangle$



$$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$$

Bernlochner, Fael, Olschwesky, Persson, van Tonder, Vos, Welsch JHEP 10 (2022) 068



Inclusive $B \to X_c \ell \bar{\nu}_\ell$

$\langle M_X \rangle$ and $\langle E_\ell \rangle$ Moments

$|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3}$ ⇨ $\rho_D^3 = (0.19 \pm 0.03)\ \mathrm{GeV}^3\ @\mathcal{O}\left(\frac{1}{m_b^3}\right)$

Bordone, Capdevila, Gambino
Phys.Lett.B 822 (2021) 136679

$\langle q^2 \rangle$ Moments

$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$ ⇨ $\rho_D^3 = (0.12 \pm 0.20)\ \mathrm{GeV}^3\ @\mathcal{O}\left(\frac{1}{m_b^4}\right)$

Bernlochner, Fael, Olschwesky,
Persson, van Tonder, Vos, Welsch
JHEP 10 (2022) 068

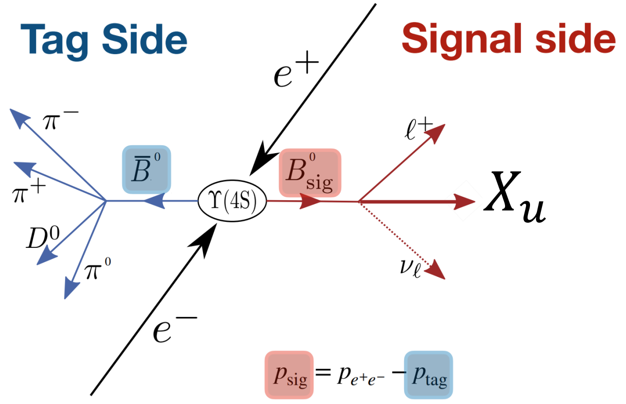
At same order of HQE

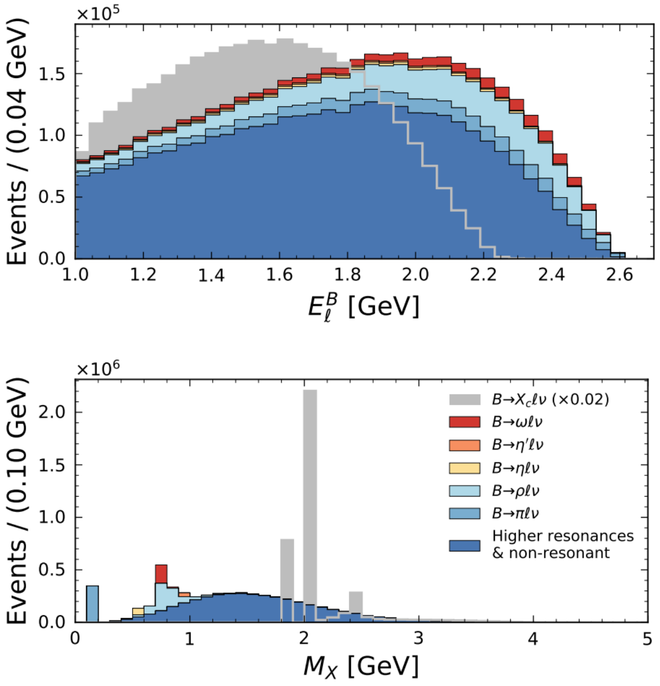
$\rho_D^3 = (0.03 \pm 0.02)\ \mathrm{GeV}^3\ @\mathcal{O}\left(\frac{1}{m_b^3}\right)$

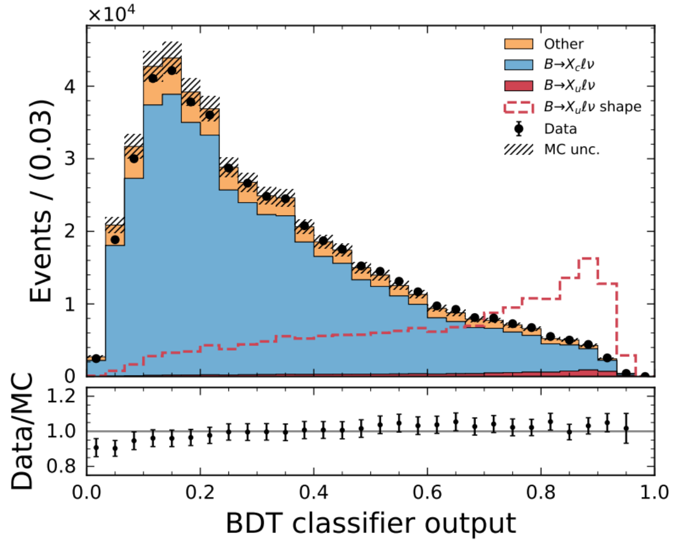
Inclusive $B \to X_u \ell \bar{\nu}_\ell$

$B \to X_u \ell \bar{\nu}_\ell$ – Measurement Strategy

Key-techniques: Hadronic tagging and background suppression







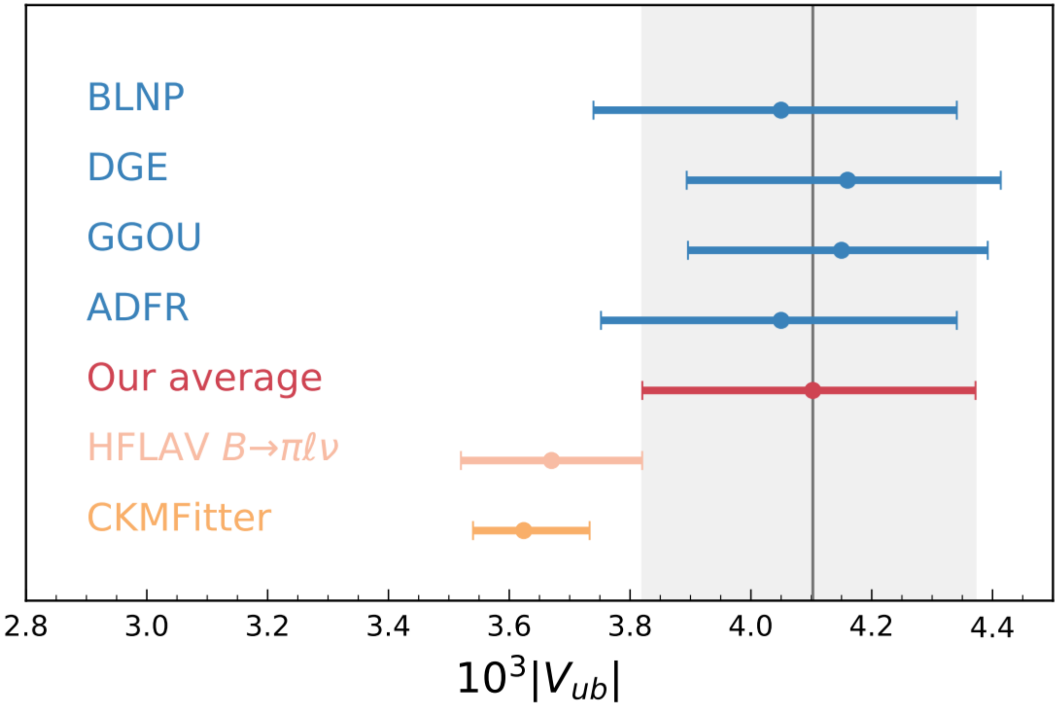
Suppress abundant $b \to c \ell \bar{\nu}_\ell$ background with an MVA

$B \to X_u \ell \bar{\nu}_\ell$

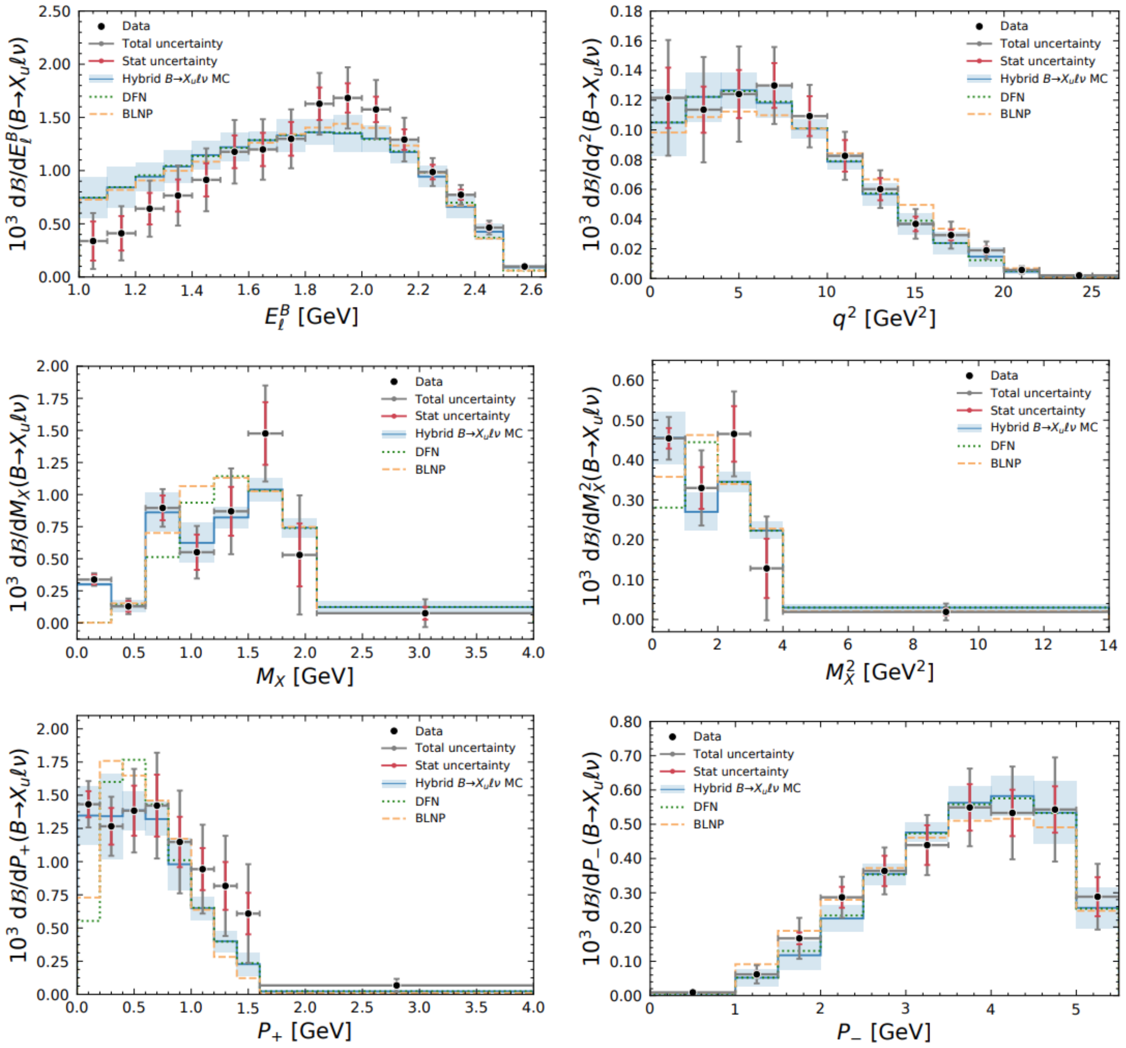
- Belle, Partial Branching Fractions
  Phys. Rev. D 104, 012008 (2021)
- Belle, Differential Branching Fractions
  Phys. Rev. Lett. 127, 261801 (2021)
- Belle, $V_{ub}^{excl}/V_{ub}^{incl}$
  2303.17309

$B \to X_u \ell \bar{\nu}_\ell$ - Results

Extraction of $|V_{ub}|$ from the measured $B \to X_u \ell \bar{\nu}_\ell$ partial rate with different theory models



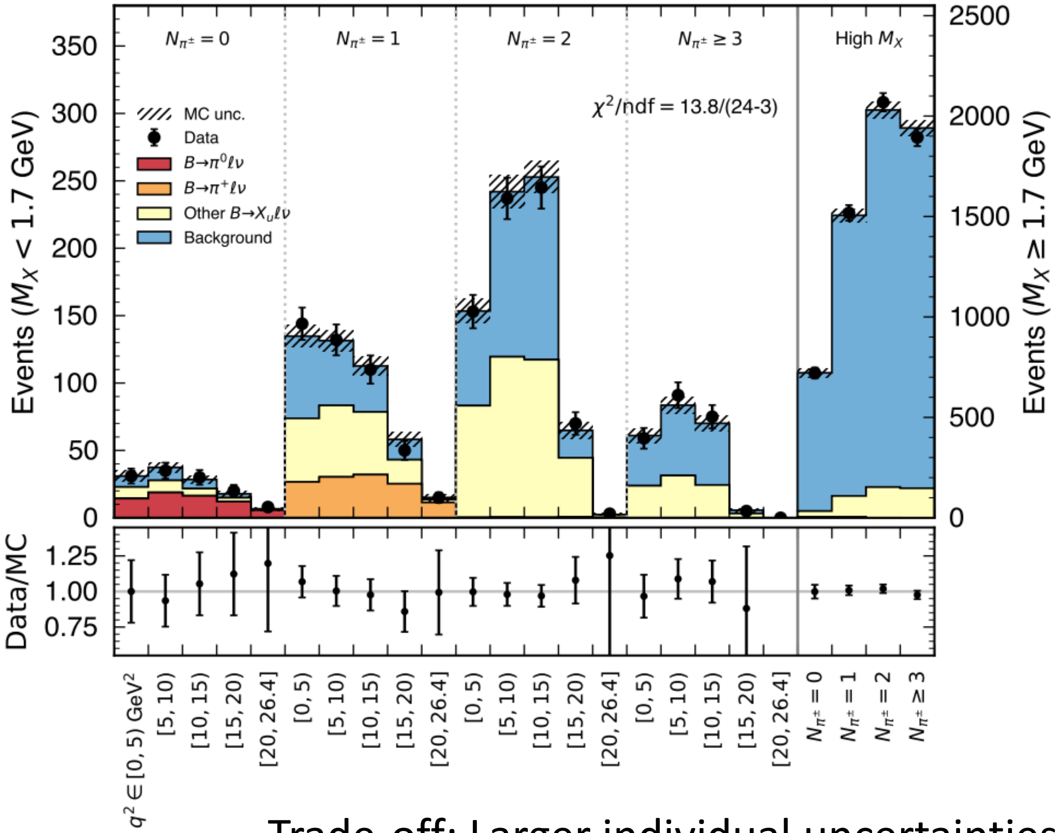
Unfolded differential spectra of $B \to X_u \ell \bar{\nu}_\ell$

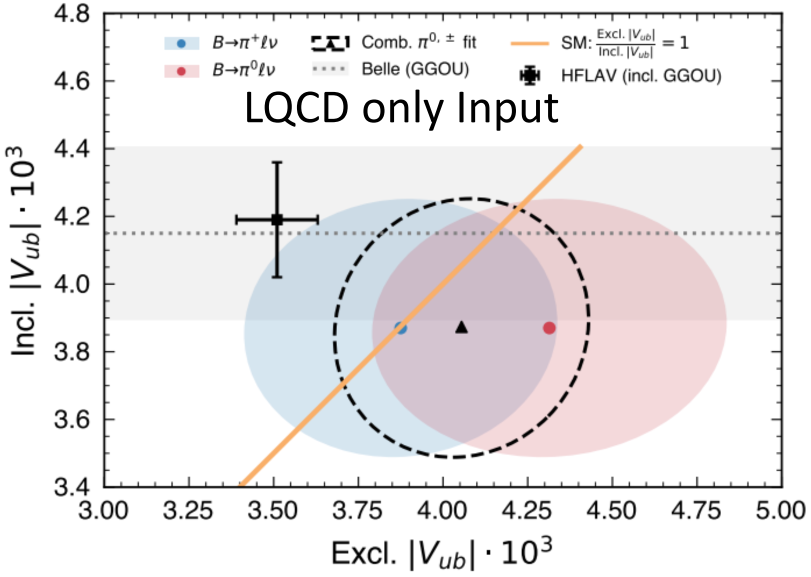


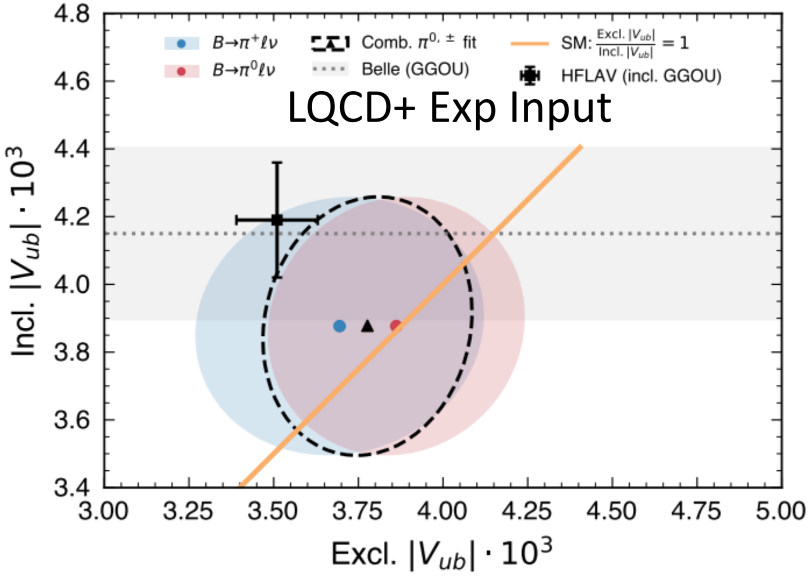
Accepted Today

$V_{ub}^{excl}/V_{ub}^{incl}$ - Results

Combined inclusive-exclusive analysis to gain insights into the inclusive-exclusive discrepancy in $|V_{ub}|$





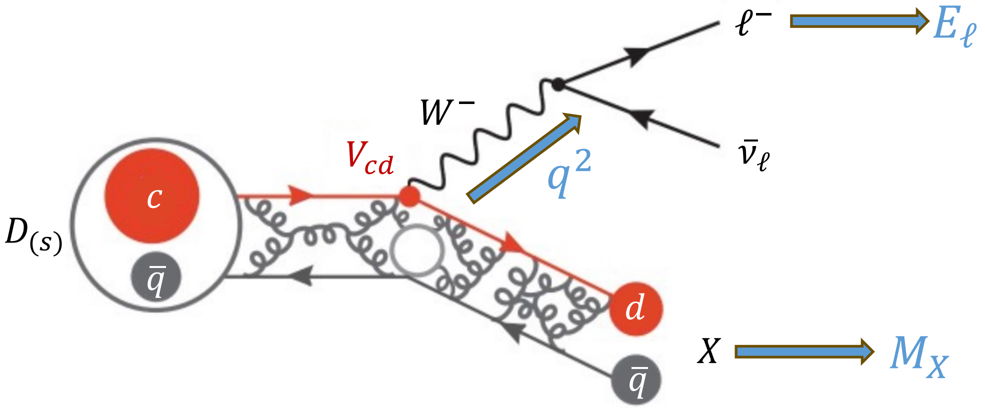


Trade-off: Larger individual uncertainties on V_{ub} excl. or incl.

$D_s \to X e \bar{\nu}_e$

by BES-III

Inclusive $D_{(s)} \to Xe\bar{\nu}_e$ - Existing Measurements



$\mathcal{B}(D_{(s)} \to Xe\bar{\nu}_e)$

- CLEO
  Phys.Rev.D 81 (2010) 052007
  Phys.Rev.D 81 (2010) 052007

11 years

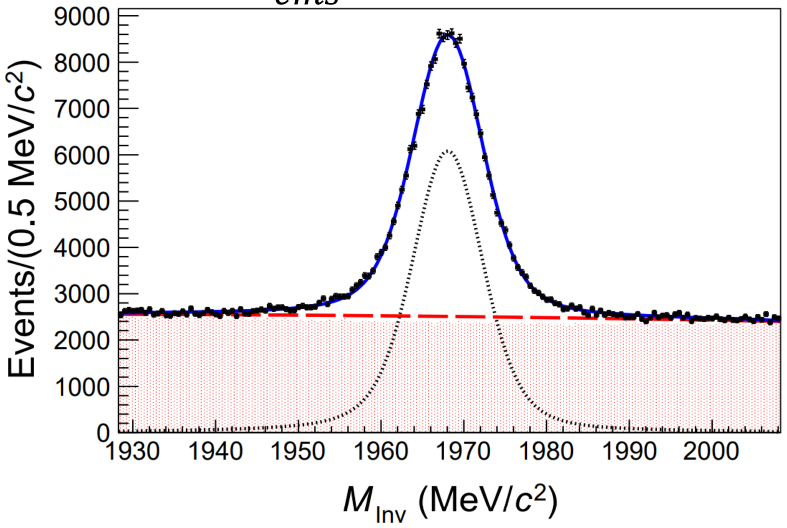
$\mathcal{B}(D_s \to Xe\bar{\nu}_e)$

- BES-III
  Phys. Rev. D 104, 012003 (2021)

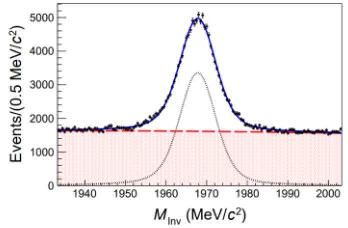
$\mathcal{B}(D_s \rightarrow X e \bar{\nu}_e)$ – Single-Tag Event Selection

- Production primarily through $e^+e^- \rightarrow D_s^{*+} D_s^-$, small contribution from $e^+e^- \rightarrow D_s^+ D_s^-$
- Chosen tag-side: $D_s^- \rightarrow K^+ K^- \pi^-$
  Sufficient statistic and well-known backgrounds
- Calculate recoil mass and require compatibility with the $D_s^{*+} D_s^-$ hypothesis
- In total 262660 ± 1137 single-tag events in the three E_{cm} regions
- → Search for the signal in the remaining recoiling tracks

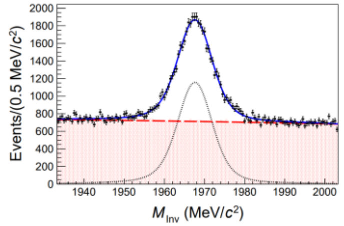
$E_{cms} = 4.178$ GeV



$E_{cms} = 4.189 - 4.219$ GeV

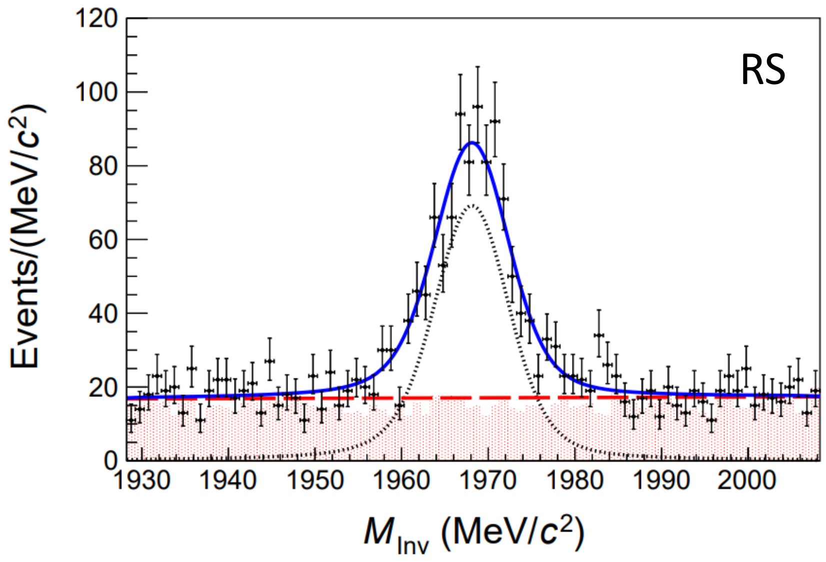


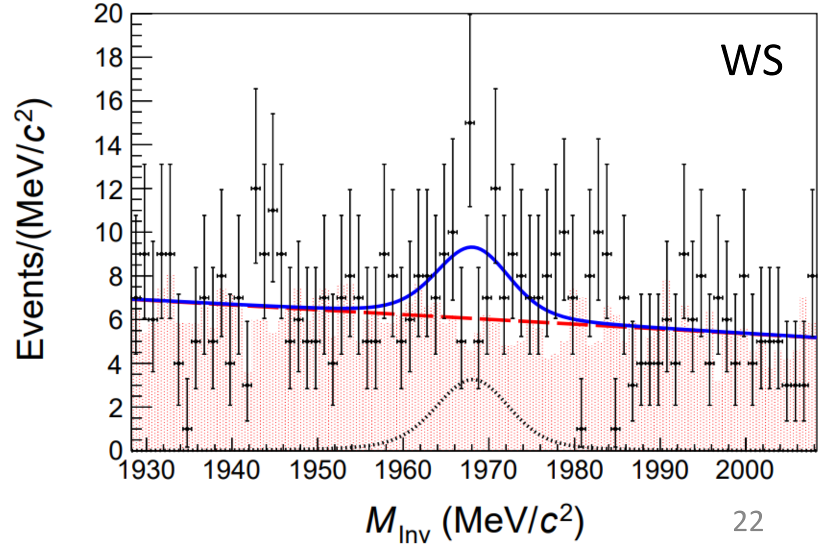
$E_{cms} = 4.225 - 4230$GeV



$\mathcal{B}(D_s \to X e \bar{\nu}_e)$ – Double-Tag Event Selection

- Sort recoil-side tracks into momentum bins
- Further, sort recoil-side tracks into bins of
  - Right-sign (RS) and Wrong-Sign (WS)
  - $e, \mu/\pi$ and K candidates
- WS sample is used to determine charge-symmetric backgrounds in the RS distribution
- Number of tracks from true signal-D_s by fitting the invariant-mass distribution of the tag-D_s
- 172 fits in total
- Yields are unfolded to correct for inefficiencies and misidentification

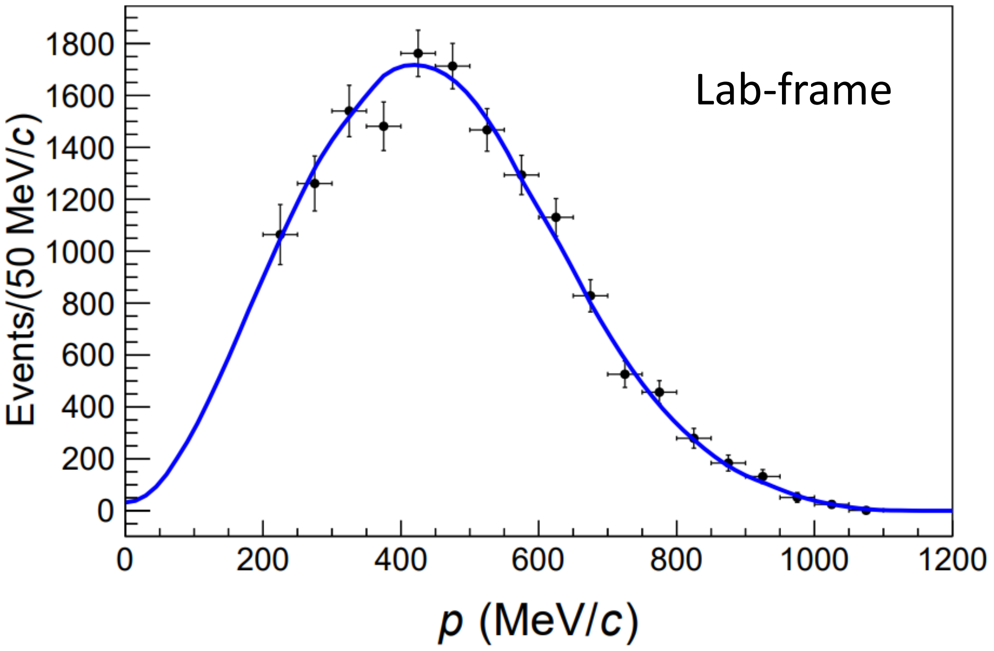




$\mathcal{B}(D_s \to X e \bar{\nu}_e)$ – Result

- $\mathcal{B}(D_s \to X e \bar{\nu}_e) = (6.30 \pm 0.13 \pm 0.10)\%$

- Lepton momentum spectrum, extrapolated to zero-recoil with a sum-of-exclusive model
  →In principle access to $\langle E_\ell \rangle$ moments



- $\mathcal{B}(D_s \to X e \bar{\nu}_e) - \sum_i \mathcal{B}(D_s \to X_i e \bar{\nu}_e) = (-0.04 \pm 0.13 \pm 0.20)\%$
  Inclusive branching fraction saturated by sum of exclusive
  → In principle access to $\langle M_X \rangle$ moments from sum-of-exclusive model

- $\frac{\Gamma(D_s \to X e \bar{\nu}_e)}{\Gamma(D^0 \to X e \bar{\nu}_e)} = 0.790 \pm 0.016 \pm 0.020$
  in agreement with prediction from an effective quark model, indicating non-spectator effects

Source	Relative Uncertainty
Tracking	0.7%
PID	0.8%
Spectrum Extrapolation	0.7%
Background Shapes	0.4%
Number of Tags	0.6%
Tag Bias	0.1%
$\mathcal{B}(D_s^+ \to \tau^+ \nu_\tau)$	0.6%
Total	1.6%

$\Lambda_c \to X e \bar{\nu}_e$

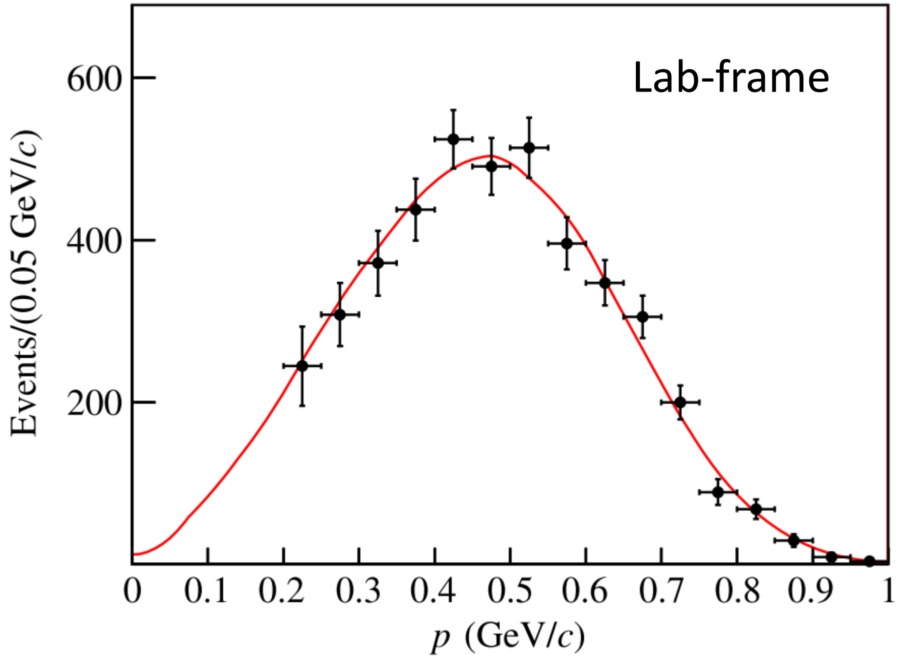
by BES-III

$\mathcal{B}(\Lambda_c \to X e \bar{\nu}_e)$

- Similar analysis strategy to the $\mathcal{B}(D_s \to X e \bar{\nu}_e)$ measurement
- But
  - multiple single-tag channels used
  - uses data taken at $\sqrt{s} = 4.600, 4.612, 4.628, 4.640, 4.661, 4.682, 4.698$ GeV
- $\mathcal{B}(\Lambda_c \to X e \bar{\nu}_e) = (4.06 \pm 0.10 \pm 0.09)\%$ with momentum spectrum
- Inclusive branching fraction **not** saturated by sum-of-exclusive decays
- $\frac{\Gamma(\Lambda_c \to X e \bar{\nu}_e)}{\Gamma(D \to X e \bar{\nu}_e)} = 1.28 \pm 0.05$
  favors 1.2 from HQE and
  disfavors 1.67 from the effective-quark-method

$\mathcal{B}(\Lambda_c \to X e \bar{\nu}_e)$

- BES-III
  Phys. Rev. D 107, 052005 (2023)



Source	Value (%)
Tracking efficiency for positron	0.4
ST signal shape	1.0
A_{PID} and A_{TRK} matrices	0.9
Momentum extrapolation	1.6
ST Yields method for RS and WS positrons	0.6
Muon contamination treatment	0.2
Total	2.2

Summary and Conclusion

$b \to c\ell\nu_\ell$

- New measurements of $\langle q^2 \rangle$, complementing existing $\langle M_X \rangle$ and $\langle E_\ell \rangle$
- Tension in the HQE fits for both approaches
- Combined study of the results is necessary for both experiment (to determine correlations) and theory (to understand ρ_D)

$b \to u\ell\nu_\ell$

- Differential rates crucial for the determination of the shape functions
- New insights into the inclusive-exclusive puzzle through combined analysis

$c \to d\ell\nu_\ell$

- The new measurements help to start investigating HQE in the charm sector
- Could already provide enough information to determine moments from the results to perform first HQE fits.