

Recent physics results from Belle II including $B^+ \rightarrow K^+ \nu \bar{\nu}$



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Apr. 11, 2024 for DIS 2024



Overview

- Quick intro. to Belle II

Part I *B decays*

- Test of LFU at Belle II

✓ Exclusive $R(D^{(*)})$

✓ Inclusive $R(X_{\tau/\ell})$

arXiv:2401.02840
submitted to PRD



arXiv:2311.07248
submitted to PRL



- $B^+ \rightarrow K^+ \nu \bar{\nu}$

arXiv:2311.14647
PRD accepted



Part II *charm baryons*

- $\Xi_c^0 \rightarrow \Xi^0 h^0$ ($h^0 = \pi^0, \eta, \eta'$)

in preparation for JHEP



Part III *Energy scan for bottomonia*

- new results on $\Upsilon(10753)$

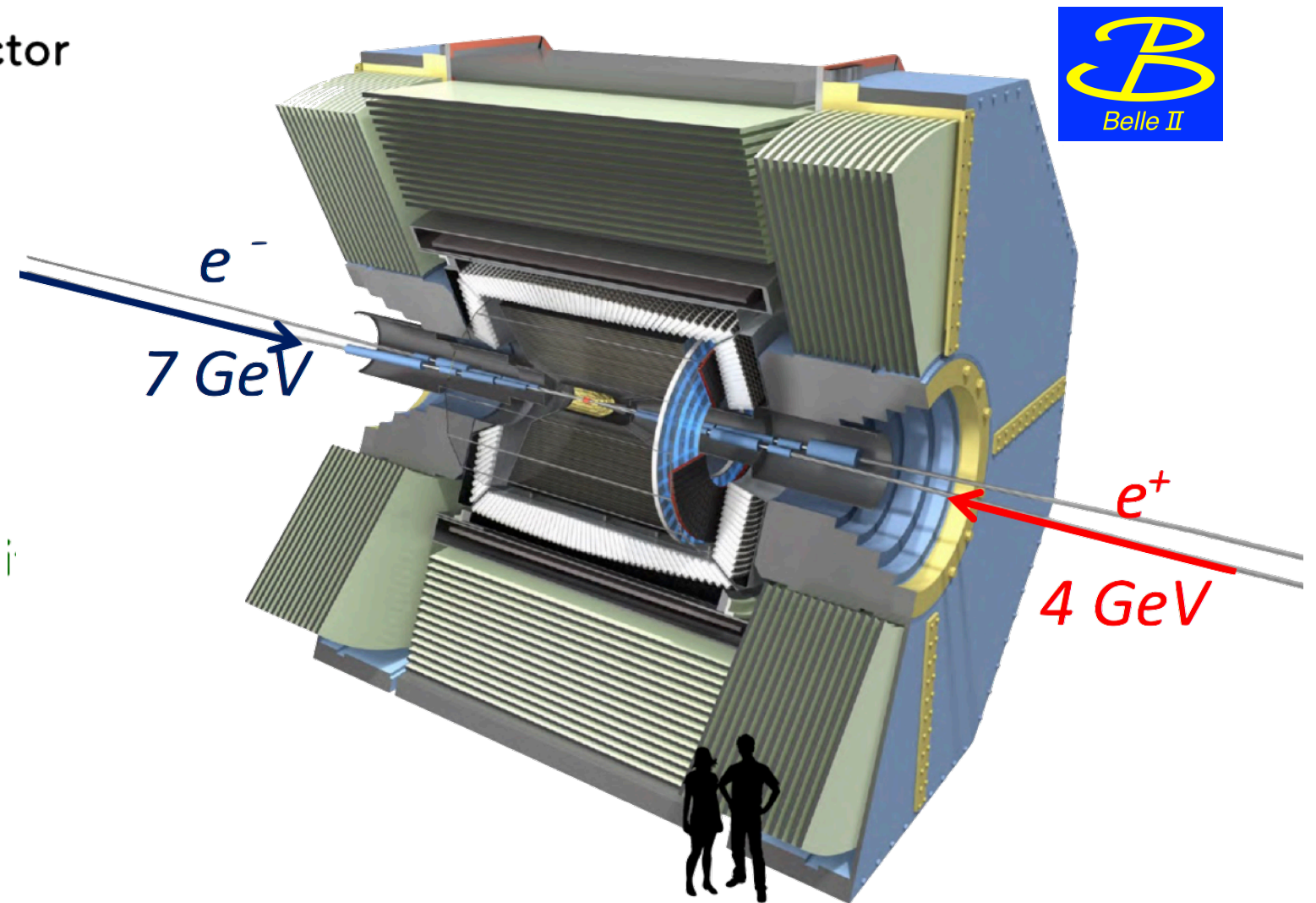
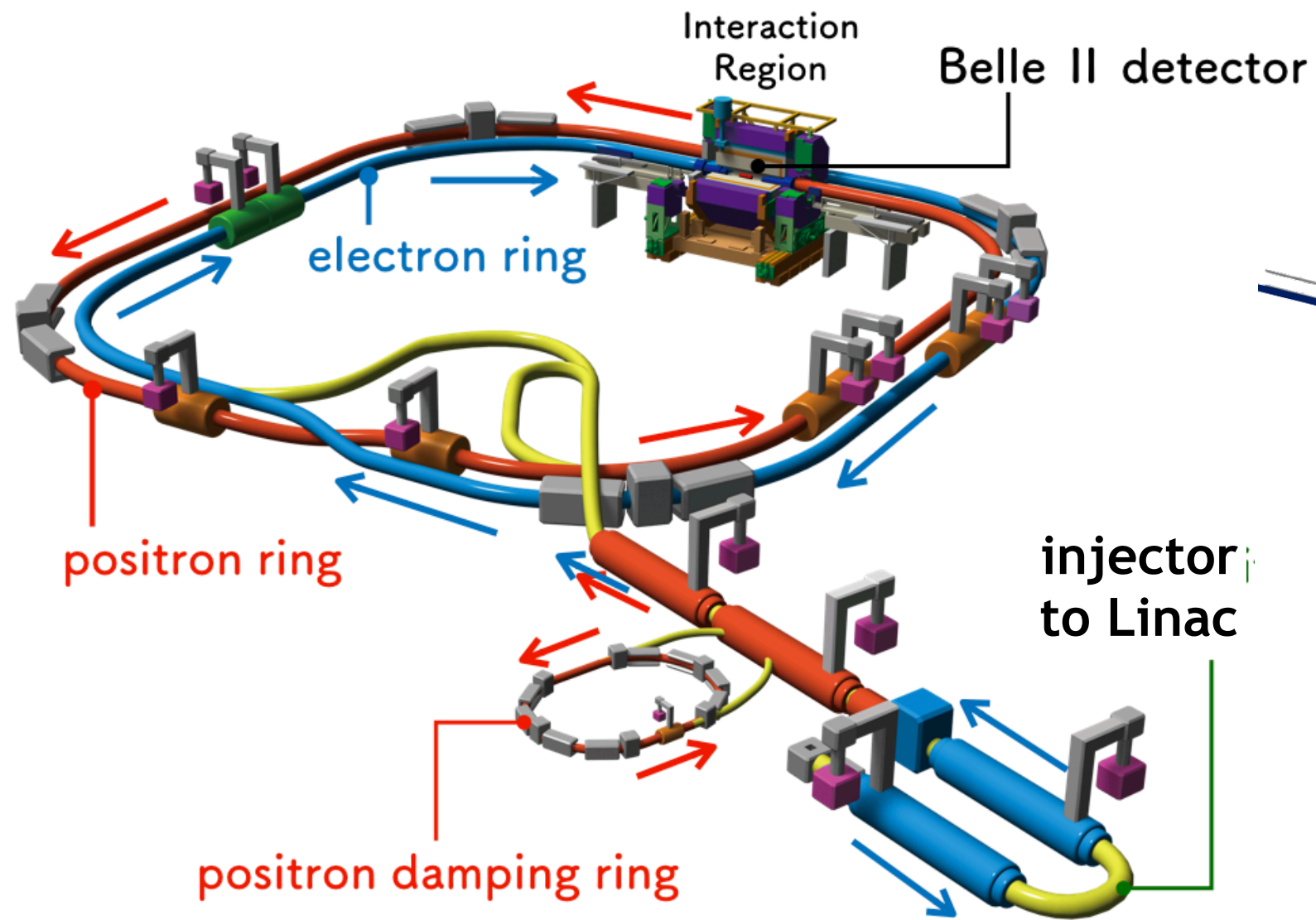
arXiv:2401.12021, JHEP submitted
arXiv:2312.13043, PRD accepted



- Closing

SuperKEKB

Belle II



$$e^- \xrightarrow{7 \text{ GeV}} (\star) \xleftarrow{4 \text{ GeV}} e^+$$

$$\sqrt{s} = 10.58 \text{ GeV} = m_{\Upsilon(4S)} c^2$$

We also have data taken off-resonance as well as energy scan around $\Upsilon(5S)$

- $\mathcal{B}(\Upsilon(4S) \rightarrow B\bar{B}) > 96\%$, with $p_B^{CM} \sim 0.35 \text{ GeV}/c$

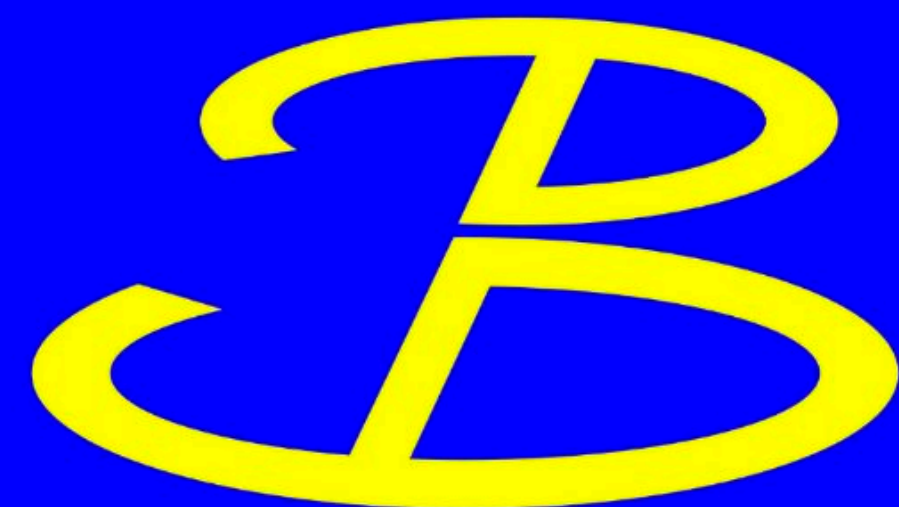
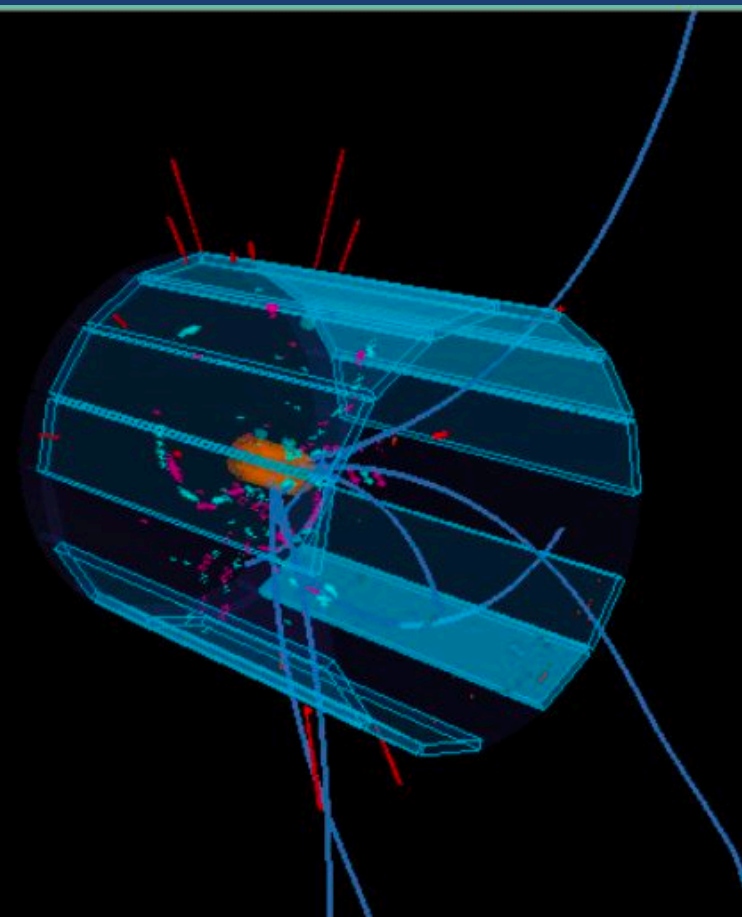
- nothing else but $B\bar{B}$ in the final state

\therefore if we know (E, \vec{p}) of one B , the other B is also constrained

See Appendix, p.35-37.

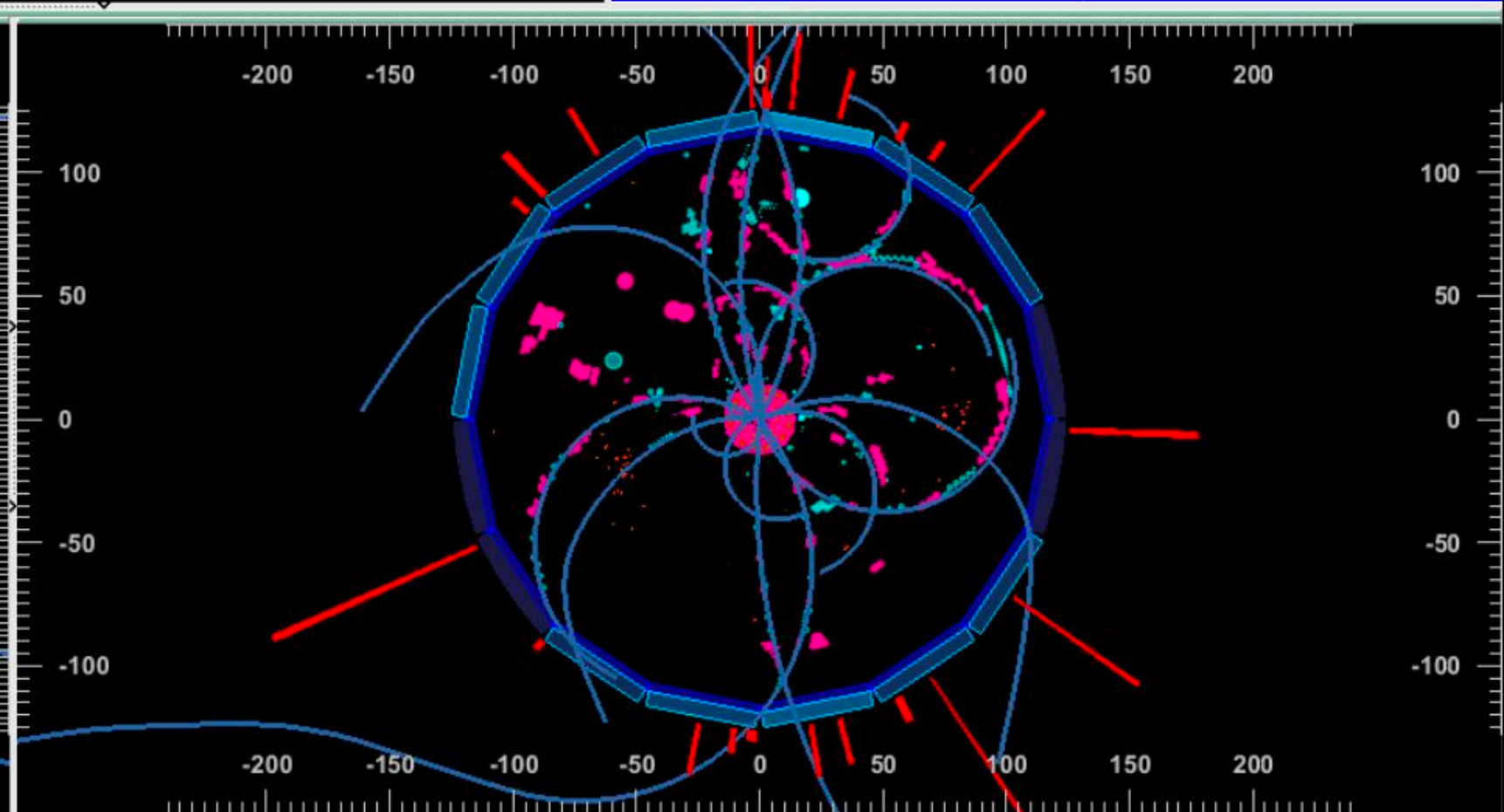
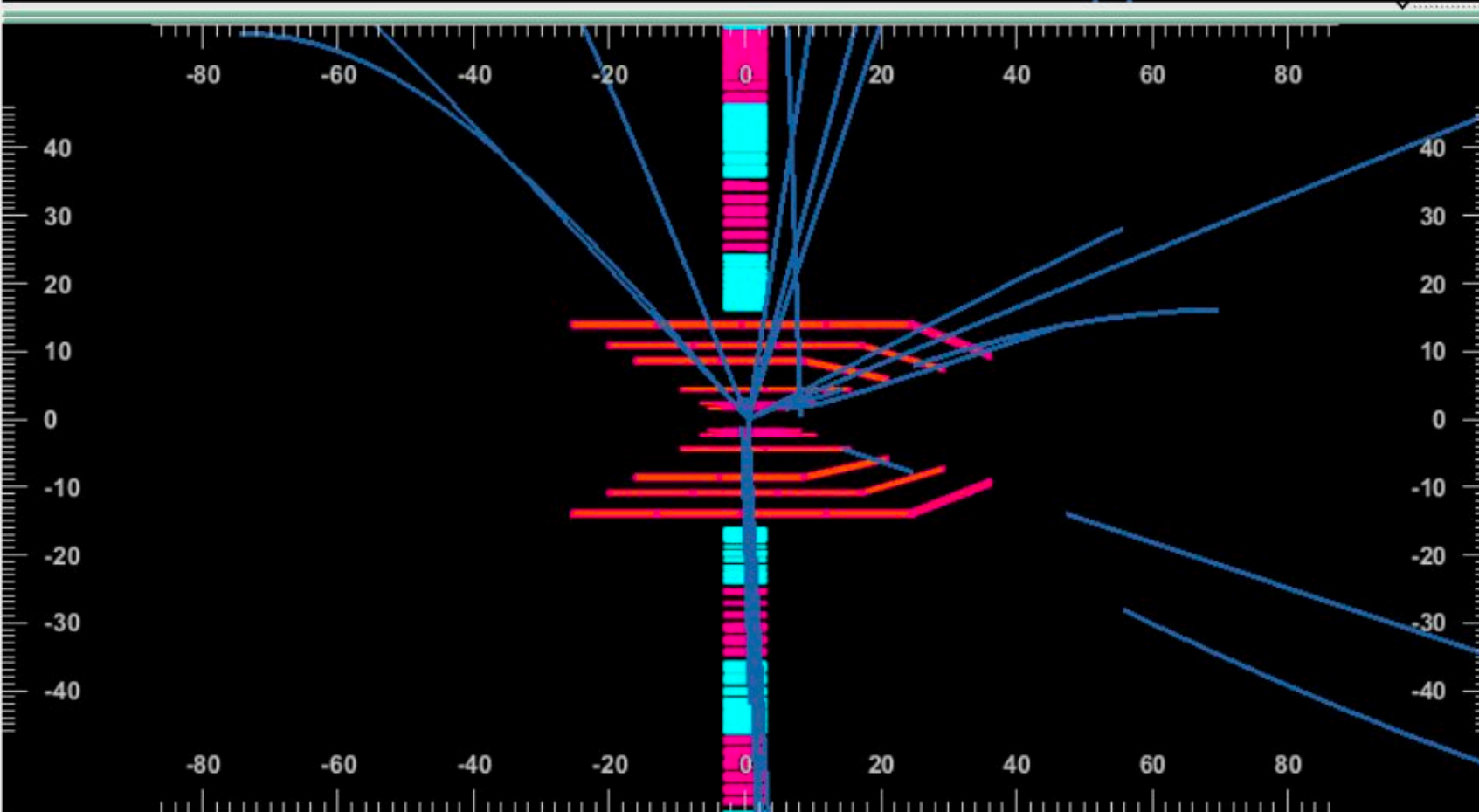
“B-tagging”

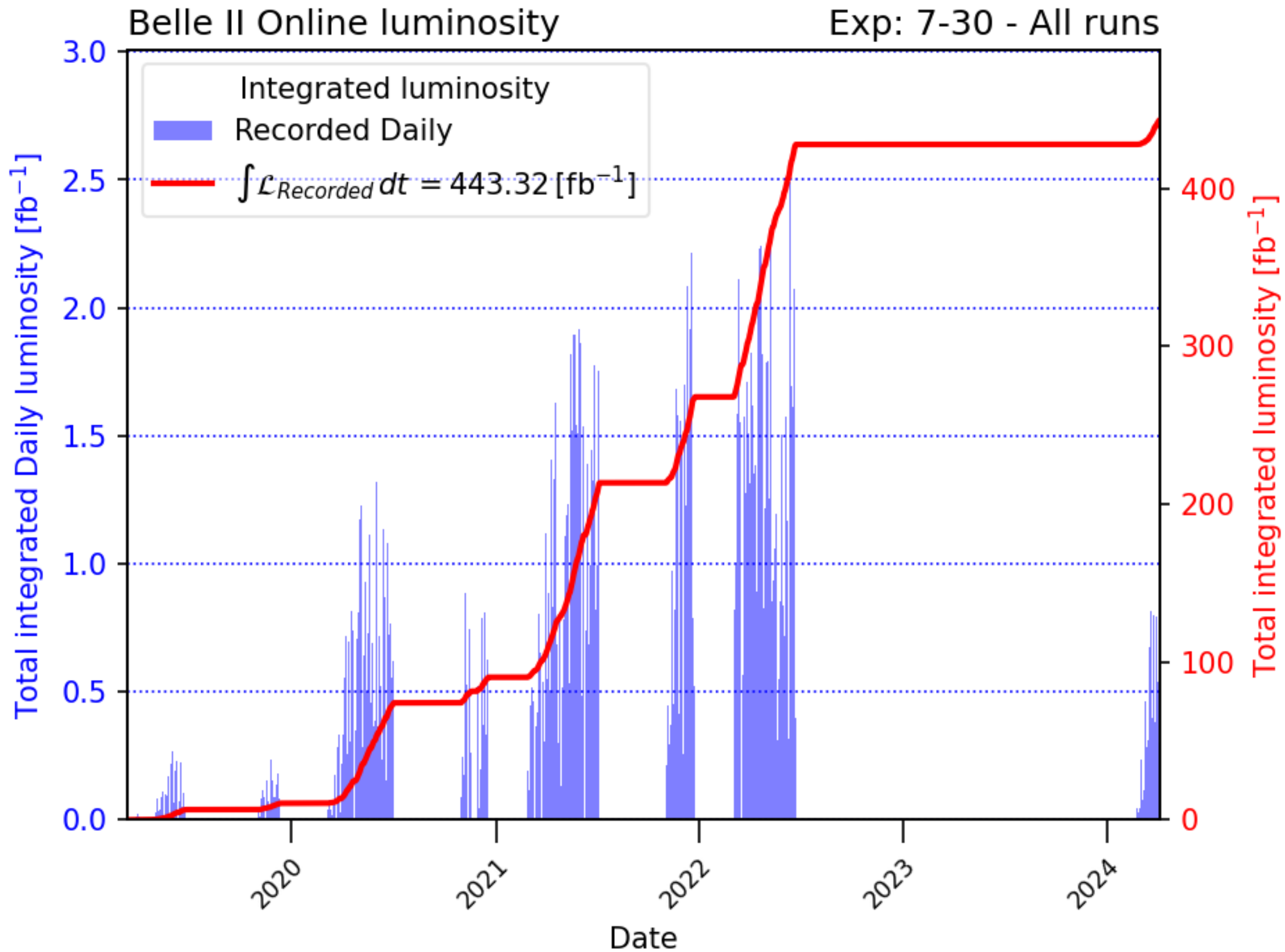
unique to e^+e^- B-factory



Belle II

First Collisions of Run 2
20th February 2024





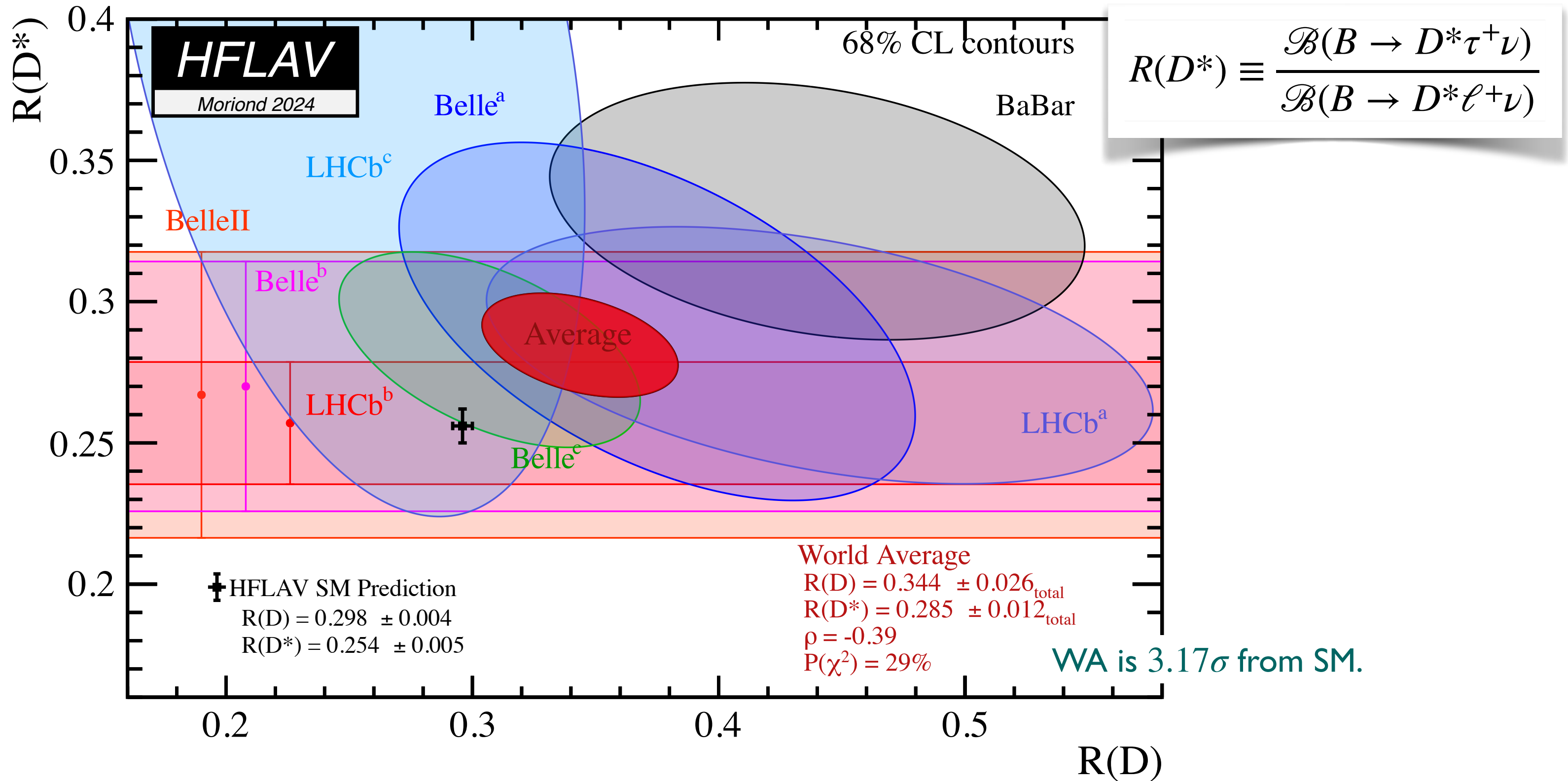
Updated on 2024/04/04 06:07 JST

Belle (1999-2010)
Luminosity

- $\int \mathcal{L}_{total} = 1039 \text{ fb}^{-1}$
980 fb⁻¹ for Ξ_c^0
- $\int \mathcal{L}_{\Upsilon(4S)} = 711 \text{ fb}^{-1}$

Part I *B* decays

LFU test via $R(D)$ vs. $R(D^*)$



For details of the Belle II $R(D^*)$ measurement, see Appendix, p.38-40.

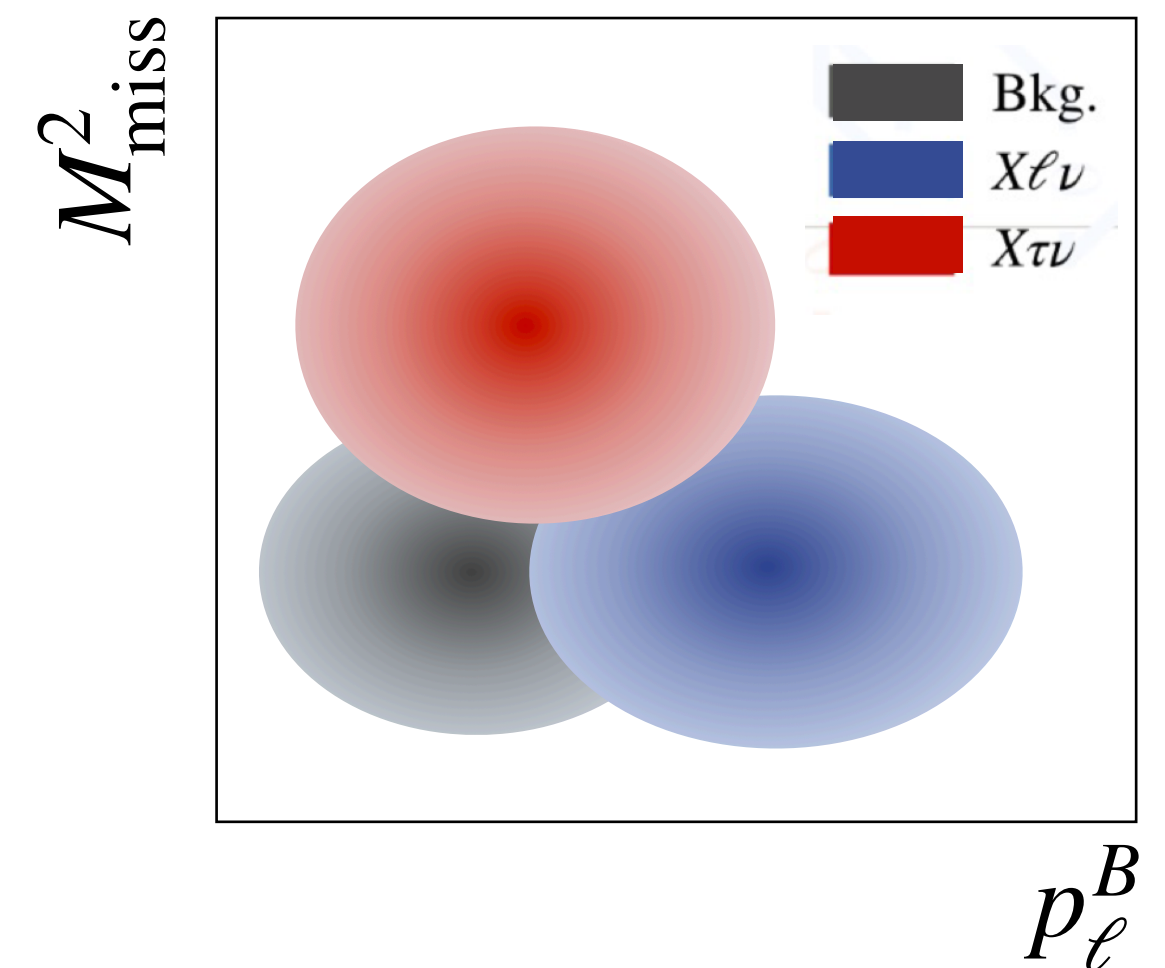
Inclusive LFU test w/ $R(X_{\tau/\ell})$

- Why measure $R(X_{\tau/\ell})$?
 - different systematics from $R(D^{(*)})$
 - hence, a complementary test of LFU

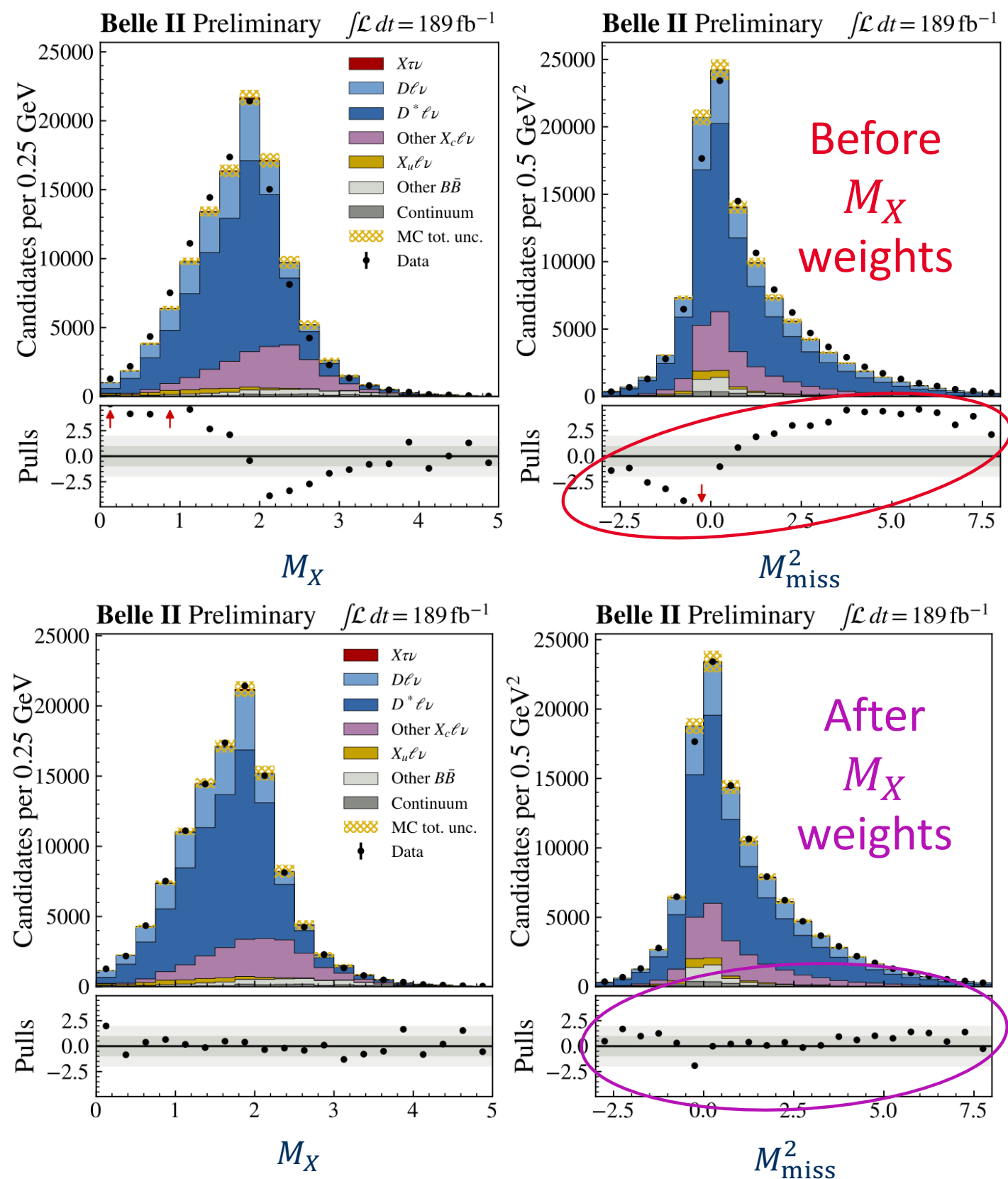
$$R(X_{\tau/\ell}) = \frac{\mathcal{B}(B \rightarrow X\tau\nu)}{\mathcal{B}(B \rightarrow X\ell\nu)}$$

- Procedure

- use $\tau \rightarrow \ell\nu_{\tau}\bar{\nu}_{\ell}$ modes
- select events with $B_{\text{tag}} + \ell$, with remaining particles attributed to X
- distinguish signal from background by using M_{miss}^2 and p_{ℓ}^B
- background mostly from $b \rightarrow c \rightarrow \ell$; some continuum and fake leptons



$R(X_{\tau/\ell})$, event distributions



- for reliable template shapes for fitting
 - make detailed adjustments to MC (FF's, B and D BF's)
 - corrections by comparing MC to data in control region: low q^2 , low M_{miss}^2 , high M_X
 - e.g. adjust M_X in $p_\ell > 1.4$ GeV sideband; using these weights also improves modeling in M_{miss}^2 and q^2

Main sources of systematic uncertainty:

- MC stat $\pm 5.7 \%$
- Bkg shape $\pm 5.5 \%$
- M_X modeling $\pm 7.1 \%$
- $B \rightarrow X_c\ell\nu$ BFs $\pm 7.7 \%$
- $B \rightarrow X_c\ell\nu$ FFs $\pm 7.9 \%$

$R(X_{\tau/\ell})$ Results

$$R(X_{\tau/\ell}) = 0.228 \pm 0.016 \pm 0.036$$

$$R(X_{\tau/e}) = 0.232 \pm 0.020 \pm 0.037$$

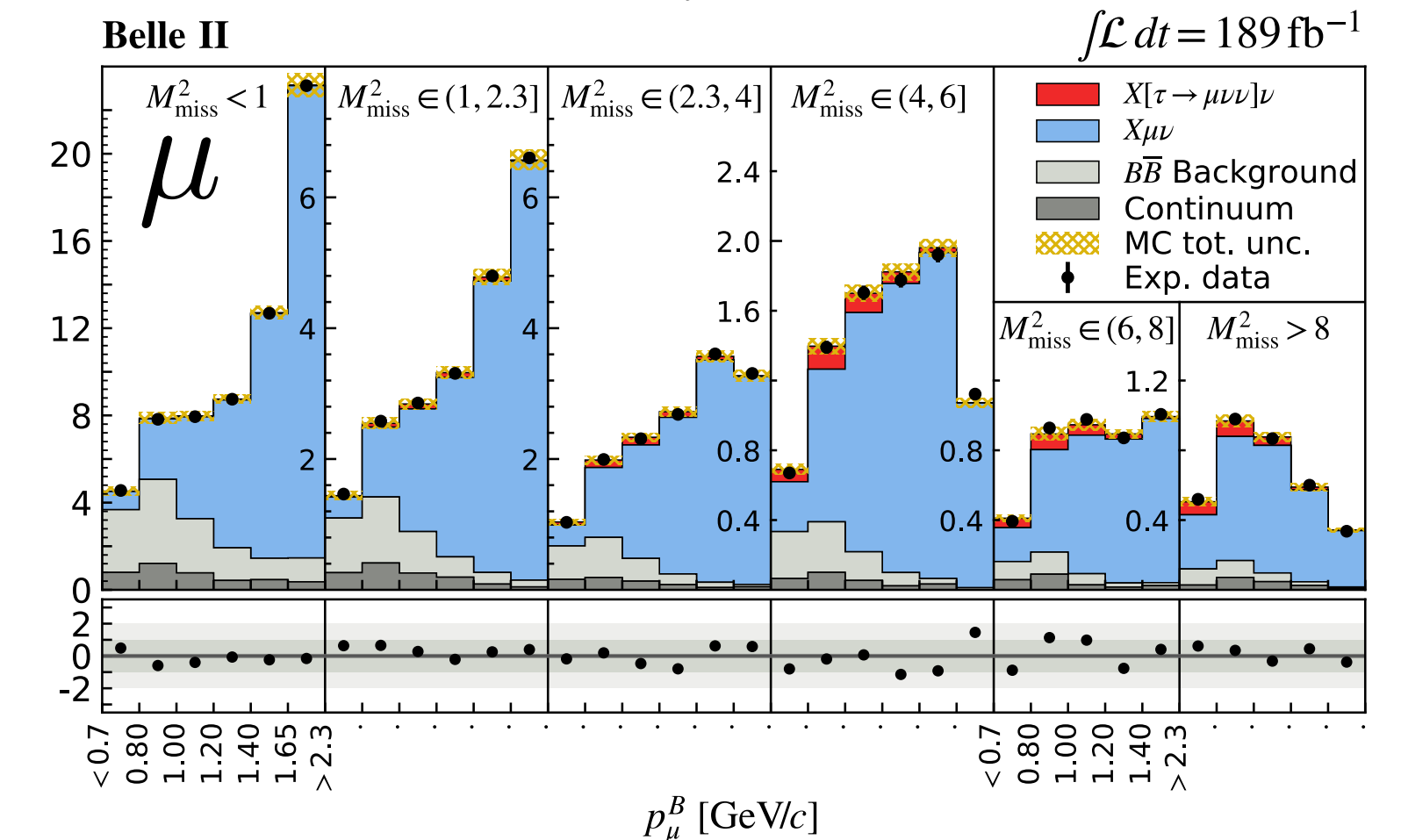
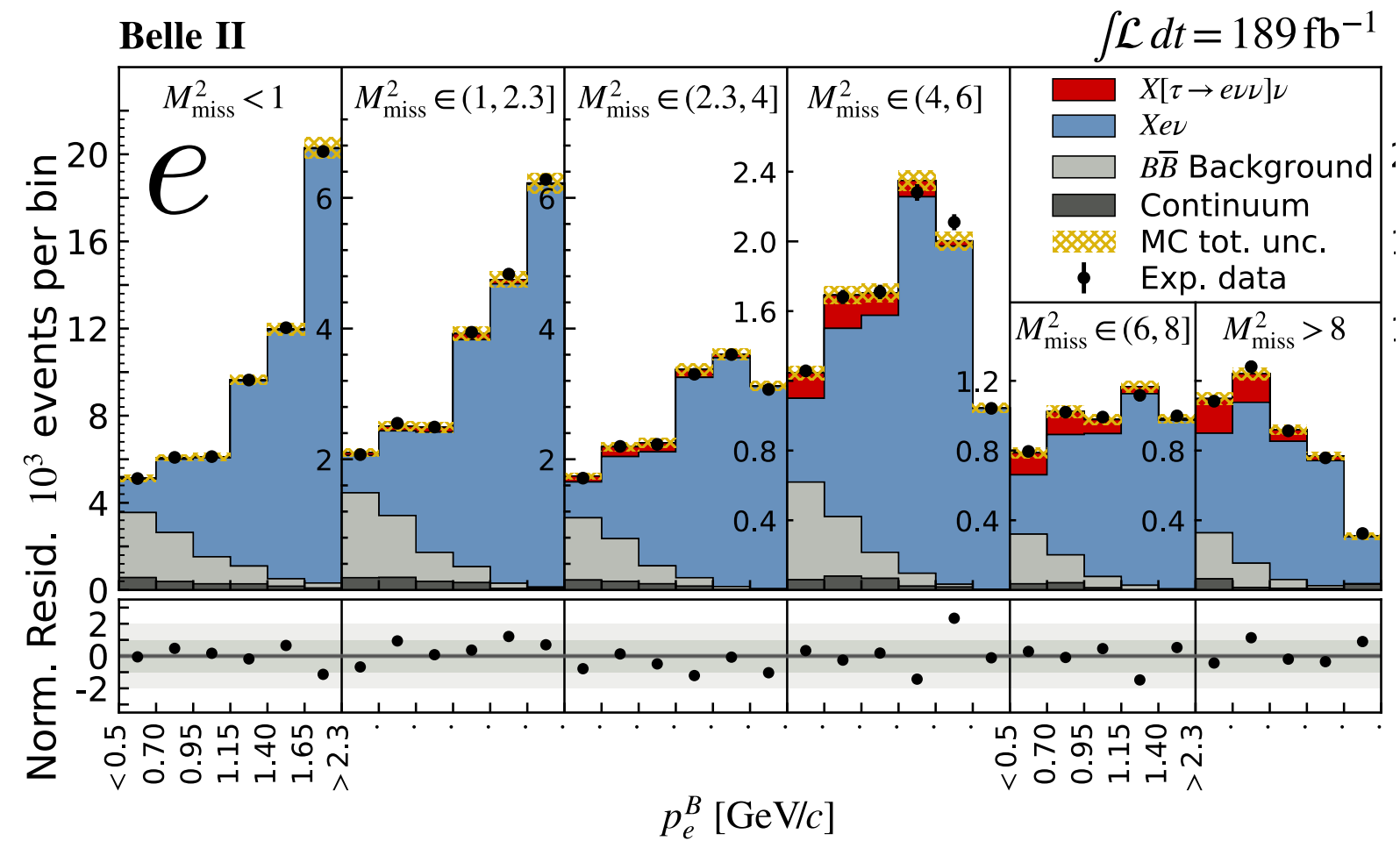
$$R(X_{\tau/\mu}) = 0.222 \pm 0.027 \pm 0.050$$

Consistent with SM: 0.223 ± 0.005

M. Freytsis et al. [PRD 92, 054018 \(2015\)](#)

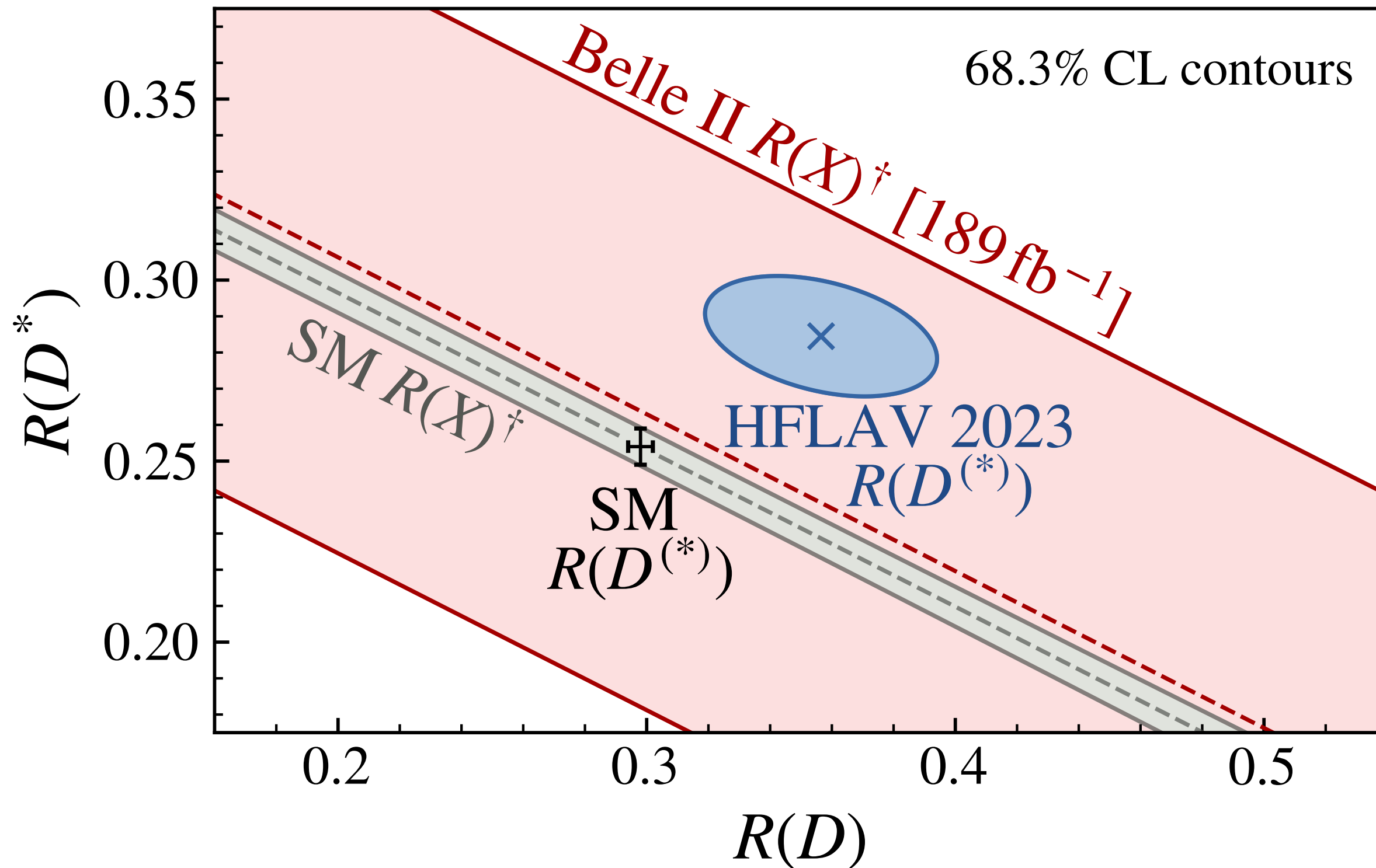
M. Rahimi, K. K. Vos, [JHEP 2022, 7 \(2022\)](#)

Z. Ligeti et al. [PRD 105, 073009 \(2022\)](#)



$R(X_{\tau/\ell})$, compared with $R(D^{(*)})$

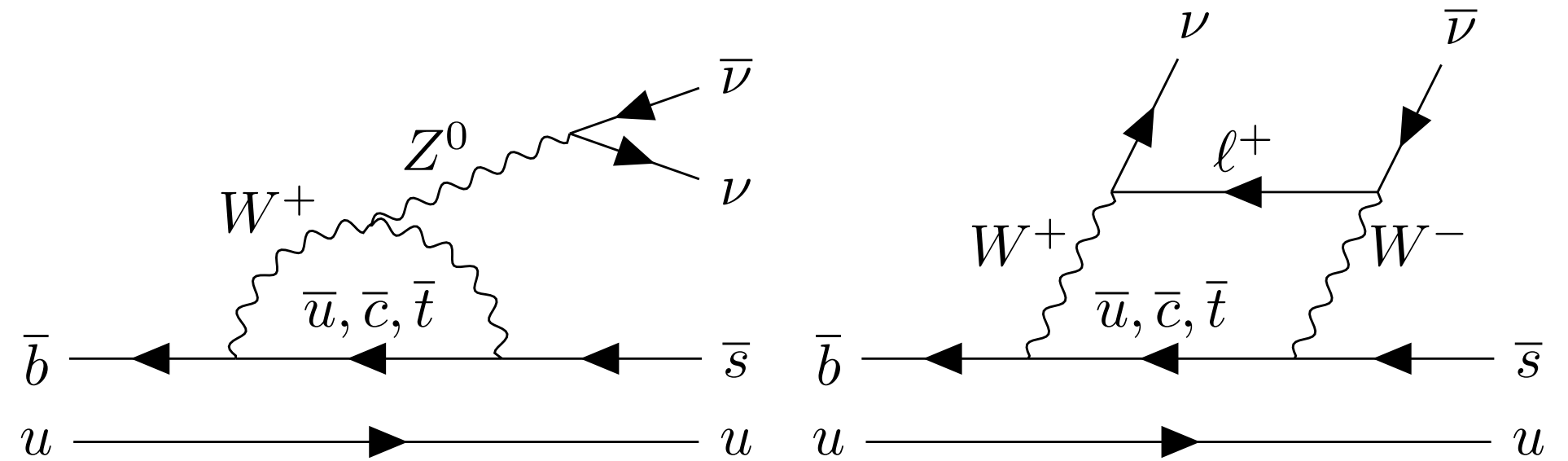
† = with expected SM contributions of $D_{(\text{gap})}^{**}$, X_u removed



Search for $B^+ \rightarrow K^+ \nu \bar{\nu}$ at Belle II

- In the SM,
 - $\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = (5.58 \pm 0.37) \times 10^{-6}$ [4]
- sensitive to new physics BSM, e.g.
 - leptoquarks,
 - axions,
 - DM particles, etc.

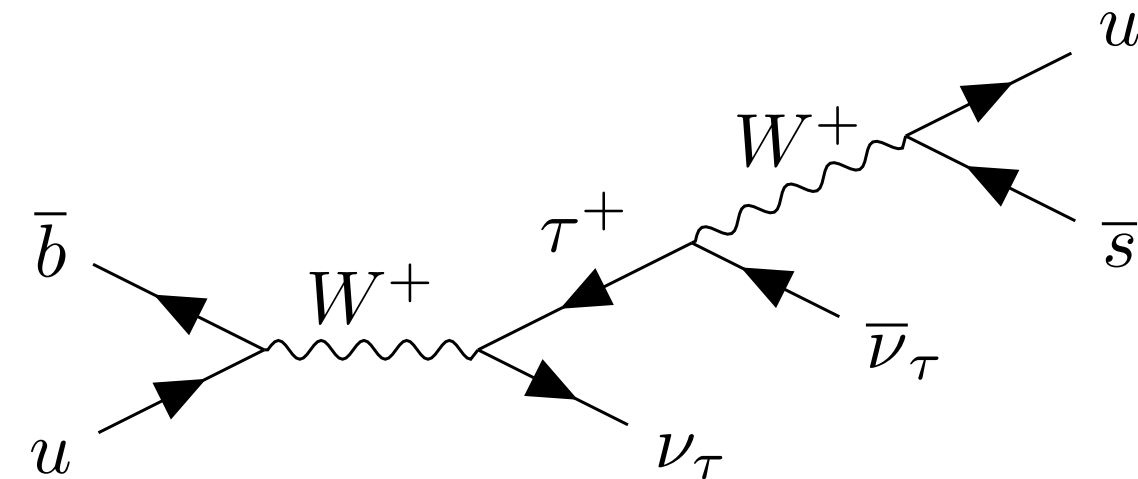
[4] W. G. Parrott et al. PRD 107, 014511 (2023)
incl. long-distance contribution from $B \rightarrow \tau \nu$



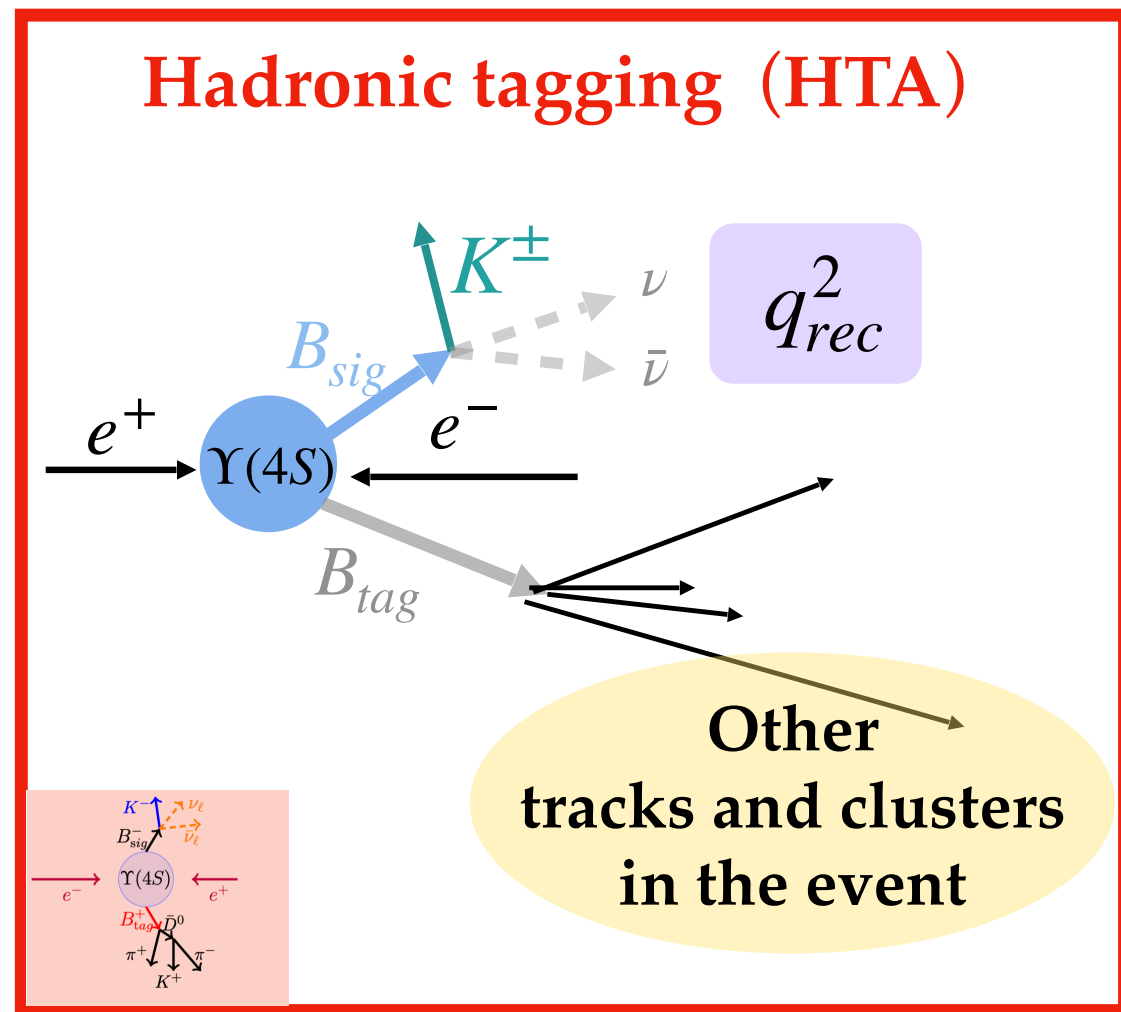
PRL 127, 181802 (2021)

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = (1.9_{-1.3}^{+1.3+0.8}) \times 10^{-5}$$

$$< 4.1 \times 10^{-5} \quad @ 90\% \text{ CL}$$



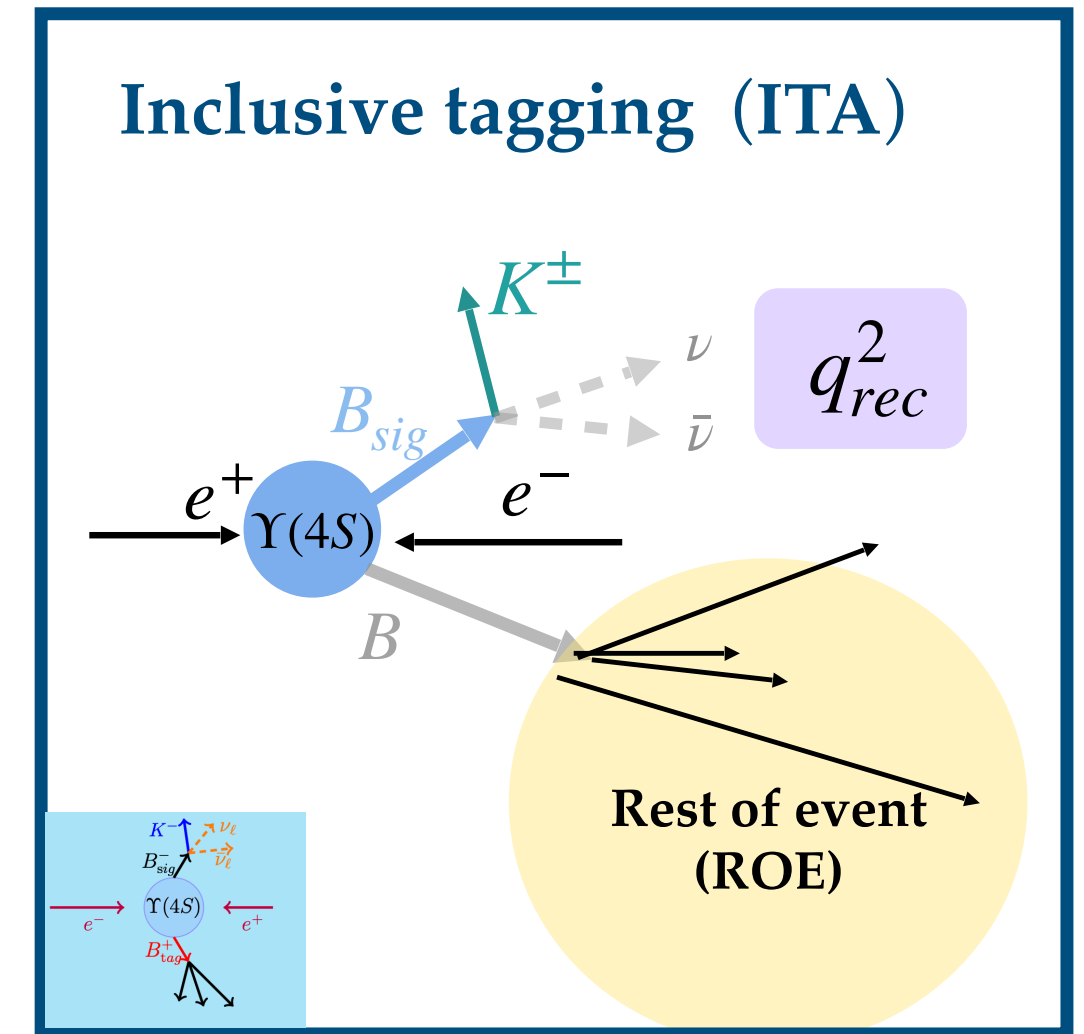
Two ways of tagging



Efficiency

q_{rec}^2 : mass squared of the neutrino pair

Purity, Resolution



● Features of HTA

- uses full decay chain information of B_{tag}
- high high purity, very low efficiency
- uses BDT for signal extraction (BDT_h)

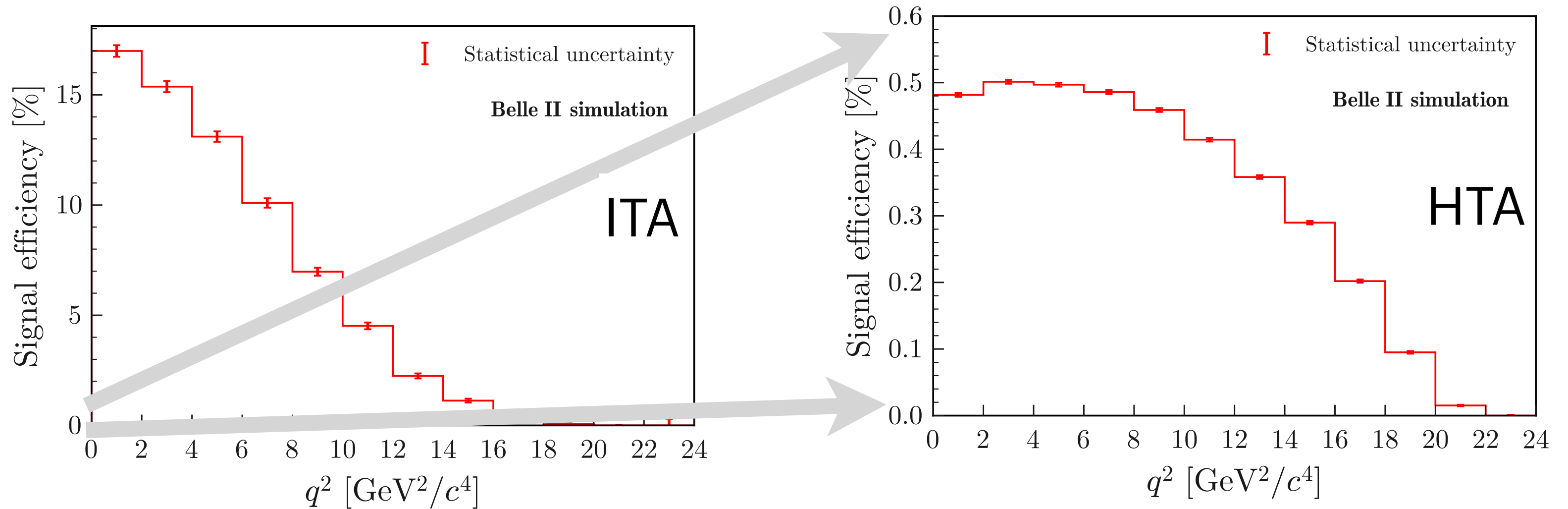
● Features of ITA

- exploits inclusive properties of B_{tag}
- high efficiency, low purity
- BDTs in two stages (BDT₁ mostly for $q\bar{q}$; BDT₂ for final signal extraction)

Signal efficiency (ITA vs. HTA)

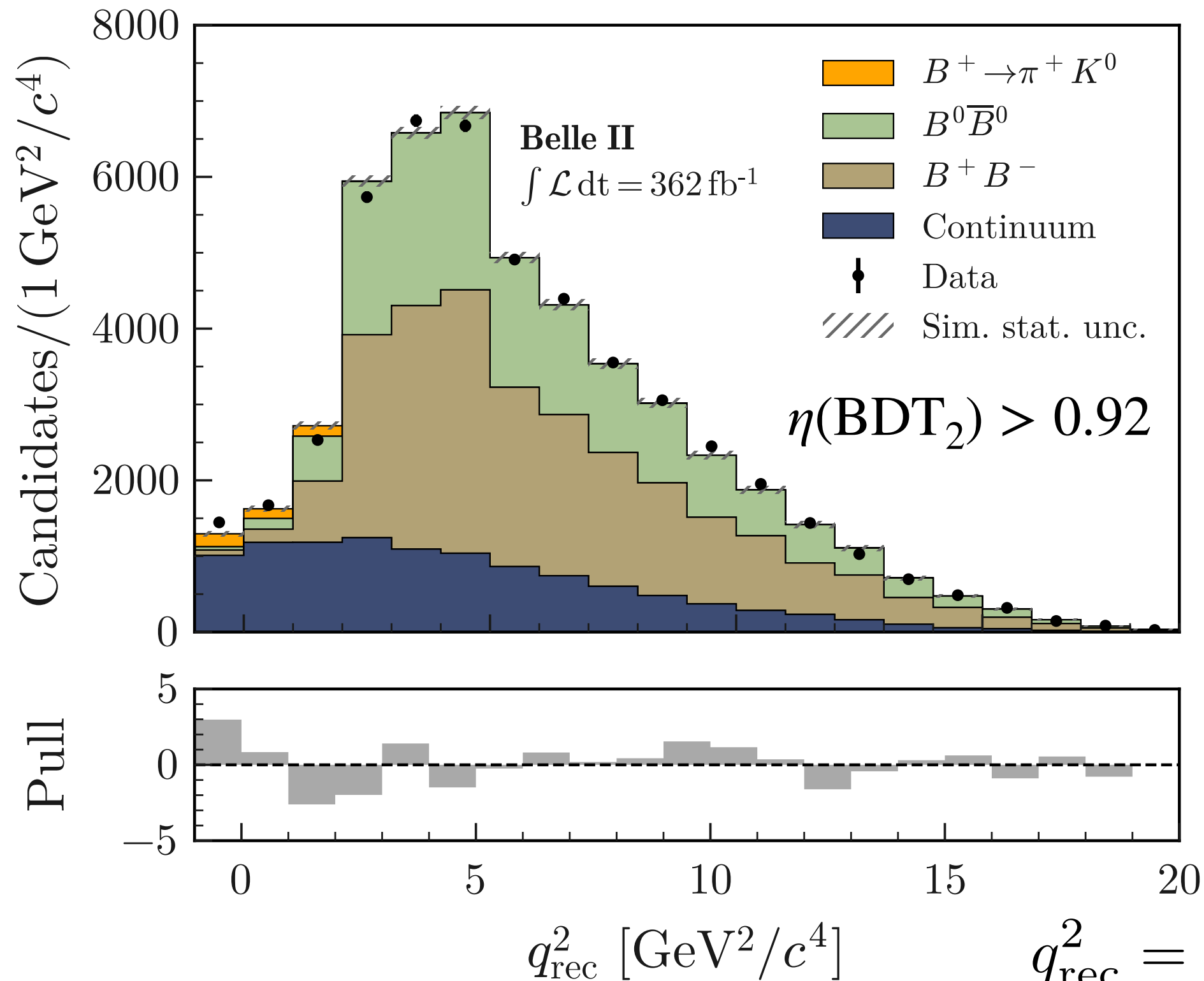
after multi-variate analysis for ROE with BDT

for BDT efficiency validation,
see p. 42 in the Appendix



$$q^2 = M(\nu\bar{\nu})^2$$

Closure test (ITA)



- Pion ID instead of kaon ID
- Different q_{rec}^2 bin boundaries
- Only on-resonance data used for fit
- Only normalization systematics included

Result:

○ $\mathcal{B}(B^+ \rightarrow \pi^+ K^0) = (2.5 \pm 0.5) \times 10^{-5}$

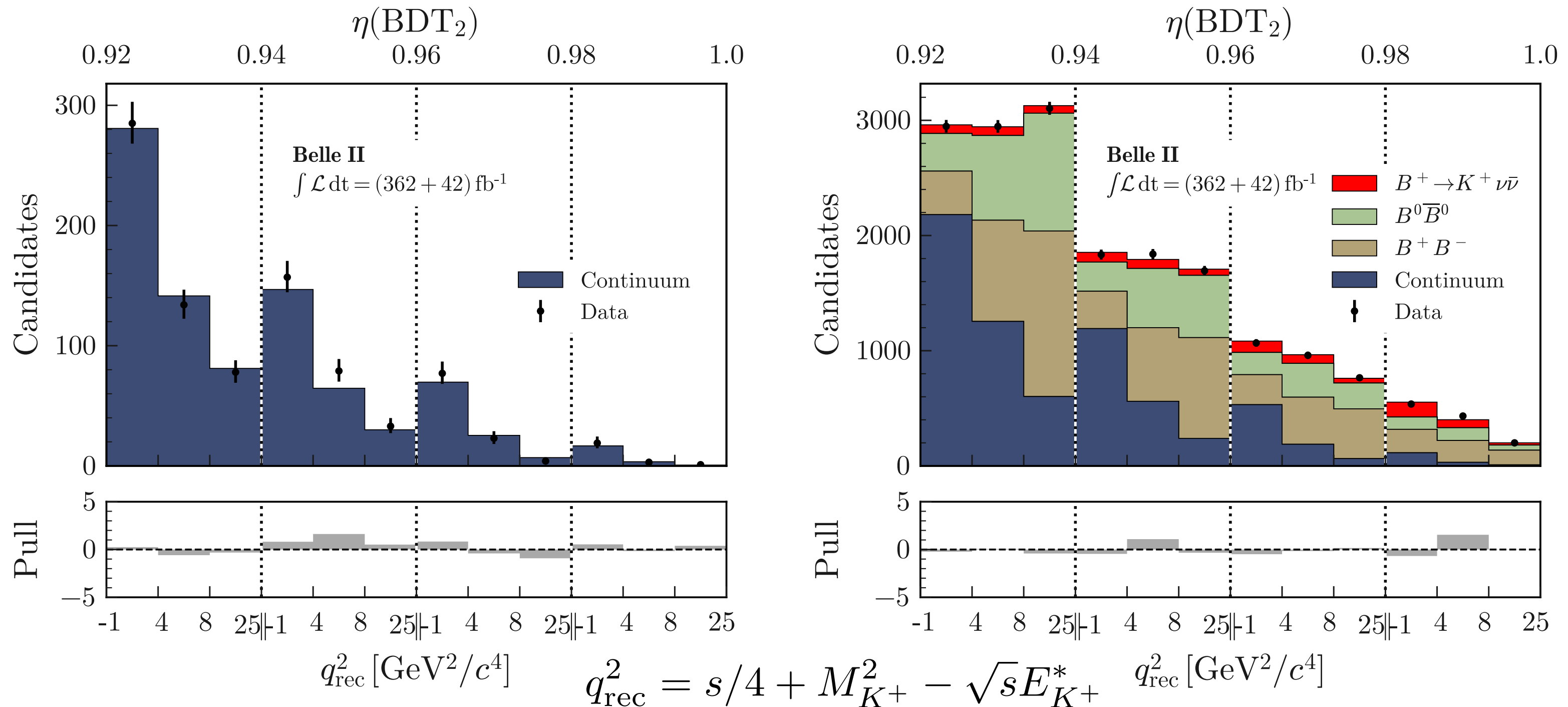
Consistent with PDG:

$\mathcal{B}(B^+ \rightarrow \pi^+ K^0) = (2.3 \pm 0.08) \times 10^{-5}$

$$q_{rec}^2 = s/4 + M_{\pi^+}^2 - \sqrt{s} E_{\pi^+}^*$$

Assume B is at rest in the $\Upsilon(4S)$ rest-frame ($c = 1$)

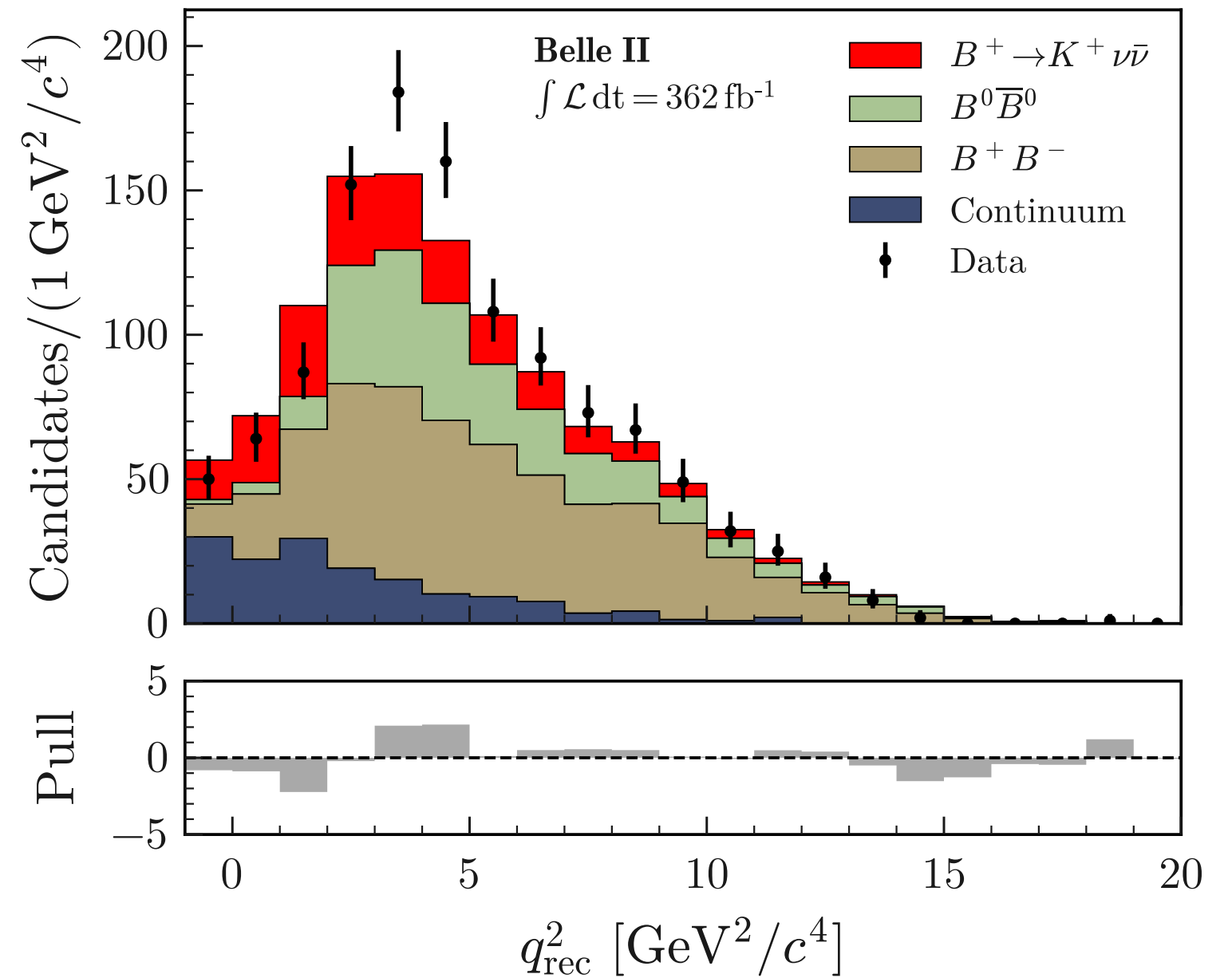
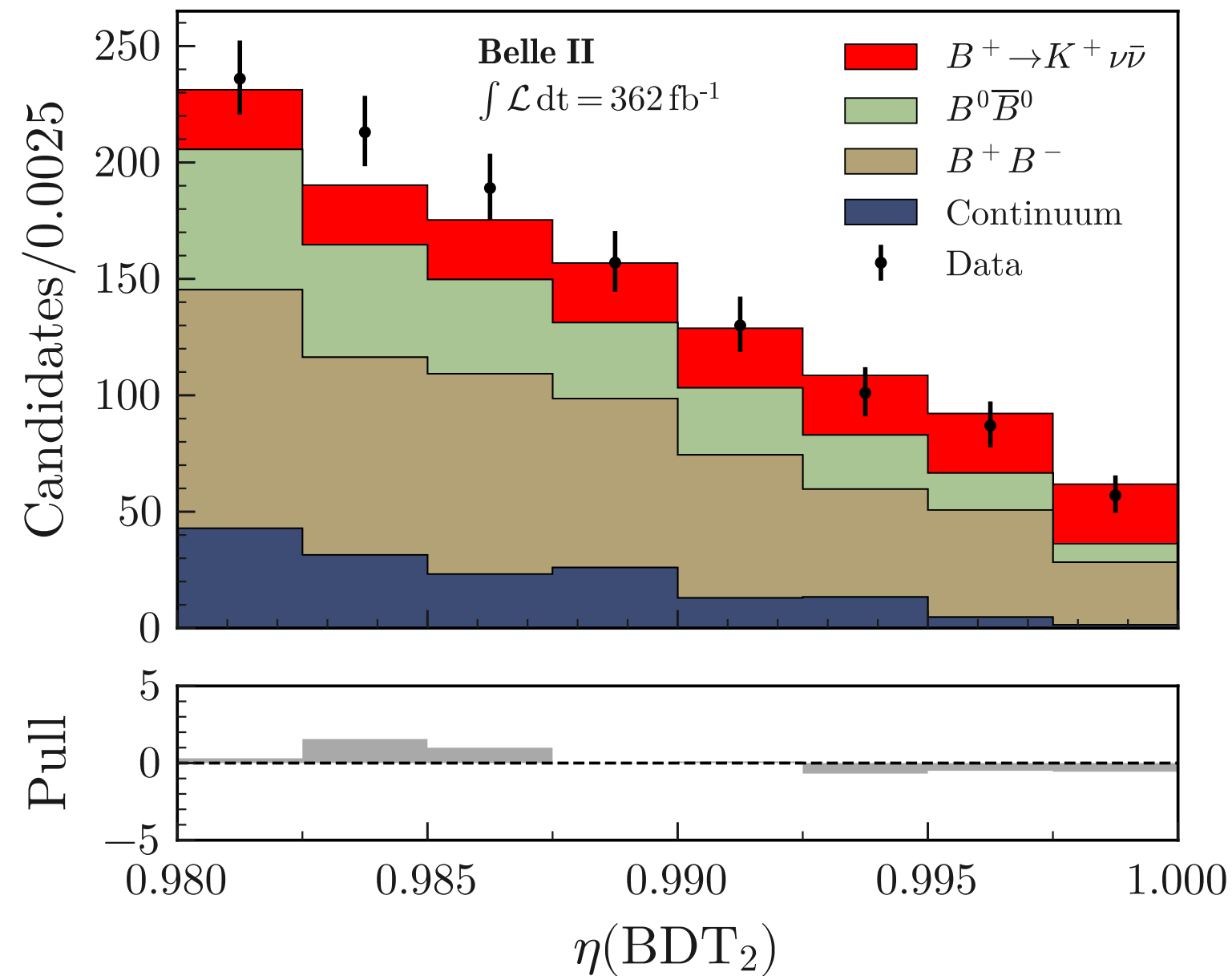
$B^+ \rightarrow K^+ \nu \bar{\nu}$ result (ITA)



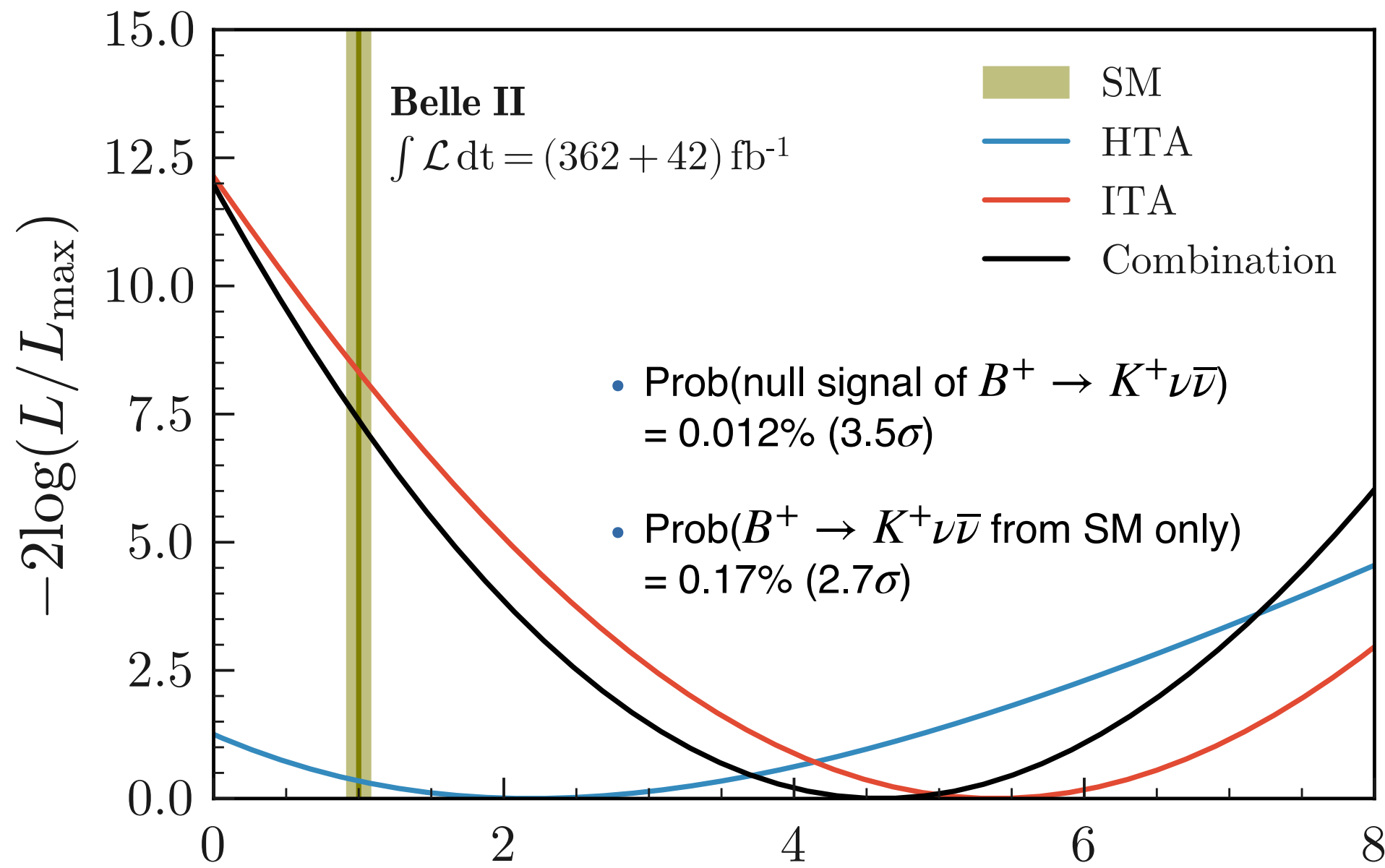
$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{ITA}} = (2.7 \pm 0.5 \pm 0.5) \times 10^{-5}$$

$B^+ \rightarrow K^+ \nu \bar{\nu}$ post-fit distributions (ITA)

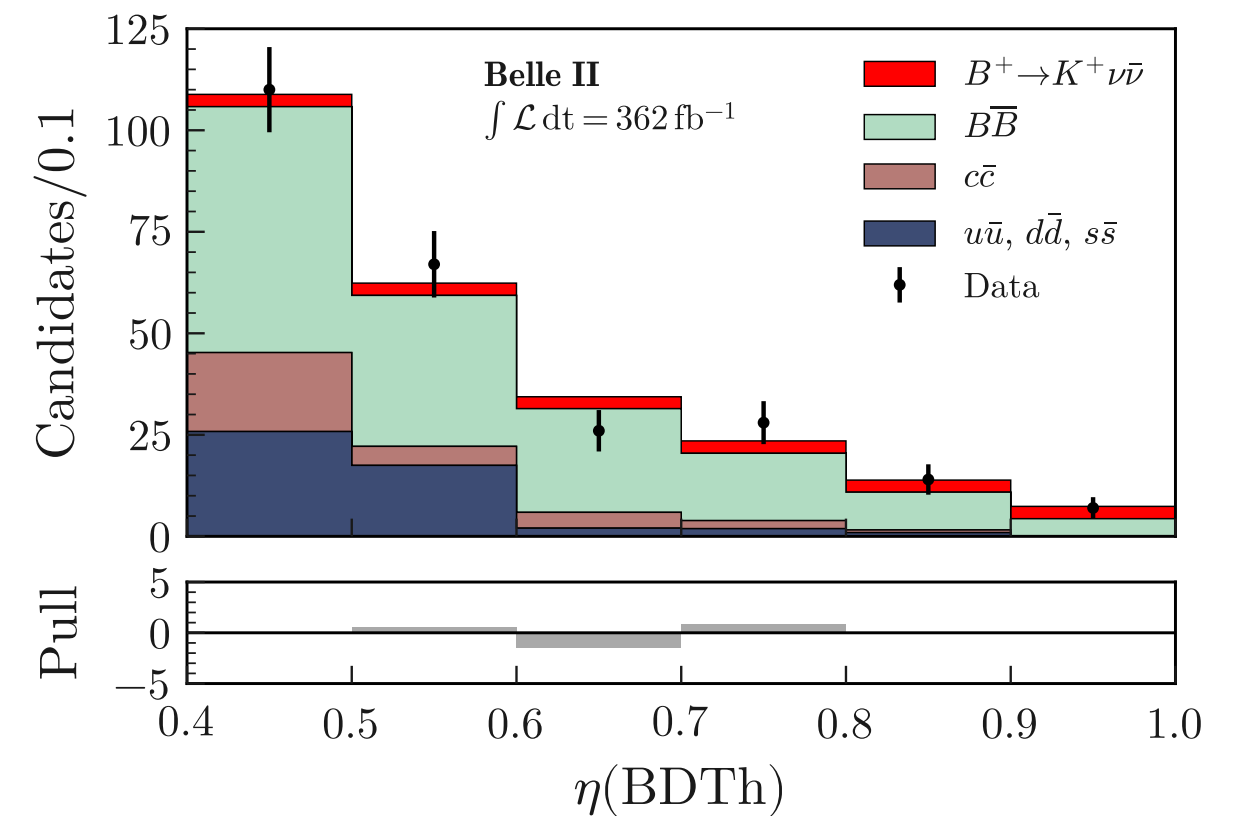
$$\eta(\text{BDT}_2) > 0.98$$



$$q_{\text{rec}}^2 = s/4 + M_{K^+}^2 - \sqrt{s} E_{K^+}^*$$



$B^+ \rightarrow K^+ \nu \bar{\nu}$ (combined)



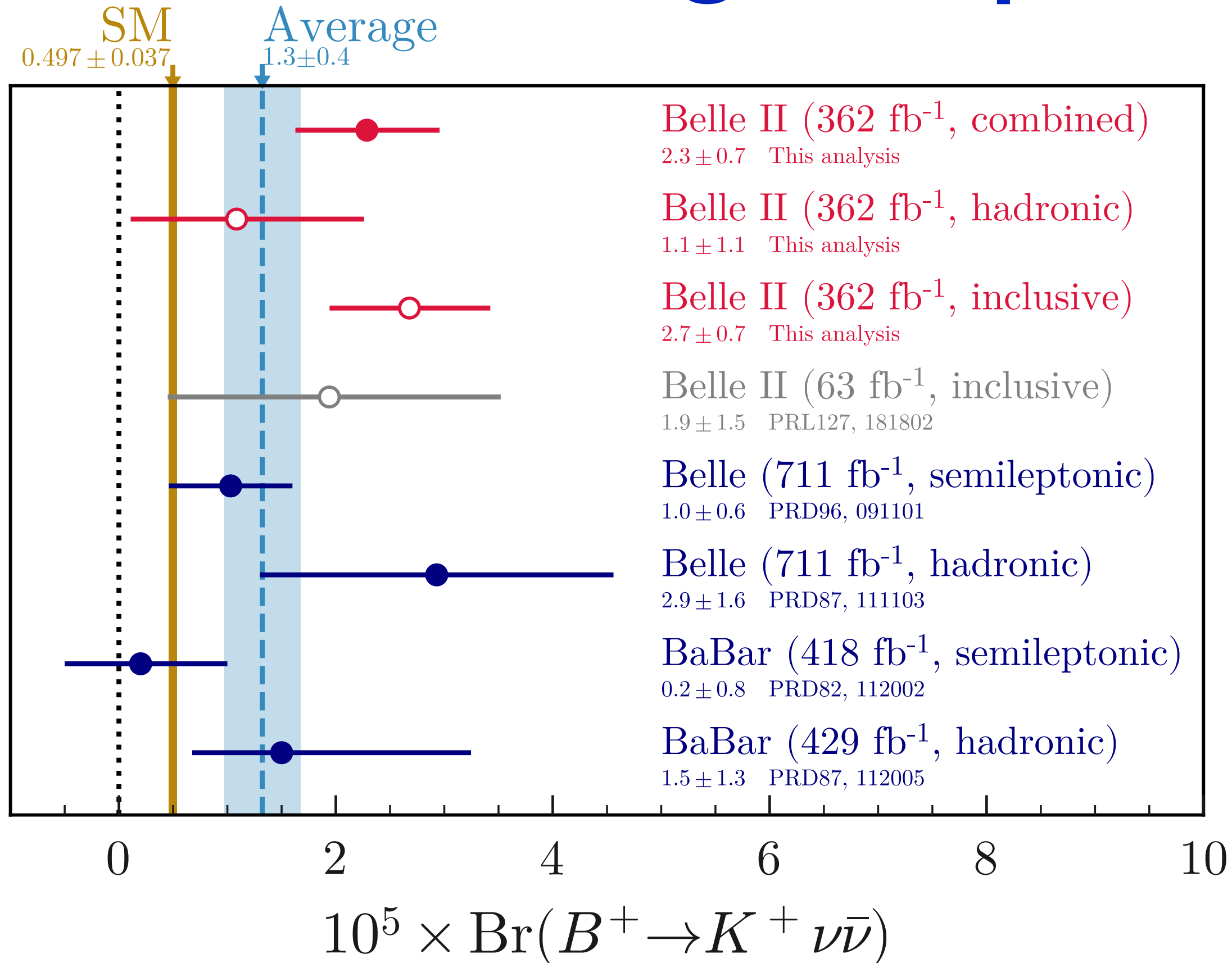
[Note] $\mu = 1 \Leftrightarrow \mathcal{B} = 4.97 \times 10^{-6} \mu$
(SM value not incl. $B \rightarrow \tau \nu$)

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{HTA}} = (1.1^{+0.9+0.8}_{-0.8-0.5}) \times 10^{-5}$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{ITA}} = (2.7 \pm 0.5 \pm 0.5) \times 10^{-5}$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{comb}} = (2.3 \pm 0.5^{+0.5}_{-0.4}) \times 10^{-5}$$

$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$ global picture



Part II *Charm baryon*

Charm baryon decays $\Xi_c^0 \rightarrow \Xi^0 h^0$

$(h^0 = \pi^0, \eta, \eta')$

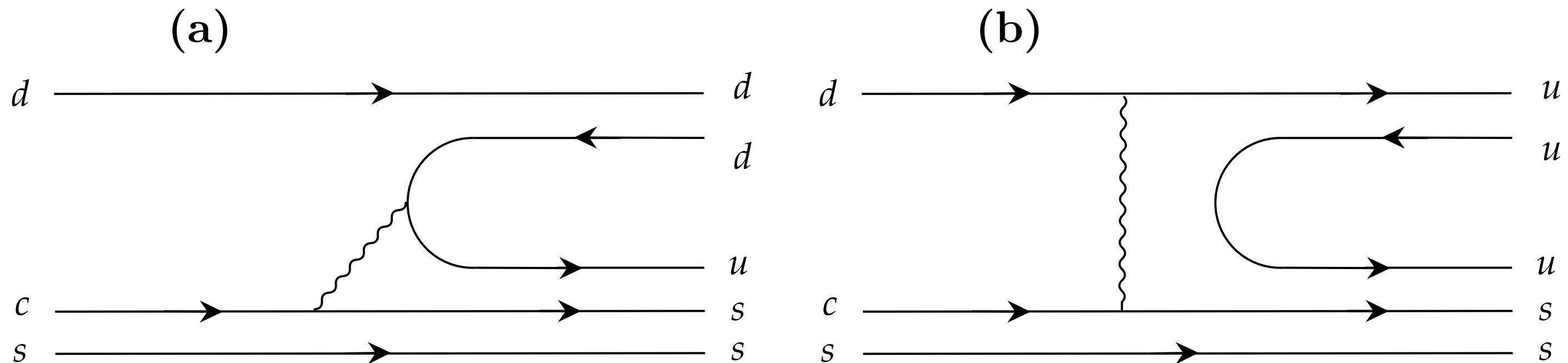
- Sensitive to (a) W-emission, and (b) W-exchange diagrams

- difficulties for theoretical predictions

Theory predictions vary in wide ranges for both BF and α
See Appendix, p.43

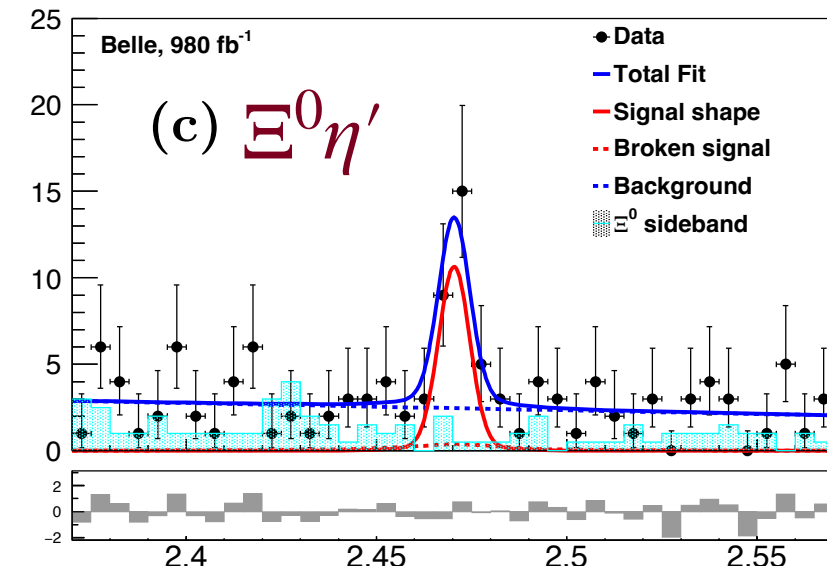
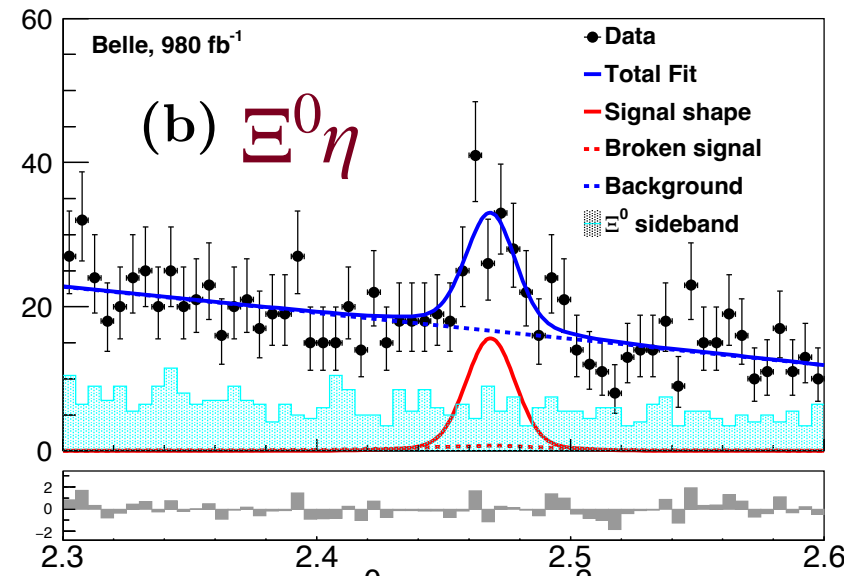
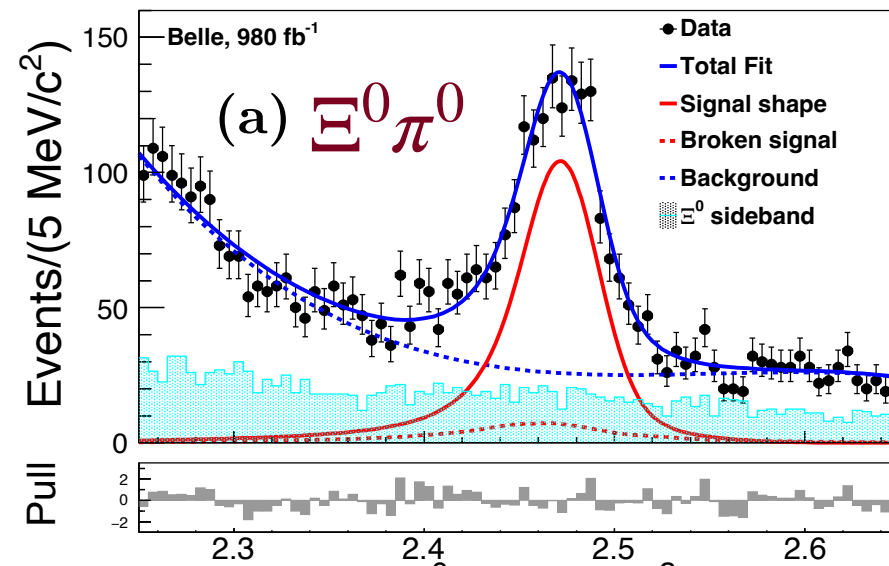
- measures BF and decay asymmetry parameter α

- in a combined data set of Belle (980/fb) + Belle II (426/fb)

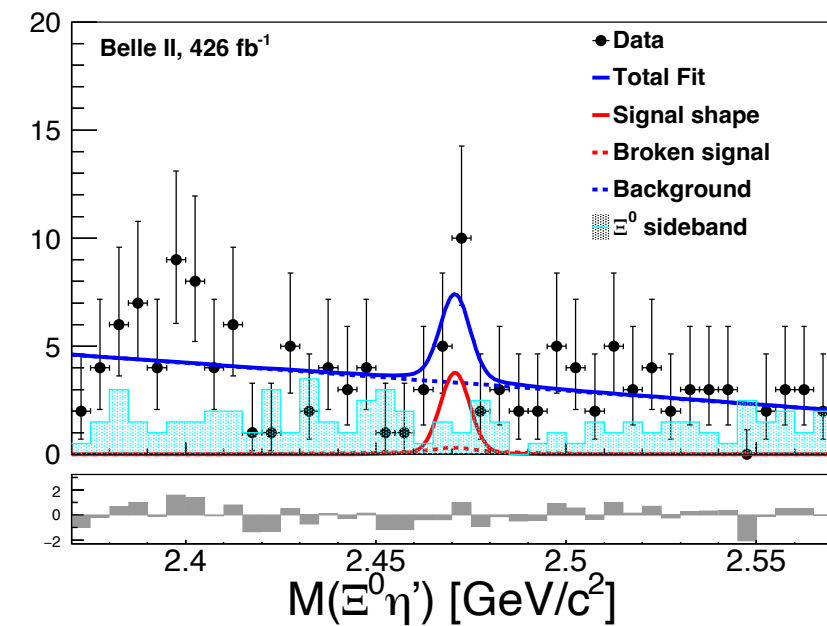
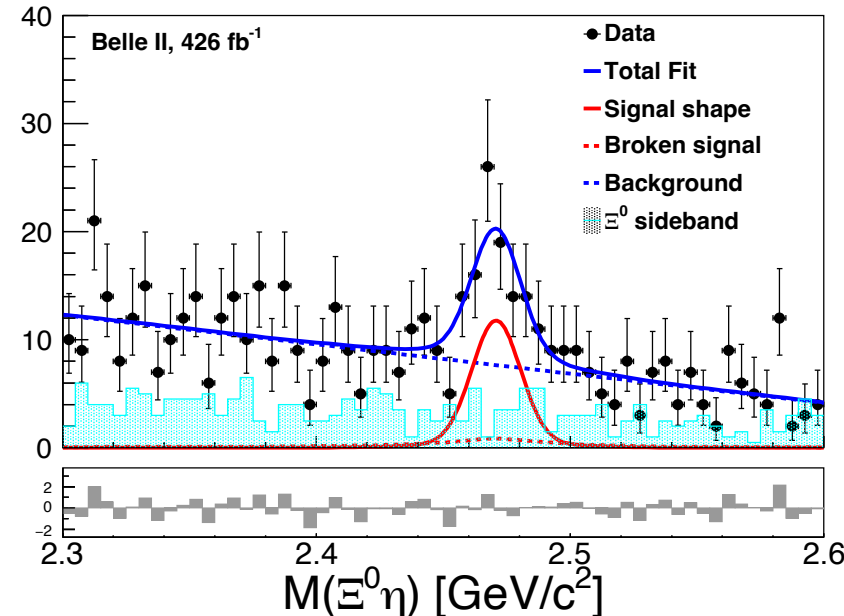
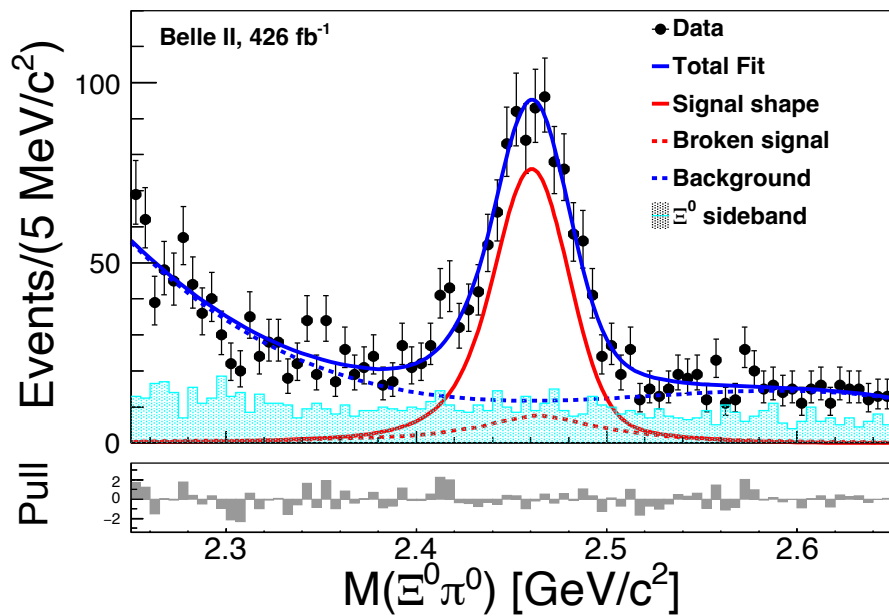


$$\frac{dN}{d \cos \theta_{\Xi^0}} \propto 1 + \alpha(\Xi_c^0 \rightarrow \Xi^0 h^0) \alpha(\Xi^0 \rightarrow \Lambda \pi^0) \cos \theta_{\Xi^0}$$

$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 h^0)$ branching fractions



Belle



Belle II

| Results | Belle | Belle II | Combined |
|---|--------------------------|--------------------------|--------------------------|
| $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ | $0.47 \pm 0.02 \pm 0.03$ | $0.51 \pm 0.03 \pm 0.05$ | $0.48 \pm 0.02 \pm 0.03$ |
| $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ | $0.10 \pm 0.02 \pm 0.01$ | $0.14 \pm 0.02 \pm 0.02$ | $0.11 \pm 0.01 \pm 0.01$ |
| $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta') / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ | $0.12 \pm 0.03 \pm 0.01$ | $0.06 \pm 0.03 \pm 0.01$ | $0.08 \pm 0.02 \pm 0.01$ |

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0) = (6.9 \pm 0.3 \pm 0.5 \pm 1.5) \times 10^{-3}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta) = (1.6 \pm 0.2 \pm 0.2 \pm 0.4) \times 10^{-3}$$

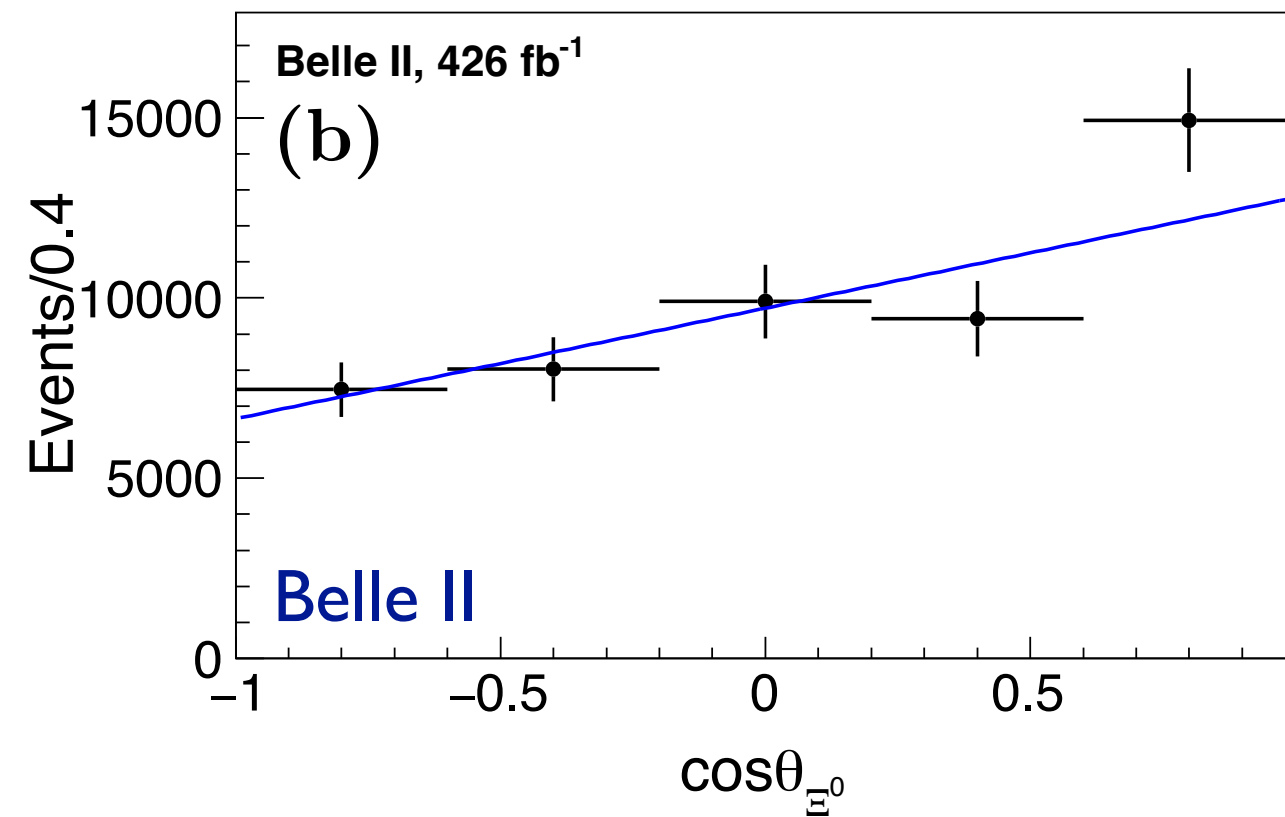
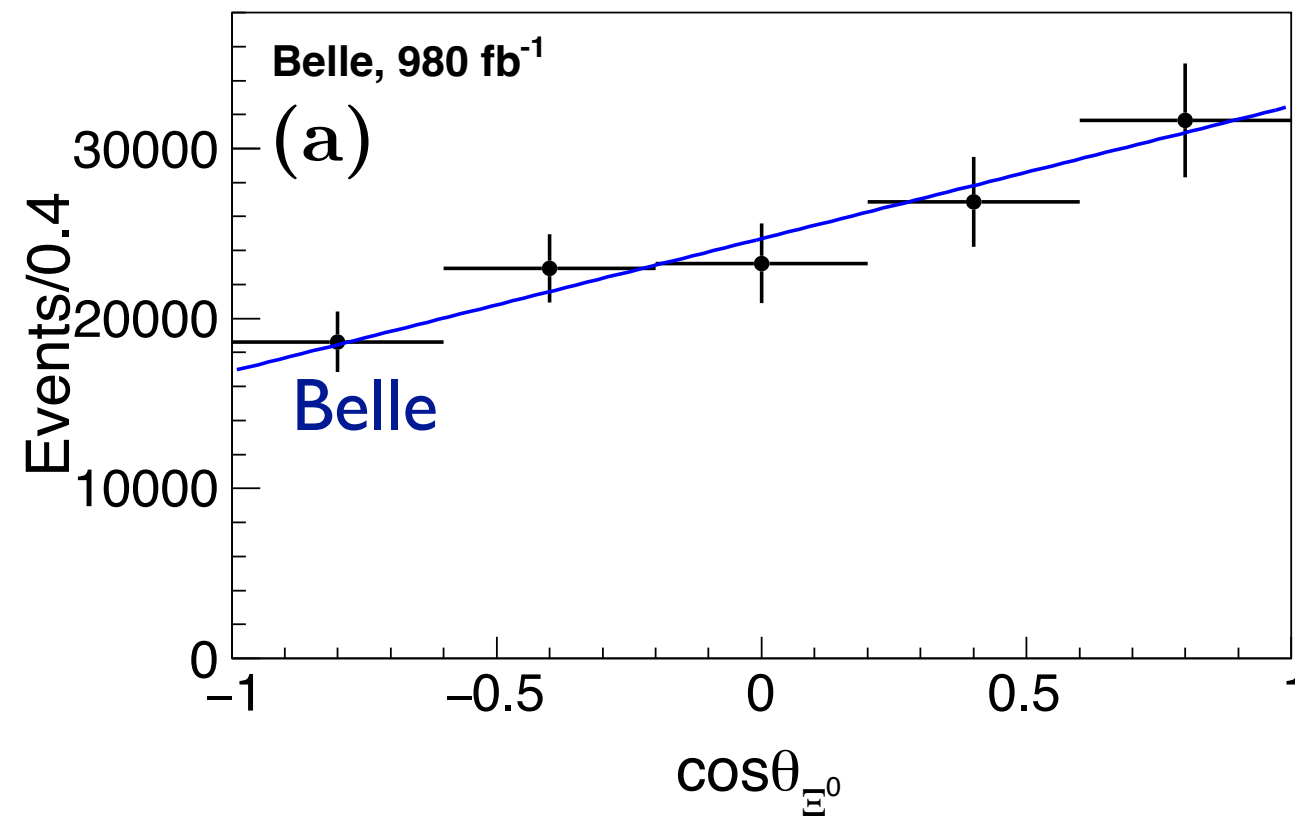
$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta') = (1.2 \pm 0.3 \pm 0.1 \pm 0.3) \times 10^{-3}$$

Belle II precision is comparable
to Belle with $\sim 1/2$ luminosity

consistent w/ Zhong et al. [JHEP (2023)]
based on $SU(3)_F$ -breaking model

$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$ decay asymmetry

$$\frac{dN}{d \cos \theta_{\Xi^0}} \propto 1 + \alpha(\Xi_c^0 \rightarrow \Xi^0 h^0) \alpha(\Xi^0 \rightarrow \Lambda \pi^0) \cos \theta_{\Xi^0}$$



$$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0) \alpha(\Xi^0 \rightarrow \Lambda \pi^0) = 0.32 \pm 0.05(\text{stat})$$

by simultaneous fits to
Belle & Belle II data sets

using $\alpha(\Xi^0 \rightarrow \Lambda \pi^0) = -0.349 \pm 0.009$ (PDG),

$$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0) = -0.90 \pm 0.15 \pm 0.23$$

consistent w/ Pole model, CA,
and SU(3)_F approaches

Part III *Energy Scan* *for Bottommoia*

Energy scan for $\Upsilon(10753)$

● $\Upsilon(10753)$

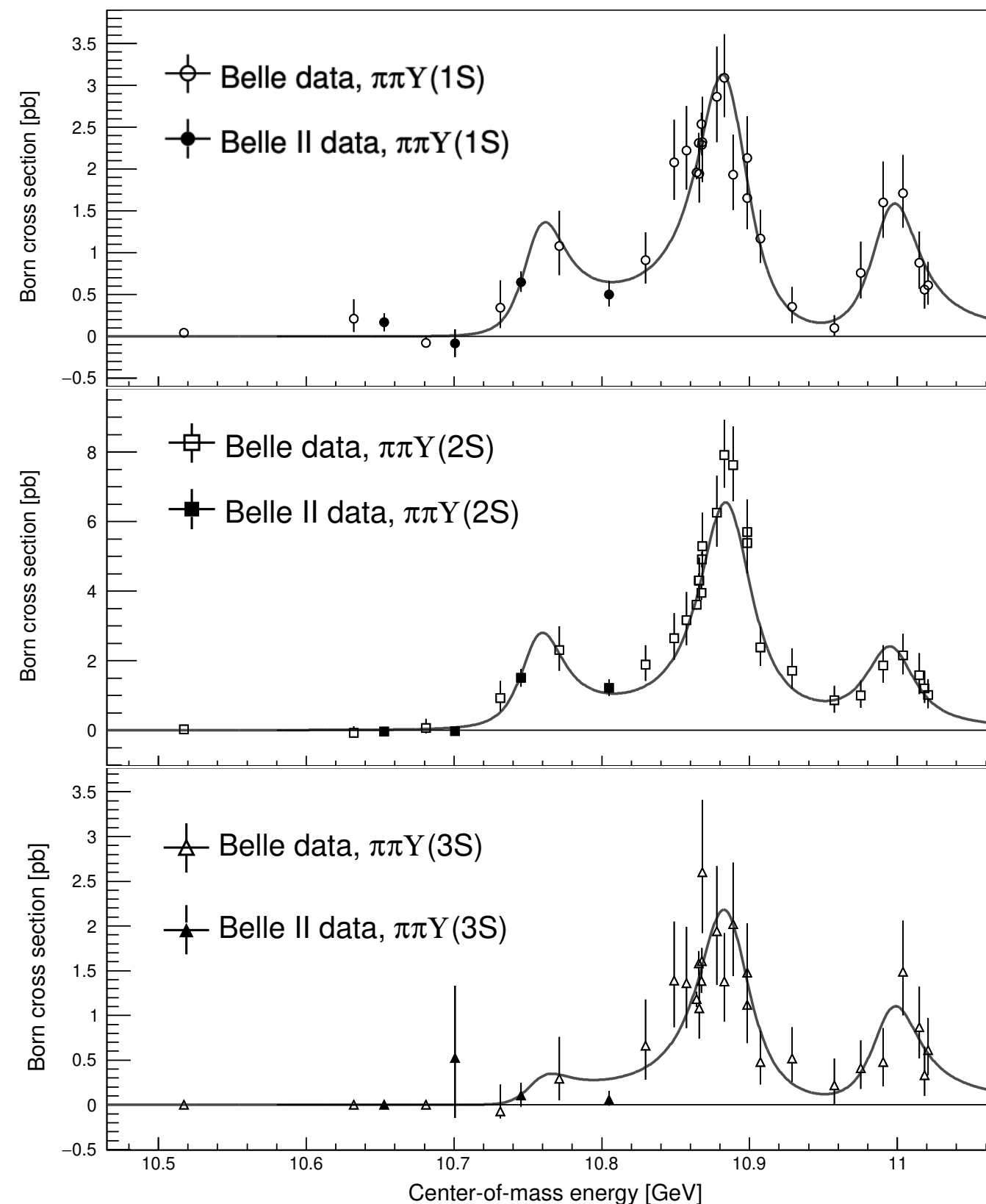
- first observed by Belle, [JHEP 10 (2019) 220] with 5.2σ
- in the energy dependence of $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$
- \exists several competing interpretations

● Belle II result

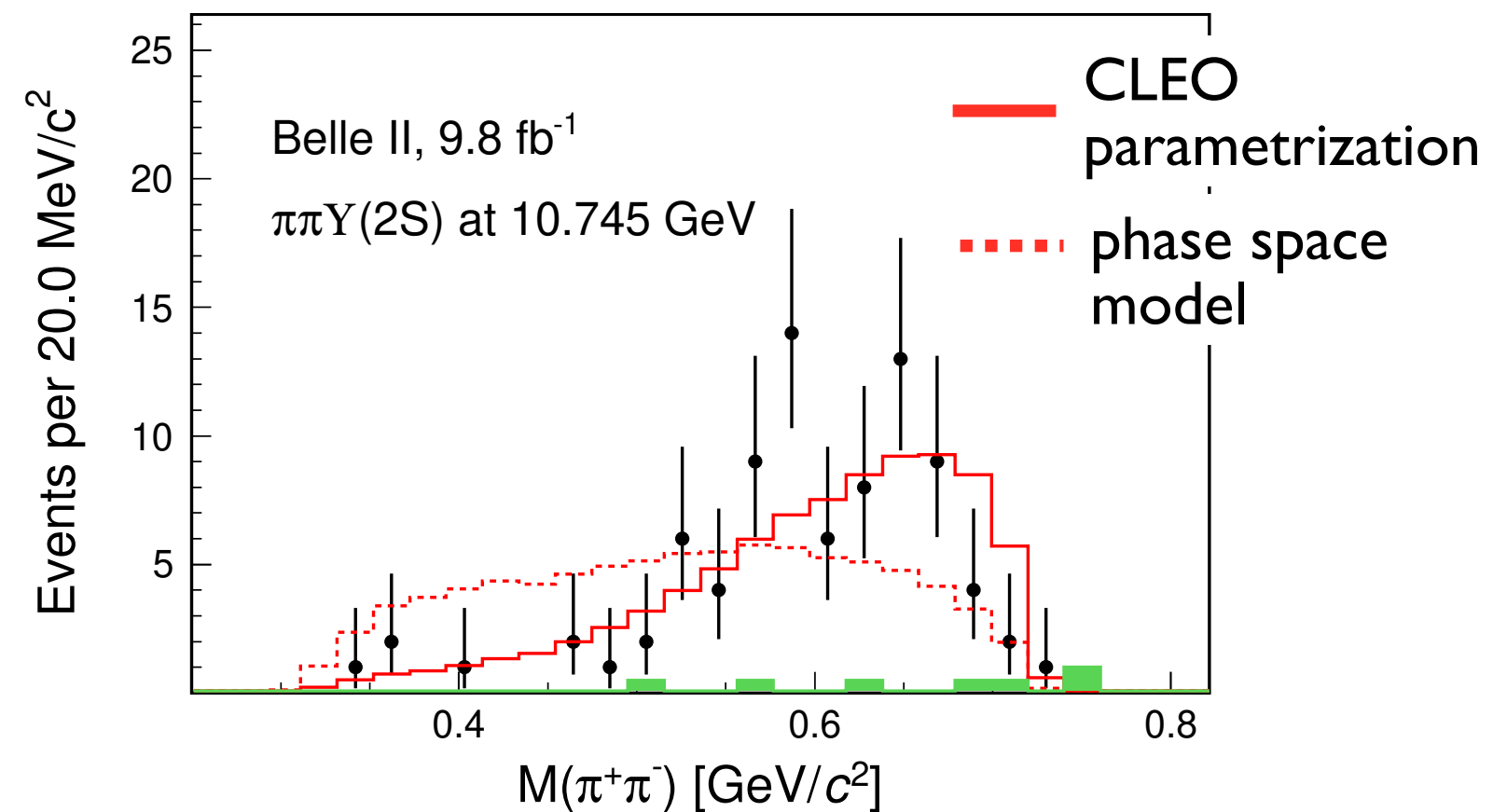
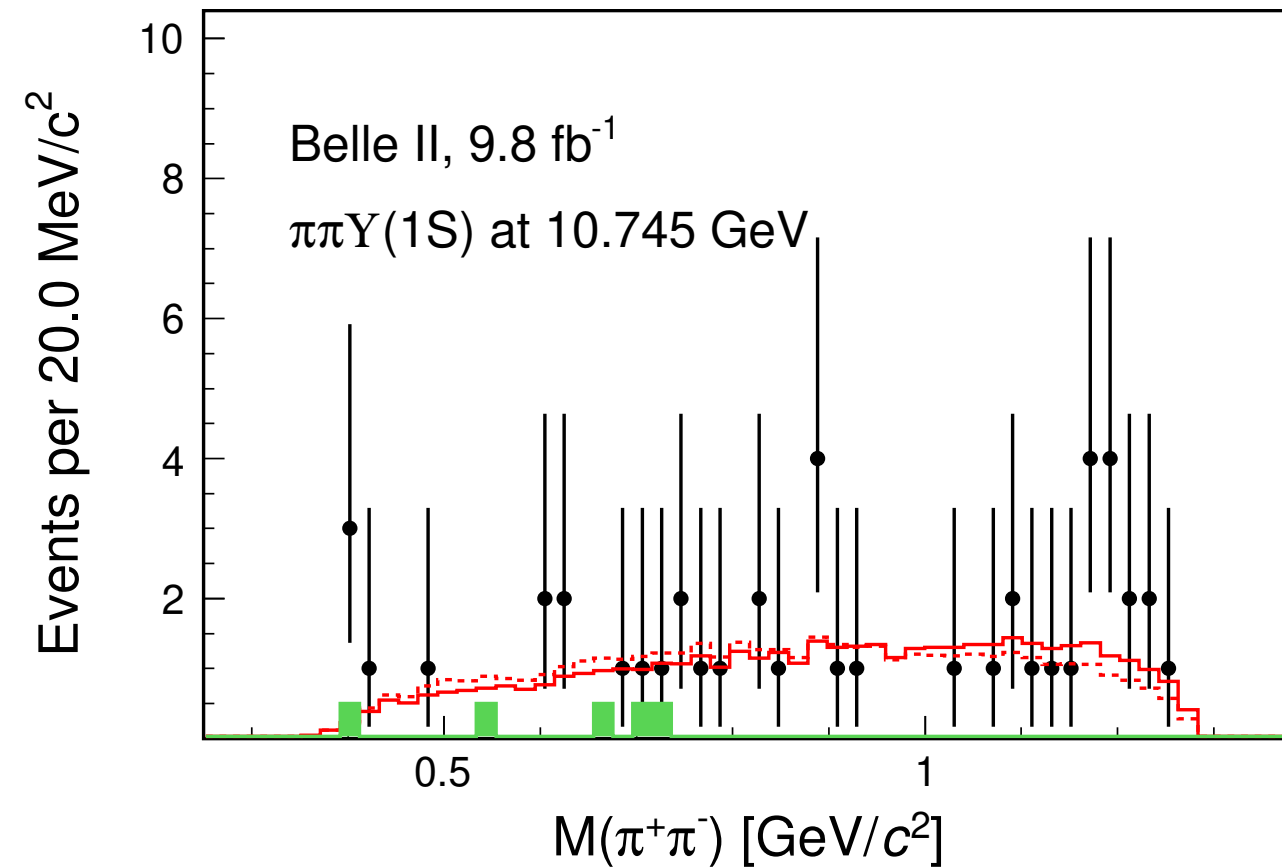
- arxiv:2401.12021
- $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ with $\Upsilon(nS) \rightarrow \mu^+\mu^-$
- confirms Belle results of $\Upsilon(10753)$

| | $\mathcal{R}_{\sigma(1S/2S)}^{\Upsilon(10753)}$ | $\mathcal{R}_{\sigma(3S/2S)}^{\Upsilon(10753)}$ |
|-------|---|---|
| Ratio | $0.46^{+0.15}_{-0.12}$ | $0.10^{+0.05}_{-0.04}$ |

small



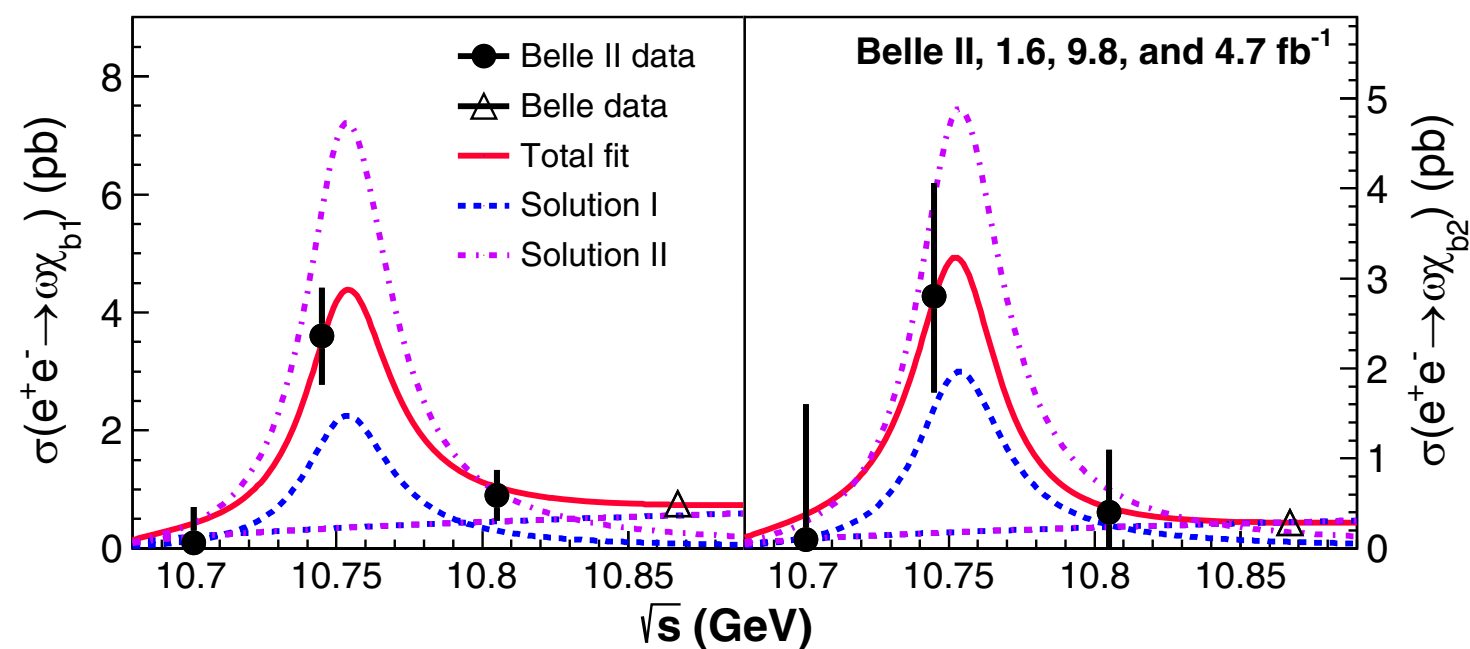
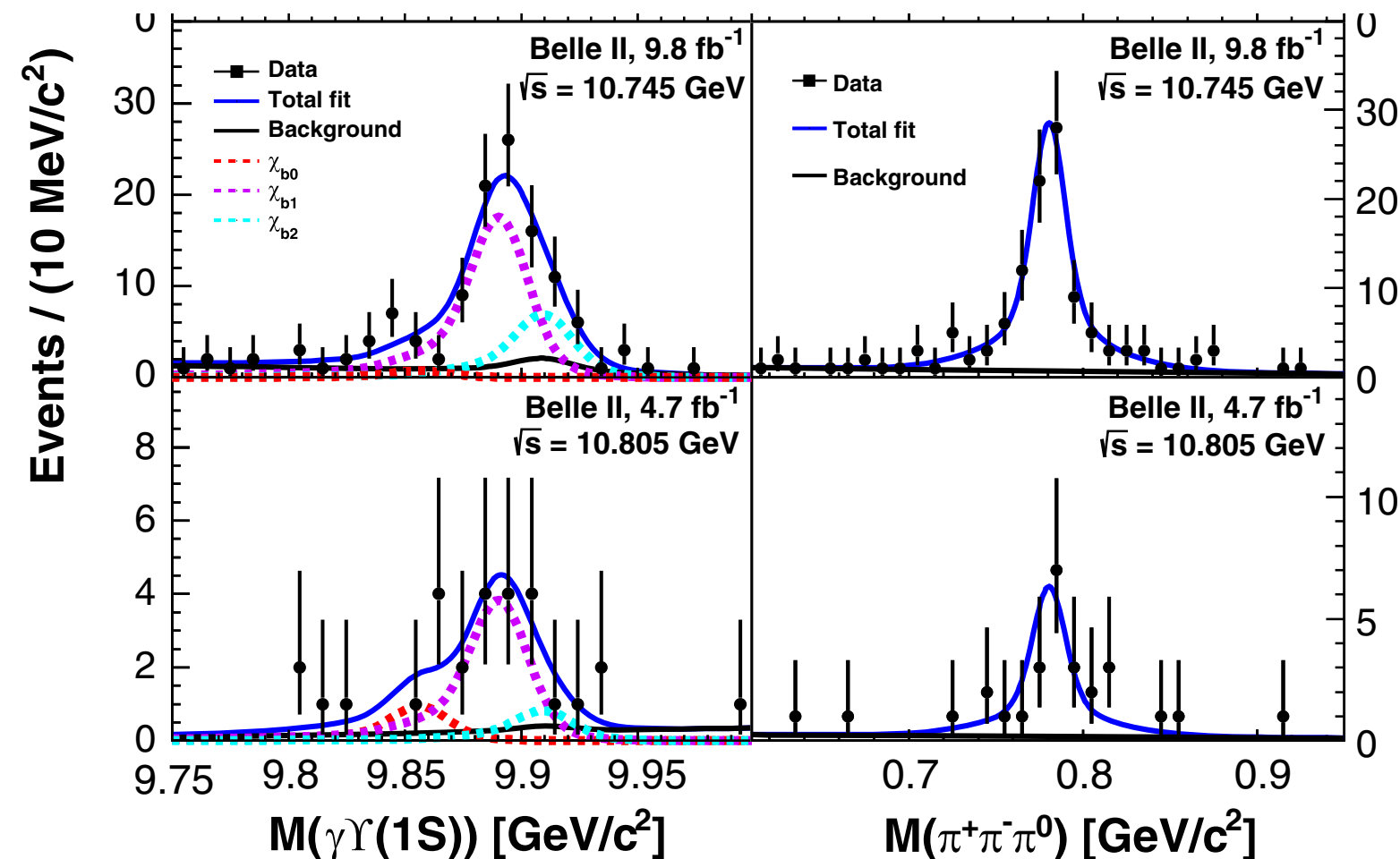
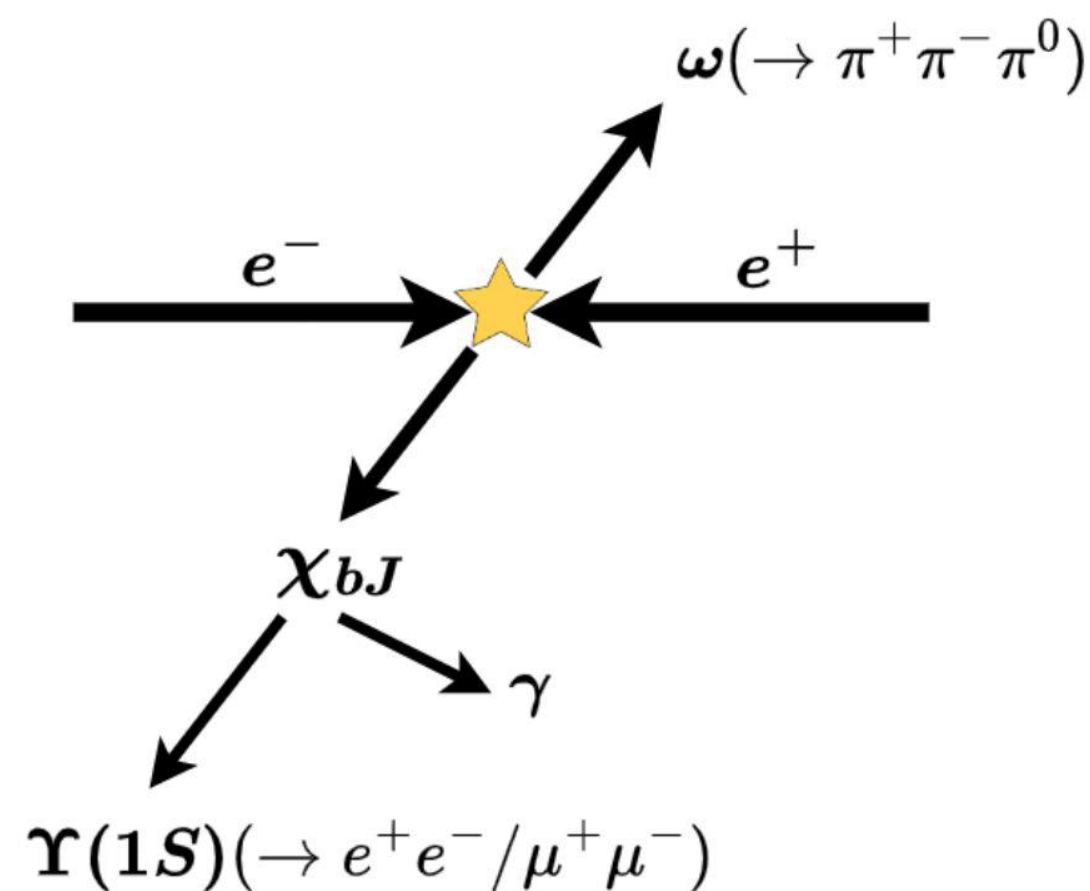
Energy scan for $\Upsilon(10753)$



dipion mass distribution

- similar to both phase-space model and $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ for $\pi^+\pi^-\Upsilon(1S)$
- but similar to $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ only for $\pi^+\pi^-\Upsilon(2S)$

$\Upsilon(10753) \rightarrow \chi_{bJ}\omega$



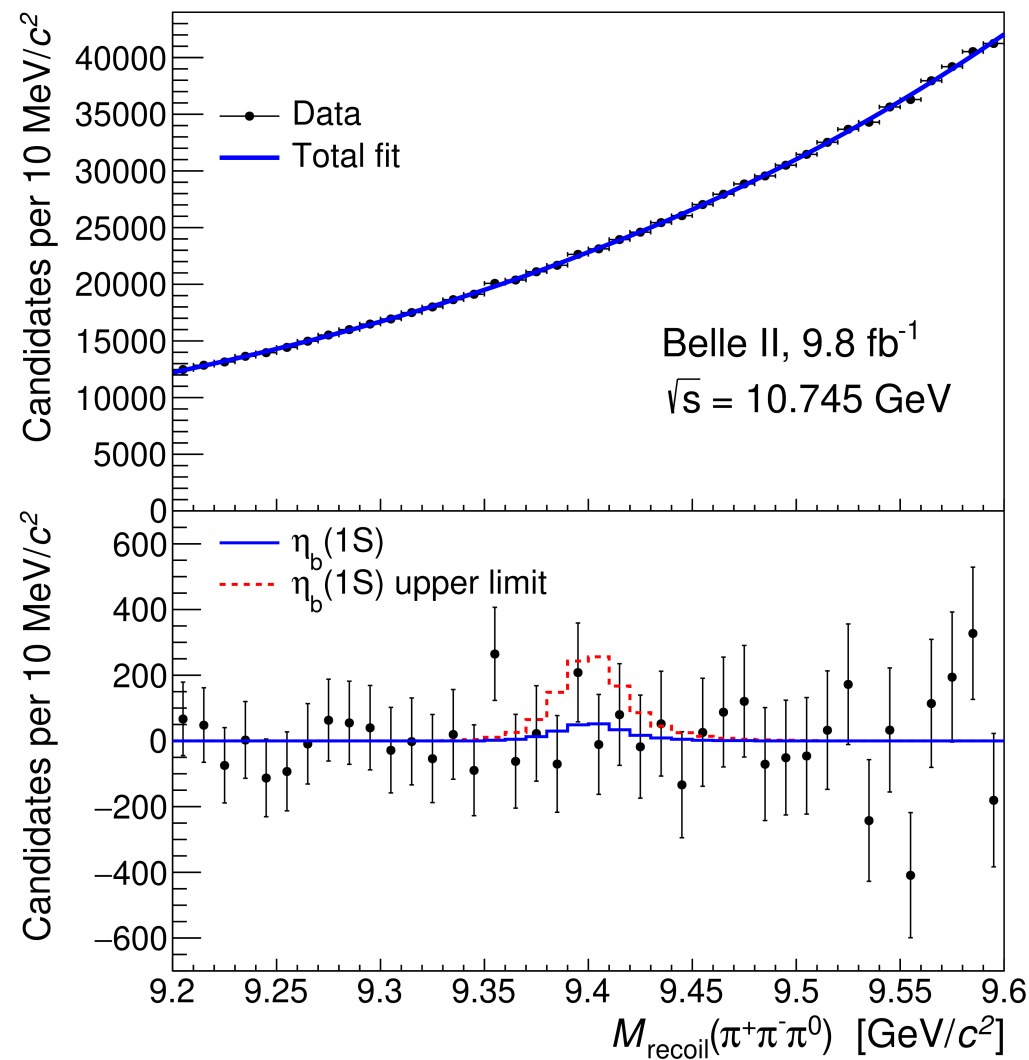
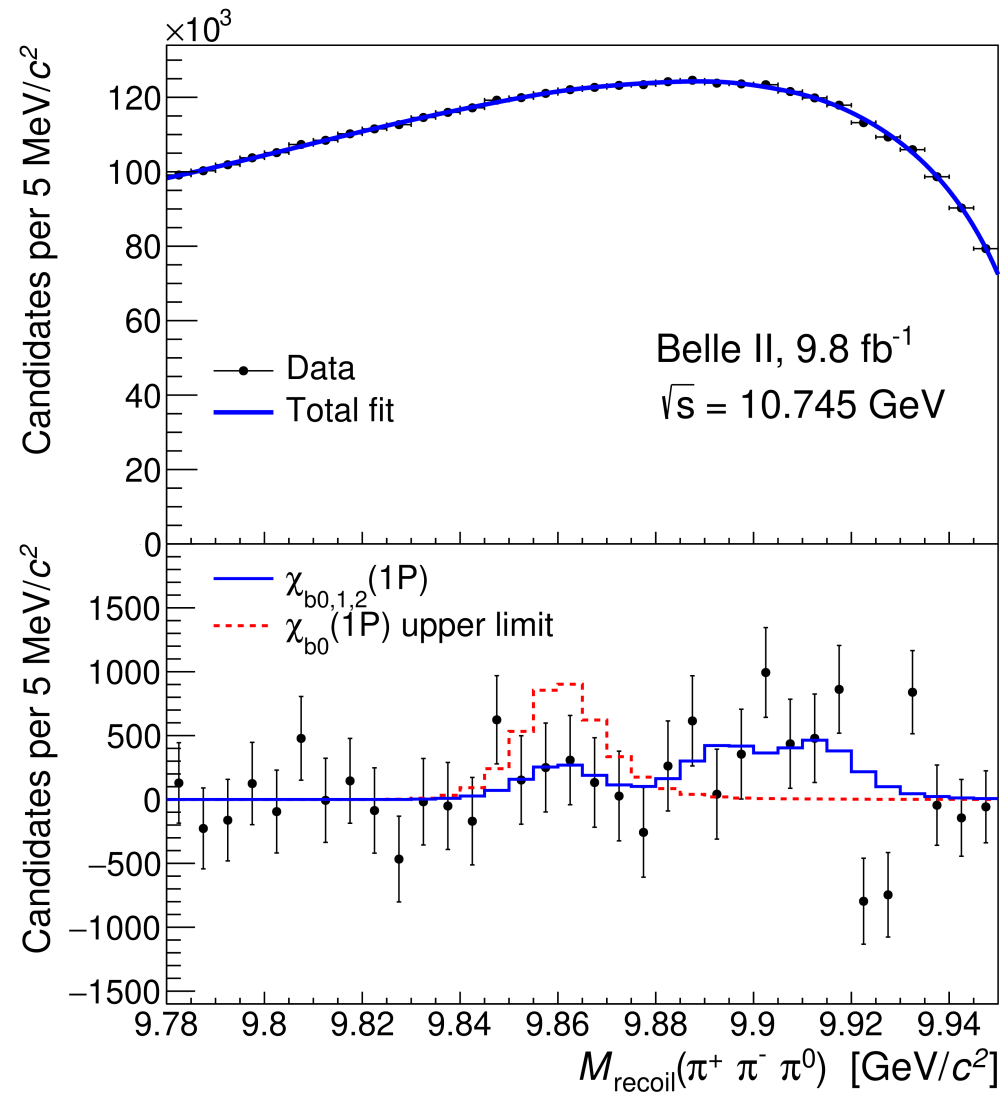
- cross section shows a peak at $\Upsilon(10753)$, hence a confirmation and a new decay channel
- the ratio $\chi_{b1}\omega/\pi\pi\Upsilon(nS) \sim$ one order of magnitude higher at $\Upsilon(10753)$ than at $\Upsilon(5S)$

$\Upsilon(10753) \rightarrow \chi_{b0}\omega$ and $\eta_b\omega$

- Tetraquark interpretation of this state predicts enhancement of $\Upsilon(10753) \rightarrow \eta_b(1S)\omega$

$$\frac{\Gamma(\omega\eta_b)}{\Gamma(\Upsilon\pi^+\pi^-)} \sim 30$$

- we measure η_b indirectly by using recoil mass $M_{\text{recoil}}(\omega) = \sqrt{(E_{\text{cm}} - E_\omega)^2 - p_\omega^2}$



$$\sigma_B(e^+e^- \rightarrow \eta_b(1S)\omega) < 2.5 \text{ pb},$$

$$\sigma_B(e^+e^- \rightarrow \chi_{b0}(1P)\omega) < 8.7 \text{ pb}.$$

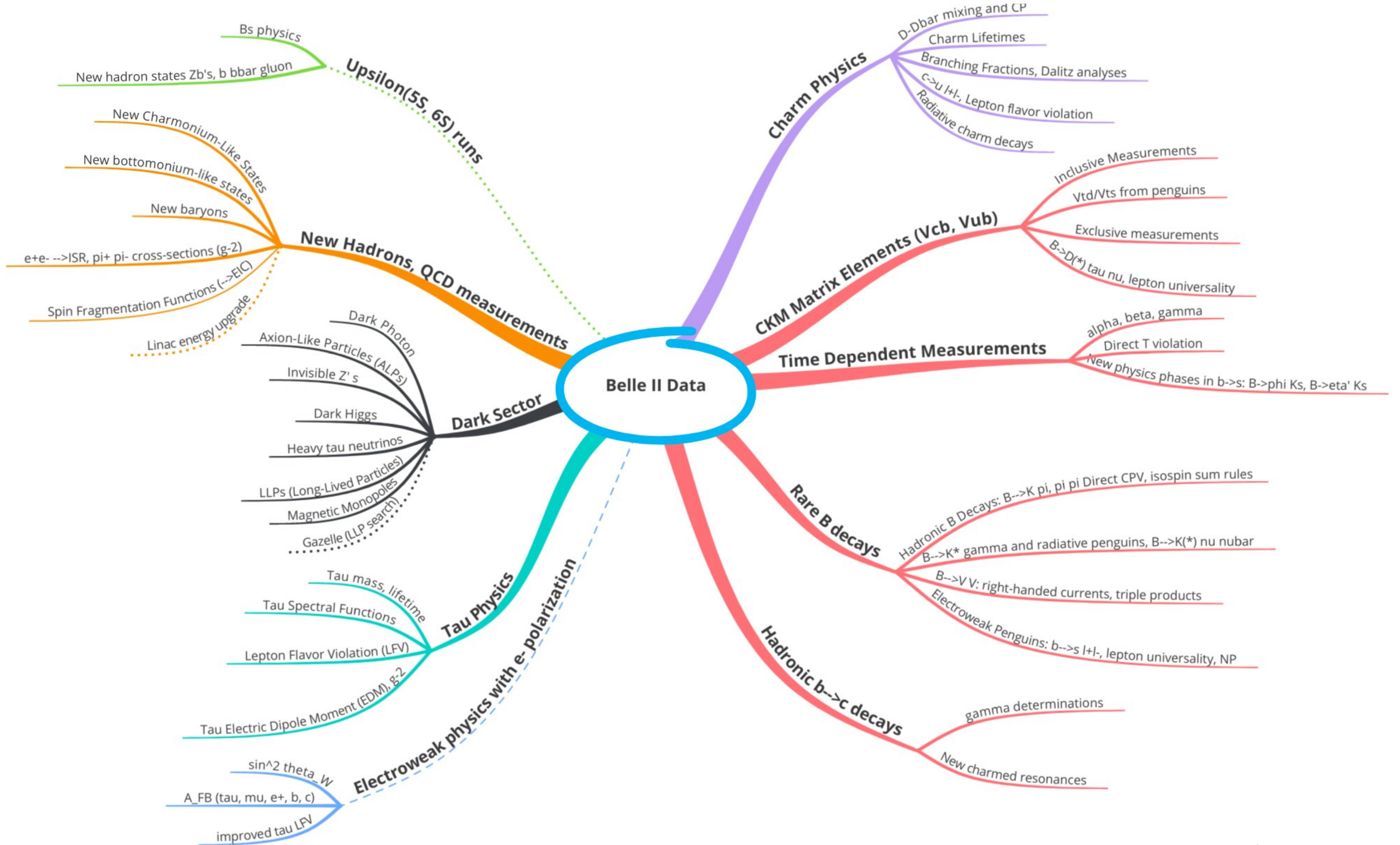
Summary

- Belle II has collected over 0.4 ab^{-1} data sample in its first 3 years of operation before LS1, and started Run 2 data taking in Feb. this year.
- With the data set of $\sim 1/2$ the size of Belle, the physics precision of Belle II results are comparable or better in many analyses.
- Recent Belle II physics highlights include first evidence for $B^+ \rightarrow K^+ \nu \bar{\nu}$, and inclusive test of LFU with $B \rightarrow X \tau \nu$.
- In addition, we have presented interesting new results in charm baryons and bottomonium spectroscopy.
- Run 2 is underway with the goal of collecting a several ab^{-1} data in the next few years.

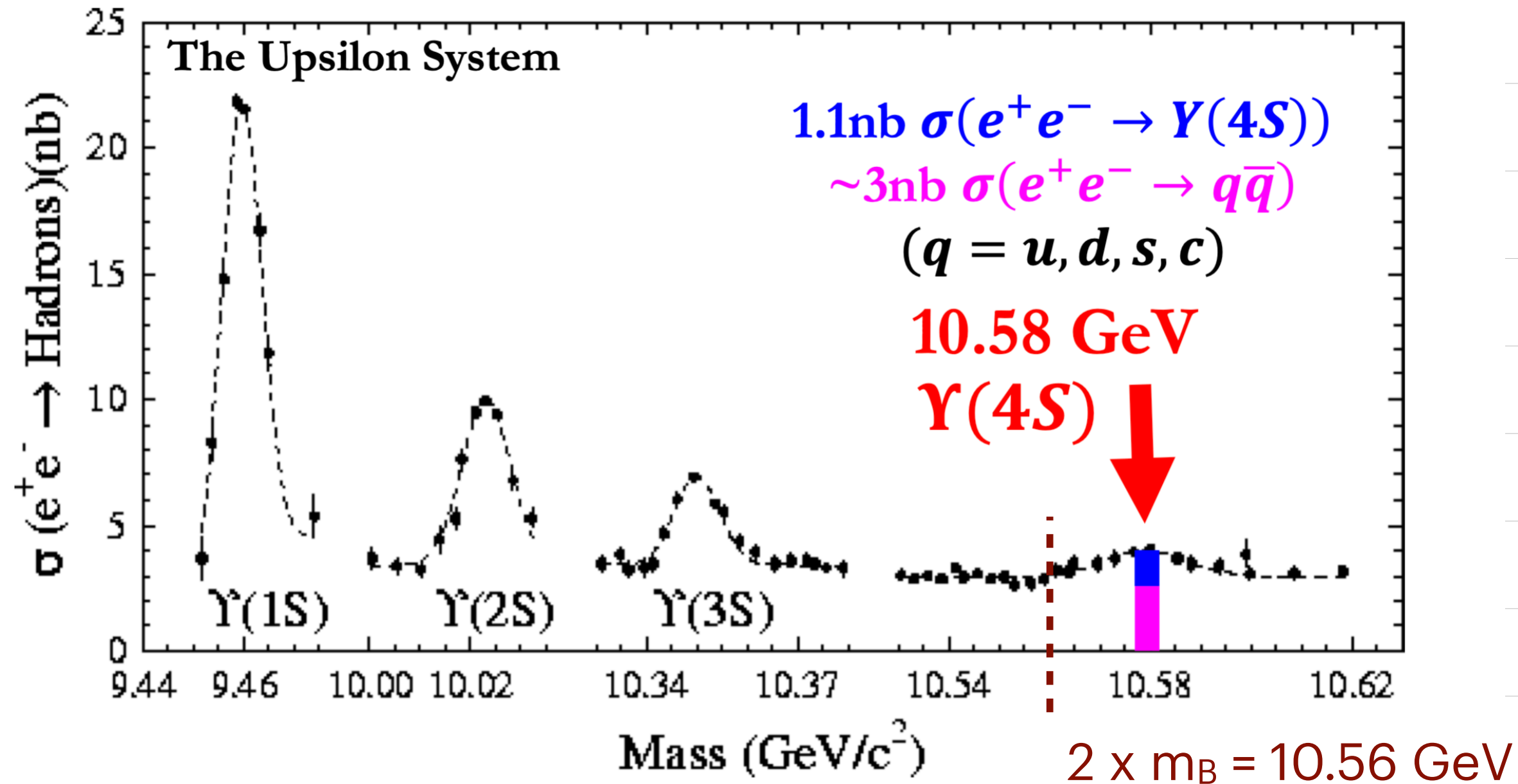
Thank you!

Appendices

Belle II Physics Mind-map



$e^+e^- \rightarrow \Upsilon(4S)$ as a B -factory

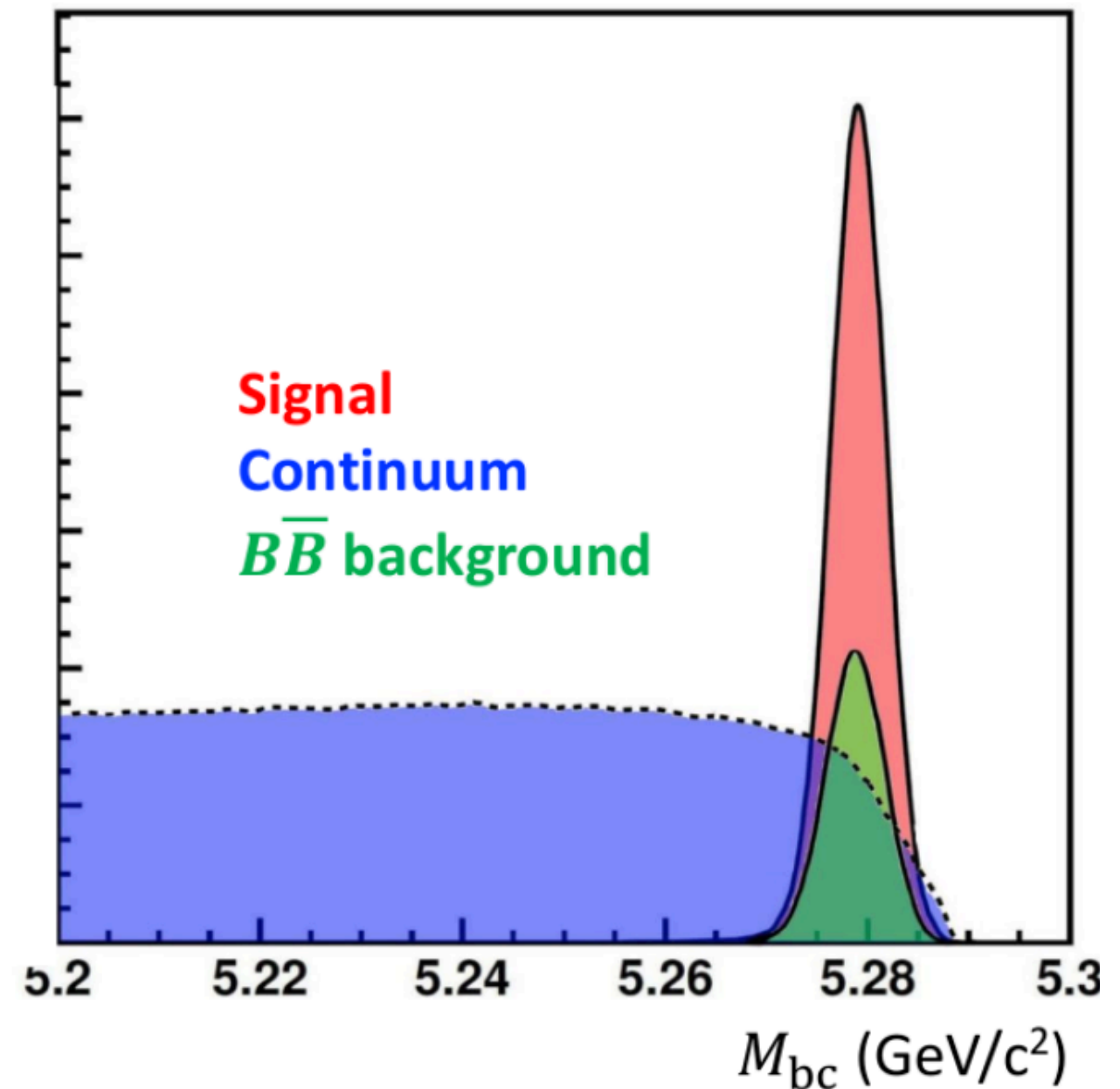
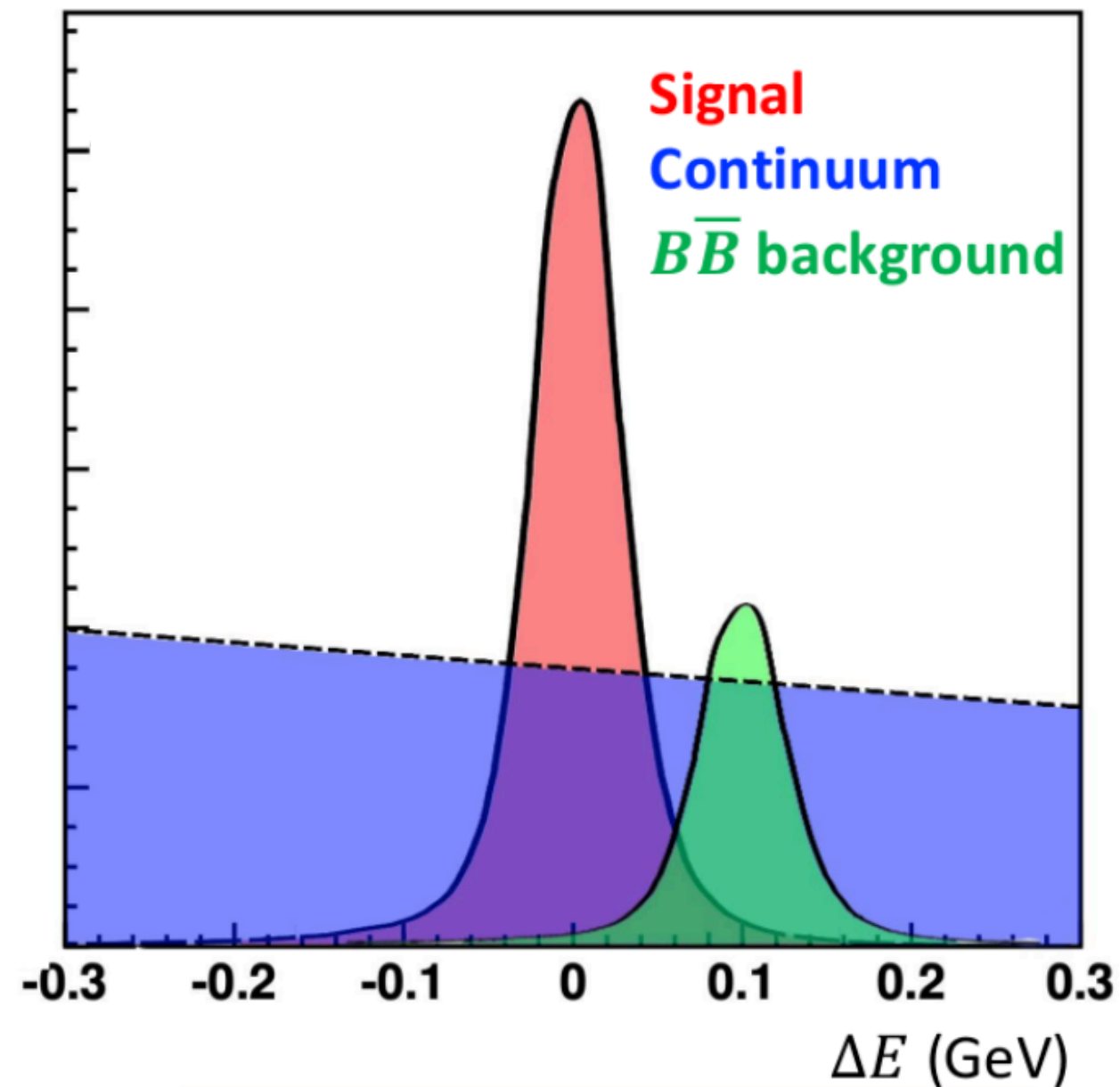


- $\mathcal{B}(\Upsilon(4S) \rightarrow B\bar{B}) > 96\%$, with $p_B^{CM} \sim 0.35 \text{ GeV}/c$
- nothing else but $B\bar{B}$ in the final state
 \therefore if we know (E, \vec{p}) of one B , the other B is also constrained

Key variables of B decays

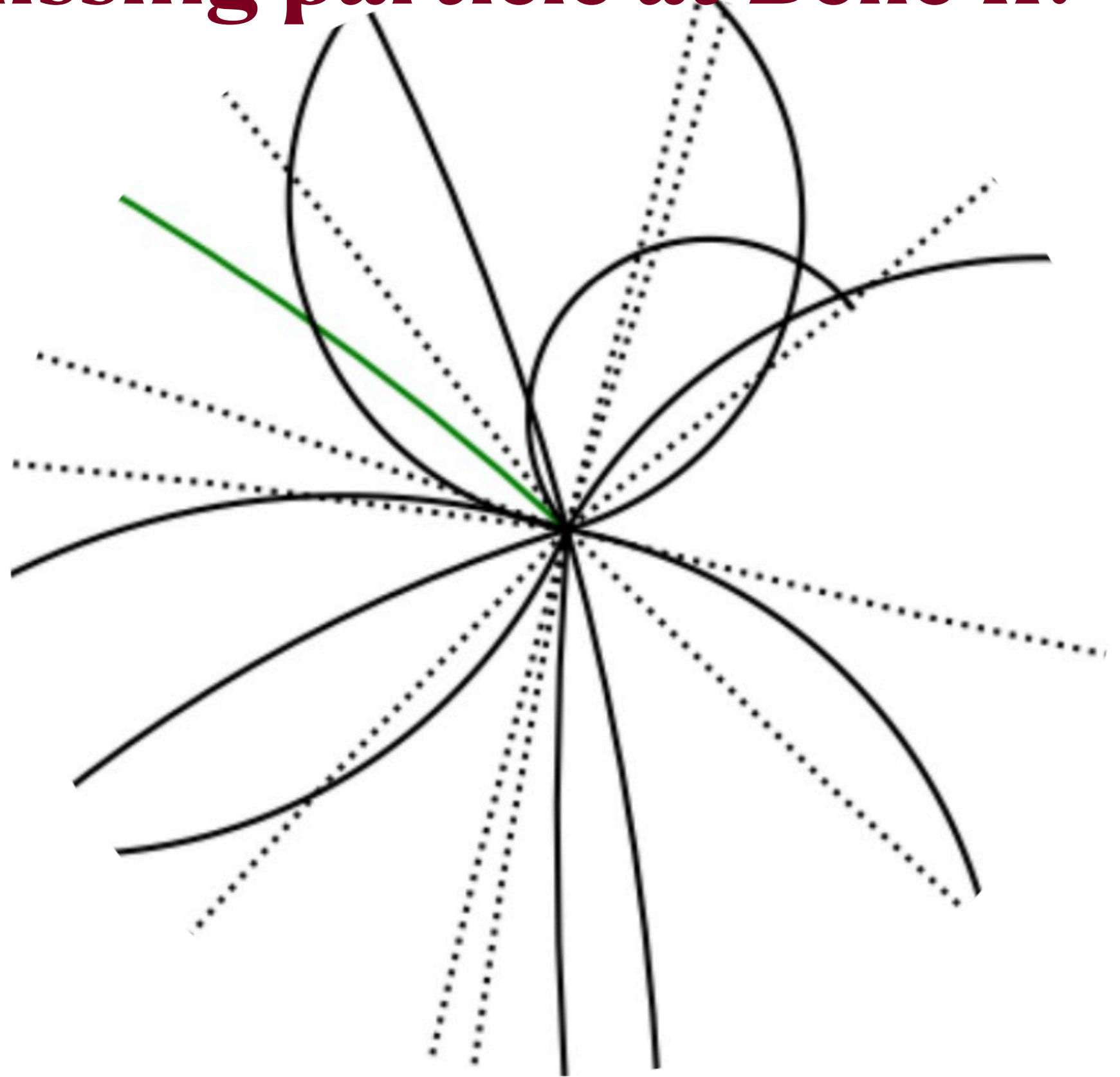
$$\Delta E = E_B^* - \sqrt{s}/2$$

$$M_{bc} = \sqrt{(\sqrt{s}/2)^2 - \vec{p}_B^{*2}}$$



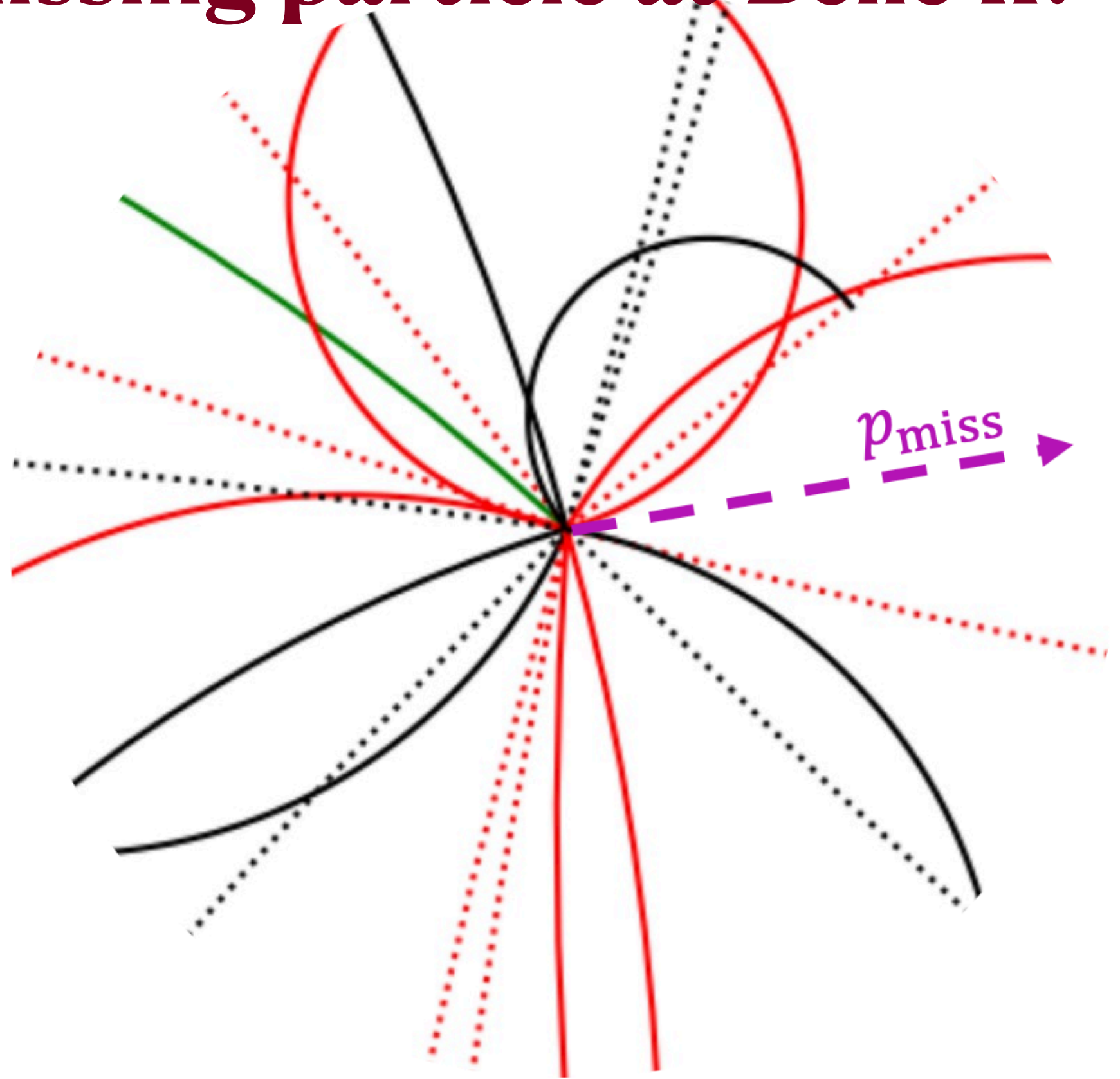
How to handle a missing particle at Belle II?

- $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
 - only two B mesons in the final state
 - Since the initial state is clearly determined, fully accounting one B (B_{tag}) makes it possible to constrain the accompanying B (B_{sig})
 - Having a single missing particle (e.g. ν) is usually as clean as getting all particles measured
 - The price to pay is a big drop of efficiency ($< \mathcal{O}(1\%)$)



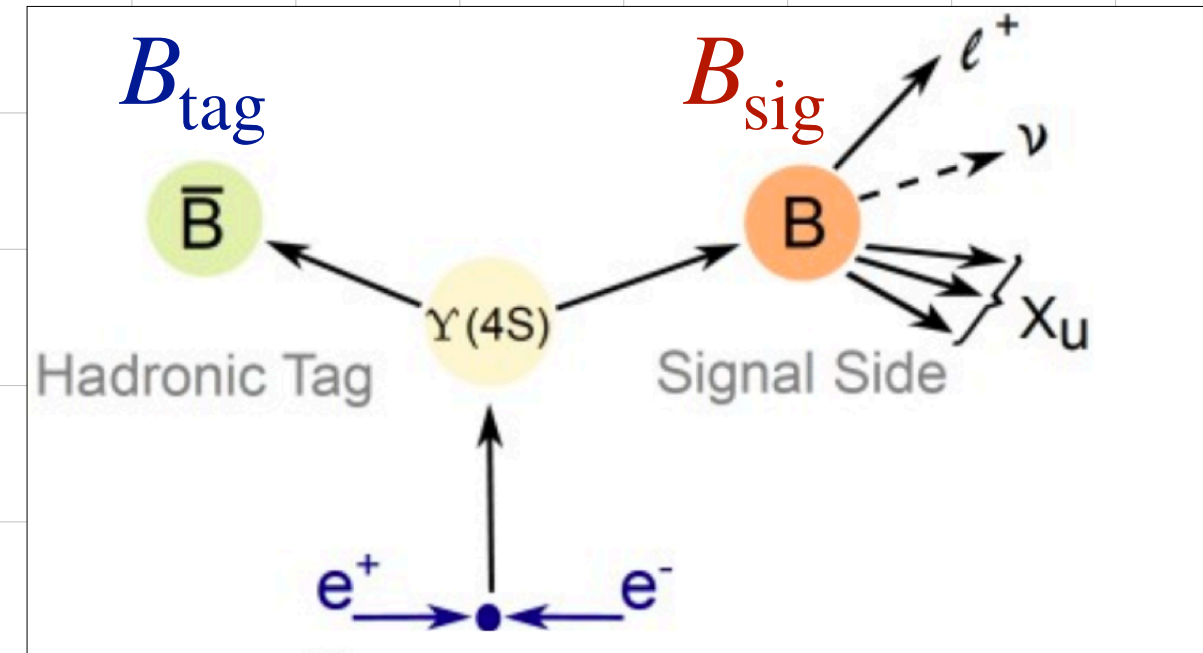
How to handle a missing particle at Belle II?

- $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
 - only two B mesons in the final state
 - Since the initial state is clearly determined, fully accounting one B (B_{tag}) makes it possible to constrain the accompanying B (B_{sig})
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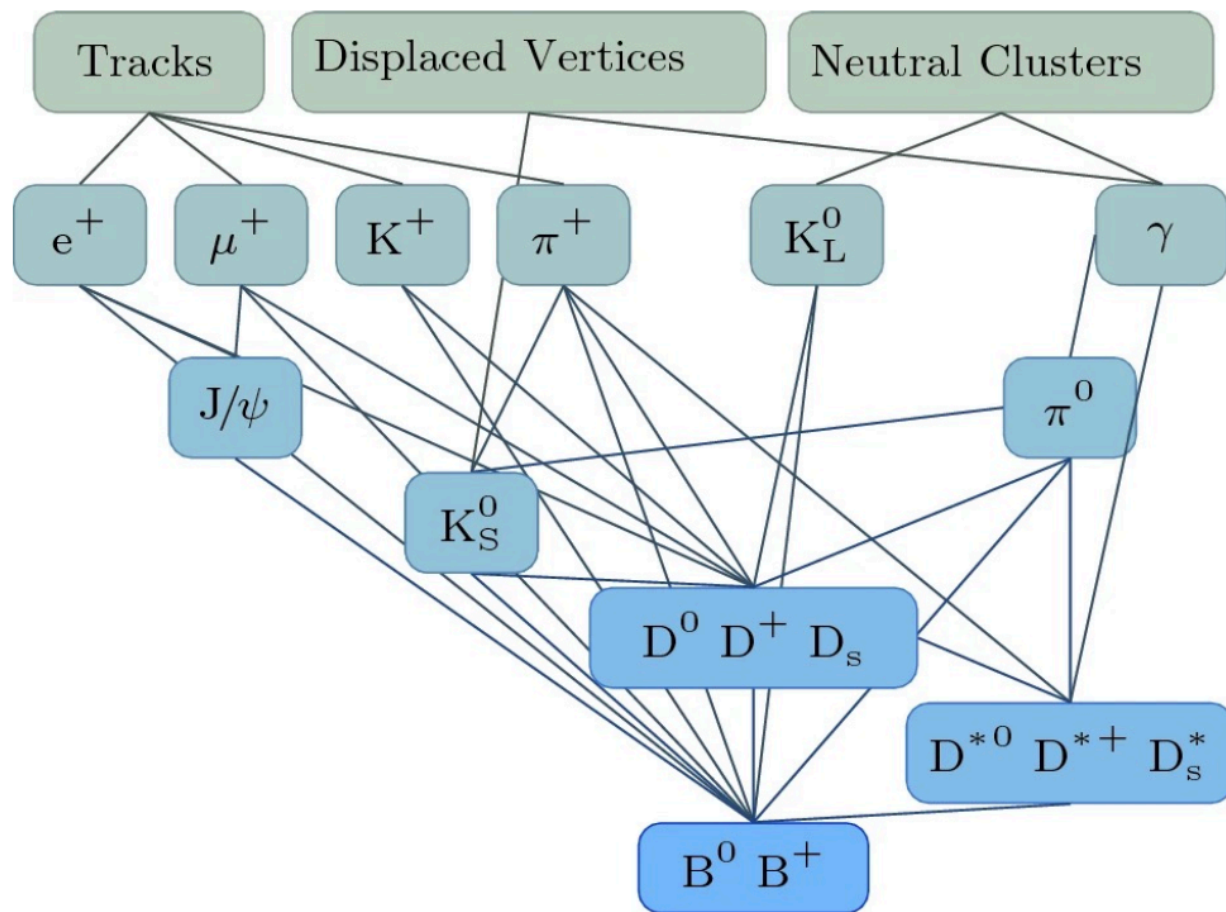


Full Event Interpretation (FEI)

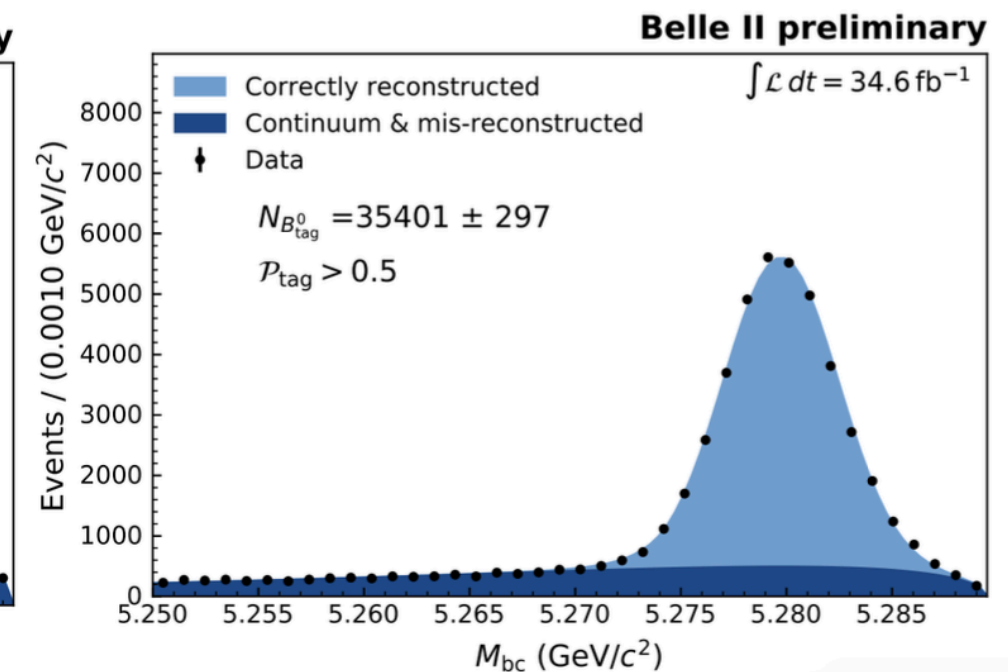
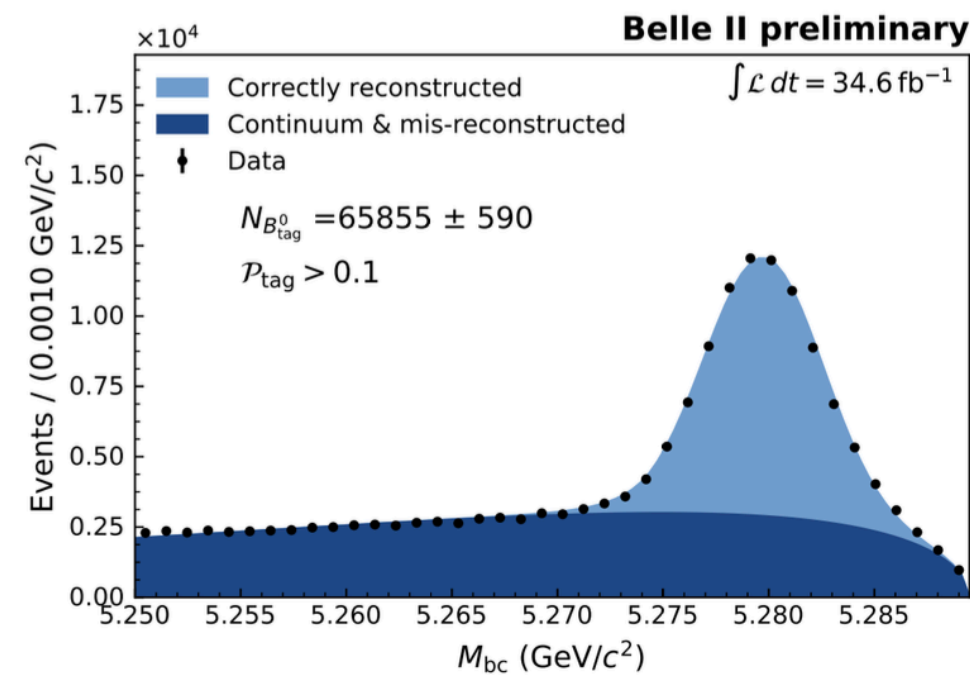
- FEI algorithm to reconstruct B_{tag}
 - uses ~ 200 BDT's to reconstruct $\mathcal{O}(10^4)$ different B decay chains
 - assign signal probability of being correct B_{tag}



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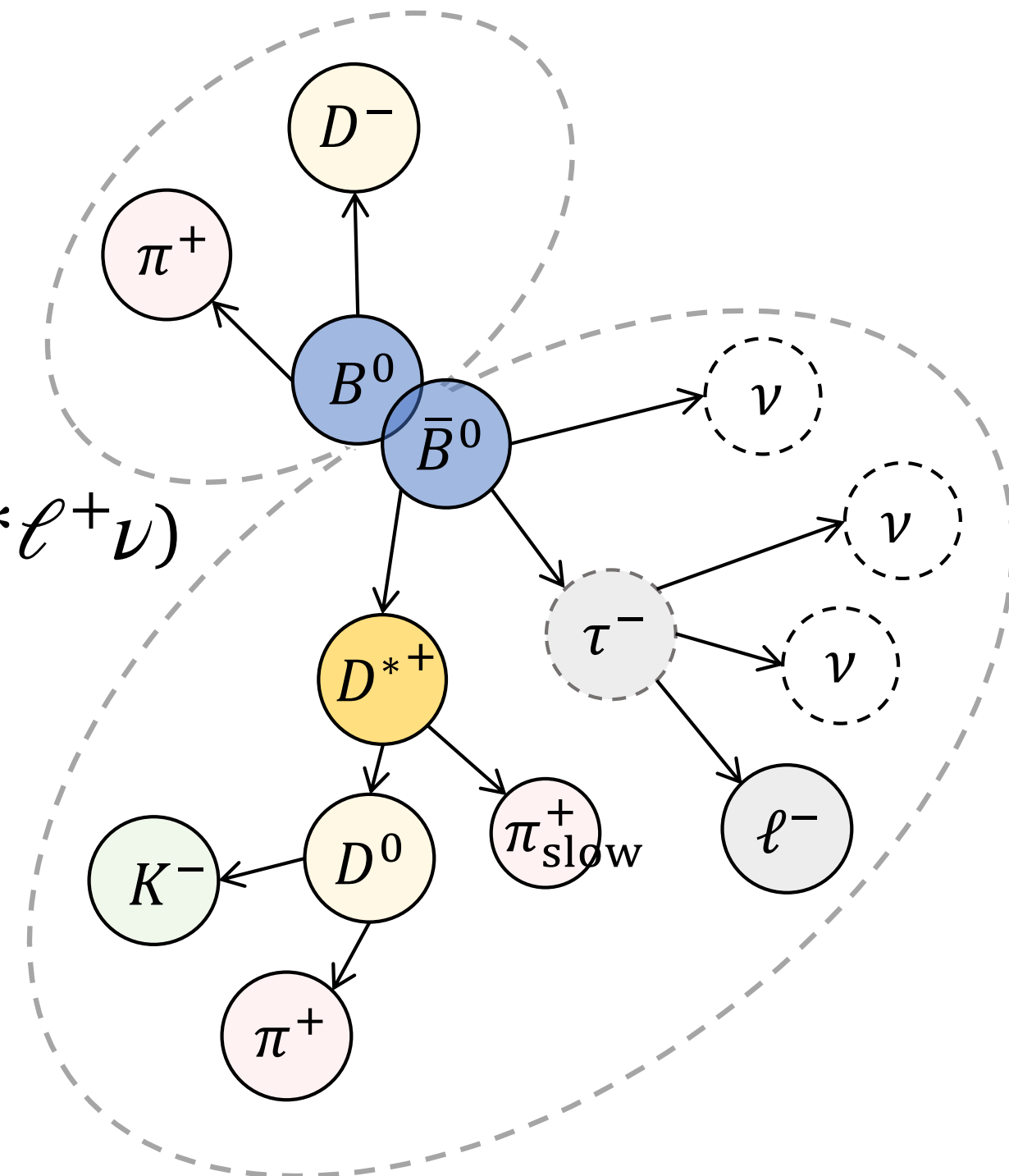
arXiv:2008.060965



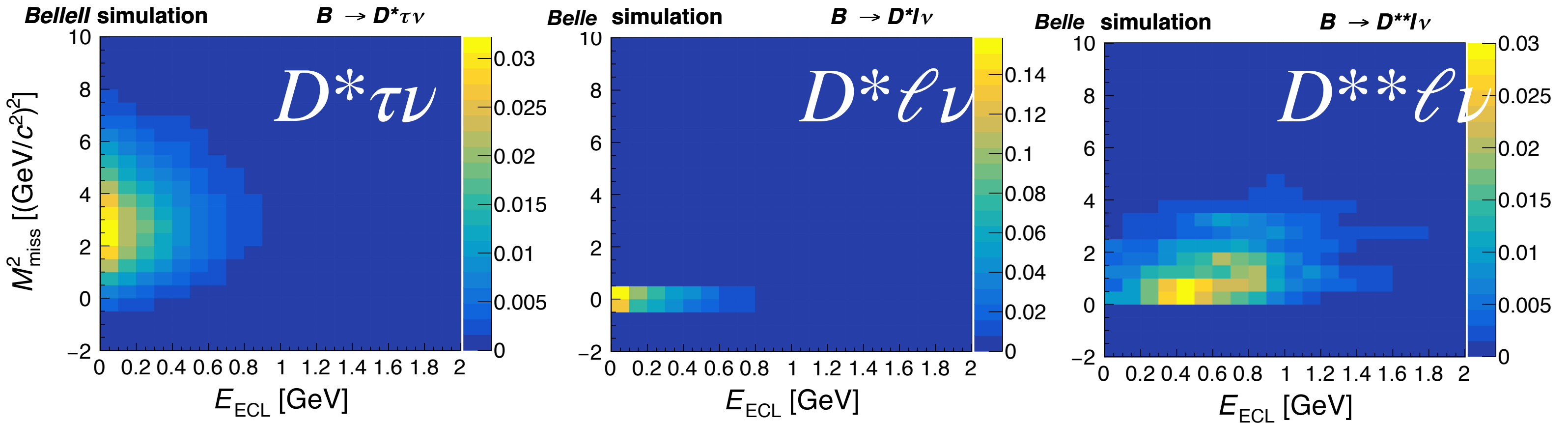
$R(D^*)$ from Belle II

- First $R(D^*)$ result from Belle II
- Analysis features
 - Use hadronic B-tagging with FEI (slide 34)
 - leptonic τ decays, $\tau^+ \rightarrow \ell^+ \nu_\ell \bar{\nu}_\tau$
 - three D^* modes: $D^{*+} \rightarrow D^0 \pi^+$, $D^+ \pi^0$ and $D^{*0} \rightarrow D^0 \pi^0$
- Signal ($B \rightarrow D^* \tau^+ \nu$) & Normalization ($B \rightarrow D^* \ell^+ \nu$)
 - extracted simultaneously
 - by fitting 2D ($M_{\text{miss}}^2, E_{\text{ECL}}$)

$$R(D^*) \equiv \frac{\mathcal{B}(B \rightarrow D^* \tau^+ \nu)}{\mathcal{B}(B \rightarrow D^* \ell^+ \nu)}$$



$R(D^*)$ from Belle II



● Signal ($B \rightarrow D^* \tau^+ \nu$) & Normalization ($B \rightarrow D^* \ell^+ \nu$)

- extracted simultaneously
- by fitting 2D $(M^2_{\text{miss}}, E_{\text{ECL}})$

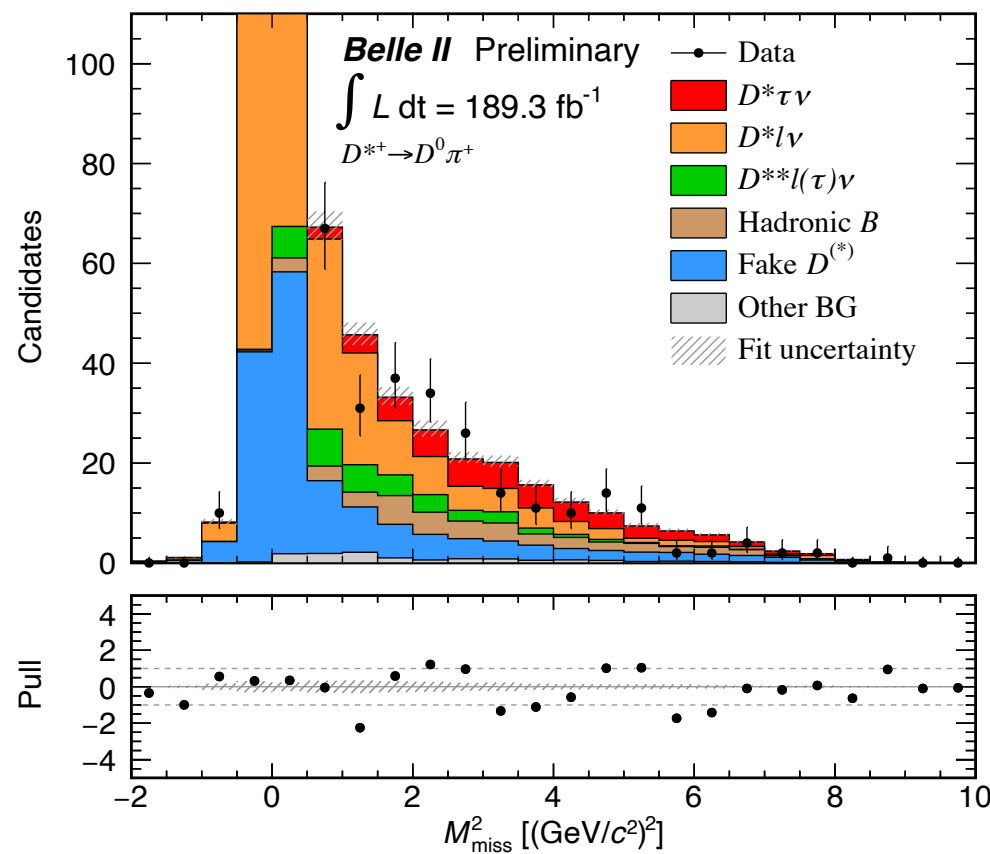
$$M^2_{\text{miss}} \equiv (p_{e^+e^-} - p_{B_{\text{tag}}} - p_{D^*} - p_{\ell})^2$$

E_{ECL} = extra energy (unmatched) in the EM calorimeter

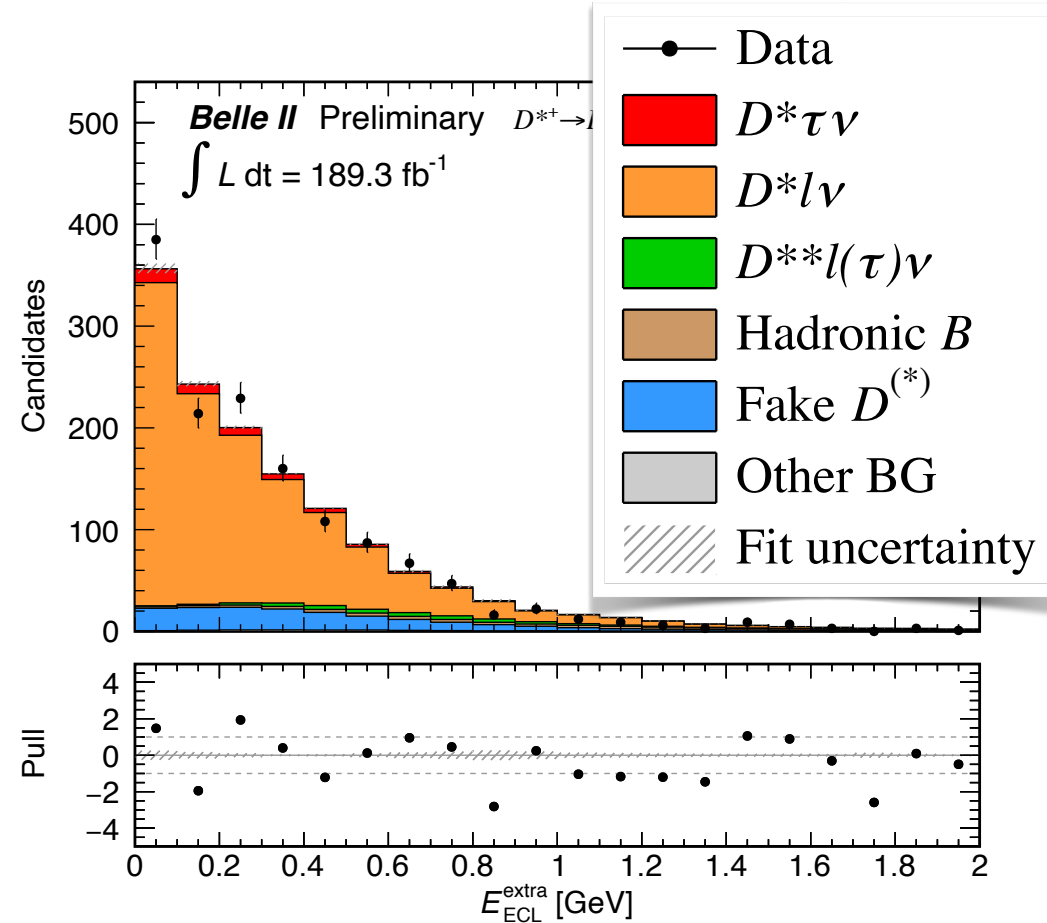
$R(D^*)$ from Belle II

● Fit projections for the sub-mode $D^{*+} \rightarrow D^0 \pi^+$

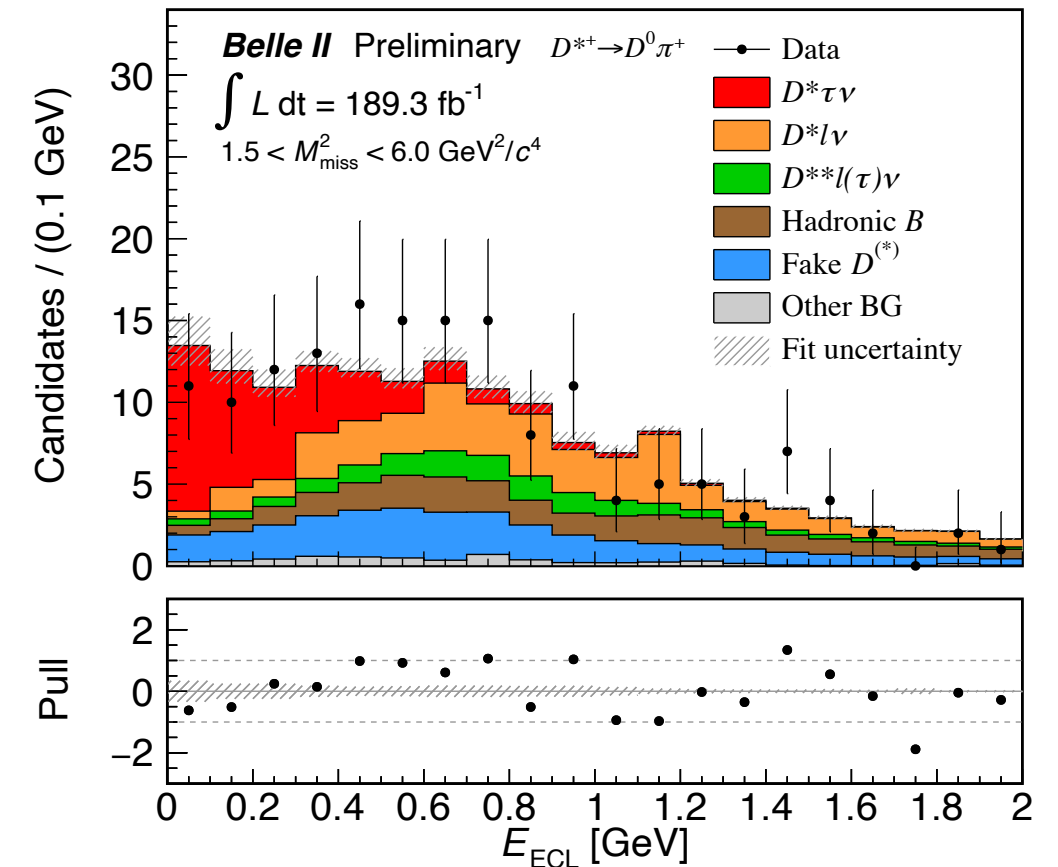
$\mathcal{L}_{\text{int}} = 189 \text{ fb}^{-1}$



M^2_{miss} (peak-bin yield $\sim O(600)$)



$E_{\text{ECL}}^{\text{extra}}$ for entire M^2_{miss} region



E_{ECL} for signal-enhanced region
 $1.5 < M^2_{\text{miss}} < 6.0 \text{ GeV}^2$

$$R(D^*) = 0.262^{+0.041+0.035}_{-0.039-0.032}$$

● Systematics

- dominant sources: E_{ECL} PDF shape, MC statistics

some corrections & validations

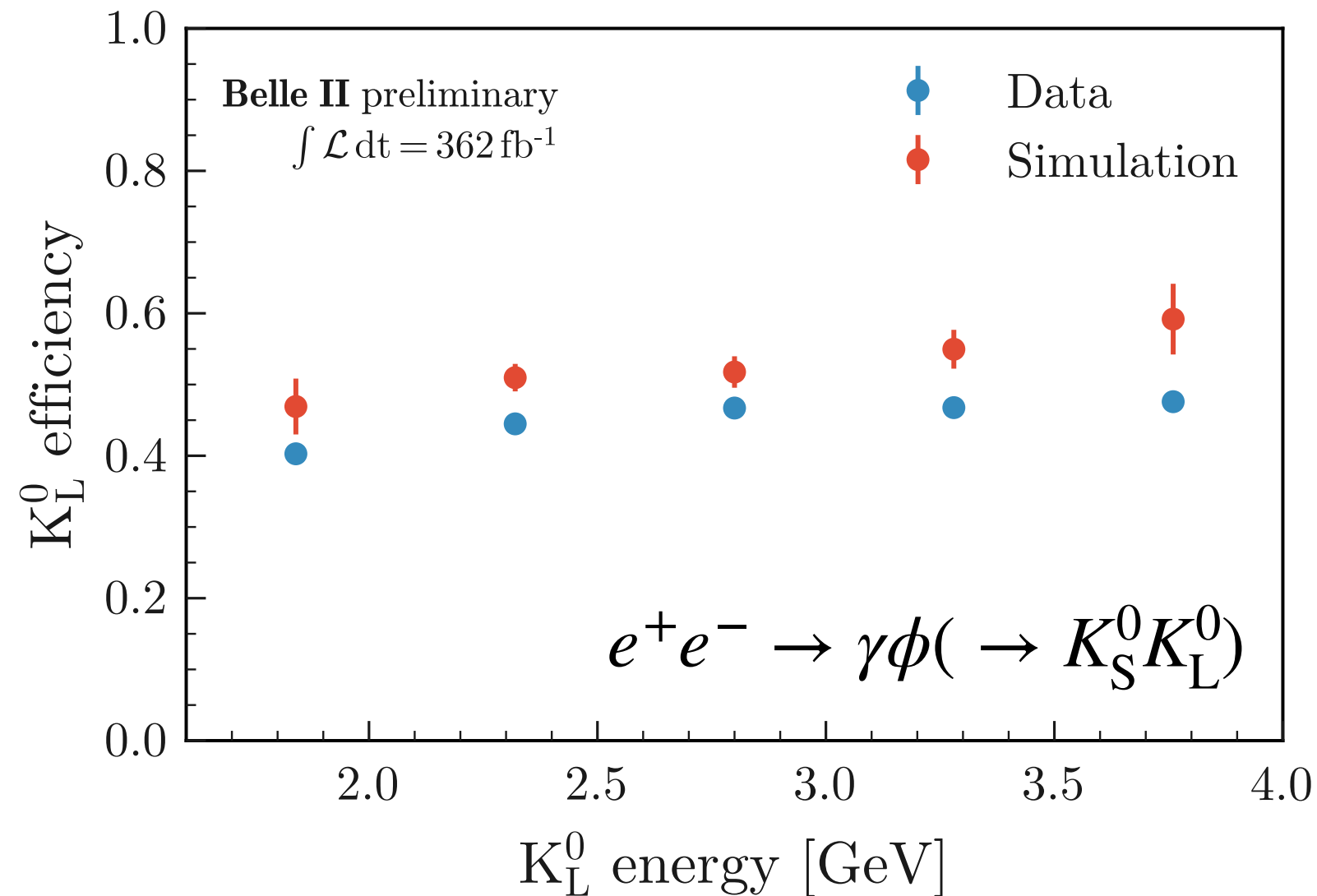


FIG. 4. Efficiency of reconstructing an energy deposit in the ECL matched to the K_L^0 direction as a function of the K_L^0 energy for data and simulation selected with the ITA analysis.

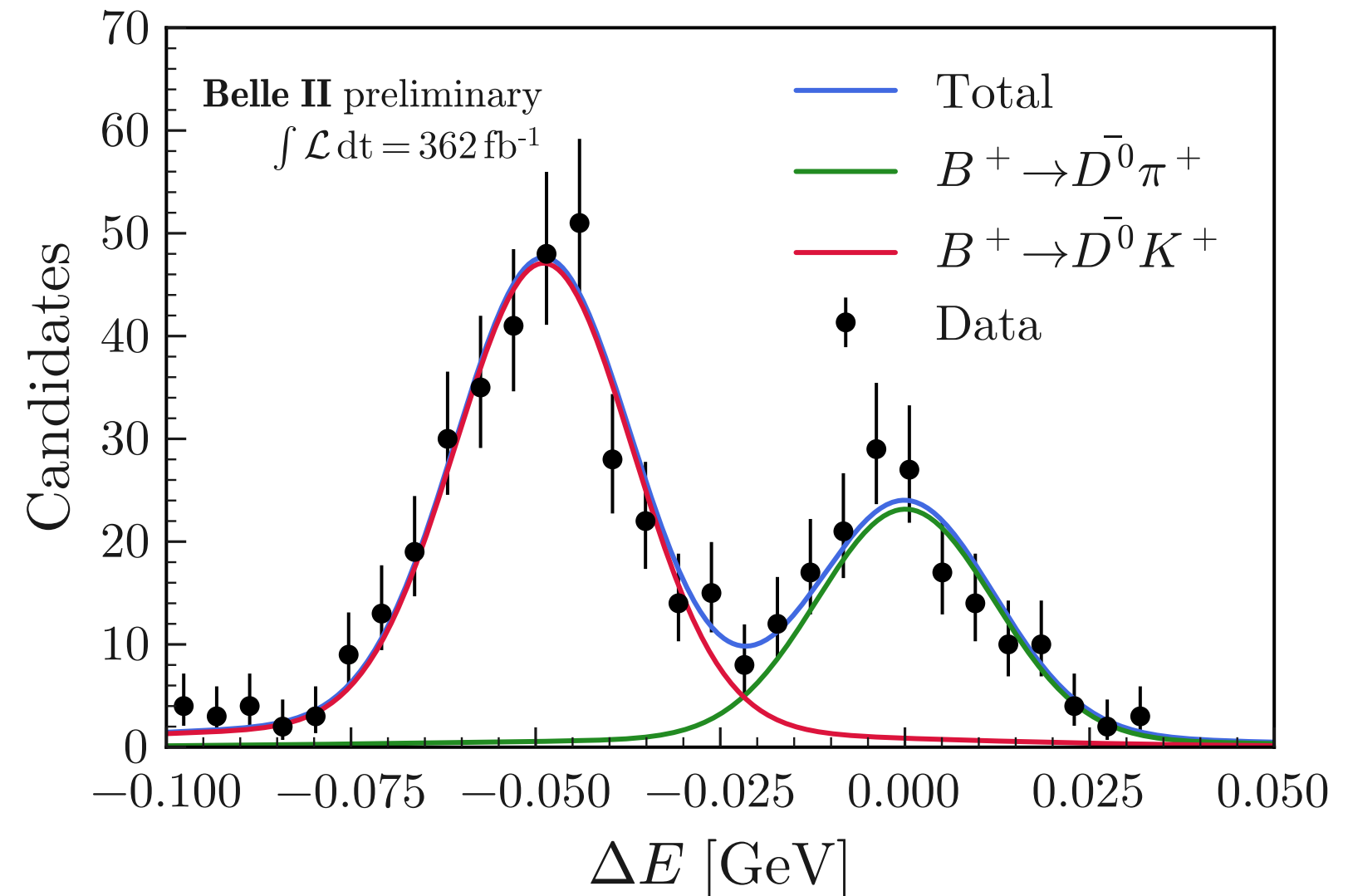
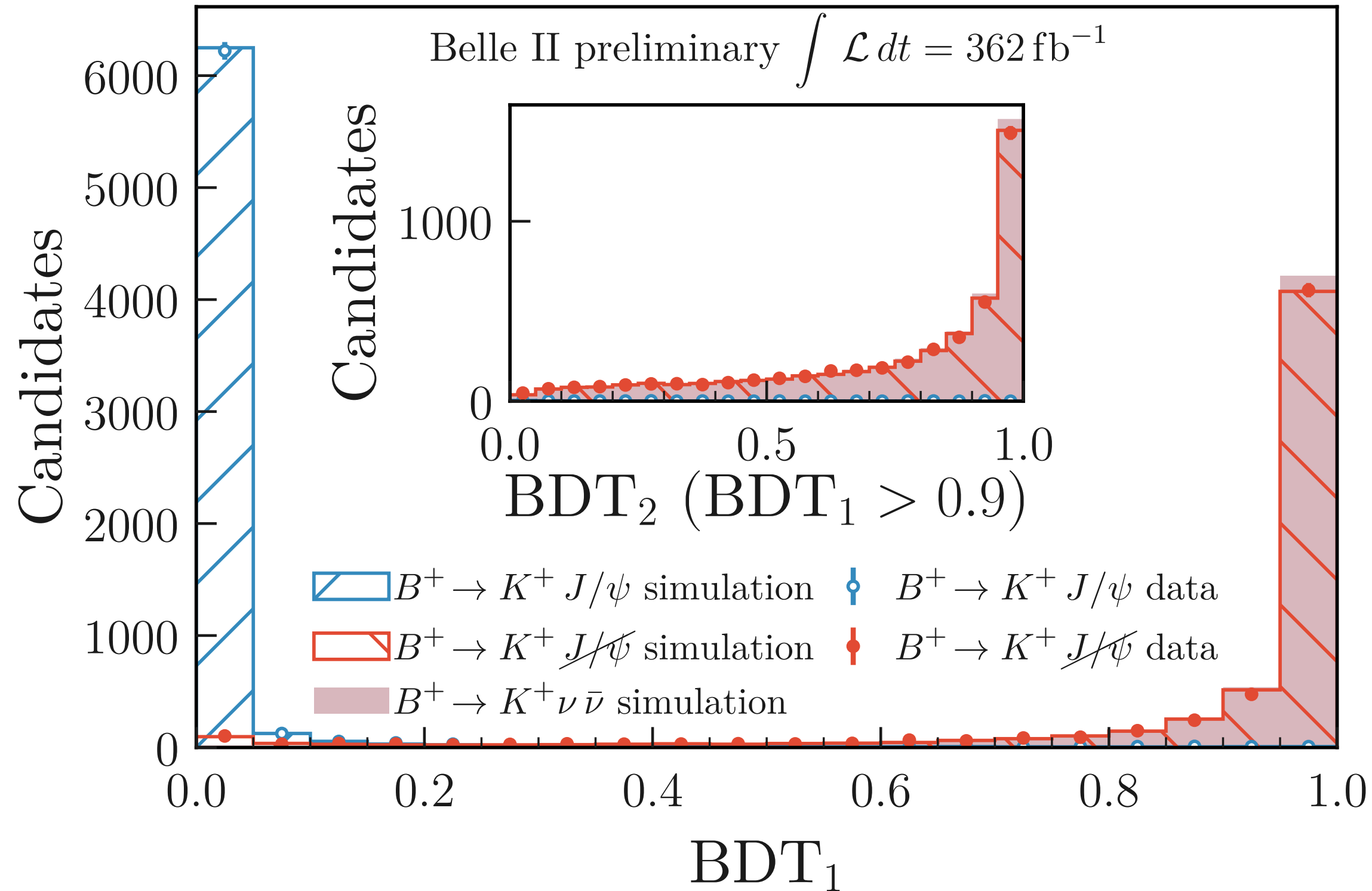


FIG. 22. Distribution of ΔE in data obtained for $B^+ \rightarrow (K^+, \pi^+) D^0$ decays reconstructed as $B^+ \rightarrow K^+ \nu \bar{\nu}$ events with the daughters from the D^0 decays removed.

The relative abundance $\bar{D}^0 K^+$ to $\bar{D}^0 \pi^+$ for data vs. MC is found to be consistent w/ expectation with 1.03 ± 0.09

Signal efficiency validation (ITA)



Charm baryon decays $\Xi_c^0 \rightarrow \Xi^0 h^0$

($h^0 = \pi^0, \eta, \eta'$)

Table 1. Theoretical predictions for the branching fractions and decay asymmetry parameters for $\Xi_c^0 \rightarrow \Xi^0 h^0$ decays. Branching fractions are given in units of 10^{-3} .

| Reference | Model | $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$ | $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta)$ | $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta')$ | $\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$ |
|--------------------------|---------------------|--|---|--|---|
| Körner, Krämer [5] | quark | 0.5 | 3.2 | 11.6 | 0.92 |
| Xu, Kamal [7] | pole | 7.7 | - | - | 0.92 |
| Cheng, Tseng [8] | pole | 3.8 | - | - | -0.78 |
| Cheng, Tseng [8] | CA | 17.1 | - | - | 0.54 |
| Żenczykowski [9] | pole | 6.9 | 1.0 | 9.0 | 0.21 |
| Ivanov <i>et al.</i> [6] | quark | 0.5 | 3.7 | 4.1 | 0.94 |
| Sharma, Verma [11] | CA | - | - | - | -0.8 |
| Geng <i>et al.</i> [12] | $SU(3)_F$ | 4.3 ± 0.9 | $1.7^{+1.0}_{-1.7}$ | $8.6^{+11.0}_{-6.3}$ | - |
| Geng <i>et al.</i> [13] | $SU(3)_F$ | 7.6 ± 1.0 | 10.3 ± 2.0 | 9.1 ± 4.1 | $-1.00^{+0.07}_{-0.00}$ |
| Zhao <i>et al.</i> [14] | $SU(3)_F$ | 4.7 ± 0.9 | 8.3 ± 2.3 | 7.2 ± 1.9 | - |
| Zou <i>et al.</i> [10] | pole | 18.2 | 26.7 | - | -0.77 |
| Huang <i>et al.</i> [15] | $SU(3)_F$ | 2.56 ± 0.93 | - | - | -0.23 ± 0.60 |
| Hsiao <i>et al.</i> [16] | $SU(3)_F$ | 6.0 ± 1.2 | $4.2^{+1.6}_{-1.3}$ | - | - |
| Hsiao <i>et al.</i> [16] | $SU(3)_F$ -breaking | 3.6 ± 1.2 | 7.3 ± 3.2 | - | - |
| Zhong <i>et al.</i> [17] | $SU(3)_F$ | $1.13^{+0.59}_{-0.49}$ | 1.56 ± 1.92 | $0.683^{+3.272}_{-3.268}$ | $0.50^{+0.37}_{-0.35}$ |
| Zhong <i>et al.</i> [17] | $SU(3)_F$ -breaking | $7.74^{+2.52}_{-2.32}$ | $2.43^{+2.79}_{-2.90}$ | $1.63^{+5.09}_{-5.14}$ | $-0.29^{+0.20}_{-0.17}$ |
| Xing <i>et al.</i> [18] | $SU(3)_F$ | 1.30 ± 0.51 | - | - | -0.28 ± 0.18 |