KEK IPNS Physics Seminar

Measurement of the $e^+e^-\to\pi^+\pi^-\pi^0$ cross section with the Belle II detector

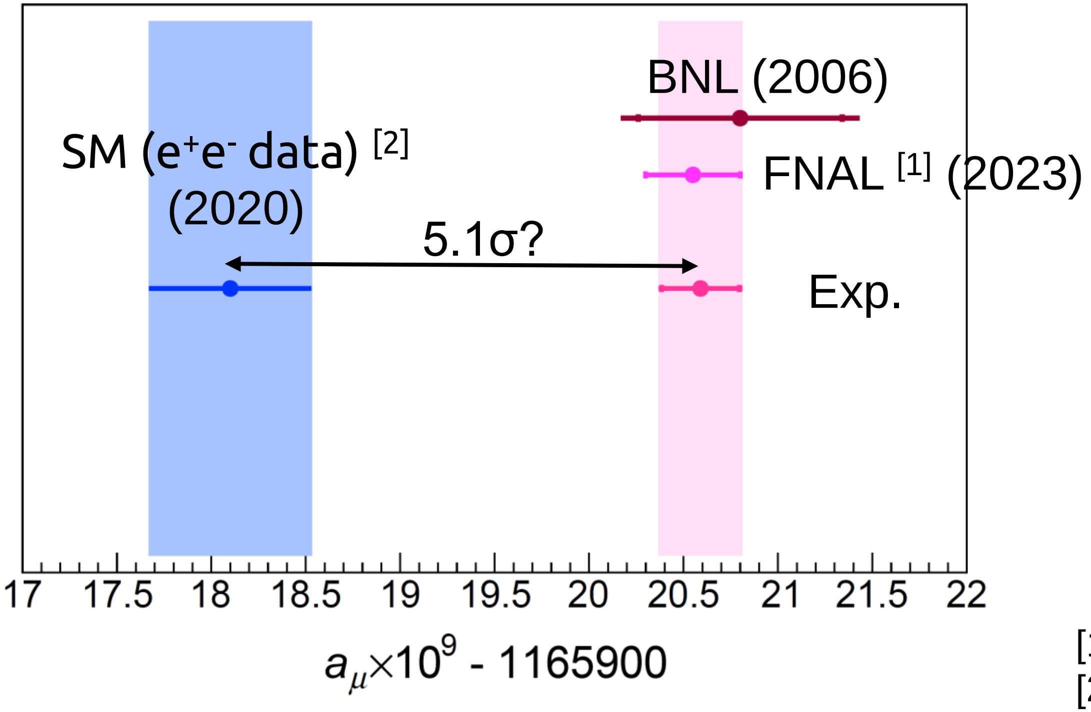
Yuki Sue, Nagoya University
on behalf of Belle II collaboration

2024.4.10

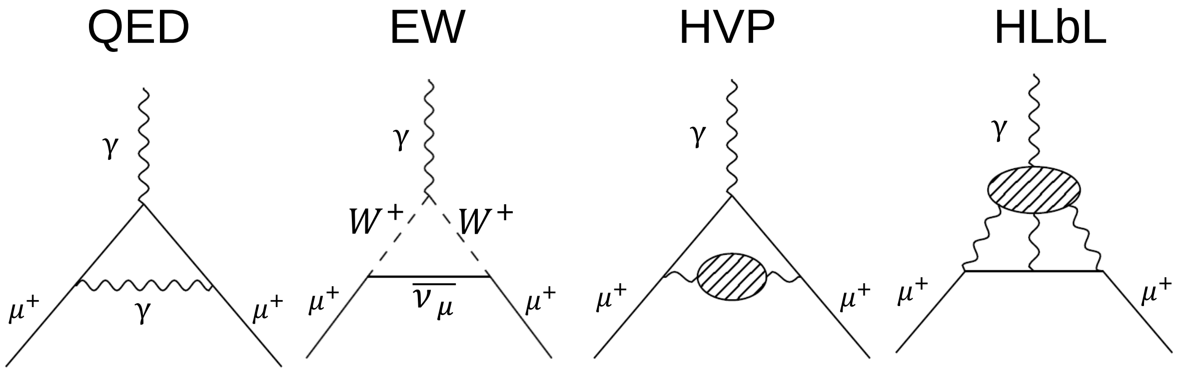
arXiv:2404.04915

Recent situation in muon g-2 anomaly

- 5σ significance through new direct measurements from Fermilab
- Non-negligible uncertainty in theoretical predictions



$$a_\mu^{\mathrm{SM}} = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$



[1] DPRL 131 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)

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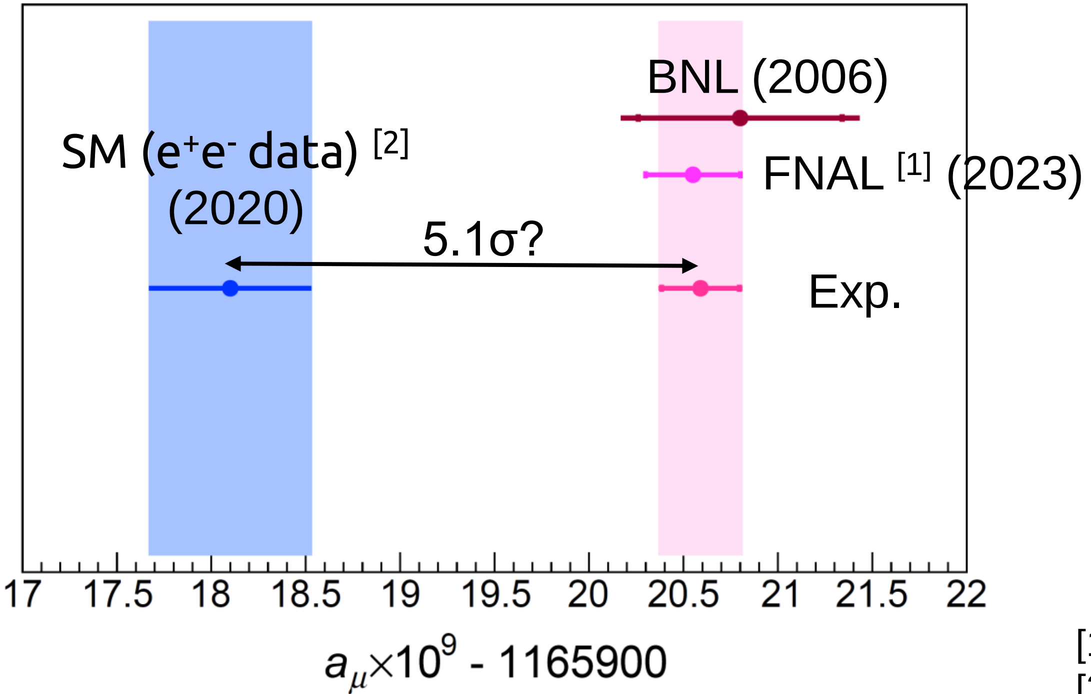

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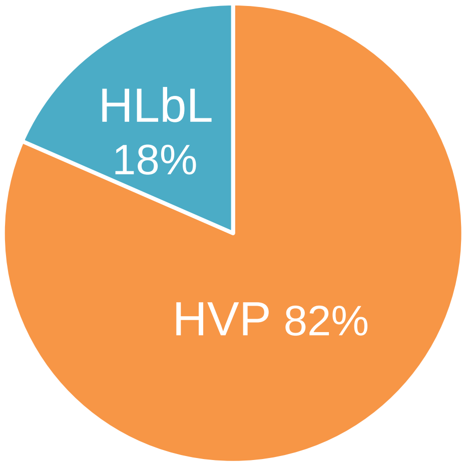
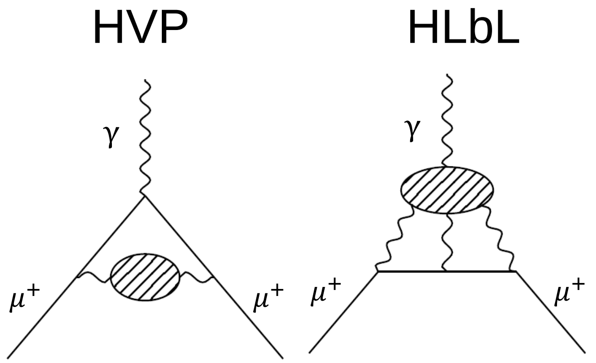
a_μ^{SM} uncertainty [2]





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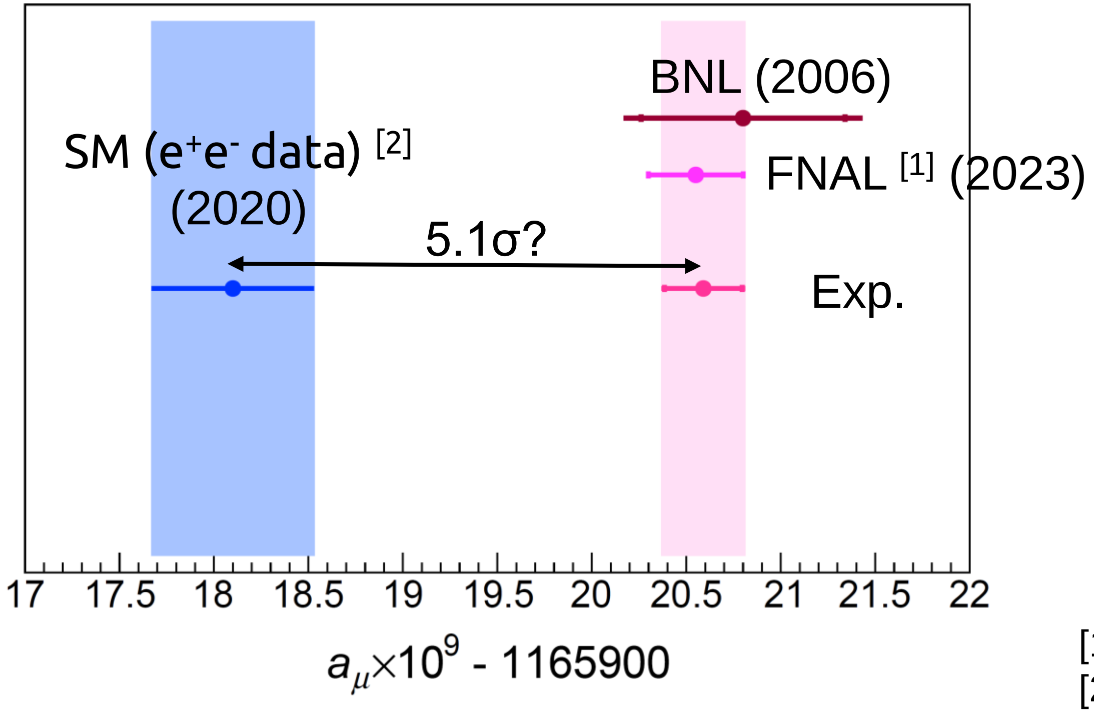
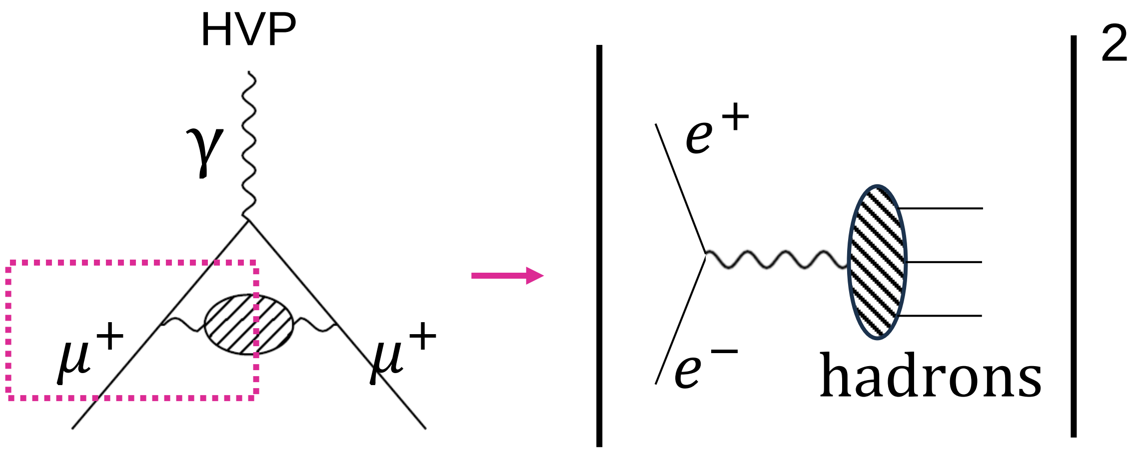


HVP calculation with ee→hadrons data



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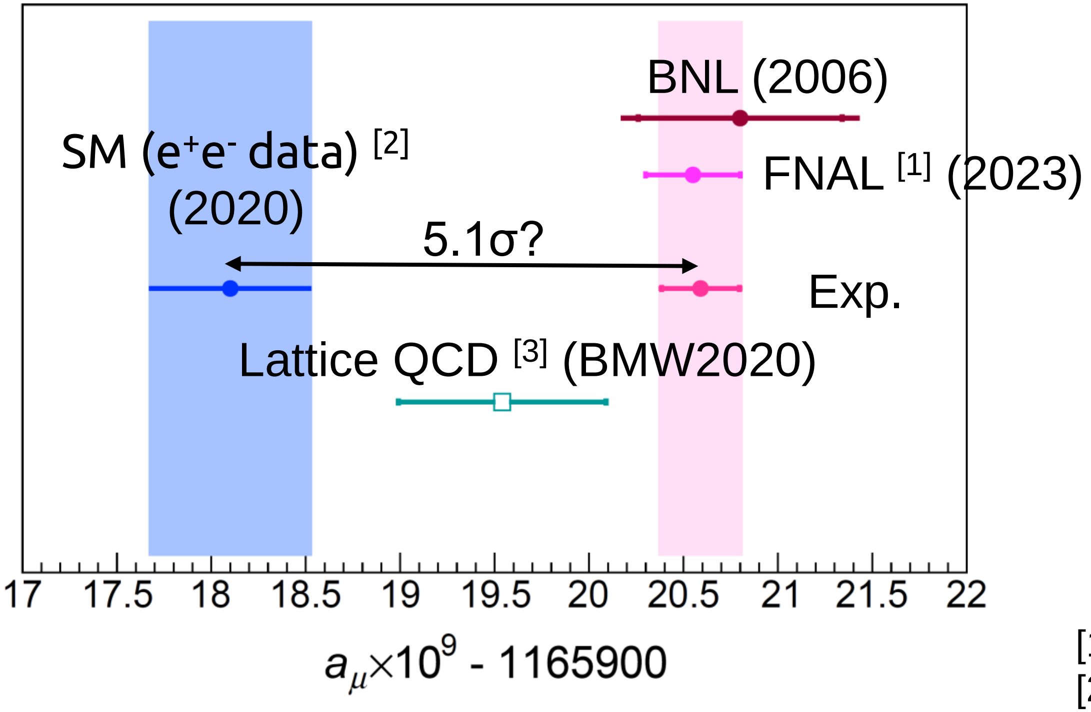
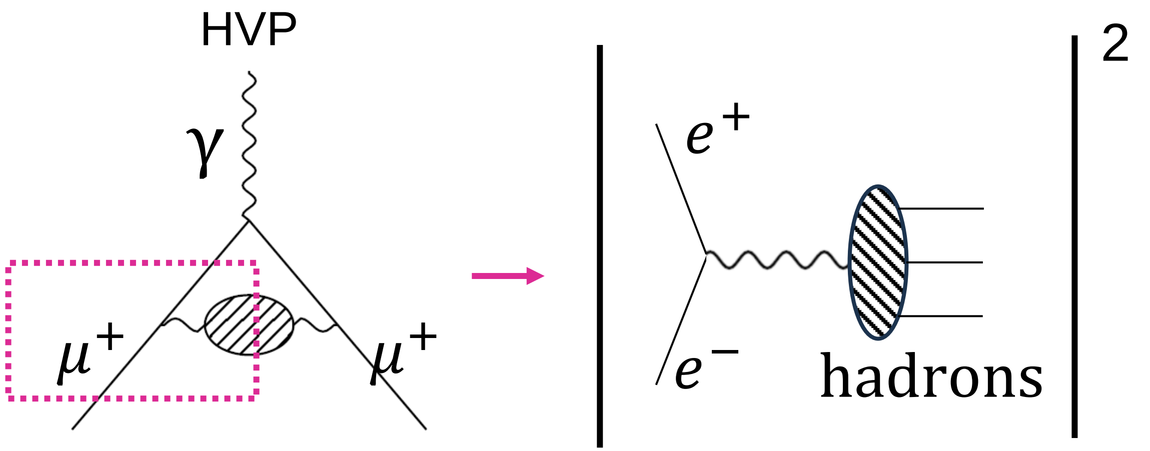
  - HVP predictions are different depending on methods: e^+e^- data vs Lattice QCD



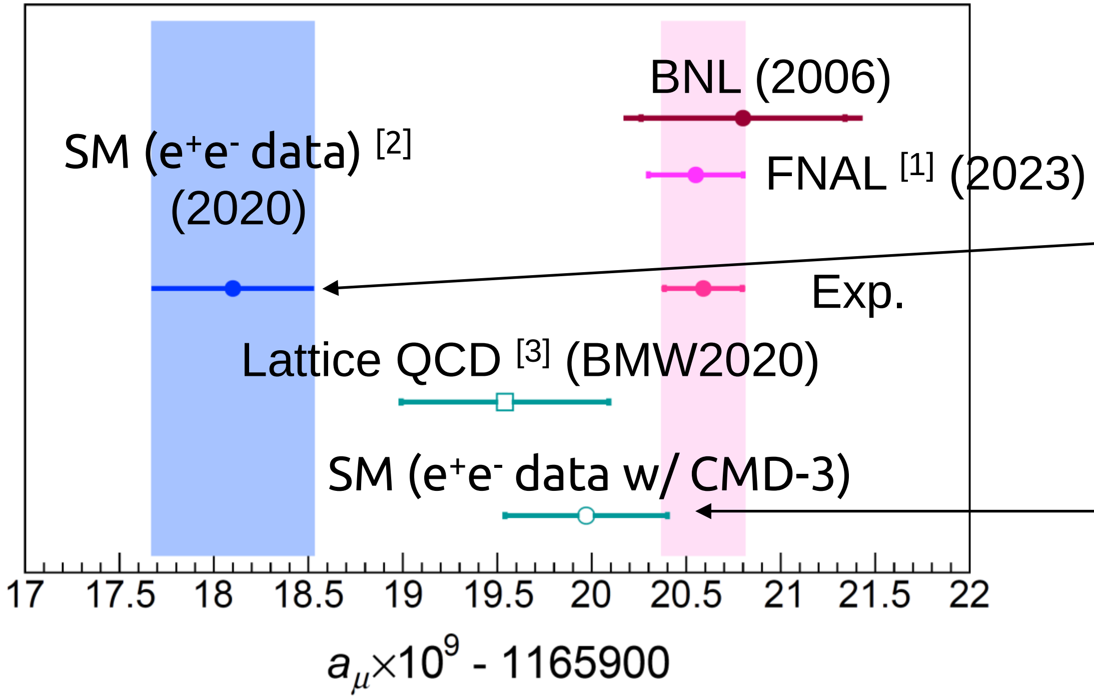
HVP calculation with ee→hadrons data



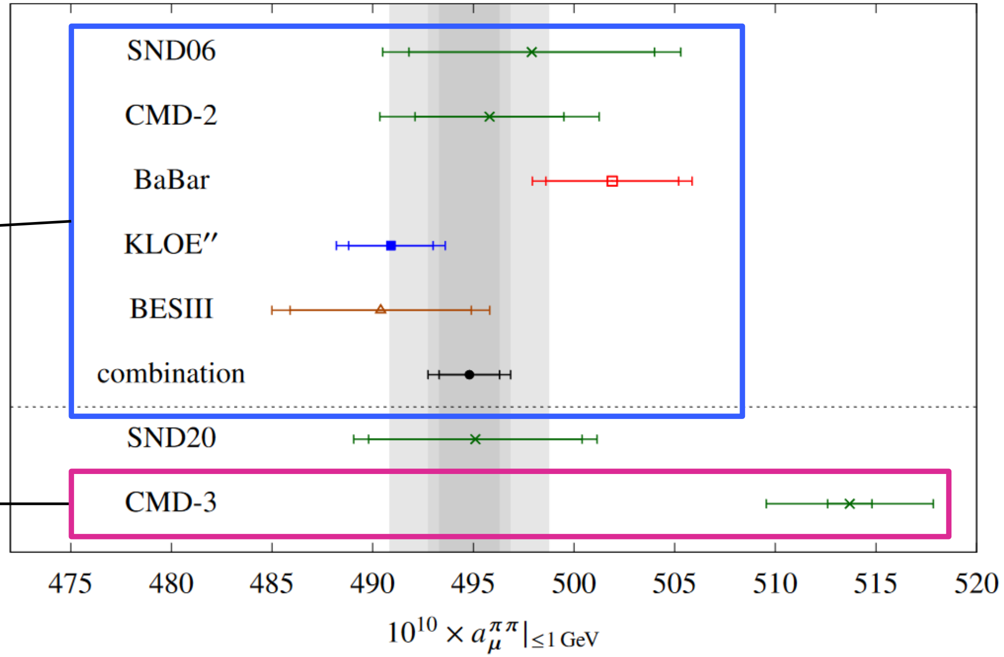
[1] DPRL 131 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)
[3] Nature 593, 7857 (2021)

Recent situation in muon g-2 anomaly

- 5σ significance through new direct measurements from Fermilab
- Non-negligible uncertainty in theoretical predictions
  - Major uncertainty is derived from Hadronic Vacuum Polarization (HVP) term
  - HVP predictions are different depending on methods: e^+e^- data vs Lattice QCD
  - Differences among e^+e^- experiments are also non-negligible
- Validation by independent experiments is important in HVP prediction



ee→ππ contribution to HVP



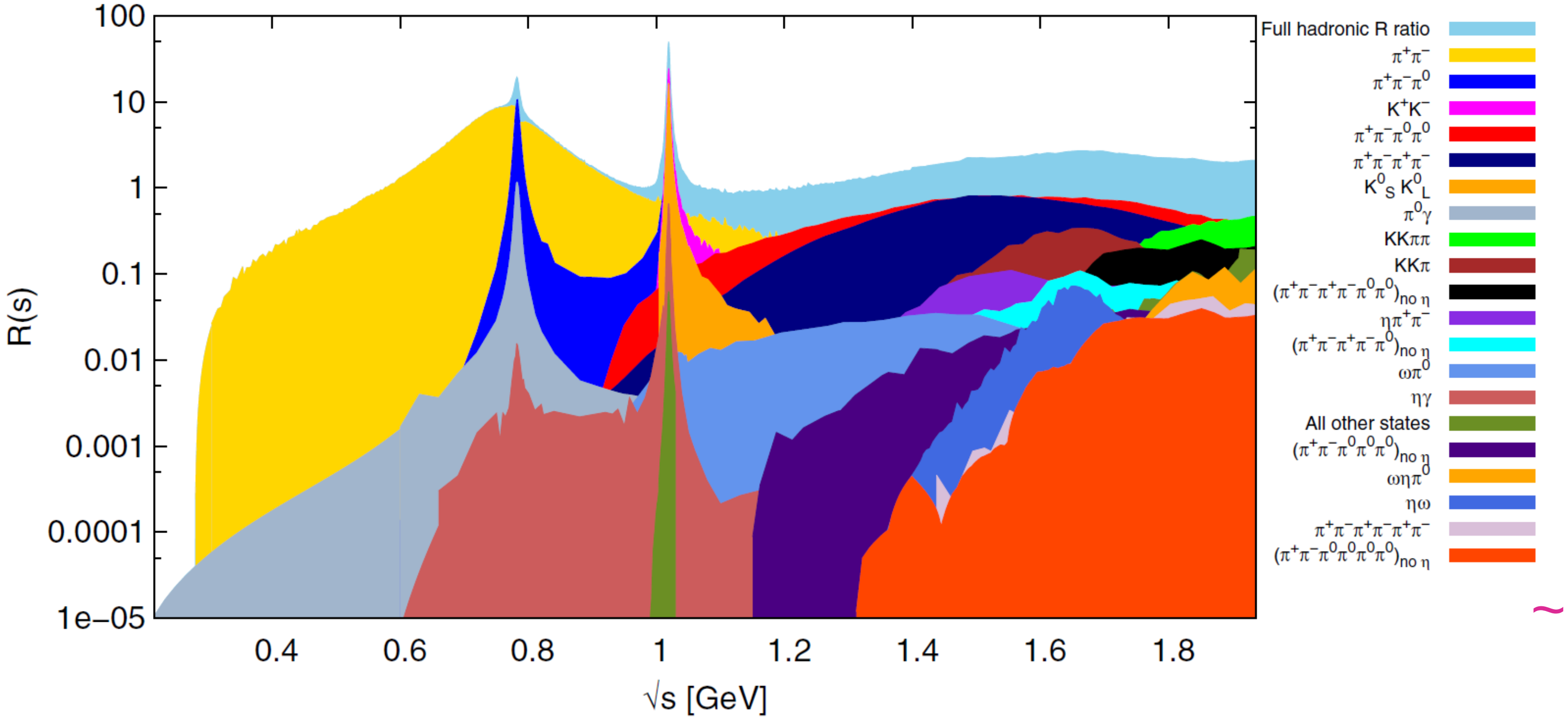
G. Colangelo *et. al*, arXiv:2308.04217

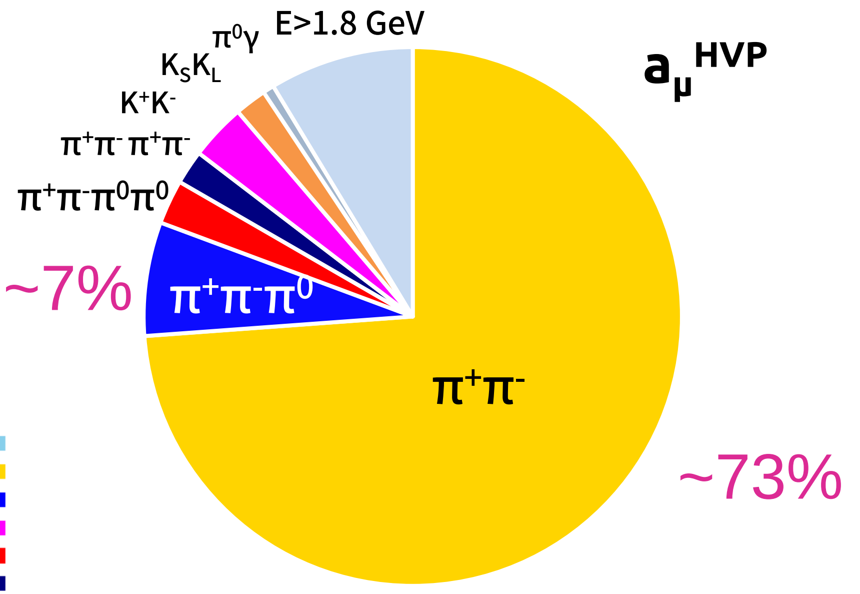
Cross section measurements of exclusive channels

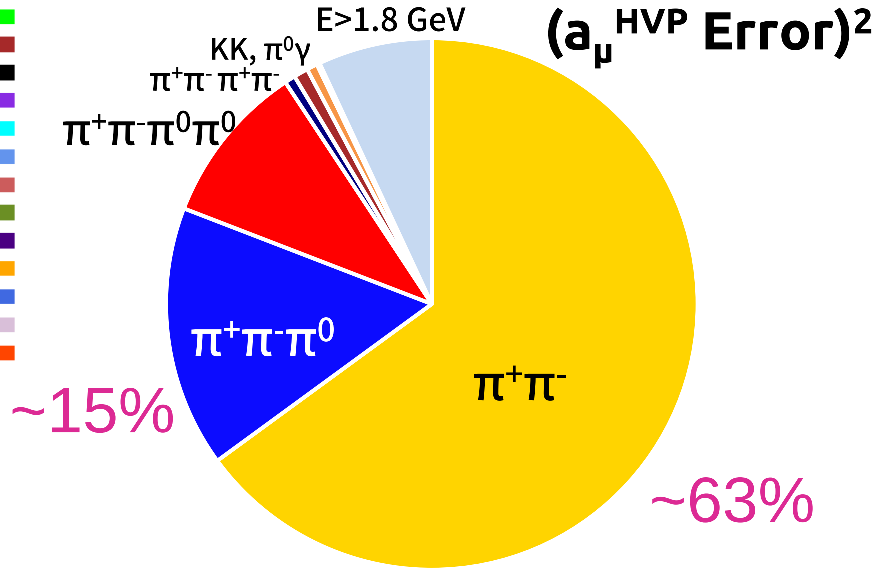
Leading order HVP contribution

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{\hat{K}(s)}{s^2} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} ds$$

- Aiming to measure and verify cross sections at Belle II
- As a first step, we begin with $e^+e^-\to\pi^+\pi^-\pi^0$ channel





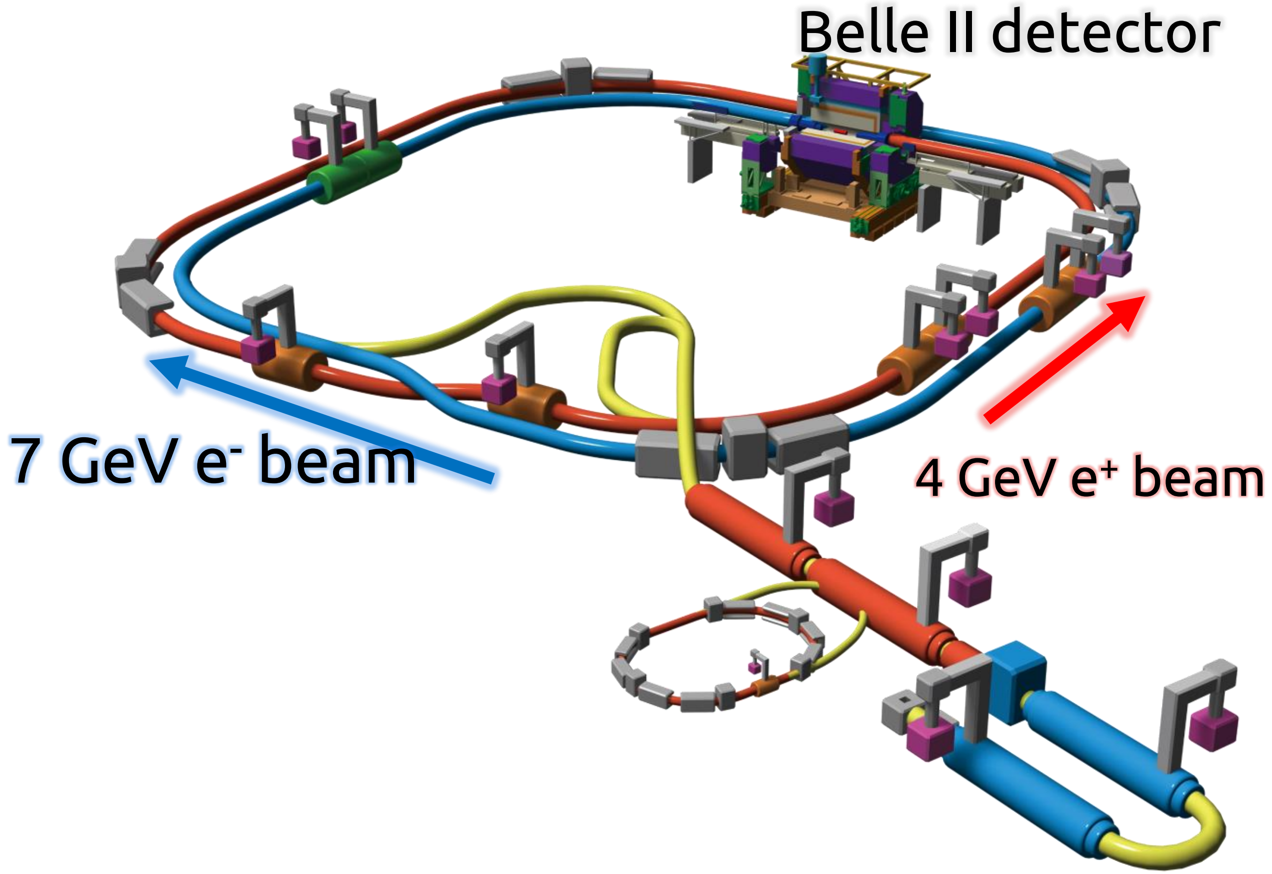


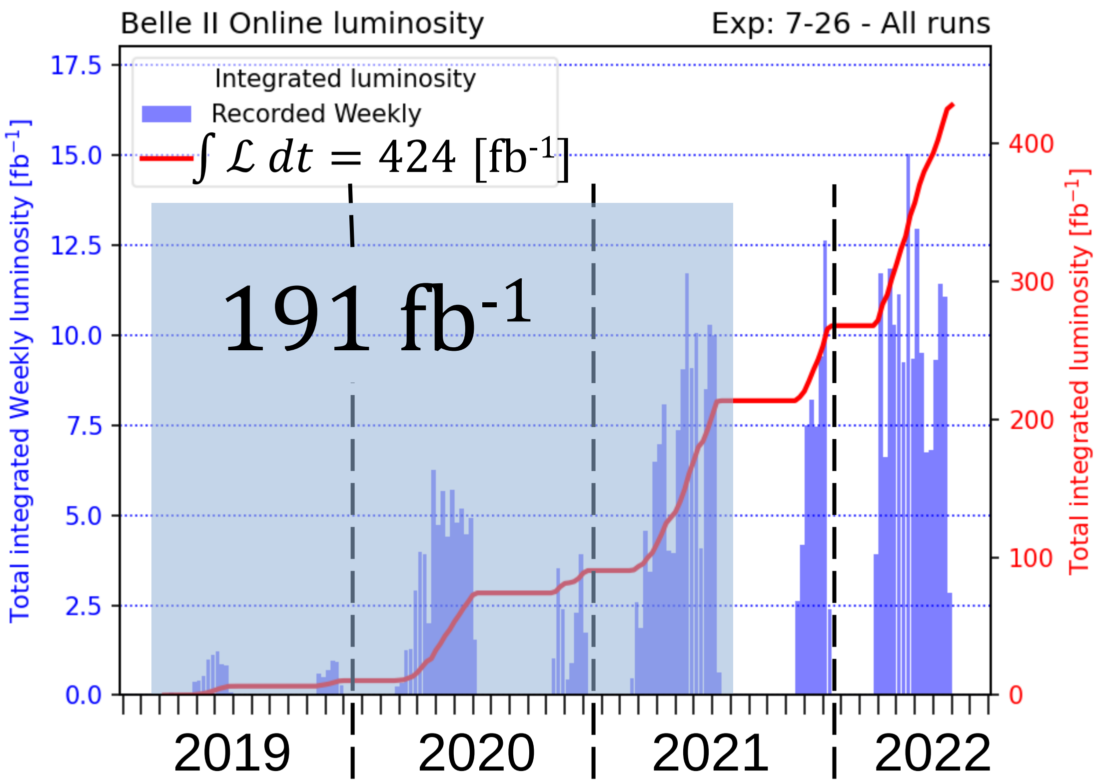
A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D 101, 014029 (2020).

SuperKEKB/Belle II experiment

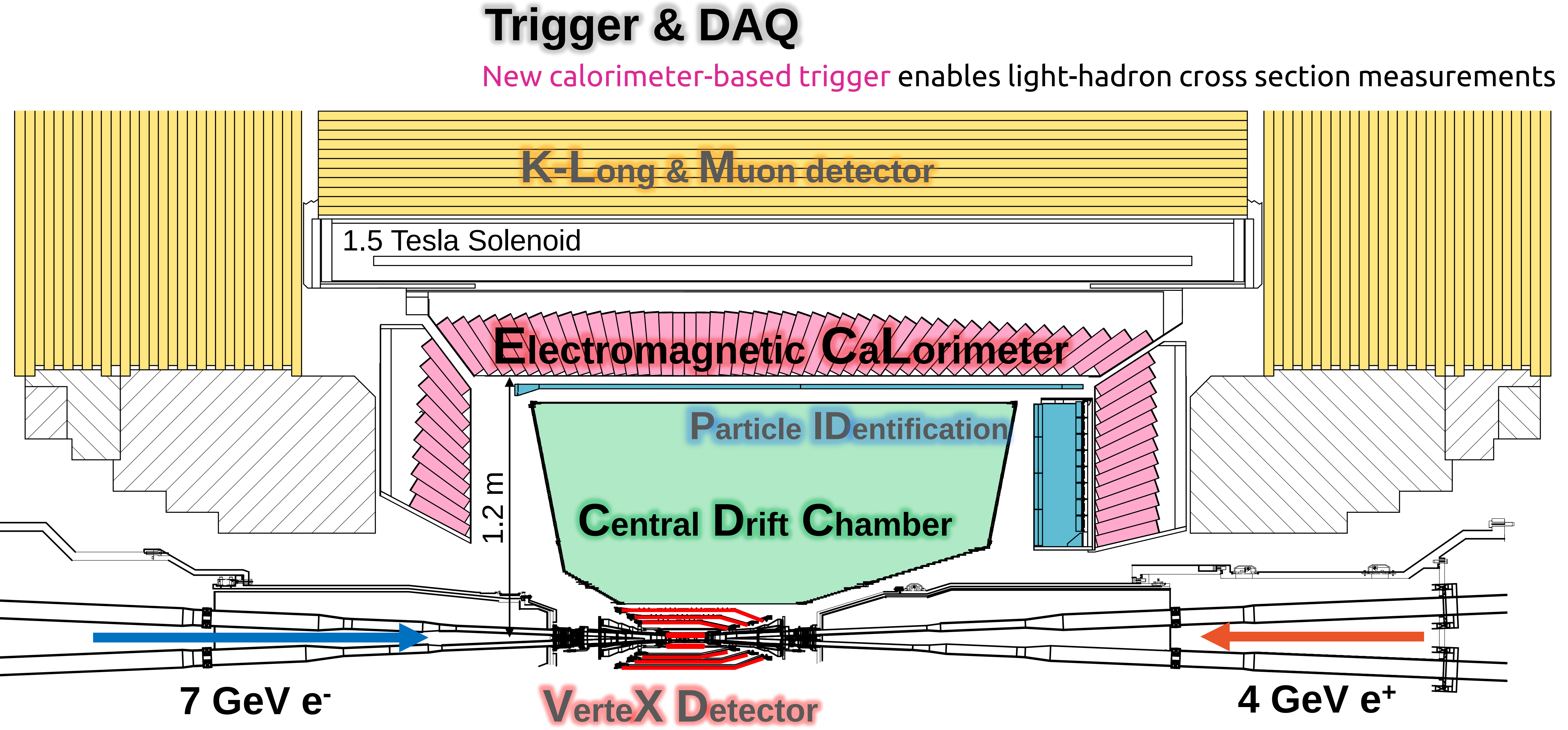
- Asymmetric e^+e^- collider at KEK
  - $\sqrt{s}$ = M(Υ(4S)) = 10.58 GeV
  - World record instantaneous luminosity : 4.7×10^{34} /cm²/s
  - ~90% data taking efficiency : 1-2 fb^{-1}/day

- Used dataset in this analysis
  - 2019 - 2021 Summer dataset
  - Integrated luminosity: 191 fb^{-1}
    - A half of the collected data, 424 fb^{-1}

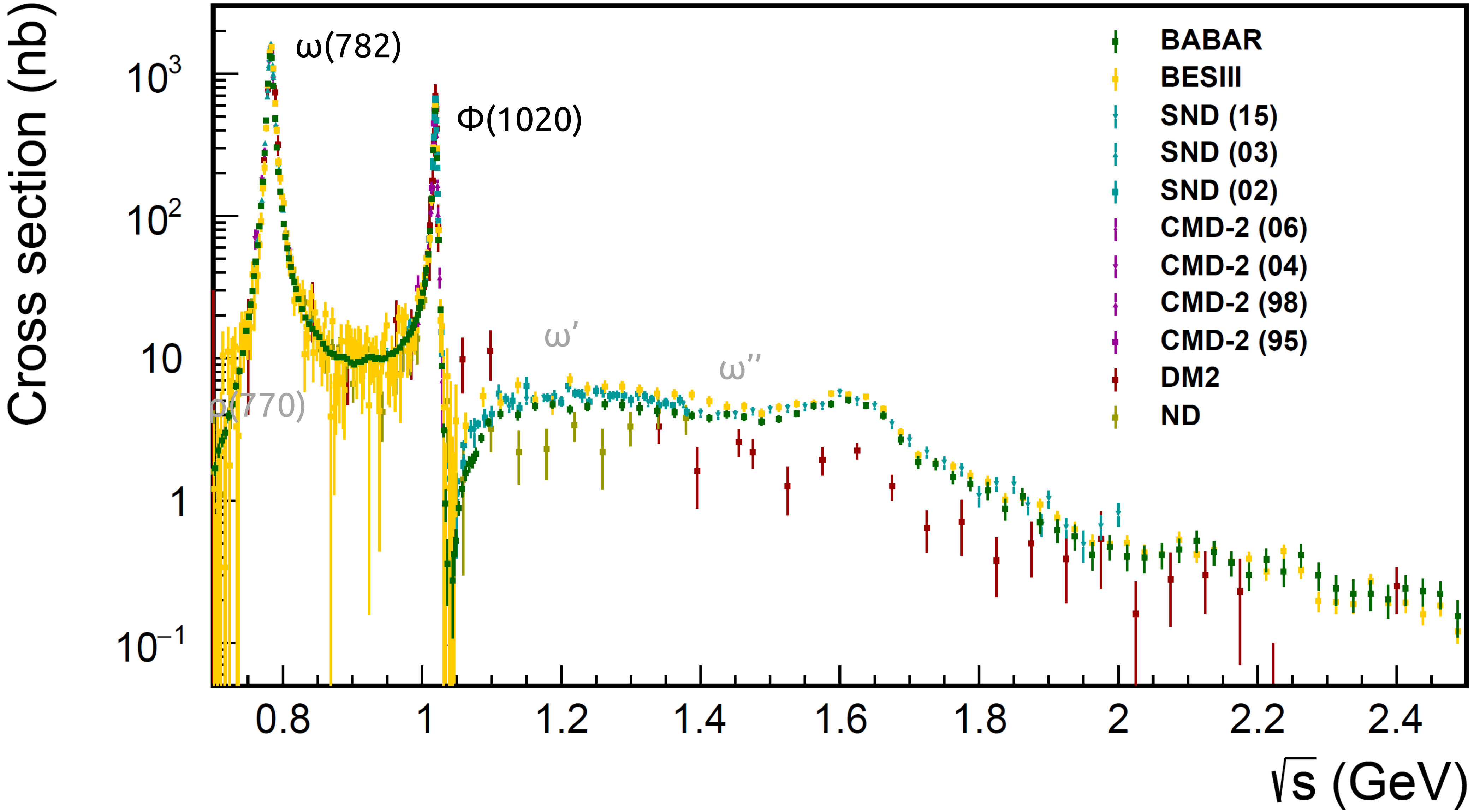




Belle II detector

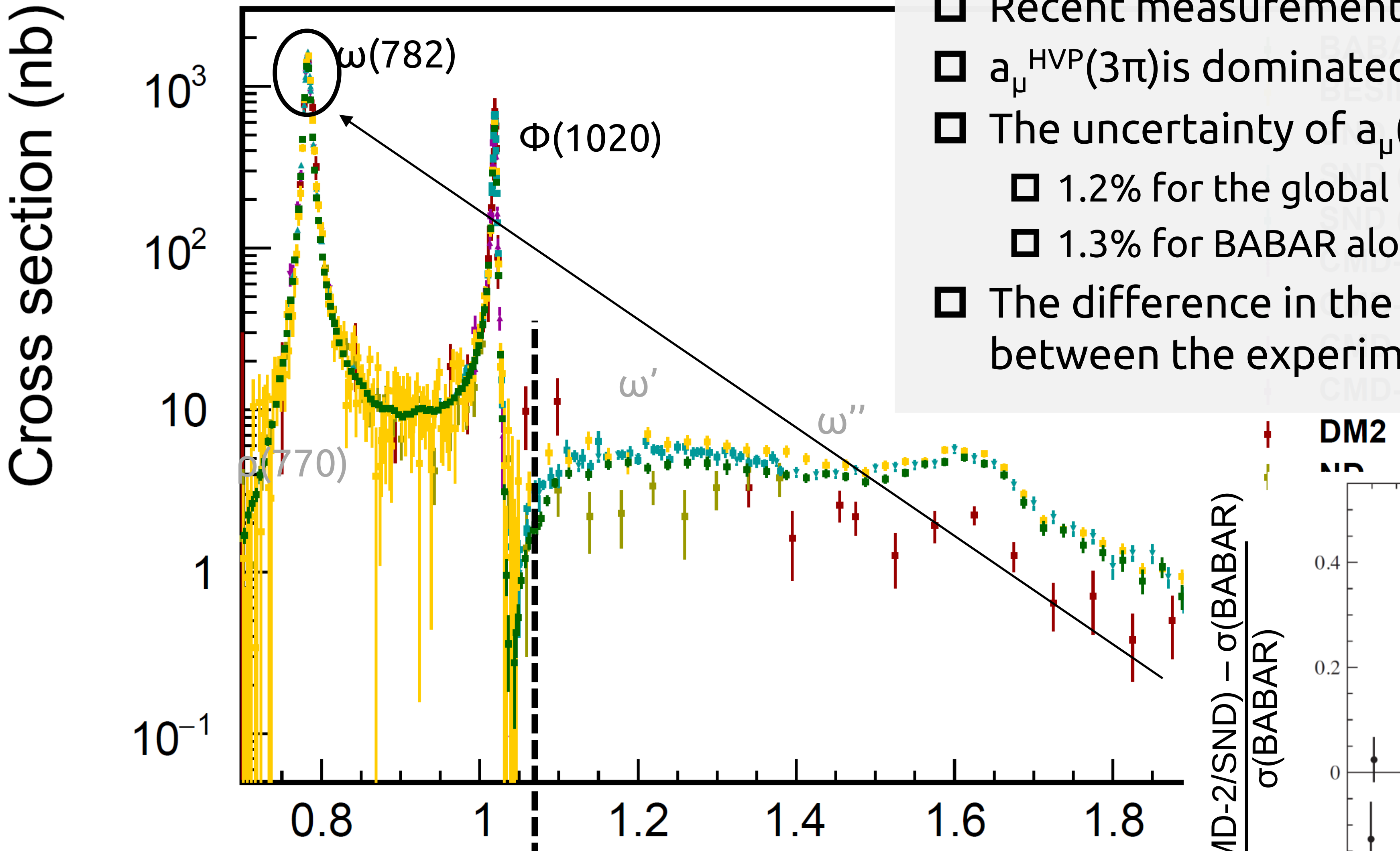


Previous measurements for $e^+e^- \to \pi^+\pi^-\pi^0$

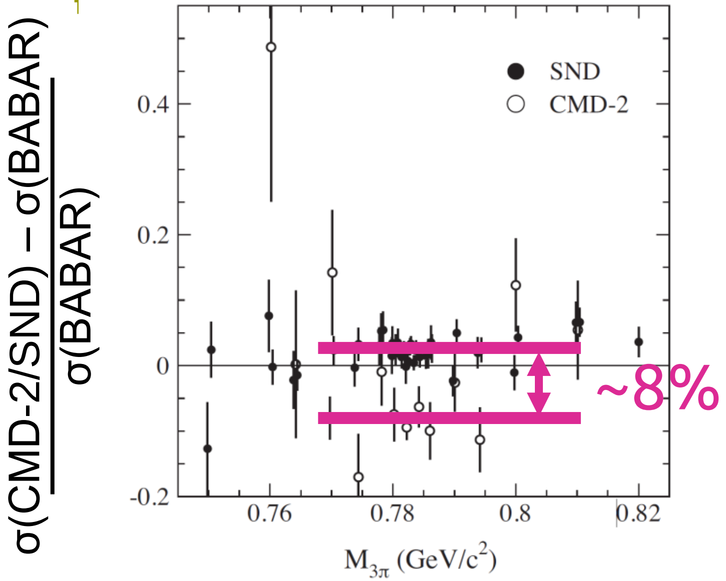


Previous measurements for $e^+e^- \to \pi^+\pi^-\pi^0$

- Recent measurements: BaBar, SND, CMD-2…
- $a_\mu^{HVP}(3\pi)$is dominated by ω and Φ resonances
- The uncertainty of $a_\mu(3\pi)$:
  - 1.2% for the global fit
  - 1.3% for BABAR alone
- The difference in the cross section between the experiments below 1.1 GeV



$a_\mu^{HVP}(3\pi)$ Contribution 93% 7%



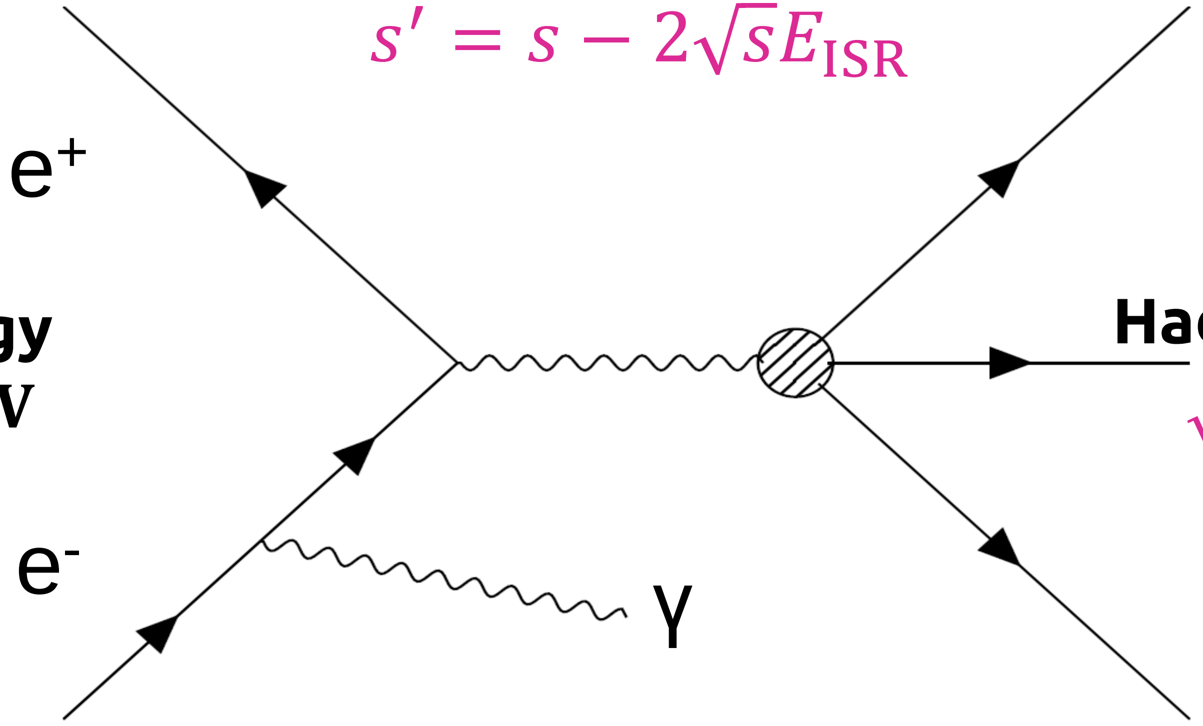
Radiative return method

- Measure the cross section in the energy range 0.4-3.5 GeV at fixed e^+e^- energy collision
- Use a process associated with energetic ISR emission
  - Only less than10% of ISR photons are emitted into detector acceptance

$$s' = s - 2\sqrt{s}E_{\mathrm{ISR}}$$

Initial e^+e^- energy

$\sqrt{s} = \mathbf{10.58\ GeV}$



Hadronic system energy

$\sqrt{s'} = \mathbf{0.4-3.5\ GeV}$

Initial-state radiation : ISR

$E_{\mathrm{ISR}} > 4.7 - 5.3\ \mathrm{GeV}$

Analysis overview

Signal spectrum

Cross section

$$\sigma_{3\pi}(M(3\pi)) = \frac{N_{\mathrm{signal}}}{\varepsilon(M(3\pi)) \cdot L_{\mathrm{eff}}(M(3\pi))}$$

3π mass

Efficiency **Integrated luminosity**

- Target : $\delta a_\mu^{3\pi}/a_\mu^{3\pi}$ ~2% with 191 fb^{-1} data
- Key items
  - Event selection to extract $e^+e^-\to\pi^+\pi^-\pi^0\gamma_{ISR}$ process
  - Background suppression and estimation
  - Unfolding to mitigate detector resolution
  - Efficiency corrections between data and simulation
- Blind analysis
  - Study of analytical methods using MC before examining data

Analysis outline

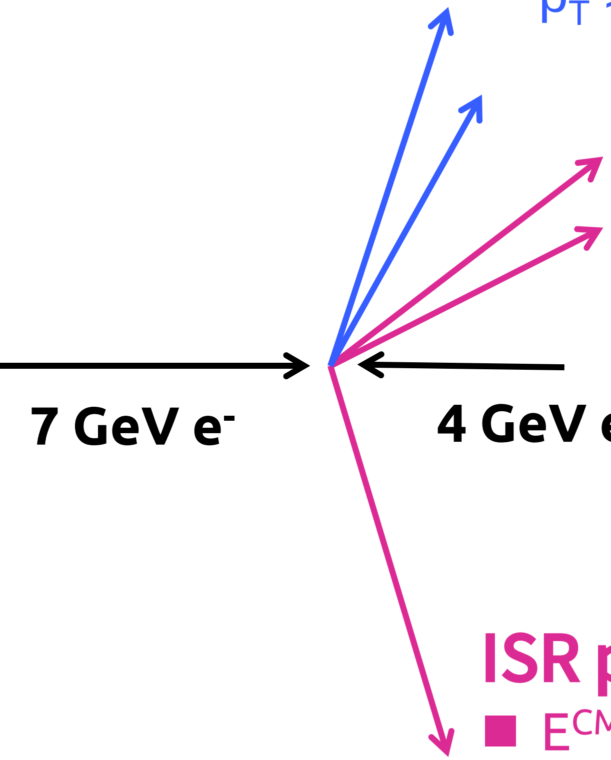
- Event selection
- Background estimation
- Signal extraction
- Unfolding
- Efficiency estimation
- Cross section and a_μ calculation

$e^+e^-\to\pi^+\pi^-\pi^0\gamma_{ISR}$ selection

Reconstruct Two tracks + three photons : $e^+e^-\to\pi^+\pi^-\pi^0\gamma_{ISR}\to\pi^+\pi^-\gamma\gamma\gamma_{ISR}$

$\pi^\pm$

- From interaction point
- Transverse momentum $p_T > 0.2$ GeV/c



π^0-decay photons

- $E > 100$ MeV
- $M(\gamma\gamma) < 1$ GeV/c^2
  [Wide mass range for π^0 mass fit]

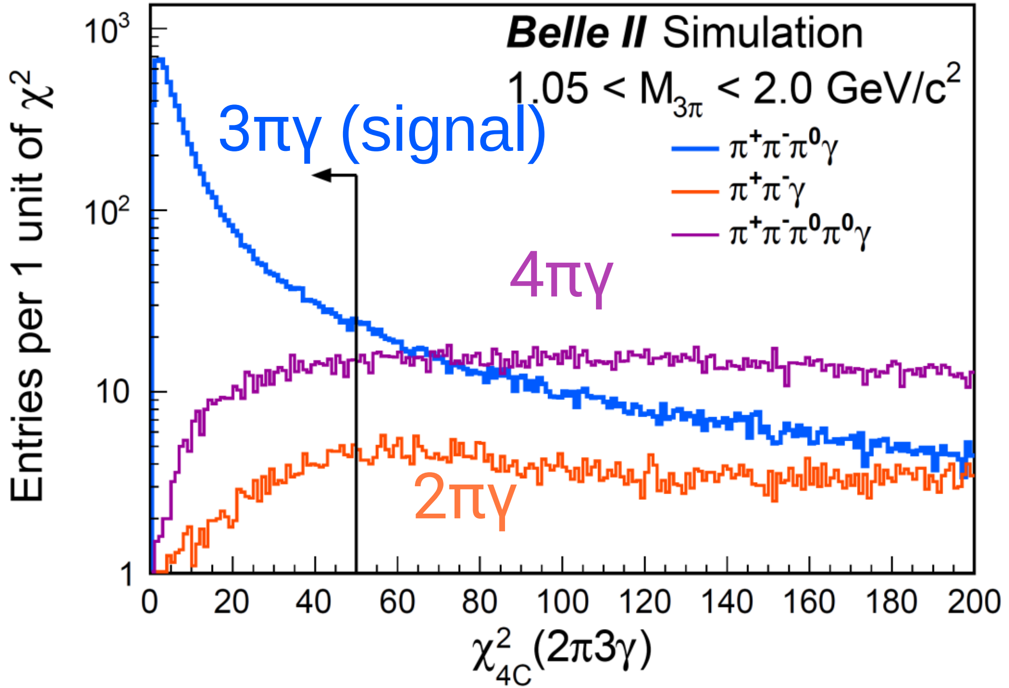
ISR photon

- $E^{CMS} > 2$ GeV
- In barrel ECL for trigger

Four-vector kinematic fit (4C-KFit)

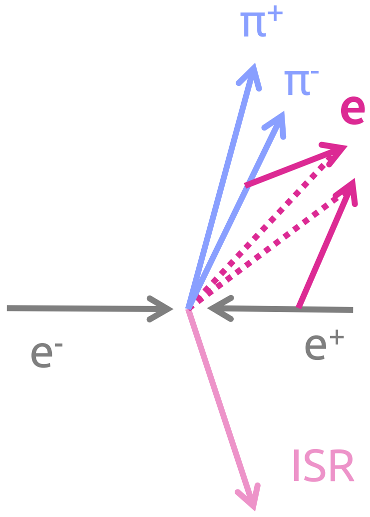
- Fit to positions and momenta
- Constrain to initial e^+e^- four-momentum
- Select small χ^2 to extract signal-like event

4C-Kfit χ^2 distribution (MC)



$e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$ selection: Background suppression

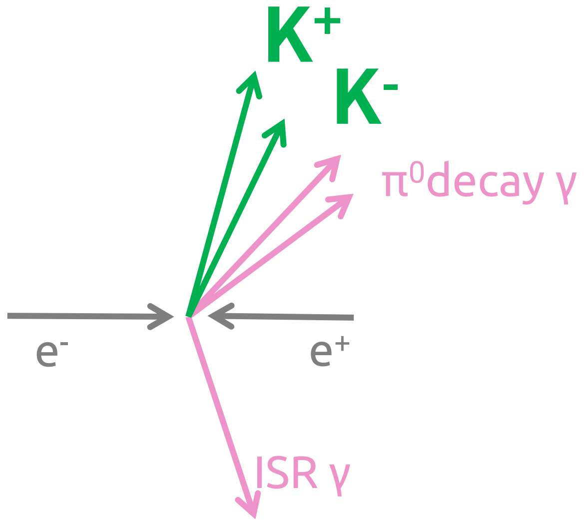
Apply background suppression criteria to reduce remaining backgrounds



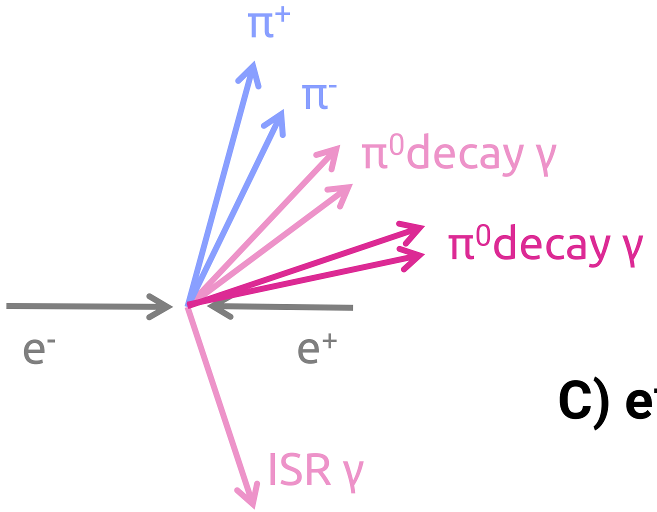
e.g. γ from detector interaction

e.g. γ from beam background

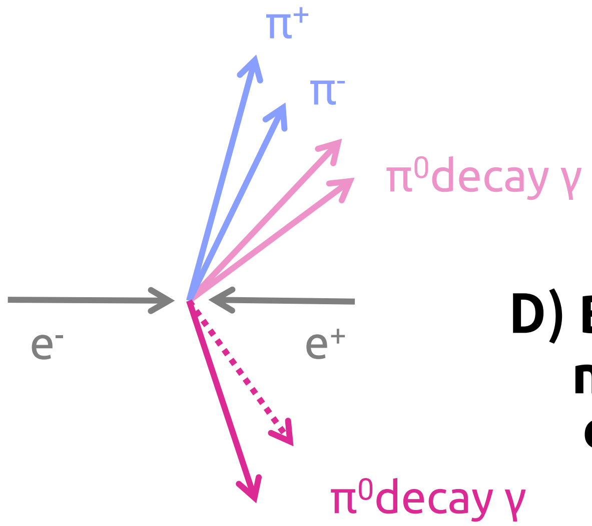
A) Background not containing real π^0:
e.g., $e^+e^- \to e^+e^-\gamma$, $\mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$



B) Kaon background:
$e^+e^- \to K^+K^-\pi^0\gamma_{ISR}$



C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{ISR}$



D) Background
not containing real ISR:
$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$, $\tau^+\tau^-$

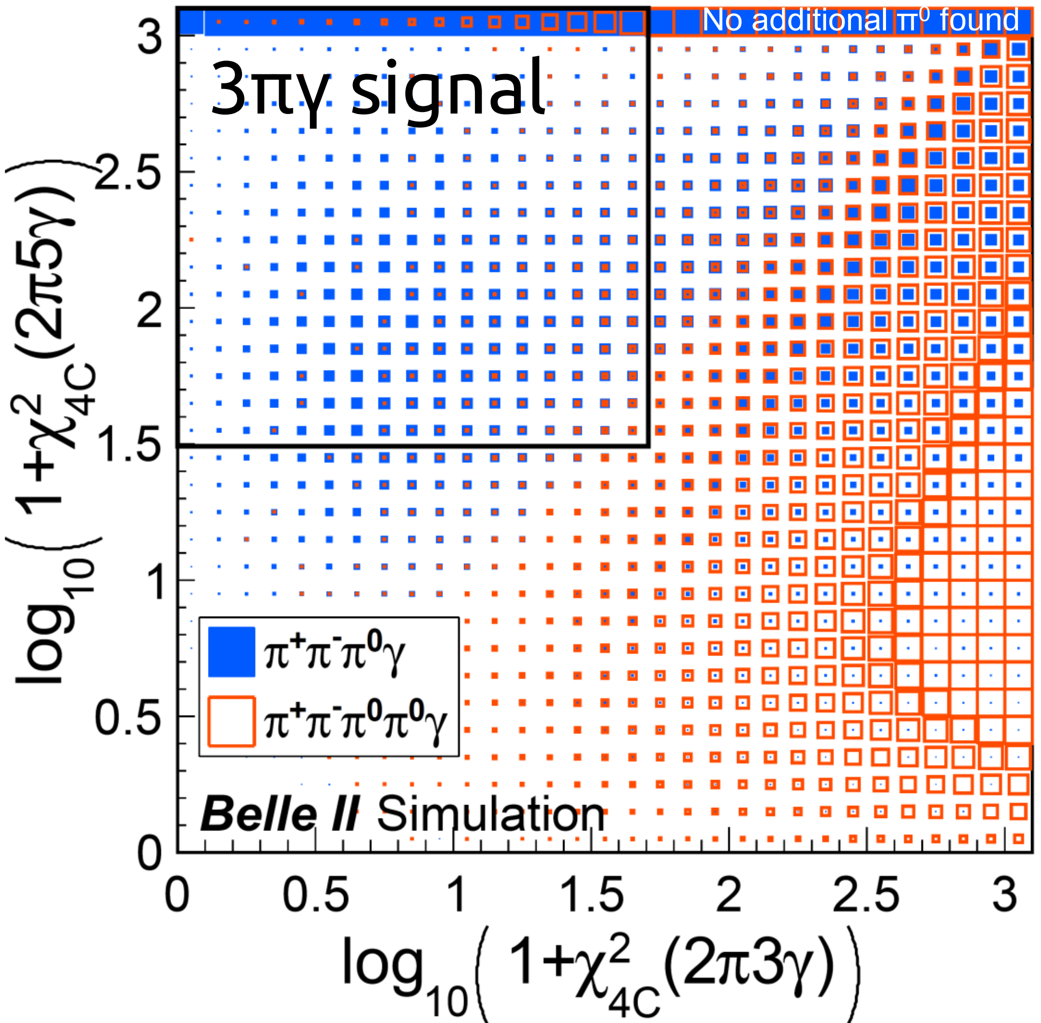
Background suppression (1)

A) Background not containing real π^0 : $e^+e^- \to e^+e^-\gamma, \pi^+\pi^-\gamma, \mu^+\mu^-\gamma$
   - Pion/Electron ID > 0.1
   - $M^2_{recoil}(\pi^+\pi^-) > 4\ GeV^2/c^4$

B) Charged kaon : $e^+e^- \to K^+K^-\pi^0\gamma$
   - Pion/Kaon ID $L(\pi/K) > 0.1$

C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$
   - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ (with additional π^0)
   - 4C kinematic fit under $\pi^+\pi^-\pi^0\pi^0\gamma$ ($2\pi5\gamma$) hypothesis, and $\chi^2_{4C}(2\pi5\gamma) > 30$

$\chi^2_{4C}(2\pi3\gamma)$ versus $\chi^2_{4C}(2\pi5\gamma)$



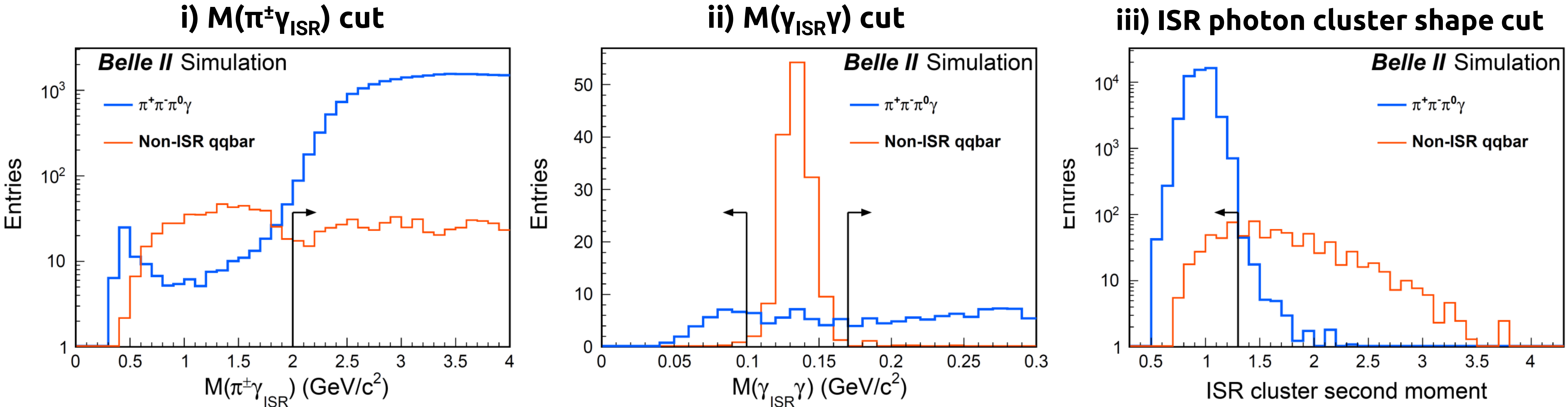
Background suppression (2)

D) Background not containing real ISR : Non-ISR qqbar (dominated by $\pi^+\pi^-\pi^0\pi^0$) and $\tau^+\tau^-$

i. $M(\pi^\pm\gamma_{ISR}) > 2$ GeV/c^2 to reduce high momentum $\rho^\pm \to \pi^+\pi^0$

ii. $M(\gamma_{ISR}\gamma)$ cut to reduce ISR candidate from π^0-decay photon

iii. Cluster shape cut to reduce ISR-like photon in which two photons from of π^0 are merged



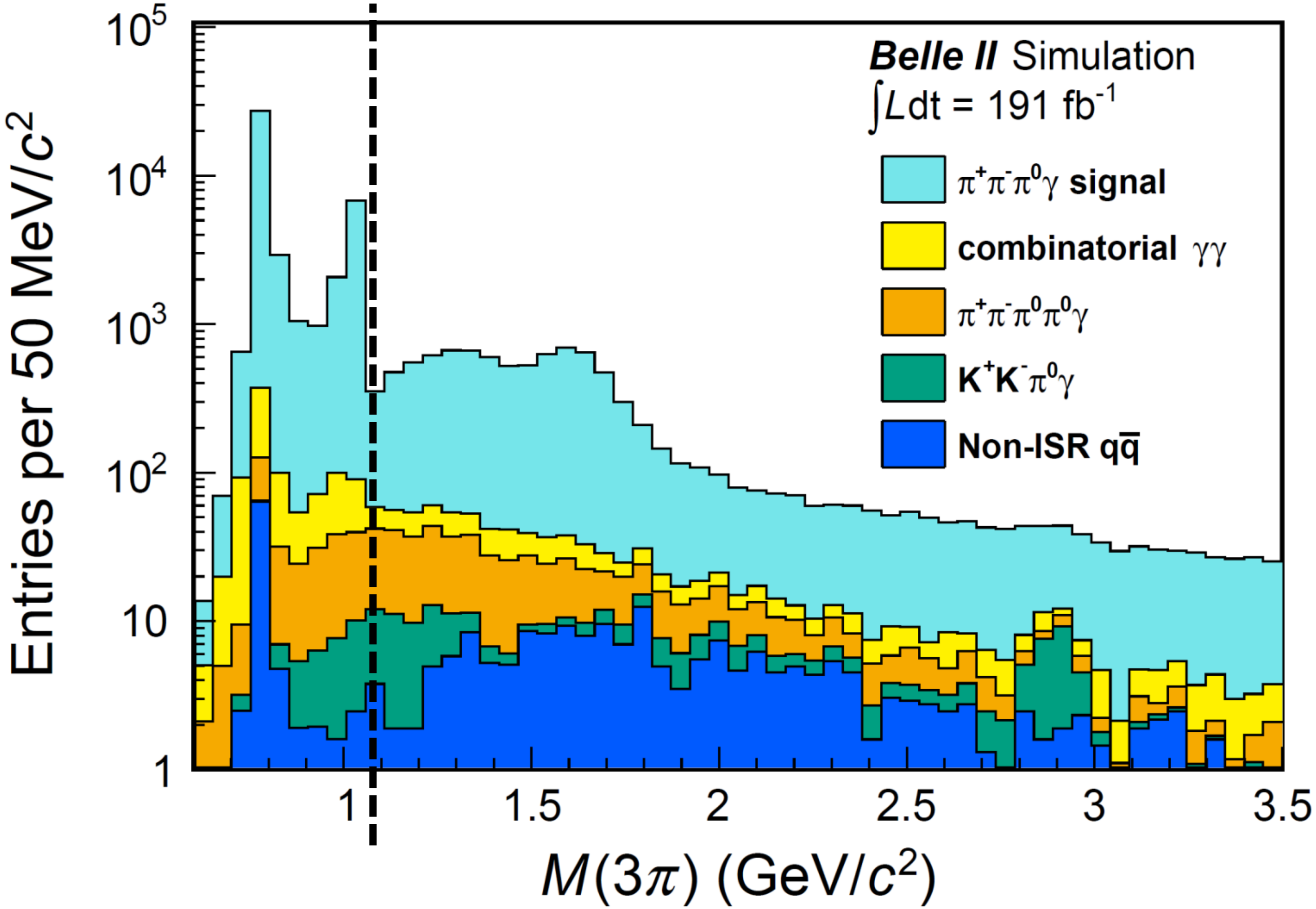
After applying all selection criteria

M(3π) < 1.05 GeV/c²

- Combinatorial γγ background is dominant bkg.
- Signal purity is 98%

M(3π) > 1.05 GeV/c²

- $\pi^+\pi^-\pi^0\pi^0\gamma$ background is dominant bkg.



Analysis outline

- ☐ Event selection
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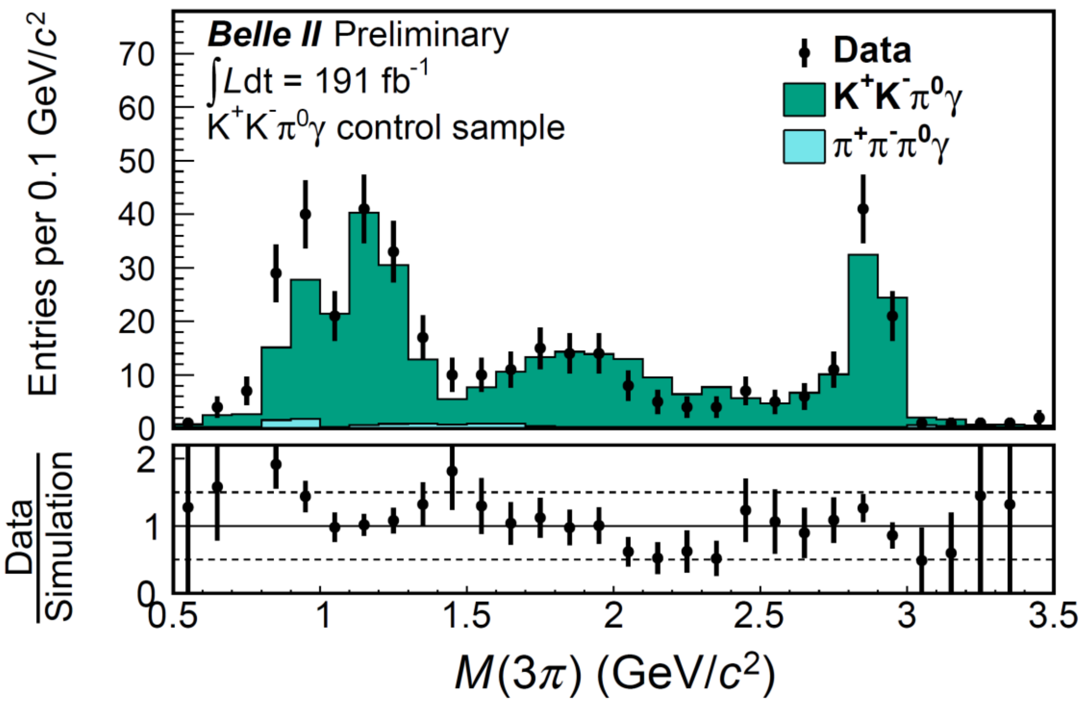
- ☐ Signal extraction
- ☐ Unfolding
- ☐ Efficiency estimation
- ☐ Cross section and a_μ calculation

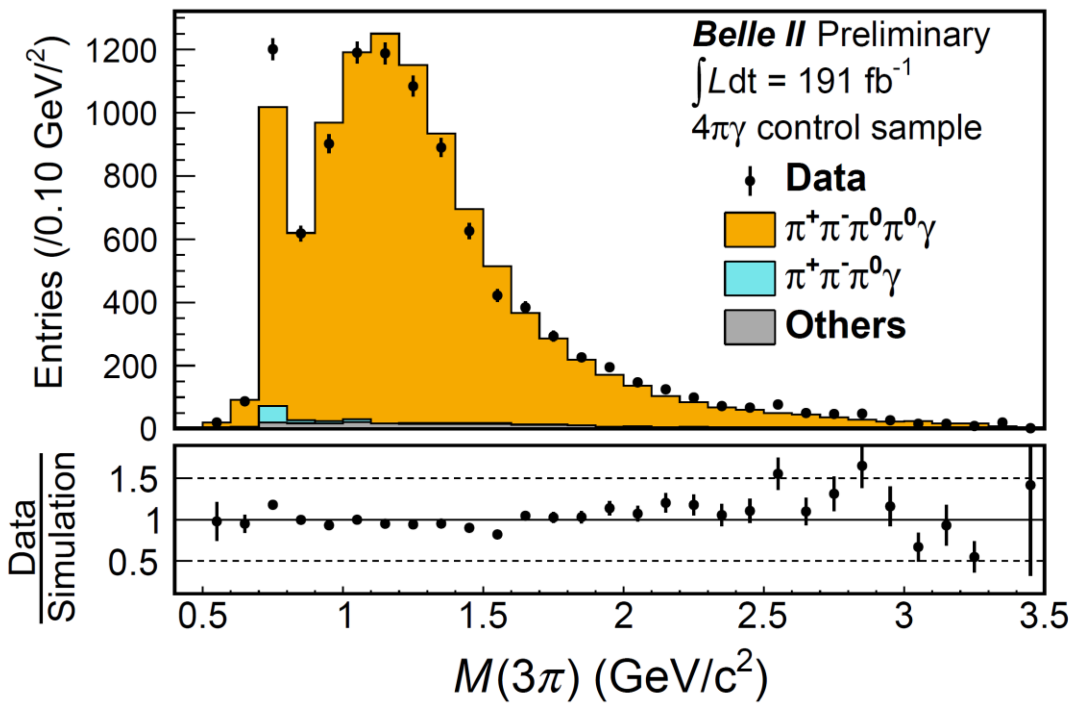
Background estimation

Estimate by determining a mass-dependent data-MC scale factor using a control sample.

$$N^{\text{data}}_{\text{Signal}} = N^{\text{MC}}_{\text{Signal}} \cdot \frac{N^{\text{data}}_{\text{Control}}}{N^{\text{MC}}_{\text{Control}}}$$

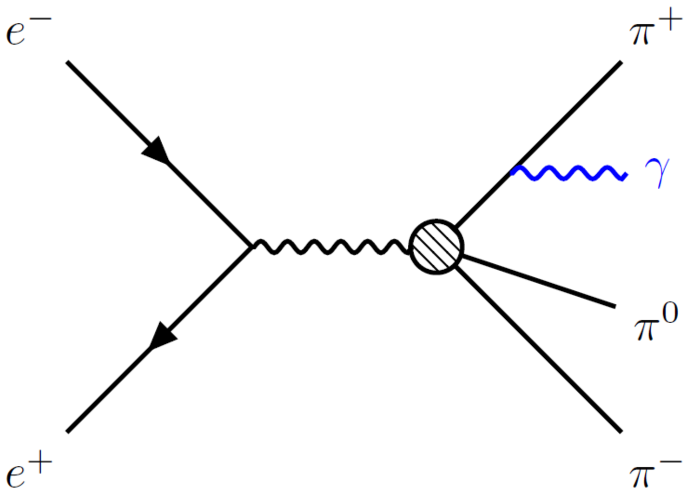
- $e^+e^- \to K^+K^-\pi^0\gamma$: Invert π/K-ID $L(\pi/K) > 0.1 \Rightarrow L(\pi/K) < 0.1$
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- Non-ISR qqbar : $0.10 < M(\gamma_{ISR}\gamma) < 0.17$ GeV / large cluster second moment





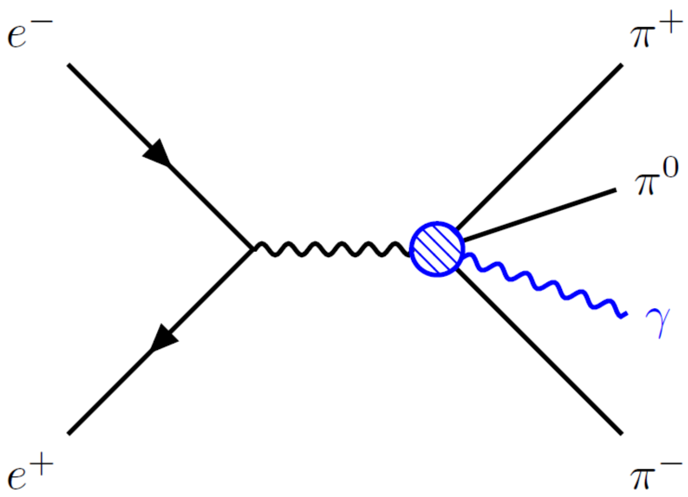
Final-state radiation background

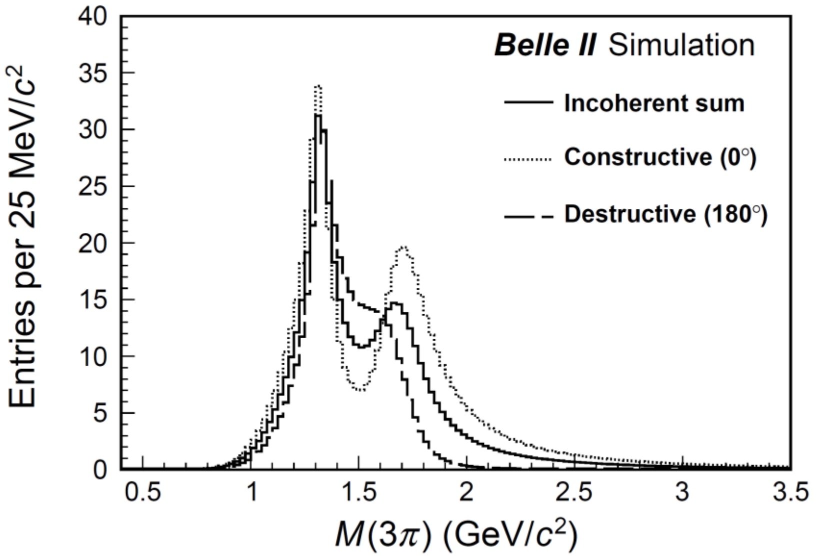
- Difficult to reject FSR background or extract control sample
- Estimate FSR background using pQCD prediction based on the BABAR previous analysis [PRD112003]



FSR emission from final-state pions

~ 0.001fb → < 1 event occur





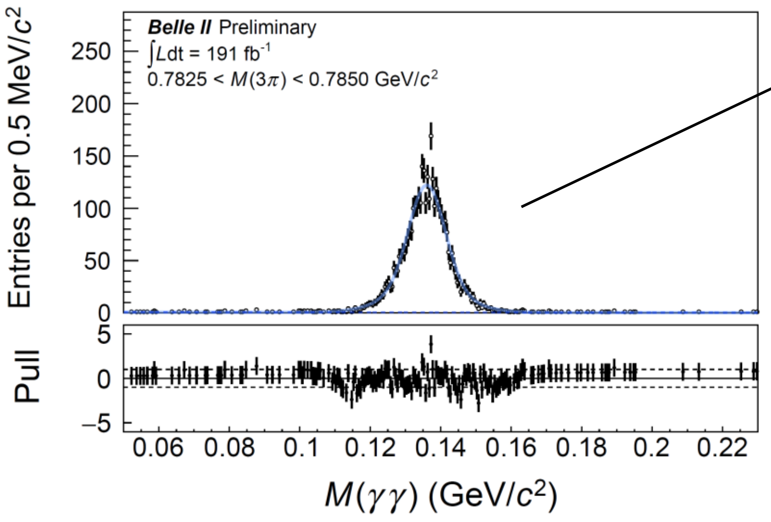
FSR emission from the quark legs

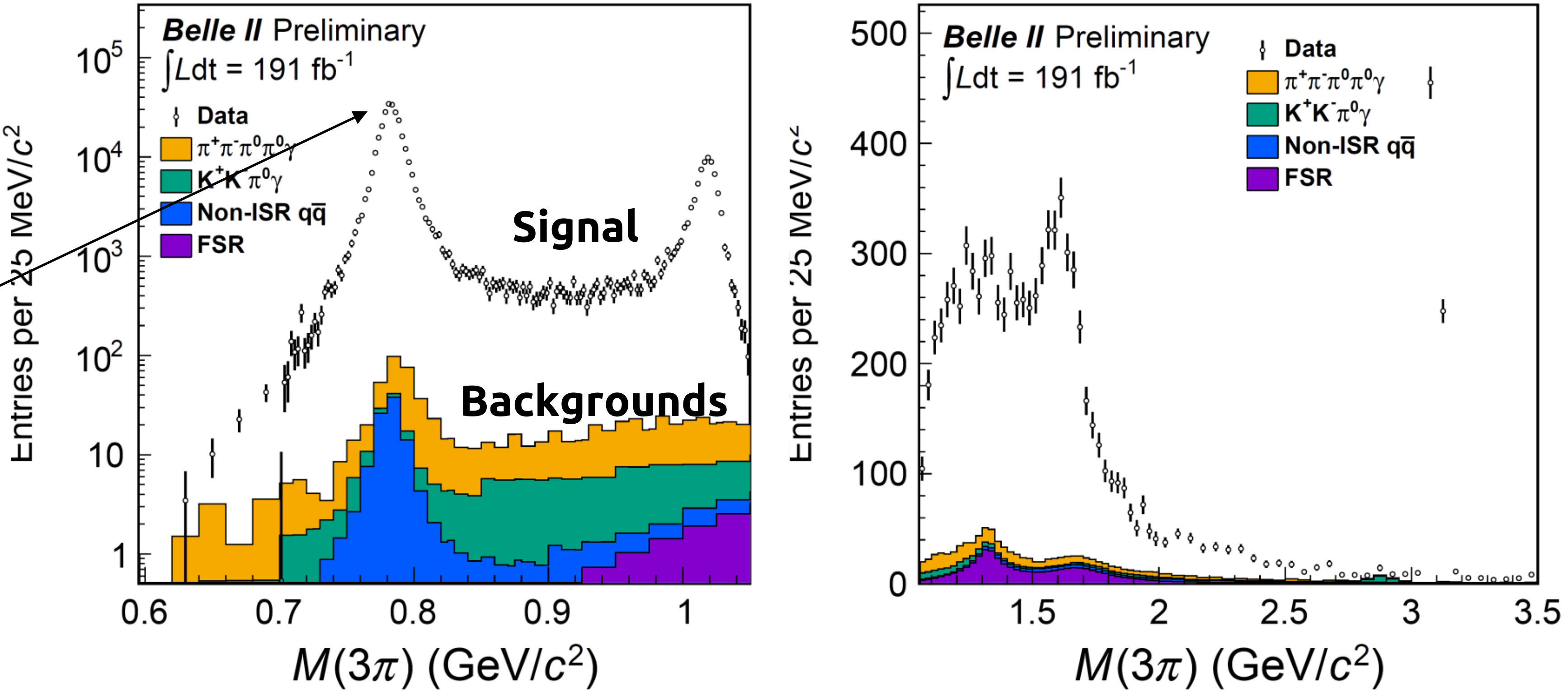
- $e^+e^- \rightarrow M\gamma_{FSR} \rightarrow \pi^+\pi^-\pi^0\gamma_{FSR}$;
  M= η, $a_1(1260)$, $a_2(1320)$, $a_1(1640)$, $a_2(1700)$, $a_1(1930)$, $a_2(2030)$

Signal extraction

- Fit M(γγ) in each M(3π) bin to remove the combinatorial background in γγ
  - Signal: Gaussian + Novosibirsk function
  - Background: linear function
- Fit each bin of M(3π) with fixed signal-shape parameters
- Signals were observed up to 0.62 GeV as the lower limit.

M(γγ) fit in one M(3π) bin





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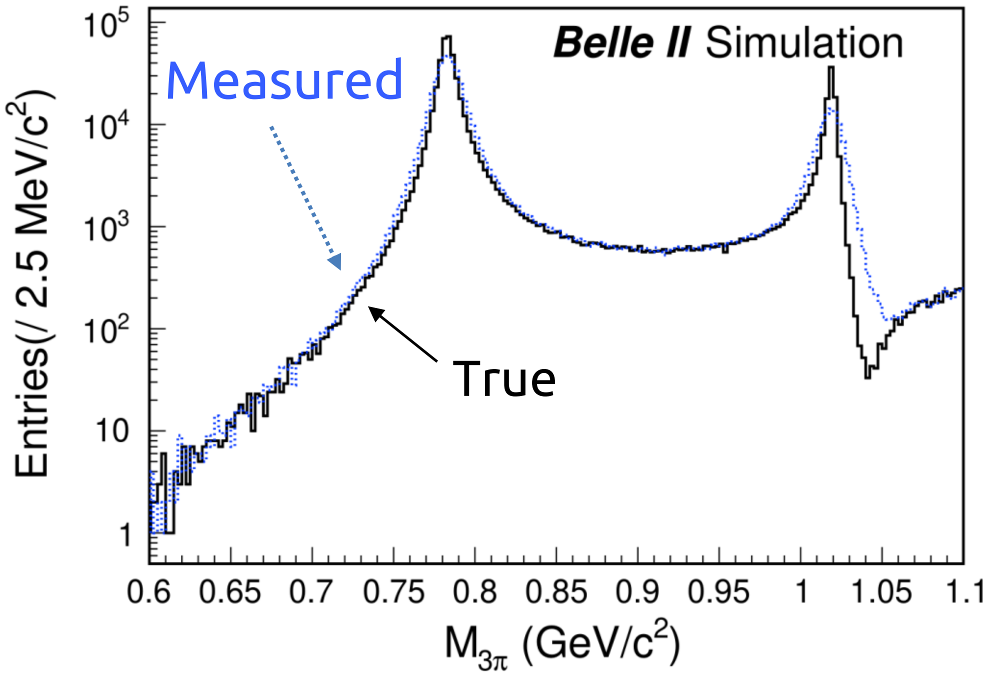
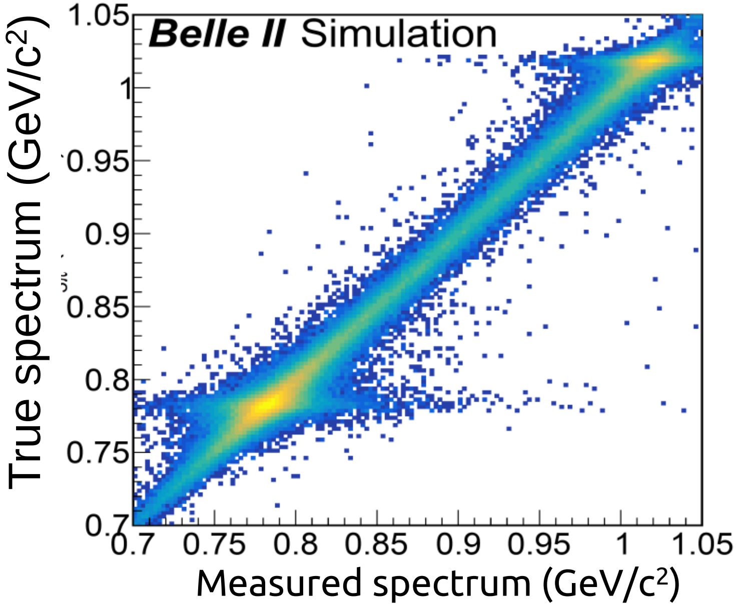
Unfolding

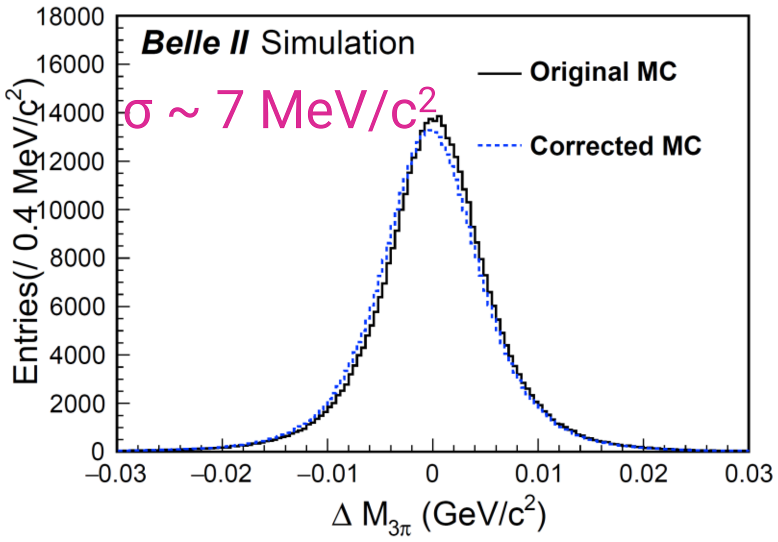
- The signal spectrum is unfolded to mitigate the effect of detector resolution
  - Typically with a mass resolution around 7-10 MeV/c^2
- The data-MC difference of mass bias and resolution is determined by a Gaussian convolution fit to the ω, Φ, and J/ψ resonances
  - Mass bias of 0.5-1.5 MeV/c^2, and resolution of about 1 MeV/c^2 is corrected



Transfer matrix



Mass resolution



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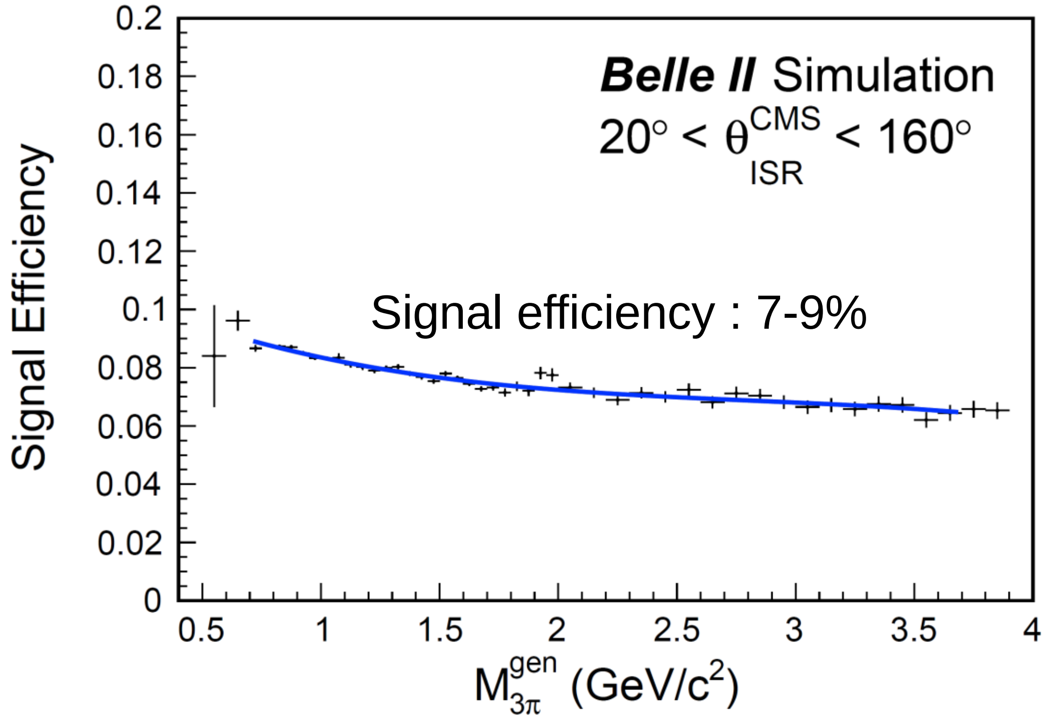
Signal efficiency and data-MC corrections

Efficiency $\varepsilon = \varepsilon_{\mathrm{MC}} \prod_i (1 + \delta_i)$ Data-MC correction δ_i ~ O(1)%

- 1st order signal efficiency is estimated using MC of the x10 larger statistics
- Possible differences between data and MC are checked in data-driven way
  - Trigger efficiency
  - Tracking efficiency
  - ISR photon efficiency
  - π^0 efficiency
  - Selection efficiency
  - Higher-order ISR effects

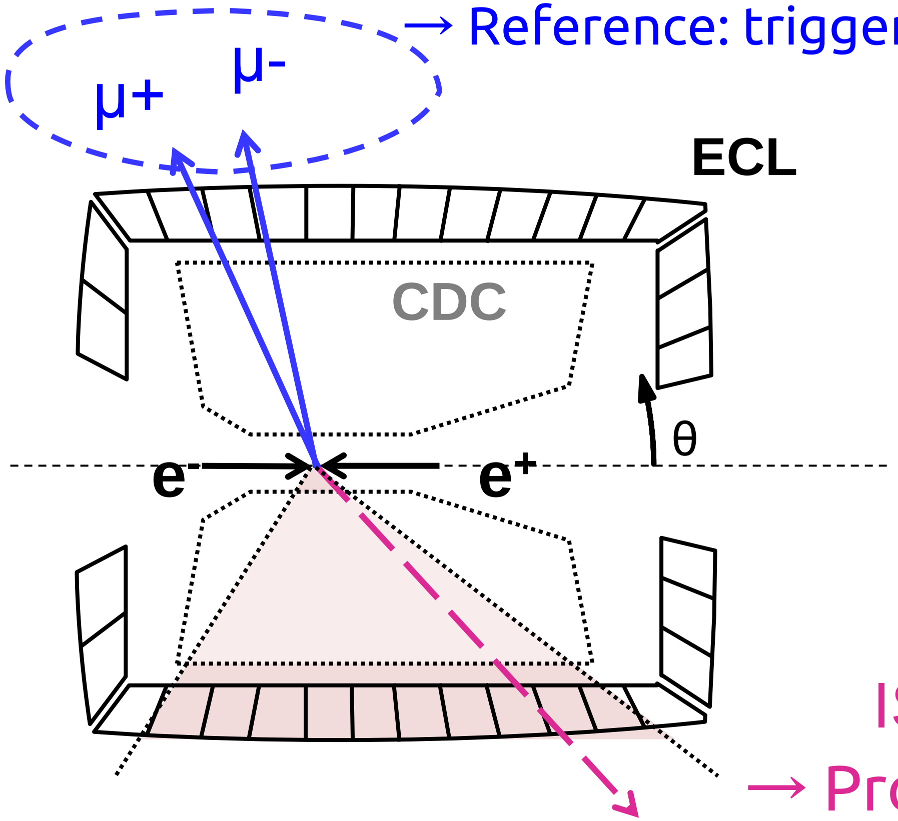
MC detection efficiency (no correction)



Trigger efficiency

- ISR events are triggered by the calorimeter
- The efficiency can be measured by using the events triggered independently by the tracker
  - Efficiency for energetic ISR in barrel region: 99.9%
- The uncertainty related is small, 0.1%
- This also benefits other final-state measurements

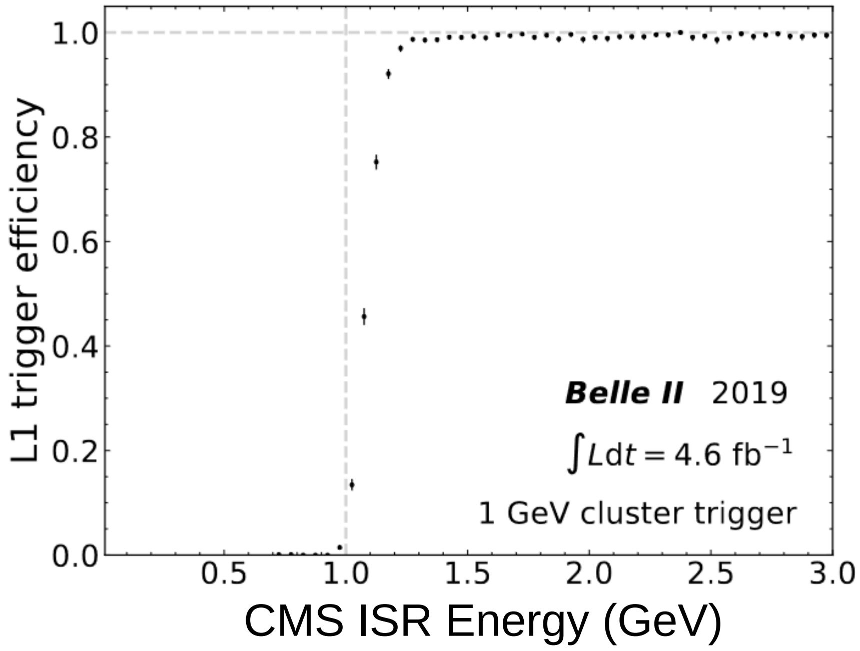
→ Reference: triggered by track trigger



ISR photon in barrel

→ Probe: fire energy trigger

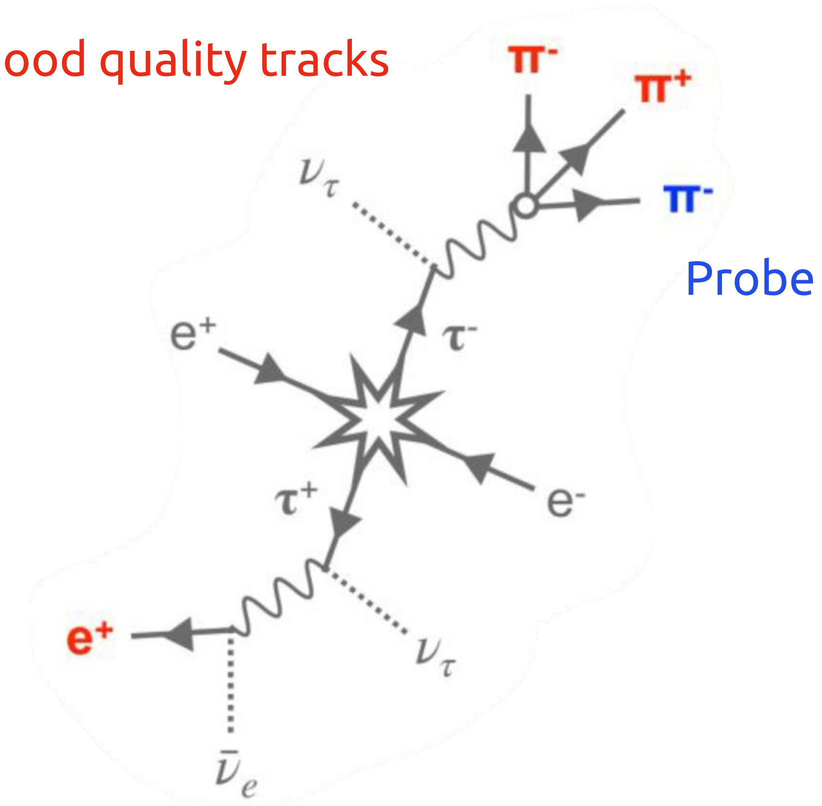
Belle II trigger efficiency measured by μμγ (data)



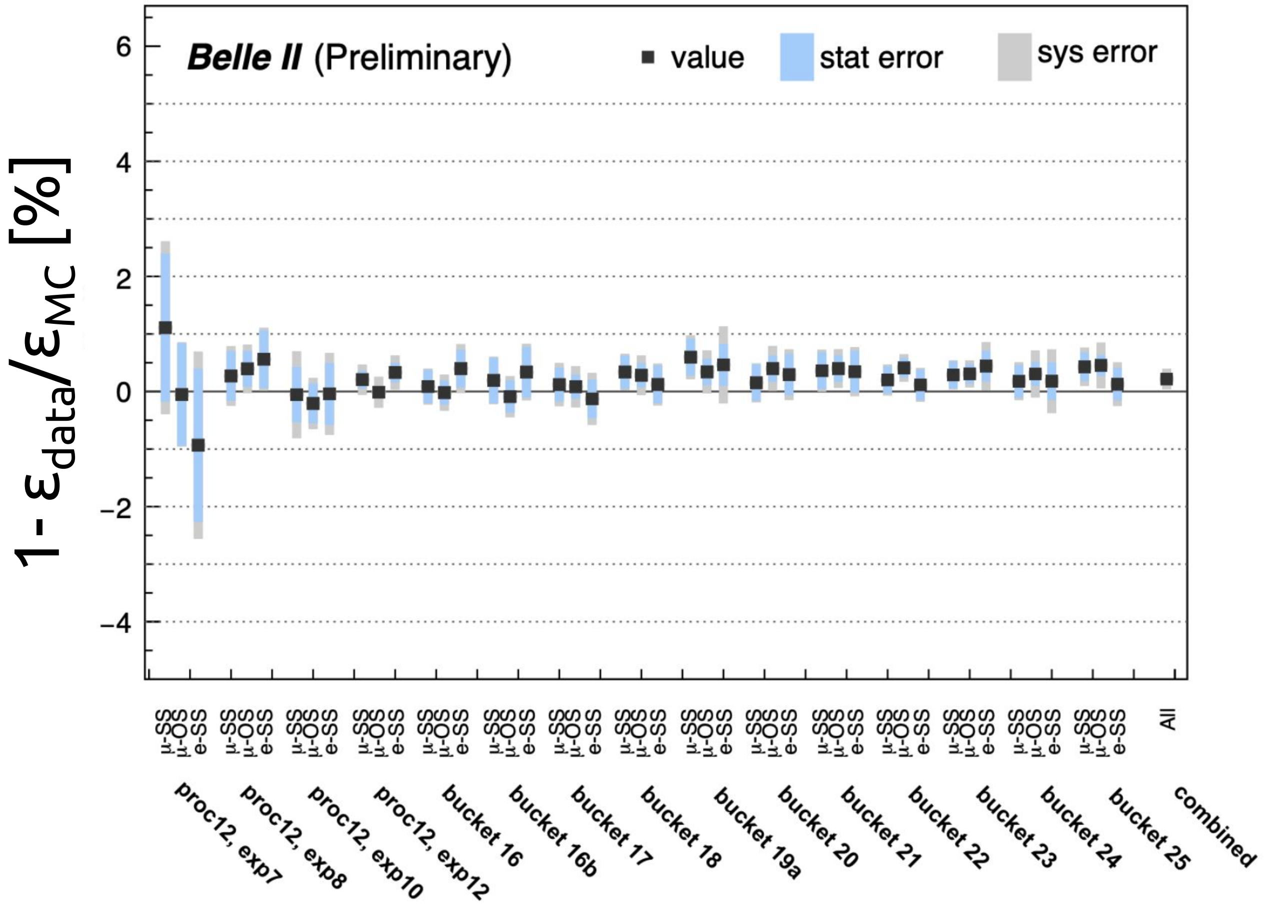
Tracking efficiency

- Tracking efficiency for pions is studied with the $e^+e^- \rightarrow \tau^+\tau^-$ process.
- Data-MC differences are confirmed to be small with 0.3% uncertainty per track.

Tag: Three good quality tracks

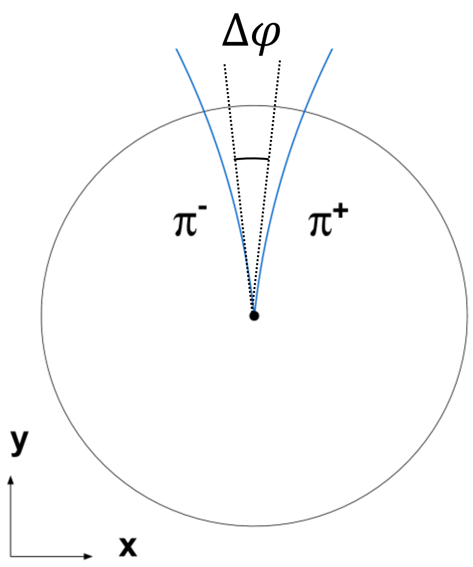


Data-MC discrepancy of tracking efficiency

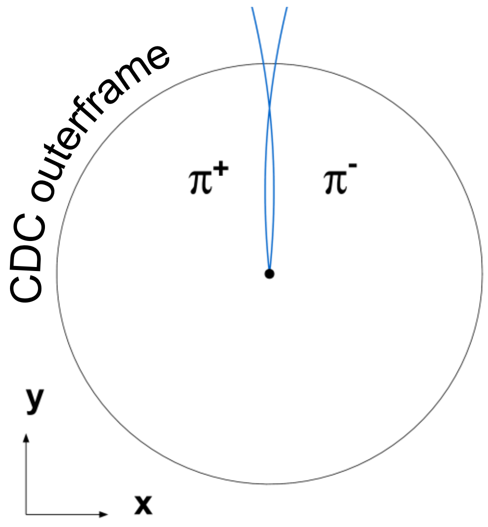


Tracking efficiency: Track loss

- Track loss due to shared hits on the drift chamber is confirmed using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$
- Define $\Delta\varphi \coloneqq \varphi(\pi^+) - \varphi(\pi^-)$
- The Inefficiency due to track loss is given by $f = \frac{N(\Delta\varphi<0) - N(\Delta\varphi>0)}{2N(\Delta\varphi<0)}$
  - The track loss is 5.0% in data and 4.0% in MC
- In total, the correction factor of tracking is (-1.4±0.8)%.
  - Dependency on no. of CDC hits and duplicated tracks are also studied.

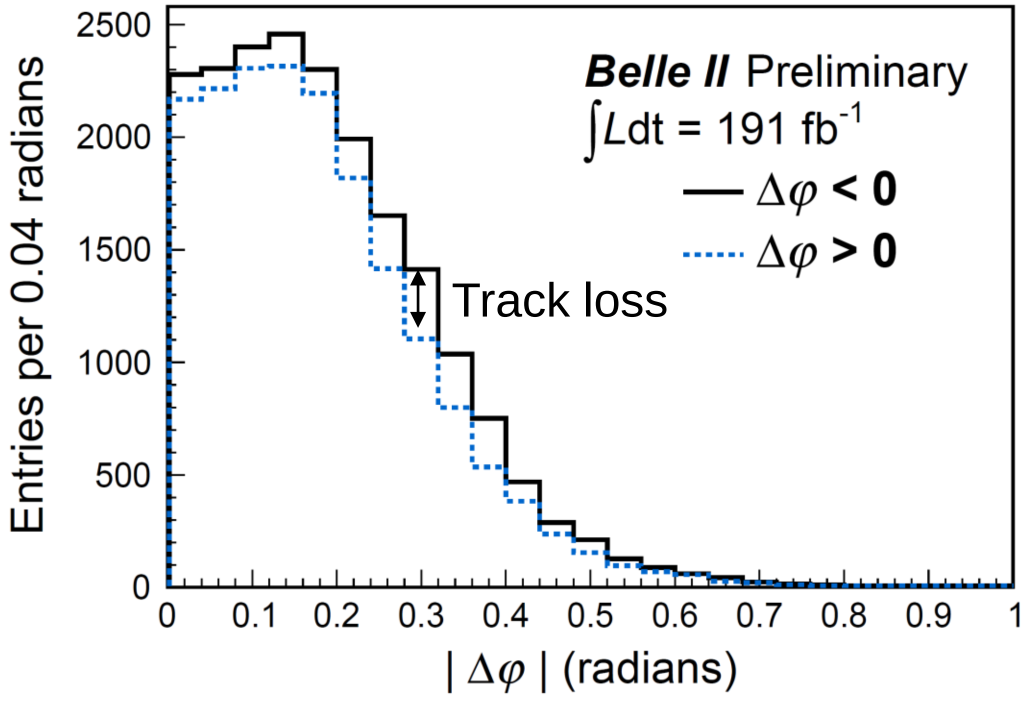


$\Delta\varphi < 0$



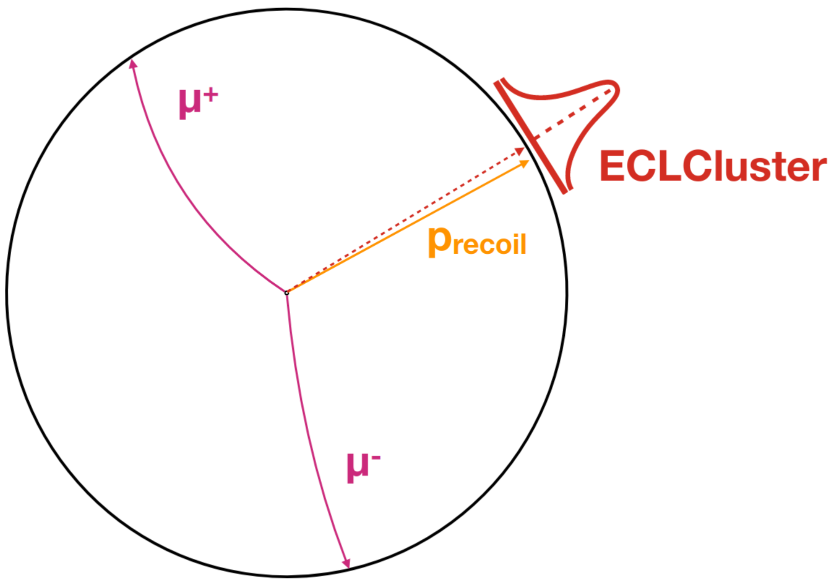
$\Delta\varphi > 0$

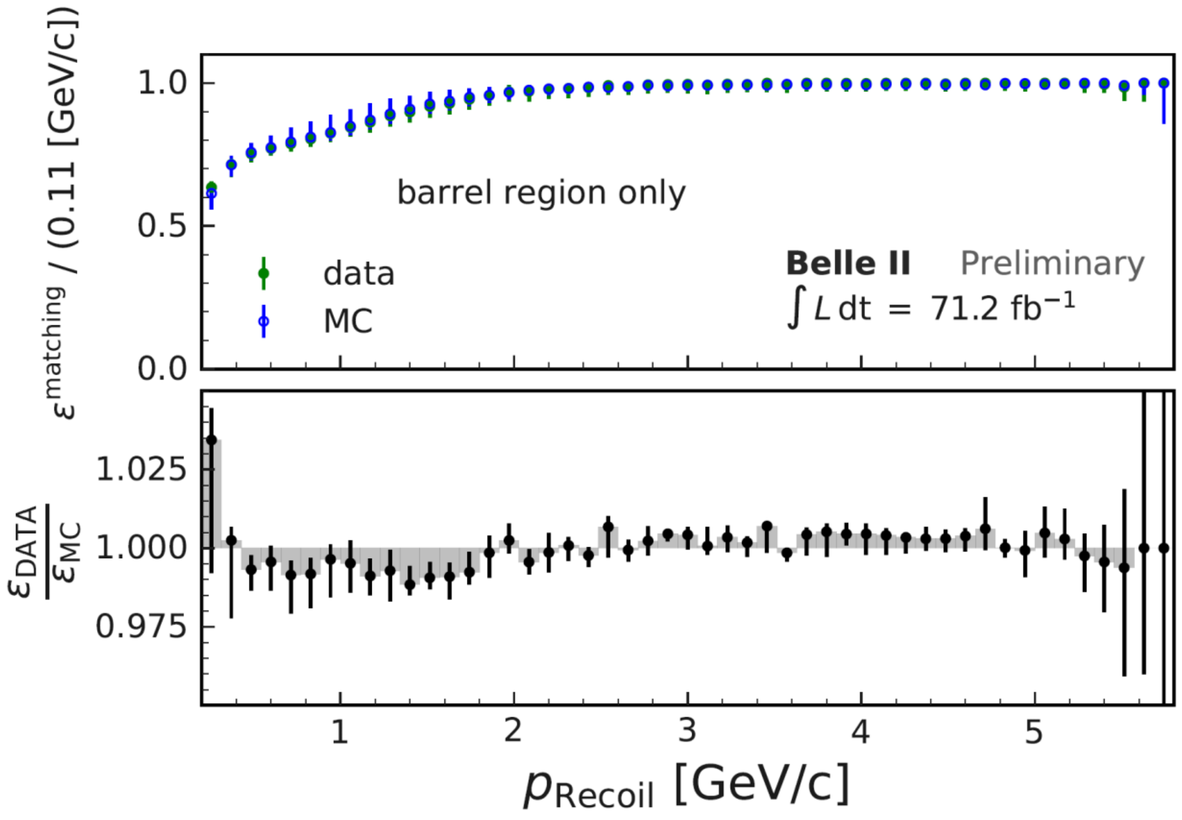
Δφ distribution in data



ISR photon detection efficiency

- Photon detection efficiency is measured using $e^+e^- \to \mu^+\mu^-\gamma$ events
  - Taking a match between a ECL cluster and the missing momentum of dimuon system
- Efficiency is in good agreement with 0.7% systematic uncertainty



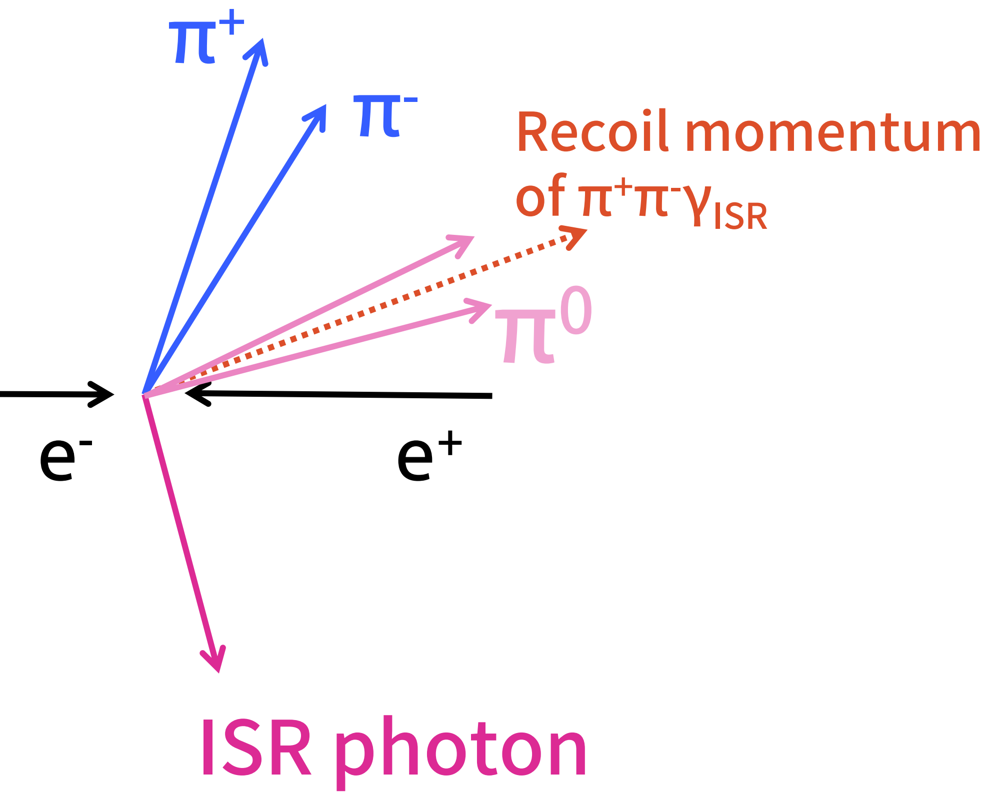


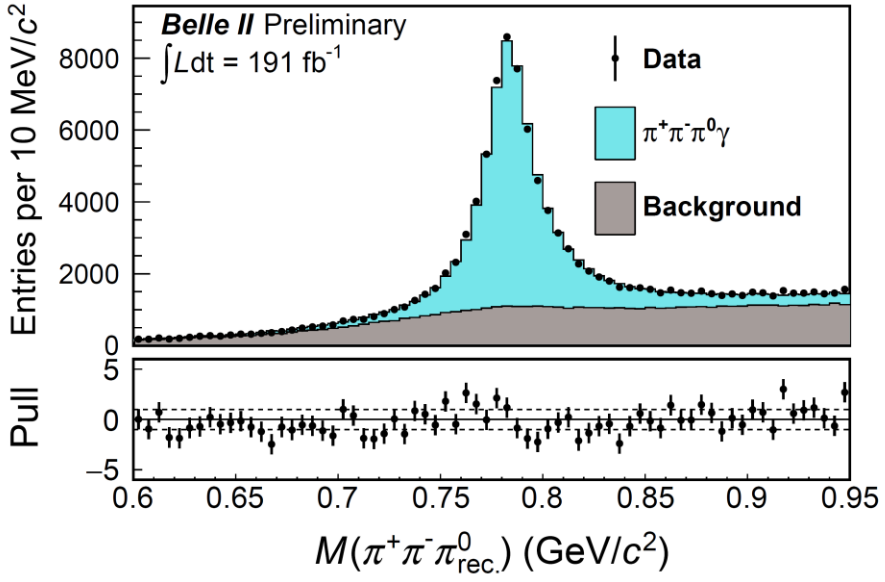
π^0 efficiency correction

- Accurate evaluation of π^0 efficiency in e^+e^- experiment is a challenging task.
  - Exclusive processes that include a π^0 are limited.
- Evaluate efficiency using the $e^+e^- \to \omega\gamma \to \pi^+\pi^-\pi^0\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

→ Count $\omega \to \pi^+\pi^-\pi^0$ decay without using π^0 information.





$$M^2(\pi^+\pi^-\pi^0_{\text{recoil}}) = (p_{\pi^+} + p_{\pi^-} + p_{\text{recoil}})^2$$

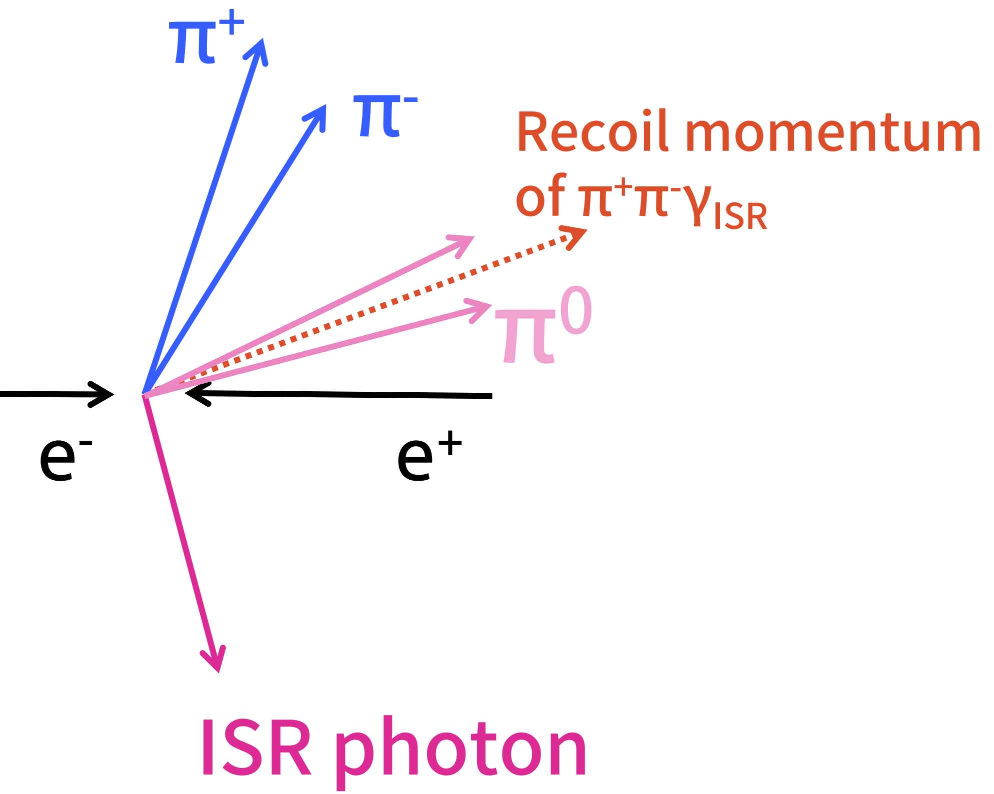
- π^0 momentum p_{recoil} is determined by kinematic fit to $\pi^+\pi^-\gamma$ with hypothesis that recoil mass equals π^0 mass

π^0 efficiency correction

- Accurate evaluation of π^0 efficiency in e^+e^- experiment is a challenging task.
  - Exclusive processes that include a π^0 are limited.
- Evaluate efficiency using the $e^+e^- \to \omega\gamma \to \pi^+\pi^-\pi^0\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction}: \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction}: \gamma_{\text{ISR}}\pi^+\pi^-)}$$

→ Count by reconstructing π^0 and fitting $M(\gamma\gamma)$



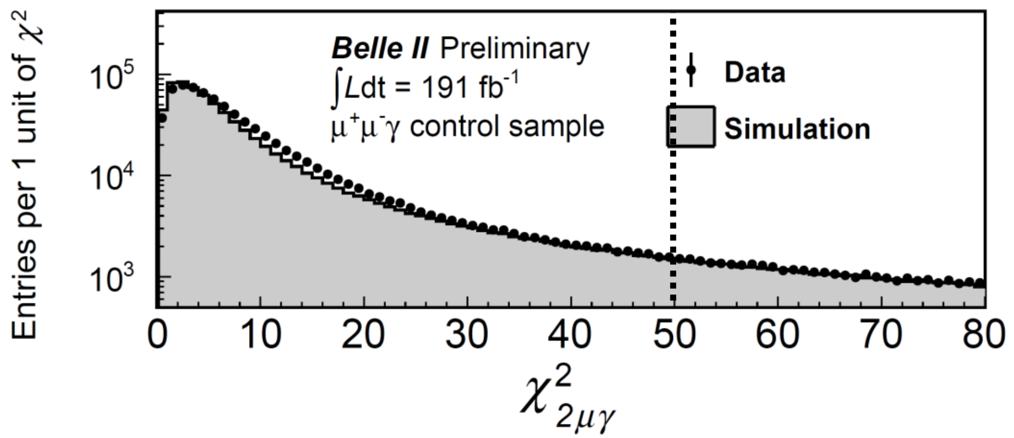
- ε_{π^0} are independently evaluated by the data and MC
  - Data/MC ratio = $0.986 \pm 0.006_{\text{stat}}$
- The systematic uncertainty related to π^0 is 1.0%
  - The uncertainty is evaluated by variations of the $M(\gamma\gamma)$ signal pdf, background pdfs, and selections

Background suppression efficiency

- Estimated by the ratio of signal yield before/after the criteria
- It is evaluated using ω and Φ, J/ψ resonances of good S/N
- In M(3π) < 1.05 GeV/c^2, efficiency is (89.5±0.2)% for data
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC}$ -1 = (-1.90±0.20)%
- M(3π) > 1.05 GeV/c^2 : the number of J/ψ was obtained by M(3π) fitting
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC}$ -1 =(-1.78±1.85)%
  - Error is due to statistical errors in the sample

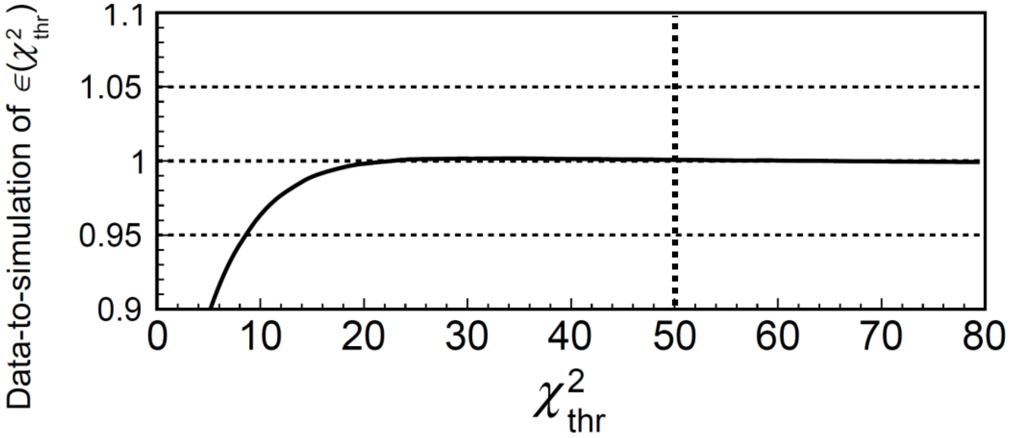
χ2 selection efficiency

- ISR and tracks χ^2-criteria efficiency is confirmed using $e^+e^-\to\mu^+\mu^-\gamma$ sample
- Confirm effects from differences in position, momentum, and energy of ISR and tracks
  - Agreement confirmed within ±0.6% uncertainty
- Dependence on multi-ISR photon calculations is discussed on the next page



$$\varepsilon(\chi^2_{\mathrm{thr}}) = \frac{N(\chi^2 < \chi^2_{\mathrm{thr}})}{N_{\mathrm{all}}}$$

Data-MC ratio $\dfrac{\varepsilon_{\mathrm{data}}(\chi^2_{\mathrm{thr}})}{\varepsilon_{\mathrm{MC}}(\chi^2_{\mathrm{thr}})}$

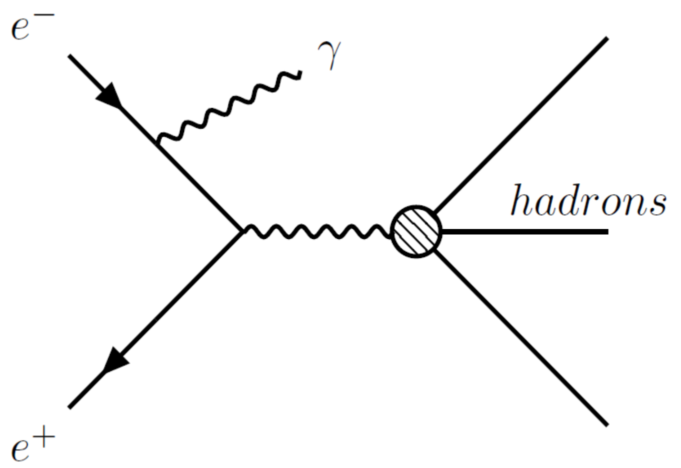


Higher-order ISR effects

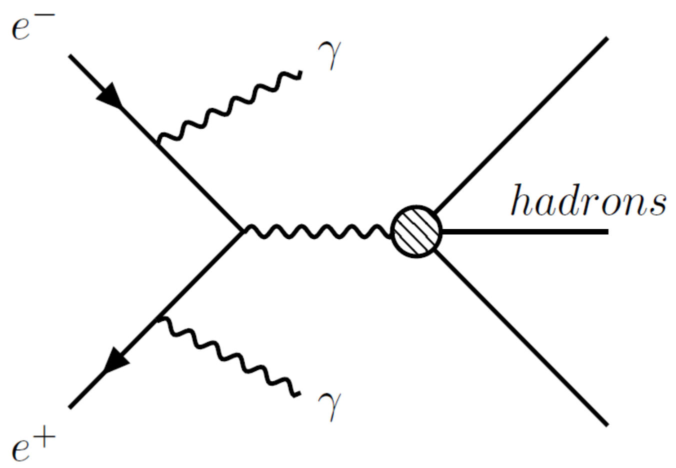
- Although a one-ISR photon emission process is set as the signal, in reality there are processes with multiple photon emissions.
- Two effects need to be considered from the existence of multiple photons:
  - A) Effective integrated luminosity L_{eff} (radiative correction): 0.5% unc.
  - B) χ2 selection efficiency due to ISR photon calculations in generator: 1.2% unc.

Signal process

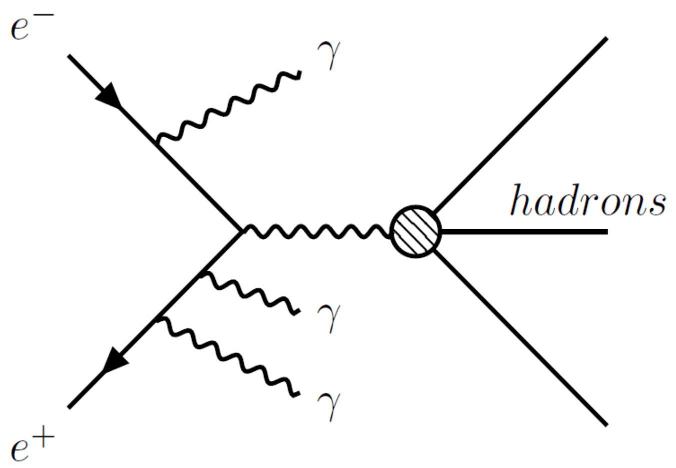
Leading-order (LO) ISR



Next-to-Leading-order (NLO) ISR



NNLO ISR



Efficiency correction : Summary

Source	Efficiency correction (%)	
	$M < 1.05$ GeV/c^2	$M > 1.05$ GeV/c^2
Trigger	-0.1±0.1	-0.1±0.1
ISR photon detection	0.2±0.7	+0.2±0.7
Tracking	-1.4±0.8	-1.7±0.8
π^0 detection	-1.4±1.0	-1.4±1.0
Background suppression	-1.9±0.2	-1.8±1.9
χ^2 distribution	0.0±0.6	0.3±0.3
MC generator	0.0±1.2	0.0±1.2
Total correction	-4.6±2.0	-4.6±2.0

Analysis outline

- Event selection
- Background estimation
- Signal extraction
- Unfolding
- Efficiency estimation
- Cross section and a_μ calculation

Systematic uncertainty for $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

- Luminosity is measured with Bhabha events and confirmed with $e^+e^- \to \gamma\gamma$ and $\mu^+\mu^-$ processes
- Major systematic uncertainty comes from MC generator, and π^0 efficiency
  - In $M(3\pi) > 1.05$ GeV, the uncertainty of selection efficiency is dominant

Source	Systematic uncertainty (%)	
	$\sqrt{s} < 1.05$ GeV^2	$\sqrt{s} > 1.05$ GeV
Trigger efficiency	0.1	0.2
ISR photon efficiency	0.7	0.7
Tracking efficiency	0.8	0.8
π^0 efficiency	1.0	1.0
χ^2 criteria efficiency	0.6	0.3
Background suppression efficiency	0.2	1.9
MC generator	1.2	1.2
Radiative correction	0.5	0.5
Integrated luminosity	0.6	0.6
Total systematics	2.2	2.8

Cross section calculation

Unfolded signal spectrum

Cross section

$$\sigma_{ee\to 3\pi}(M_i(3\pi)) = \frac{N_{\mathrm{unfolded},i}}{\varepsilon(M_i(3\pi)) \cdot L_{\mathrm{eff}}(M_i(3\pi)) \cdot r_{\mathrm{rad}}}$$

Radiative correction

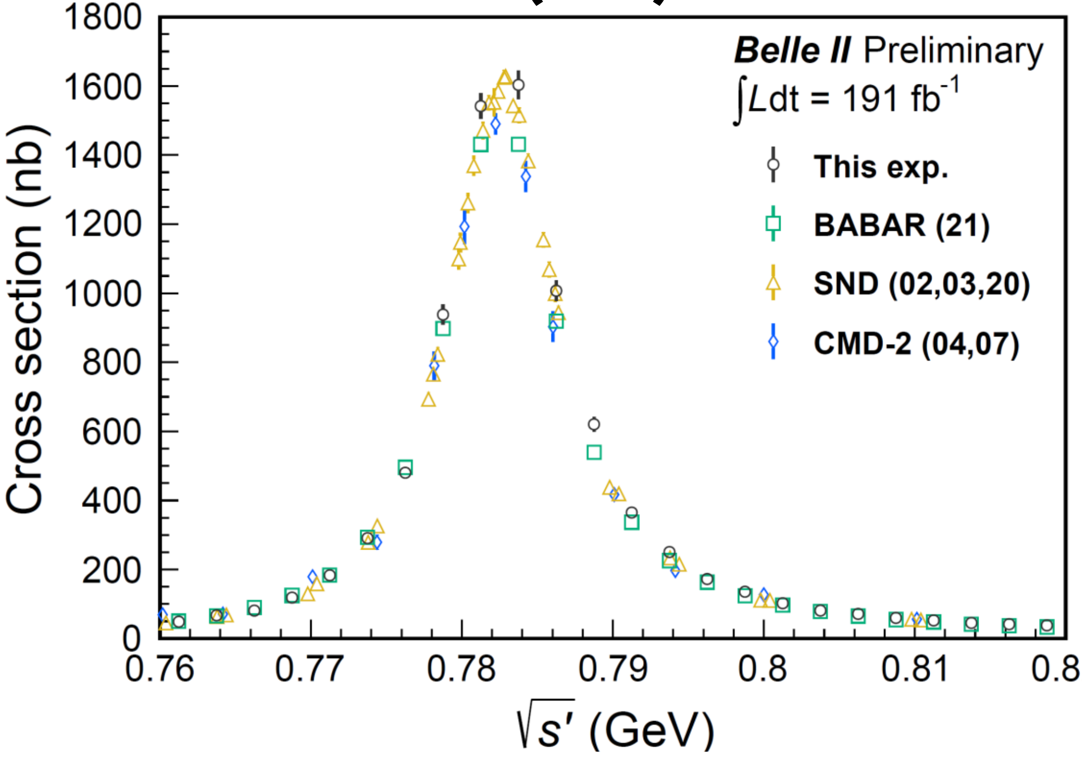
3π mass at i-th bin

Effective luminosity

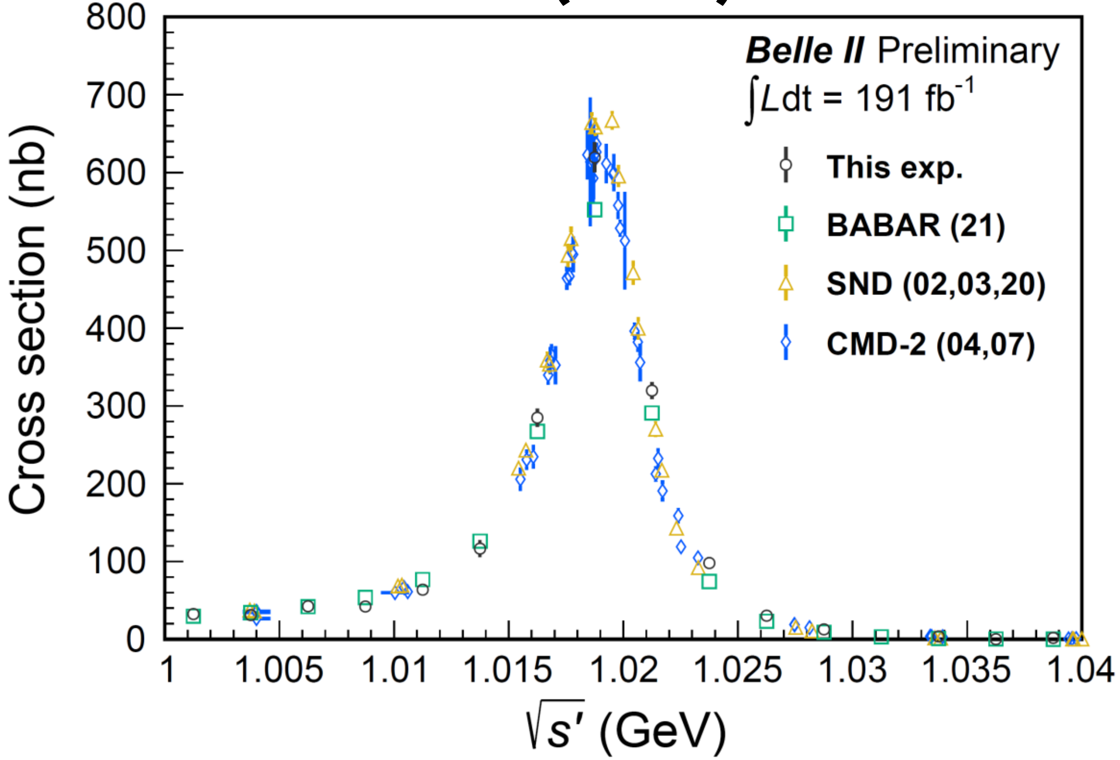
Corrected Efficiency

Result: cross section below 1.05 GeV

ω(782)

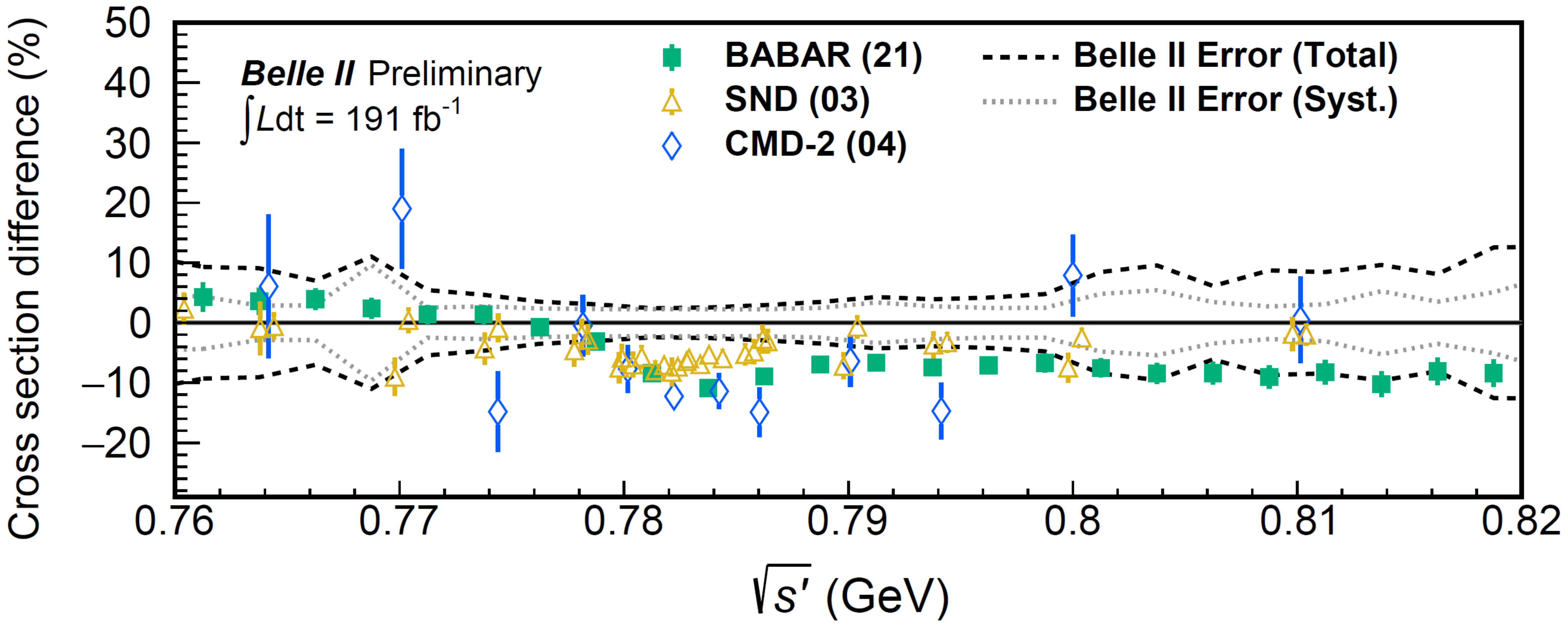


Φ(1020)



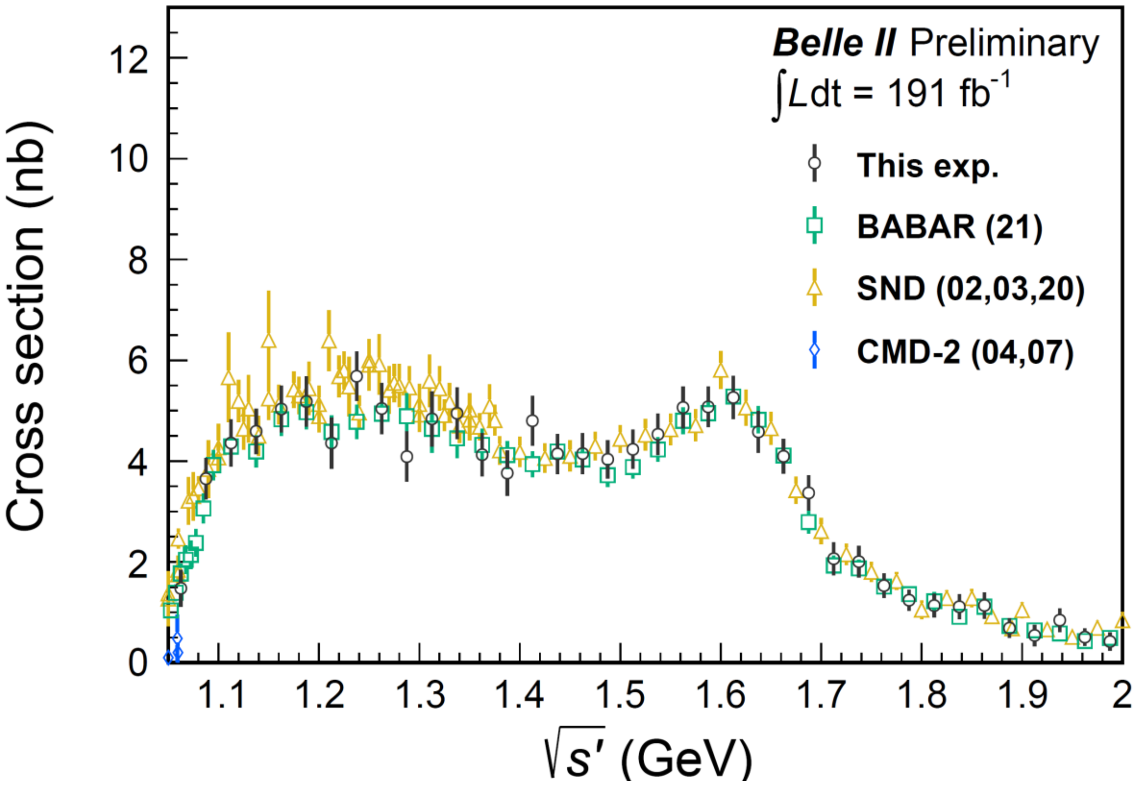
Result: cross section below 1.05 GeV

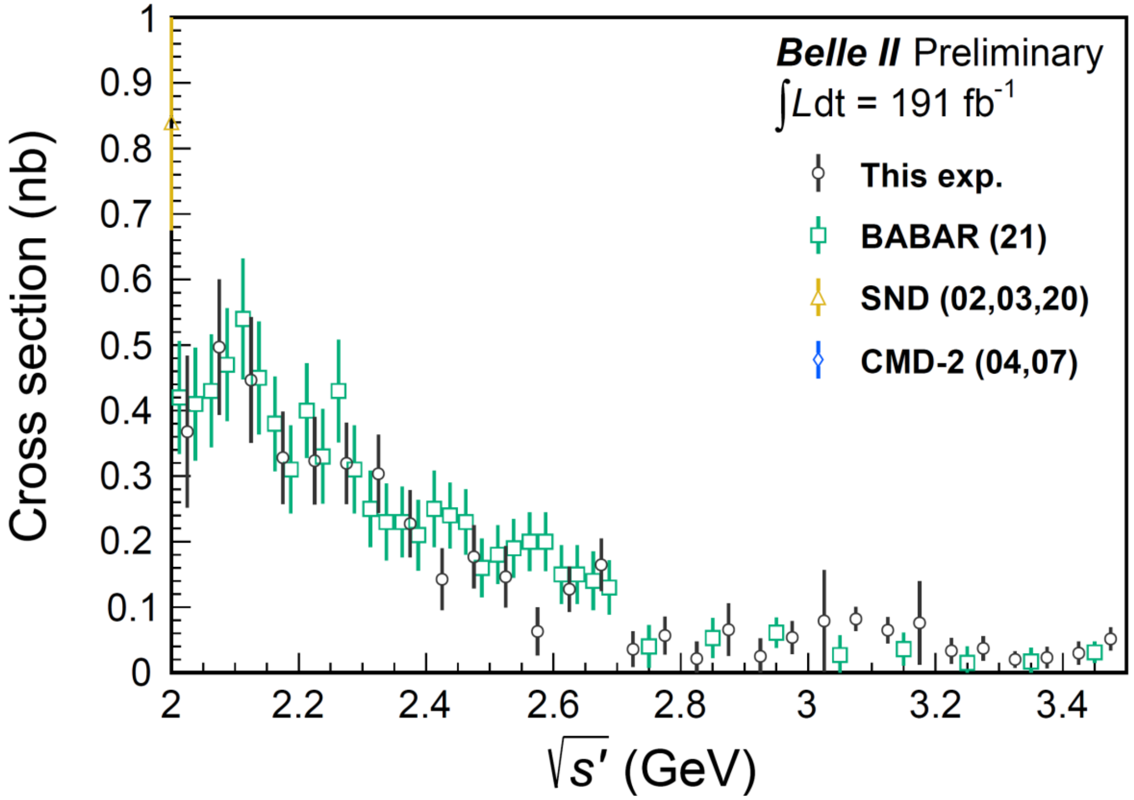
- Cross section at ω resonance is 5-10% higher than SND, BABAR, and CMD-2



Result: cross section above 1.05 GeV

- Good agreement with BABAR result





Results: 3π contribution to a_μ HVP

$$a_\mu^{\mathrm{LO,HVP},3\pi}(0.62\text{- }1.8\ \mathrm{GeV}) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$$

	$a_\mu(3\pi)\times10^{10}$	Difference$\times10^{10}$
BABAR alone [PRD104 11 (2021)]	45.86 ± 0.14 ± 0.58	-3.2±1.3 (6.9%)
Global fit [JHEP08 208 (2023)]	45.91 ± 0.37 ± 0.38	-3.0±1.2 (6.5%)

- 6.5% higher than the global fit result with 2.5σ significance
- This difference 3×10^{-10} corresponds 10% of $\Delta a_\mu = a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = 25\times10^{-10}$

Systematic uncertainty for a_μ

Source	Systematic uncertainty (%)
Efficiency corrections	1.63
Monte Carlo generator	1.20
Integrated luminosity	0.64
Simulated sample size	0.15
Background subtraction	0.02
Unfolding	0.12
Radiative corrections	0.50
Vacuum polarization corrections	0.04
Total	2.19

Next: $e^+e^-\to\pi^+\pi^-$ at Belle II

- Target precision: 0.5% of $a_\mu(2\pi)$
- Trying to follow BABAR methods as a baseline
- Systematics uncertainty dominant analysis
  - BABAR : 232 /fb [Phys. Rev. D 86 (2012), 032013]
  - We can use large dataset to control systematic uncertainties
- Design of data-driven efficiency corrections for tracking, trigger and $\pi/\mu/K$ ID is ongoing

Summary

- Cross-section measurements are ongoing at the SuperKEKB/Belle II experiment
  - Good trigger efficiency thanks to the upgrade is confirmed
  - Further channel analysis can be expected in the future
- We measured the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section with systematic uncertainty of 2.2%
  - The second largest contribution to HVP term
  - The largest uncertainty arises from NLO/NNLO calculation in MC generator
- Our results are about 2.5σ greater than BABAR and global fit
  - $a_\mu^{\mathrm{LO,HVP,}}(3\pi) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$
- The paper is available on arXiv:2404.04915 and has been submitted to PRD