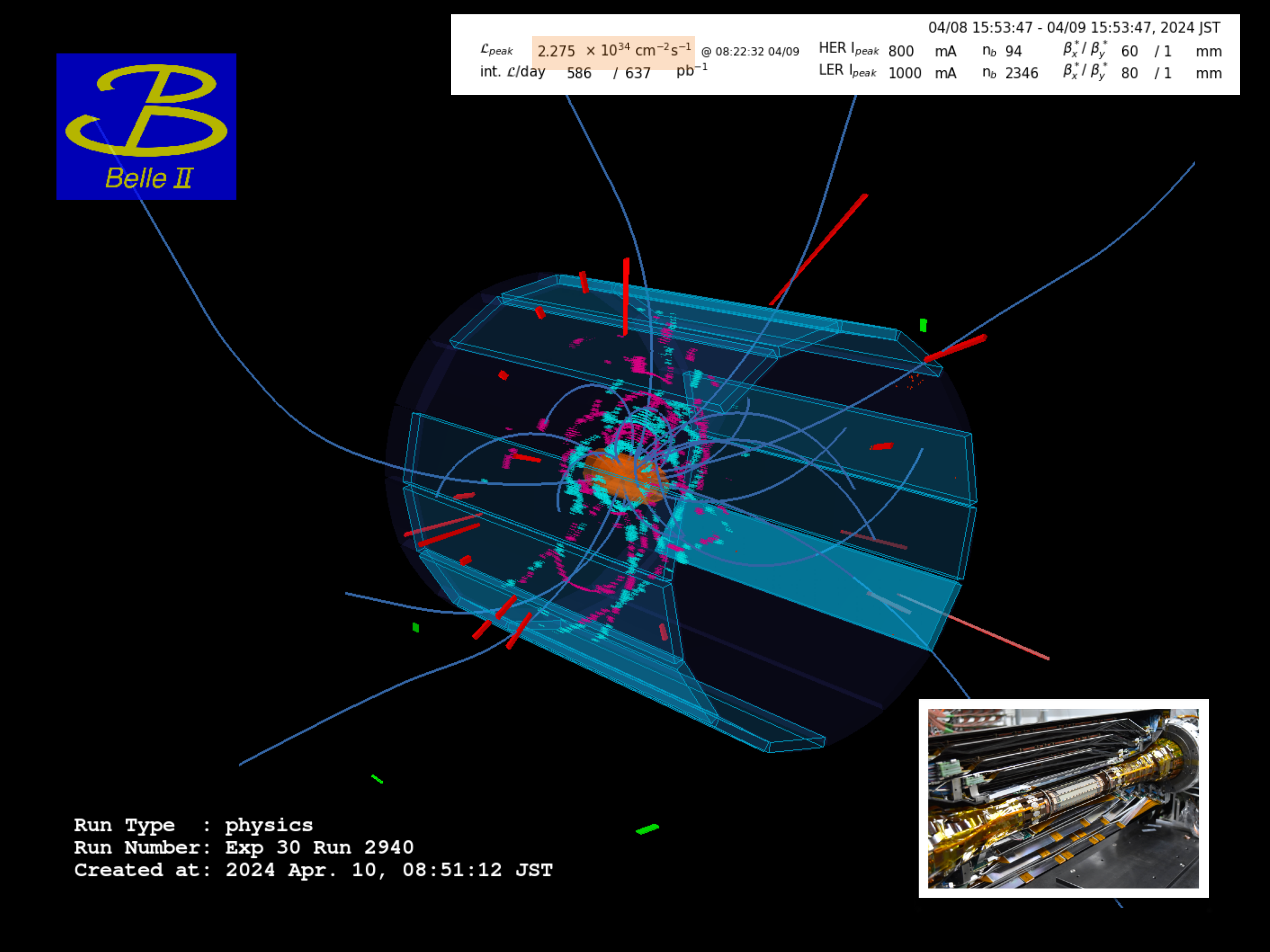
# New developments on **inclusive** $V_{cb}$

florian.bernlochner@uni-bonn.de

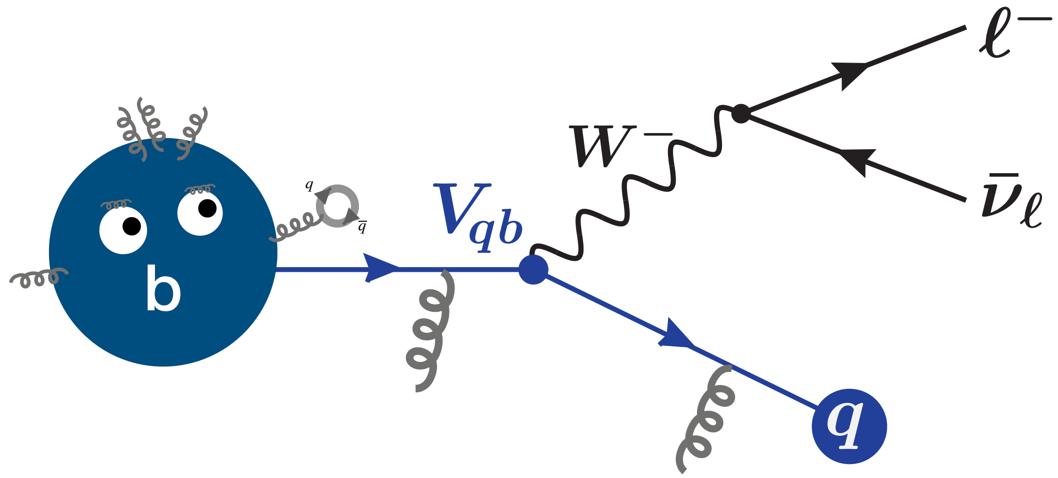
Many thanks to feedback from

Keri Vos and Markus Prim


04/08 15:53:47 - 04/09 15:53:47, 2024 JST
$\mathcal{L}_{peak}$ 2.275 × $10^{34}$ $cm^{-2}s^{-1}$ @ 08:22:32 04/09
int. $\mathcal{L}$/day 586 / 637 $pb^{-1}$
HER $I_{peak}$ 800 mA $n_b$ 94 $\beta_x^* / \beta_y^*$ 60 / 1 mm
LER $I_{peak}$ 1000 mA $n_b$ 2346 $\beta_x^* / \beta_y^*$ 80 / 1 mm
Belle II
Run Type : physics
Run Number: Exp 30 Run 2940
Created at: 2024 Apr. 10, 08:51:12 JST

# Puzzles…

It may look cute, but that might be deceiving…



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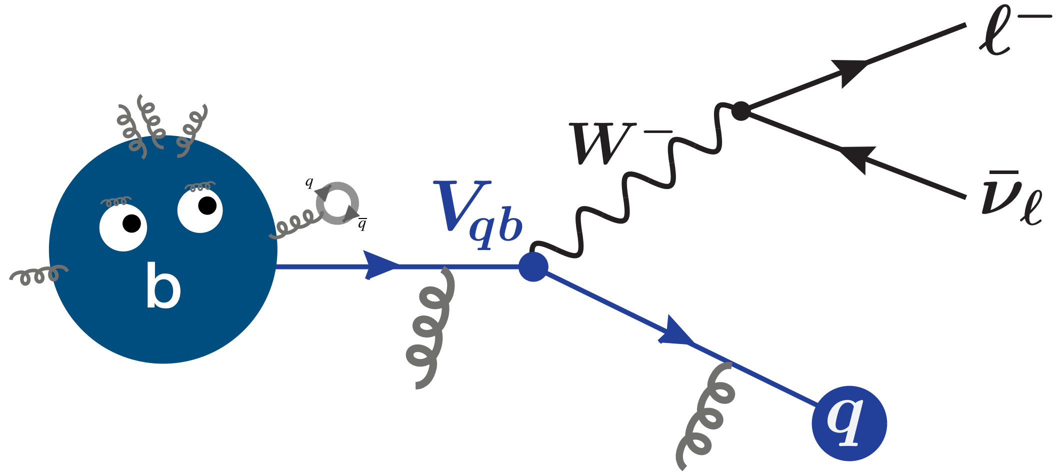
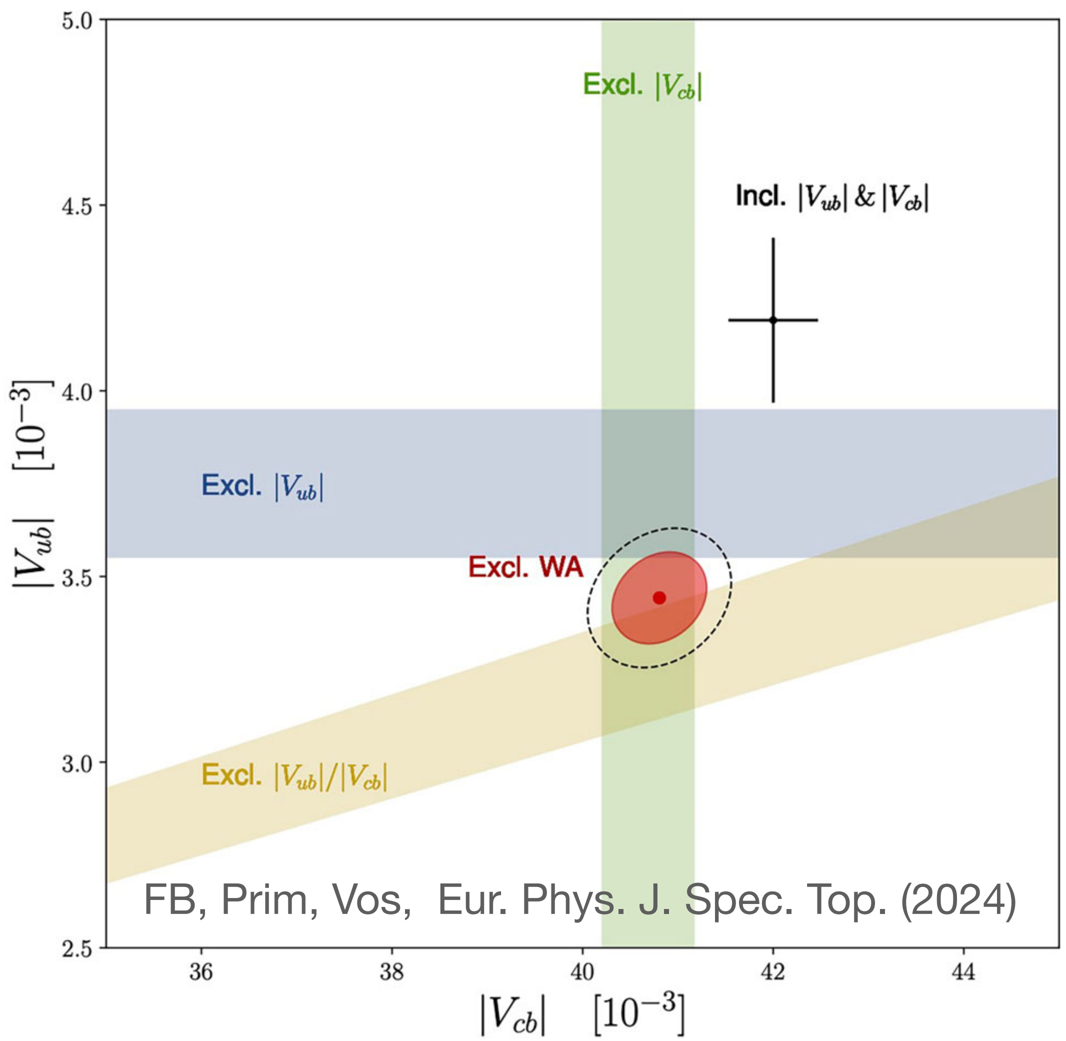
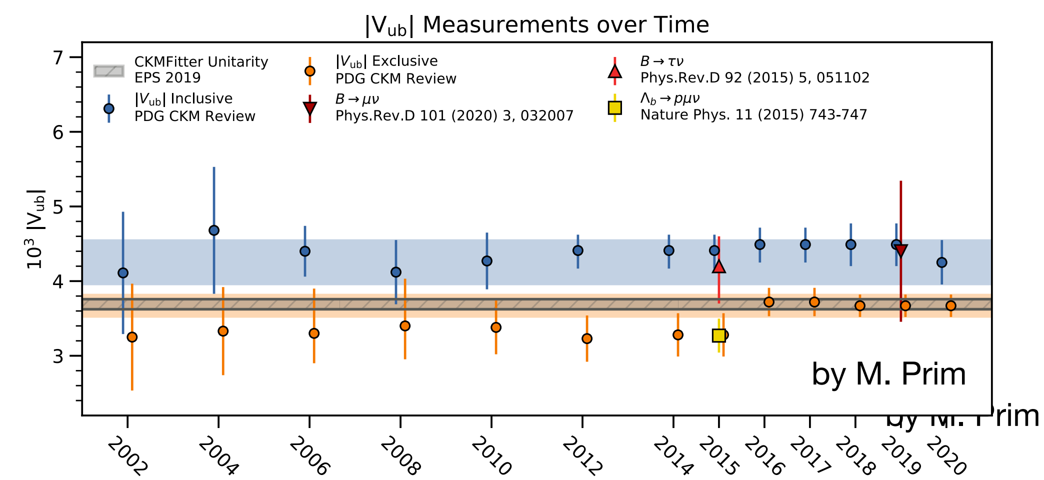



… Long-standing discrepancy since about a decade



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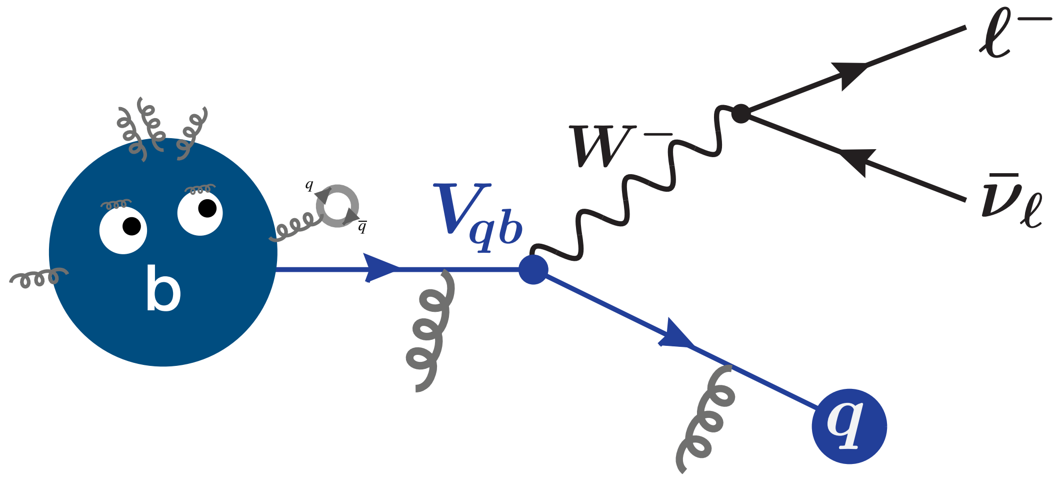


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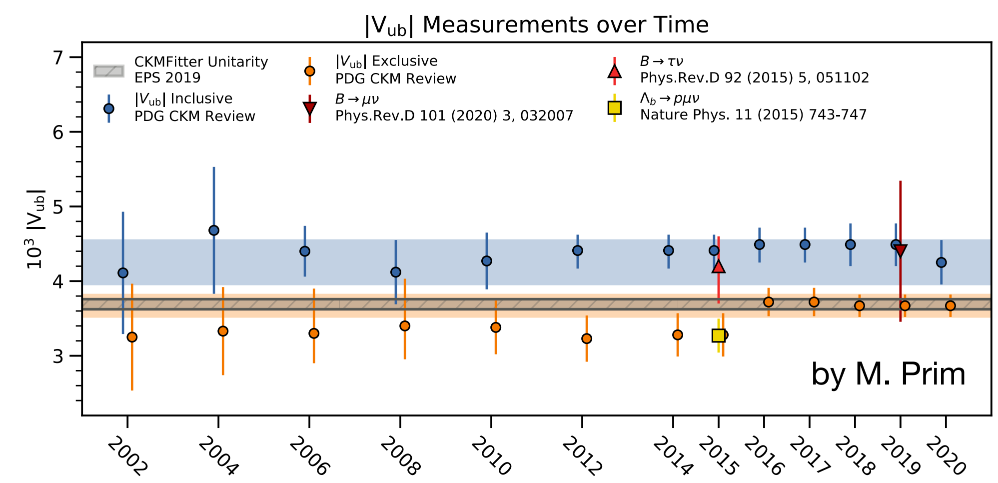


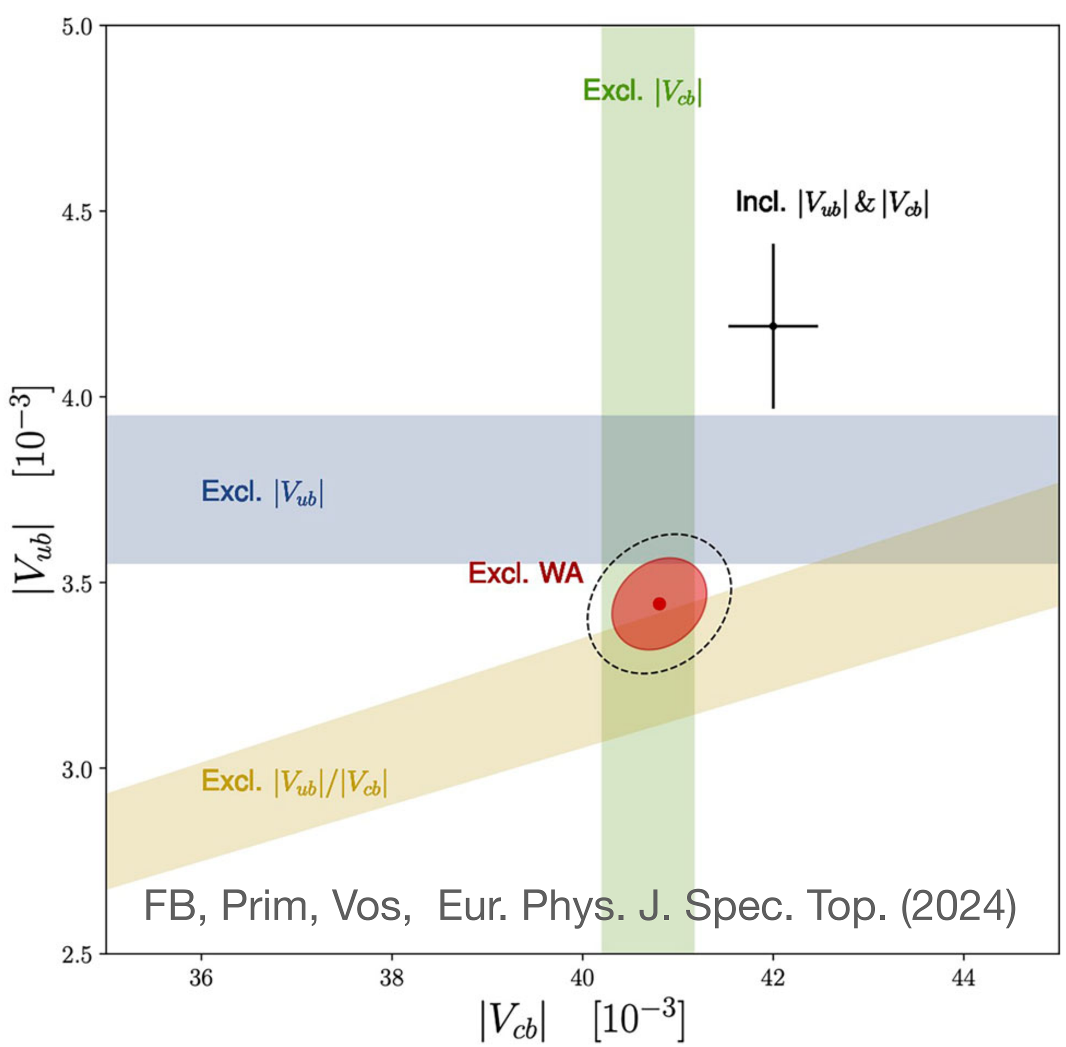


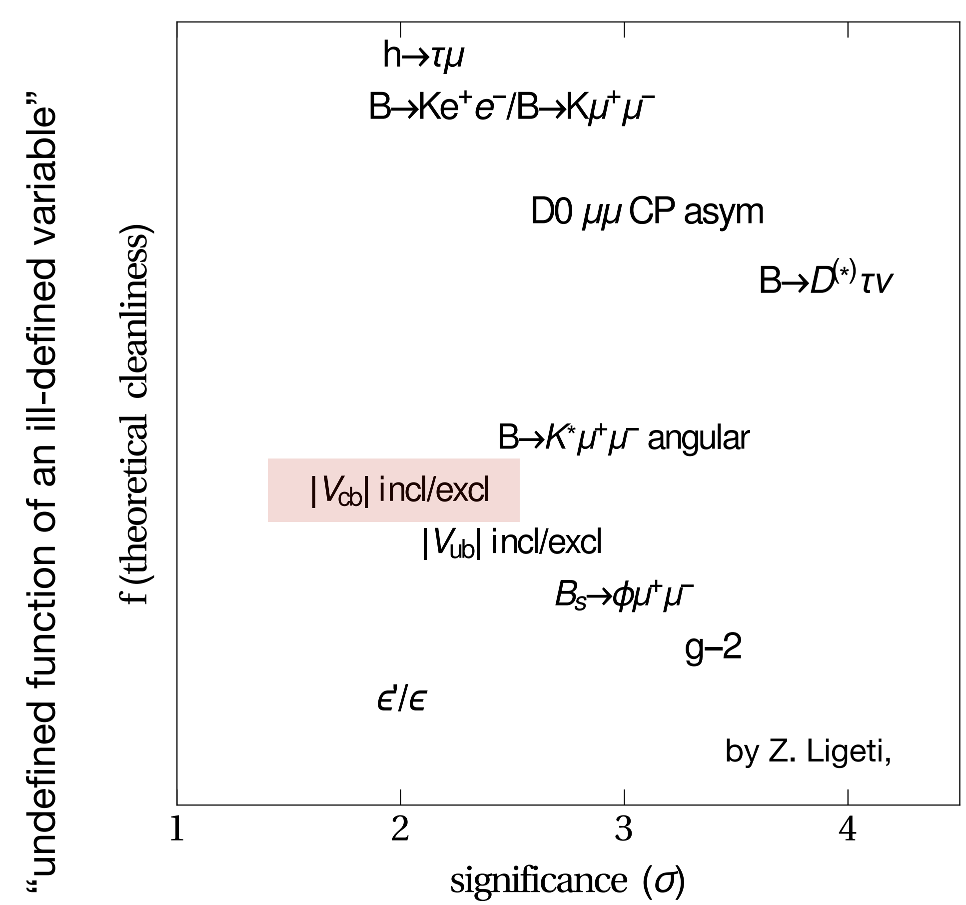


# **How** to inclusive $V_{cb}$

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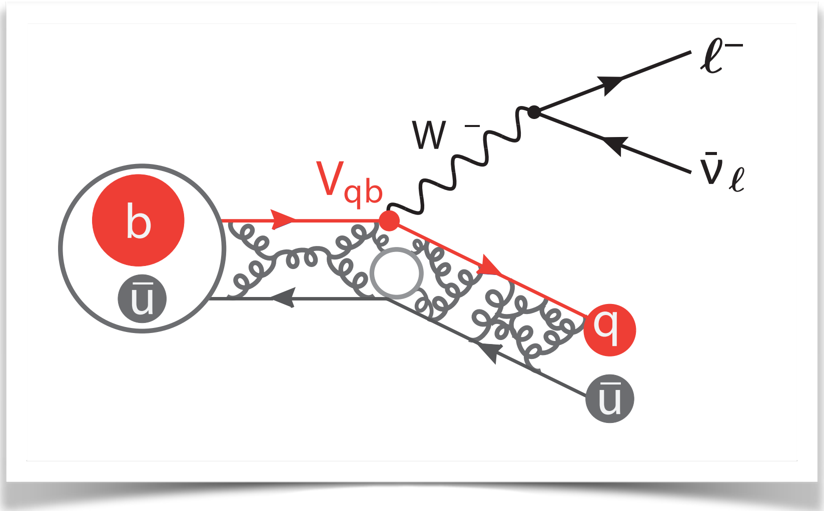



Inclusive $|V_{cb}|$

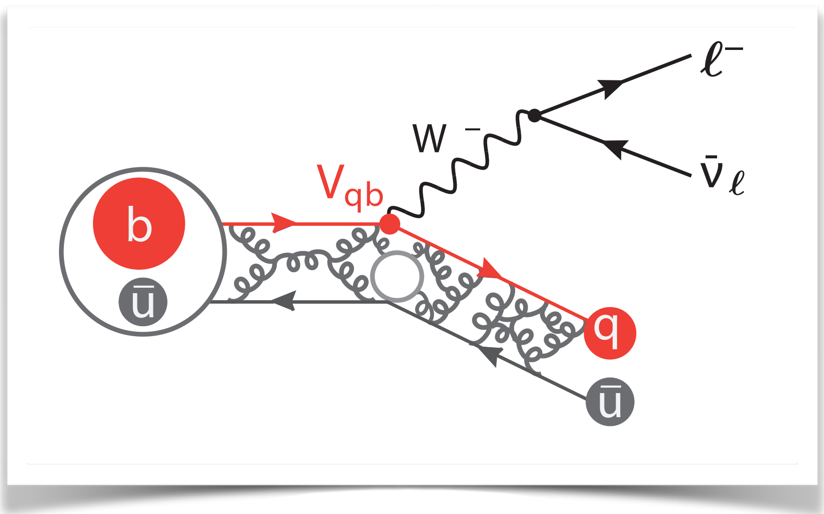
$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[ \Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

# How to inclusive $V_{cb}$



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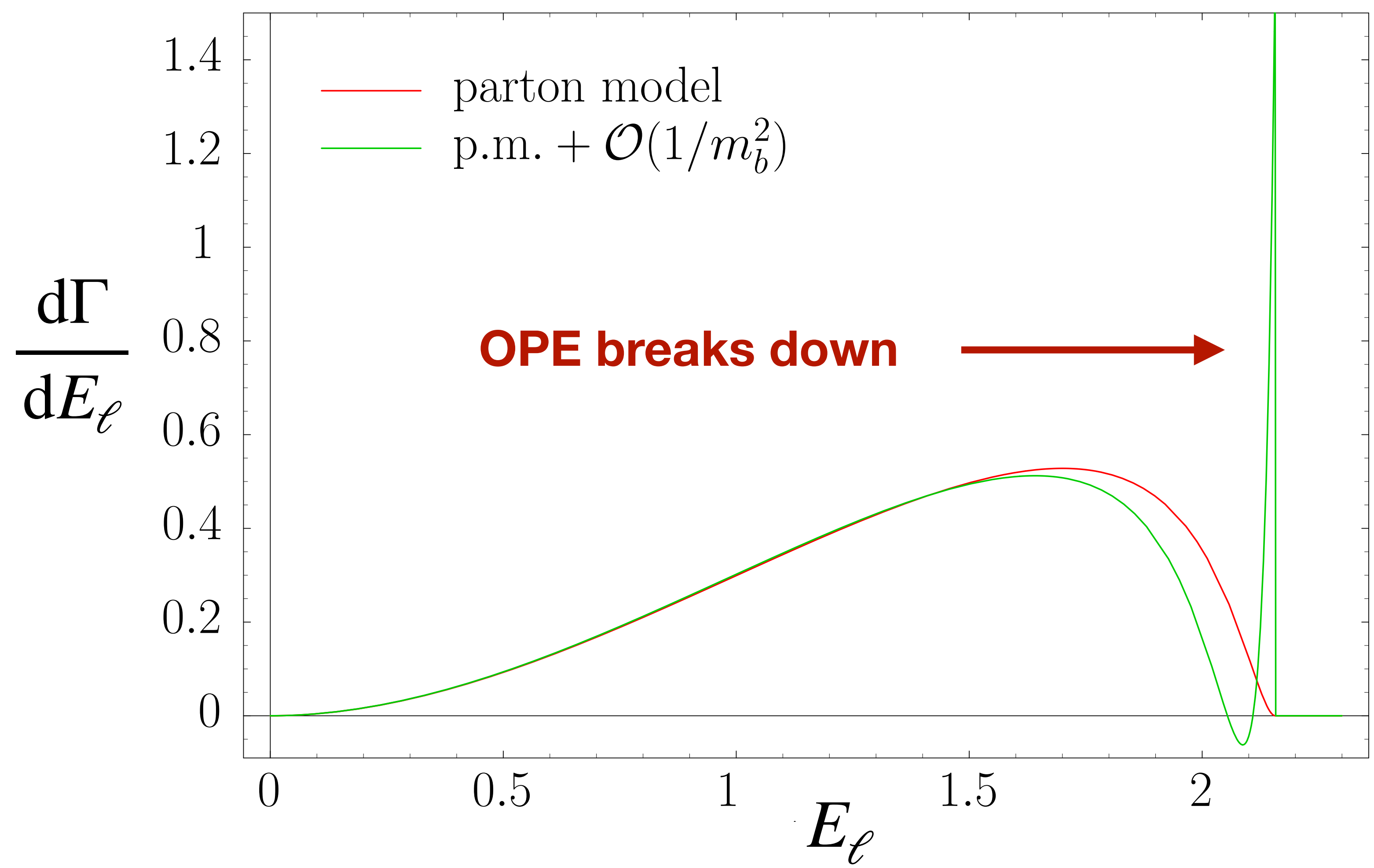
**Other complication: OPE does not allow point-by-point predictions**



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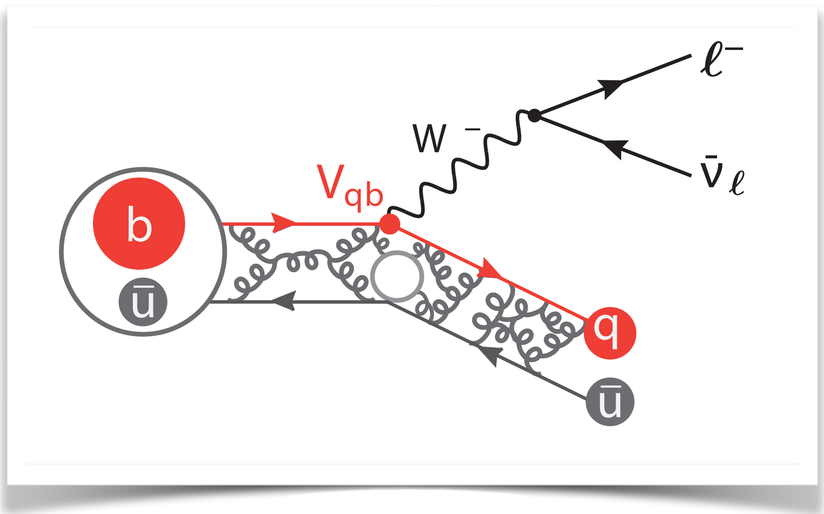


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**Other complication: OPE does not allow point-by-point predictions**

**But converges if integrated over large parts of phase space**

$v = p_B / m_B$

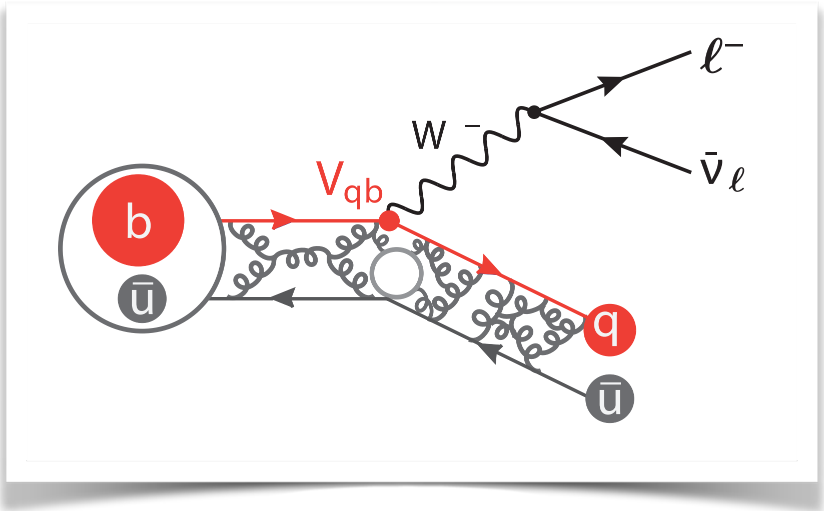
$$\int w^n(v, p_\ell, p_\nu) \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi} \, \mathrm{d}\Phi$$

weight function

**Example** weight functions

| | | |
|---|---|---|
| $w = (p_\ell + p_\nu)^2 = q^2$ | | four-momentum transfer squared |
| $w = (m_B v - q)^2 = M_X^2$ | | invariant mass squared |
| $w = (v \cdot p_\ell) = E_\ell^B$ | | Lepton Energy |

# How to inclusive $V_{cb}$




Inclusive $|V_{cb}|$

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Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[ \Gamma(b \to q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

**Established approach:** Use **spectral moments** (hadronic mass moments, lepton energy moments etc.) to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

$d\Gamma$ are calculated perturbatively

→ Available at $\mathcal{O}(\alpha_s^3)$
Fael, Schönwald, Steinhauser
Phys. Rev. D 104, 016003 (2021)

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ encapsulate non-perturbative dynamics

→ HQE parameters must be extracted from data (currently! more about that later)

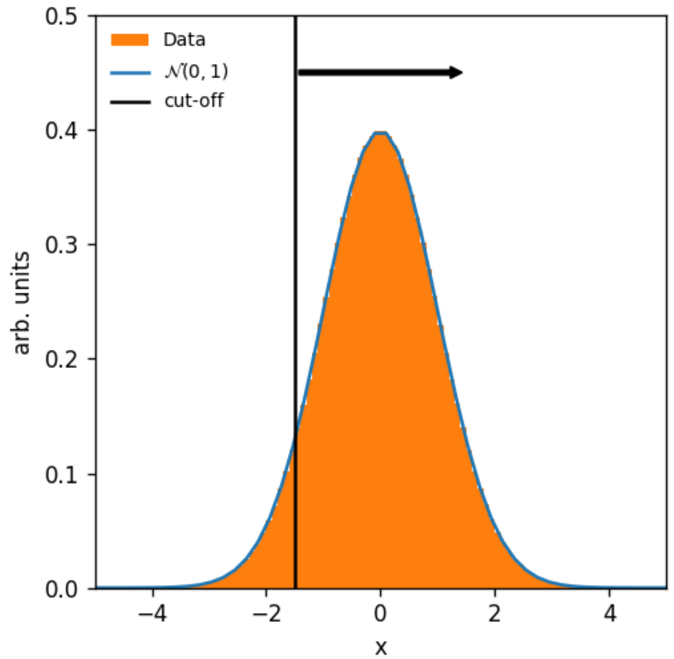
→ requires the spectral moments of $B \to X_c \ell \nu$

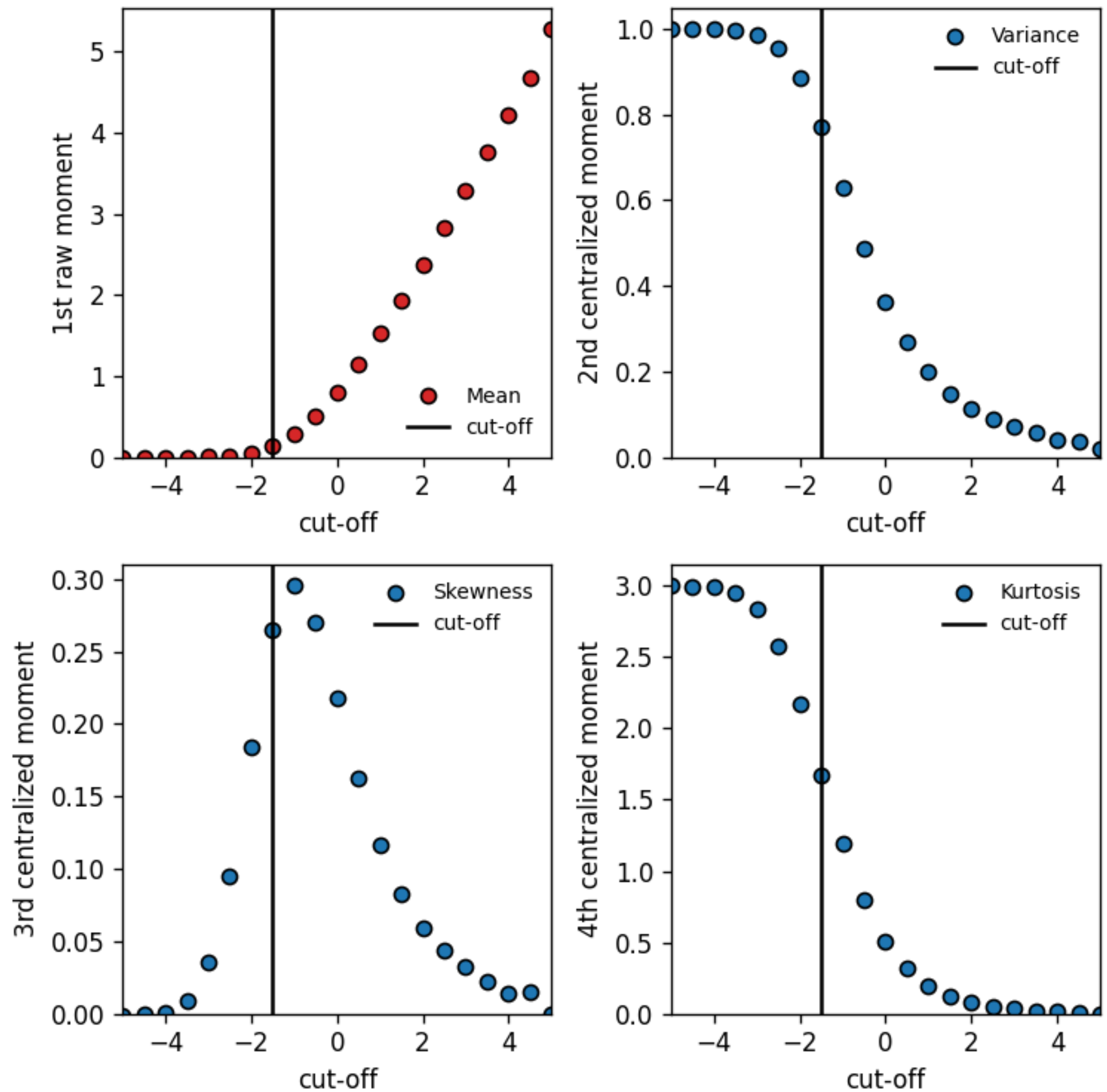
Challenge: Proliferation of HQE parameters at higher order

**Bad news:** # of matrix elements significantly increases if one increases expansion in $1/m_{b,c}$

# Let’s take a moment or two…

Illustrations by Markus Prim




$$\mu_n = \int_{-\infty}^{-\infty} (x-c)^n f(x) dx$$

Raw moment: $c = 0$
Central moment: $c = \text{Mean}$

First raw moment: Mean
*Measures the location*

Second central moment: Variance
*Measures the spread*

Third central moment: Skewness
*Measures asymmetry*

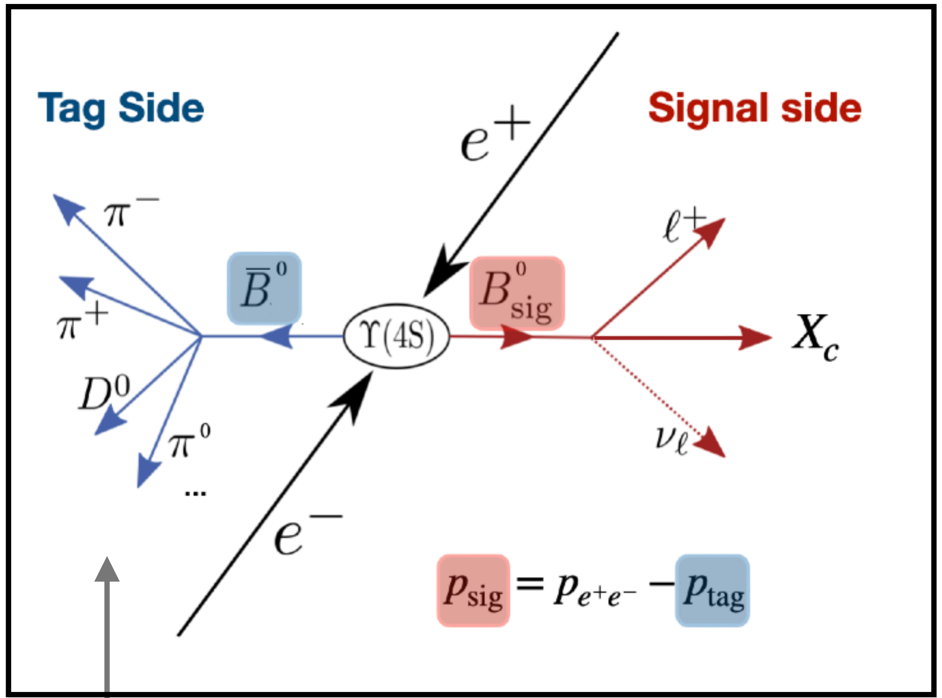
Fourth central moment: Kurtosis
*Measures “tailedness”*

Moments are measured with progressive cuts in the distribution
→ **highly correlated measurements**

# How to measure spectral moments

[PRD 107, 072002 (2023), arXiv:2205.06372]

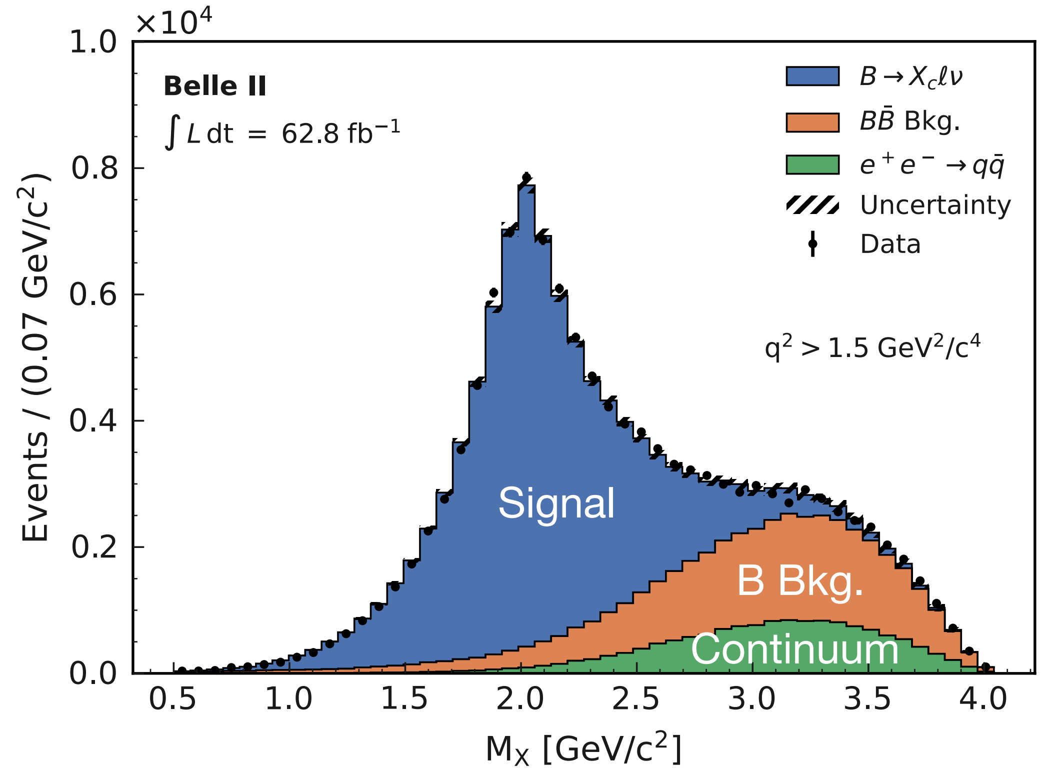
**Key-technique**: hadronic tagging

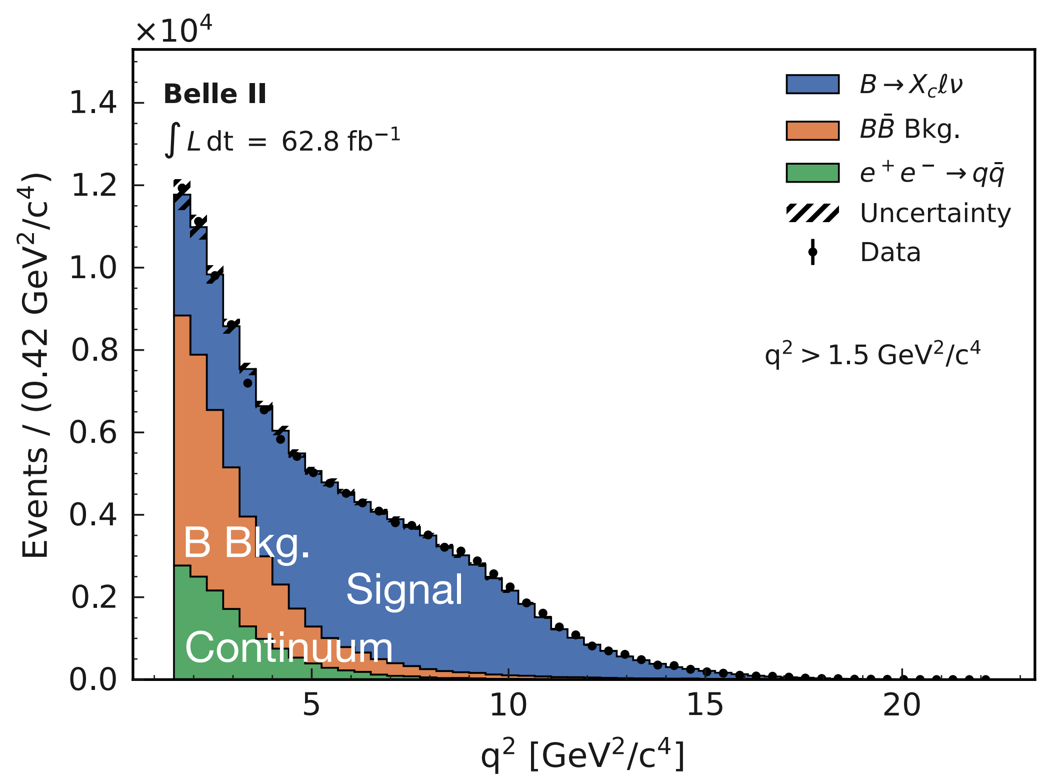



**Can identify $X_c$ constituents**

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$




**Hadronic Tagging**
with **Belle II** algorithm (FEI)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

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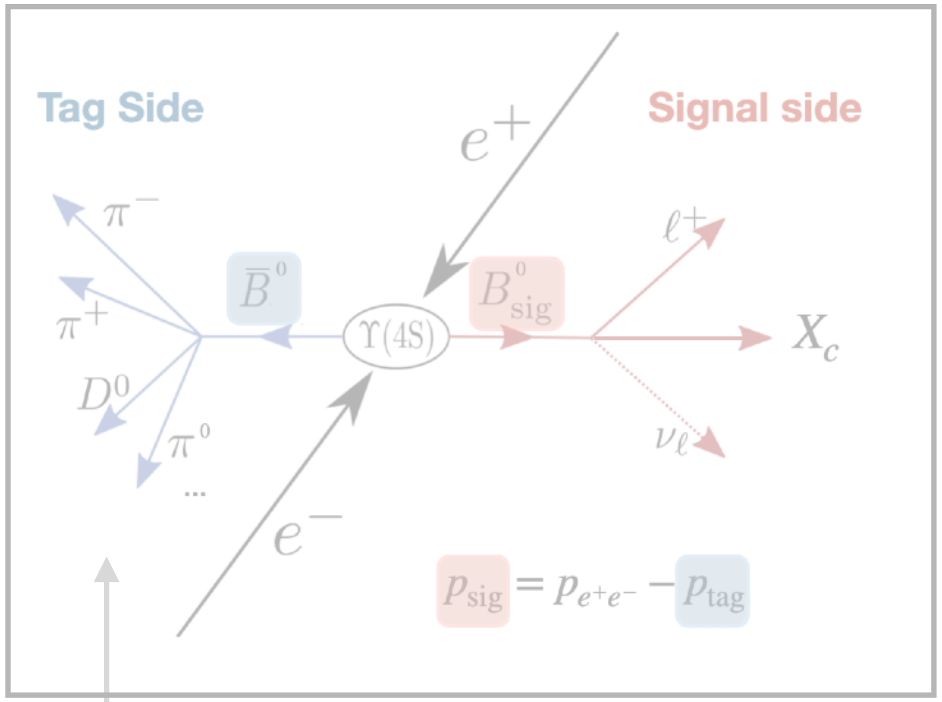


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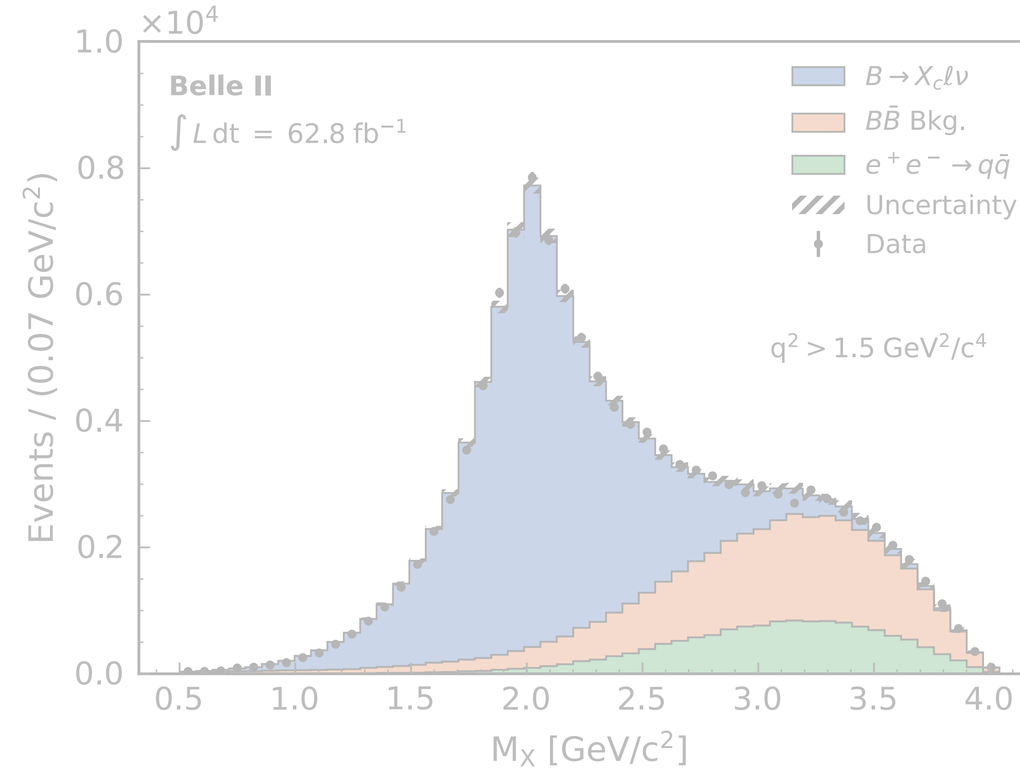
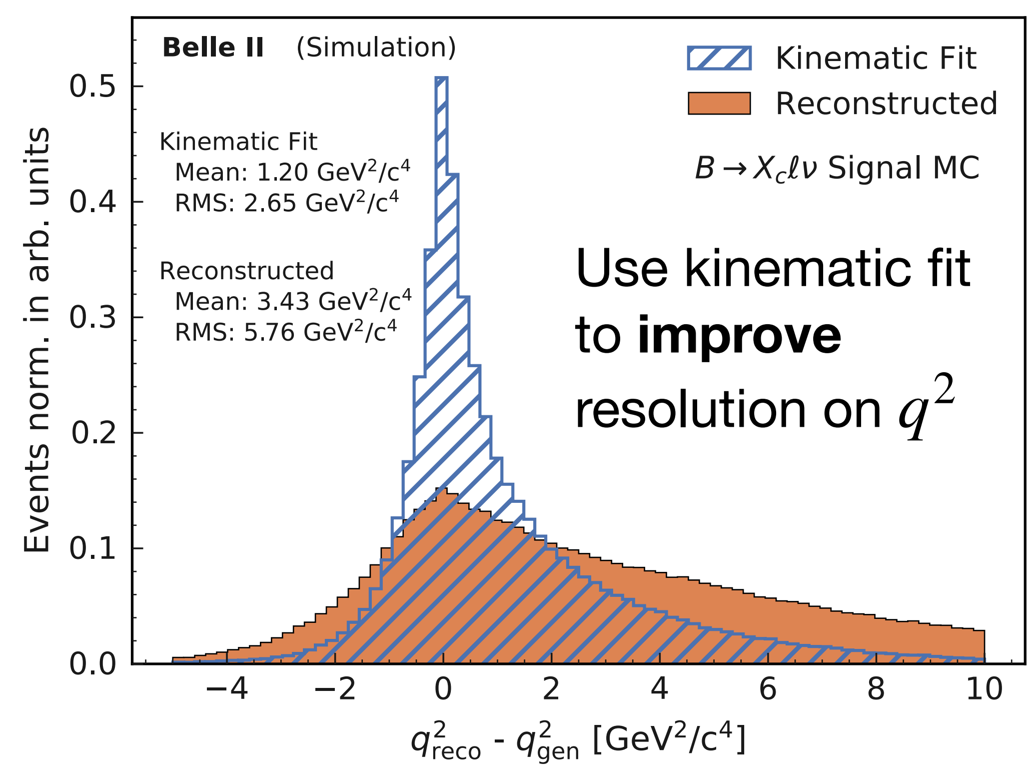
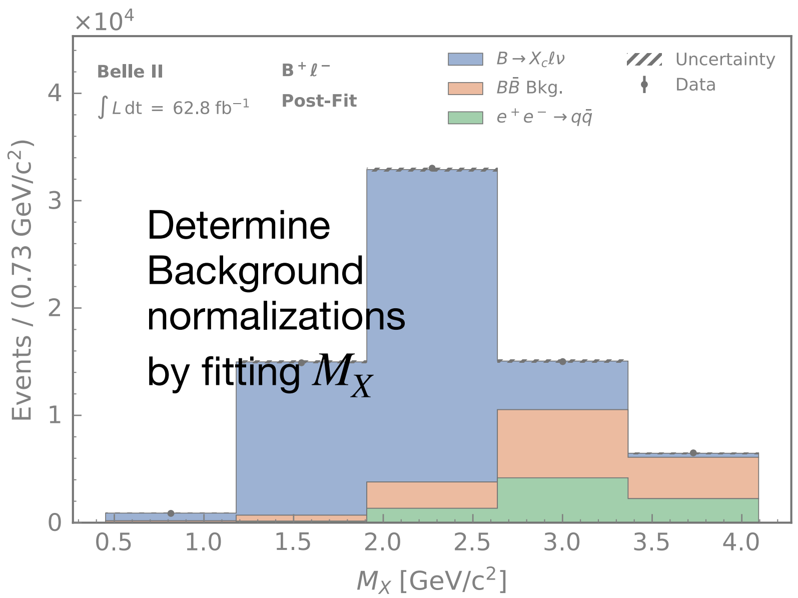
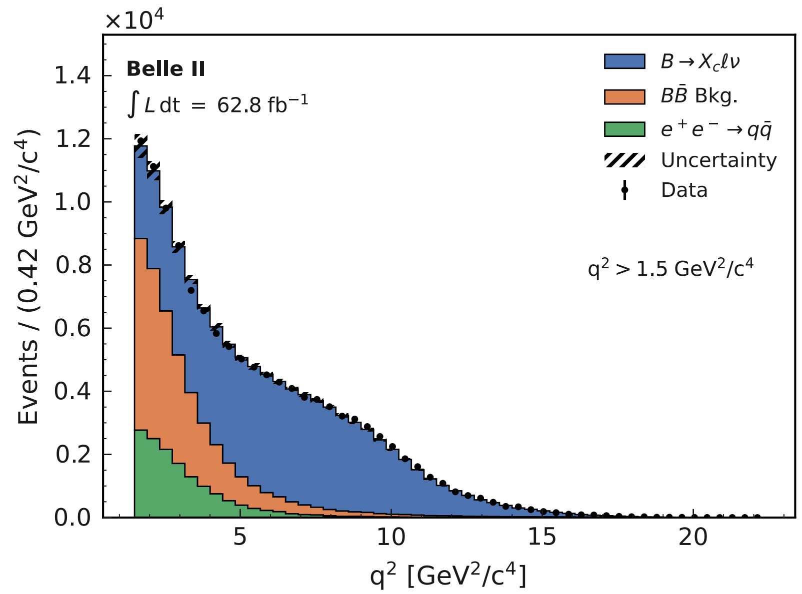


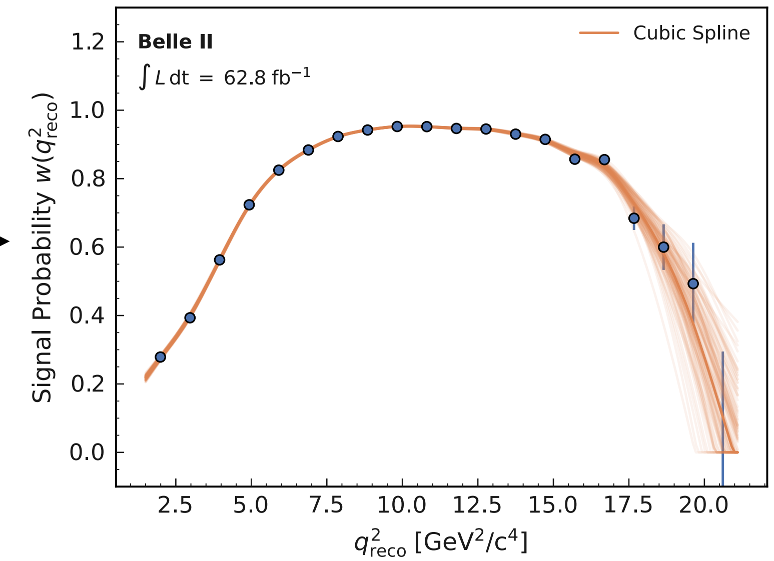




Use kinematic fit to **improve** resolution on $q^2$

# Measurement in a nutshell




Step #1: Subtract Background

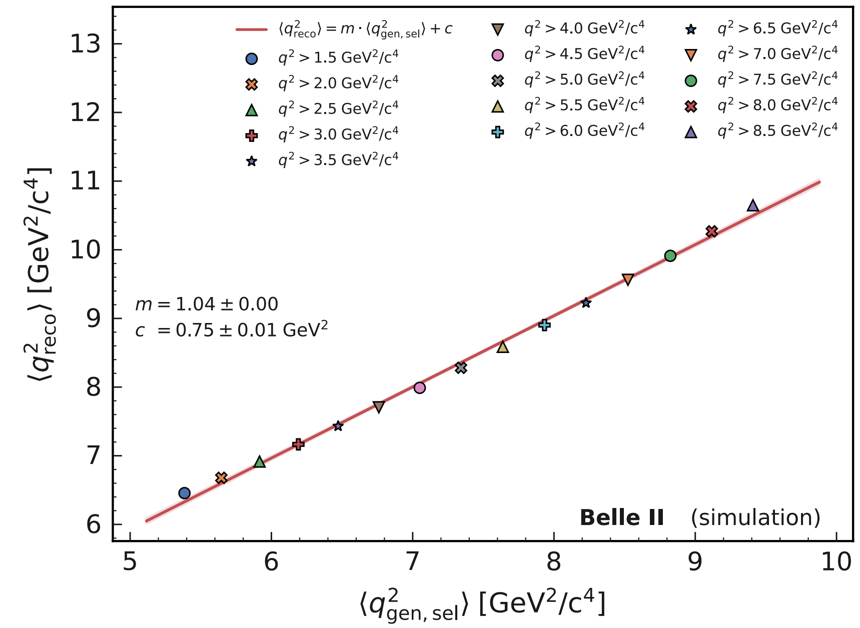
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\rm data}} w(q^2_{\rm reco,i}) \times q^{2n}_{{\rm calib},i}}{\sum_j^{N_{\rm data}} w(q^2_{\rm reco,j})} \times \mathcal{C}_{\rm calib} \times \mathcal{C}_{\rm gen} \,,$$

# Measurement in a nutshell

Exploit linear dependence between rec. & true moments

$$q^{2m}_{\mathrm{cal}\,i} = \left(q^{2m}_{\mathrm{reco}\,i} - c\right)/m$$

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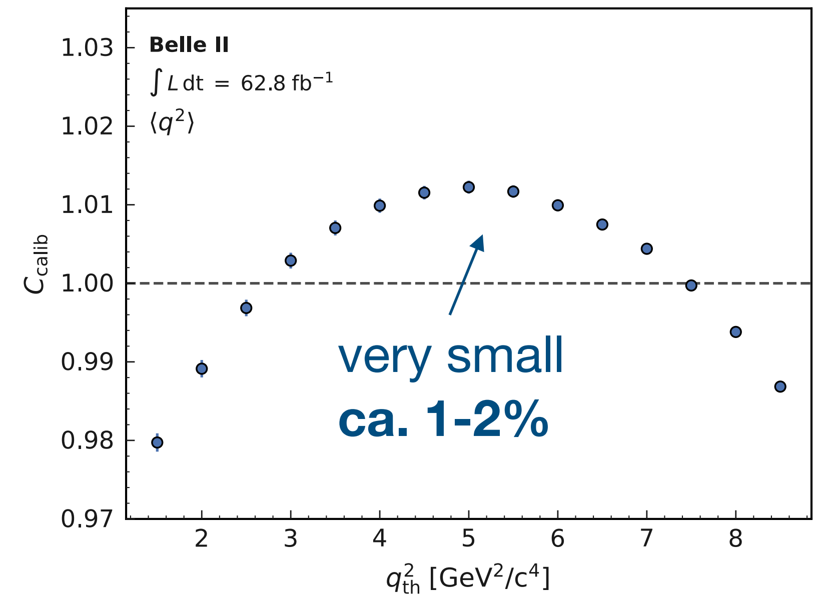
# Measurement in a nutshell


Very small deviation from linear behavior between reconstruct and true $q^2$

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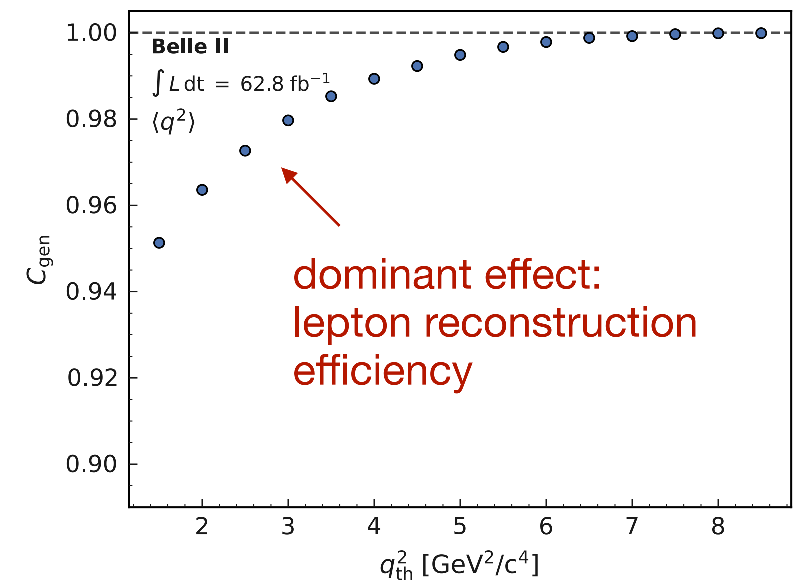
Step #3: If you fail, try again

# Measurement in a nutshell


Account for efficiency & acceptance effects

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# Measurement in a nutshell


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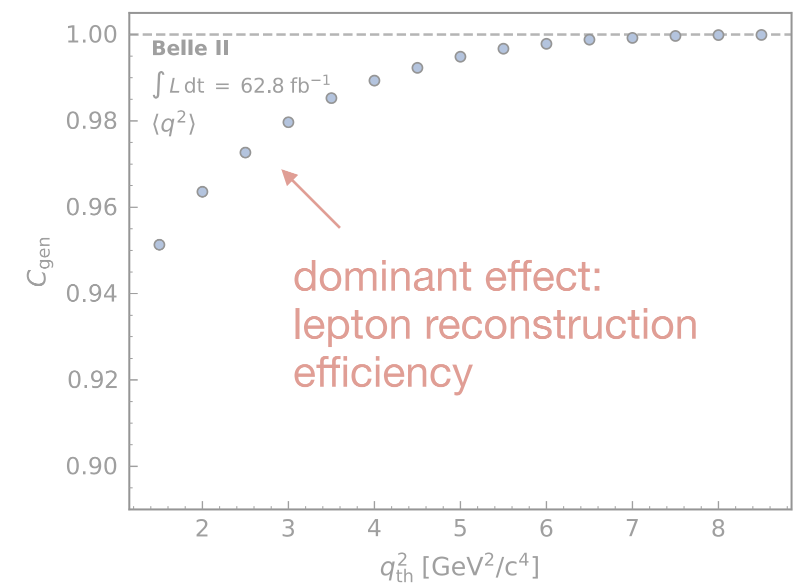


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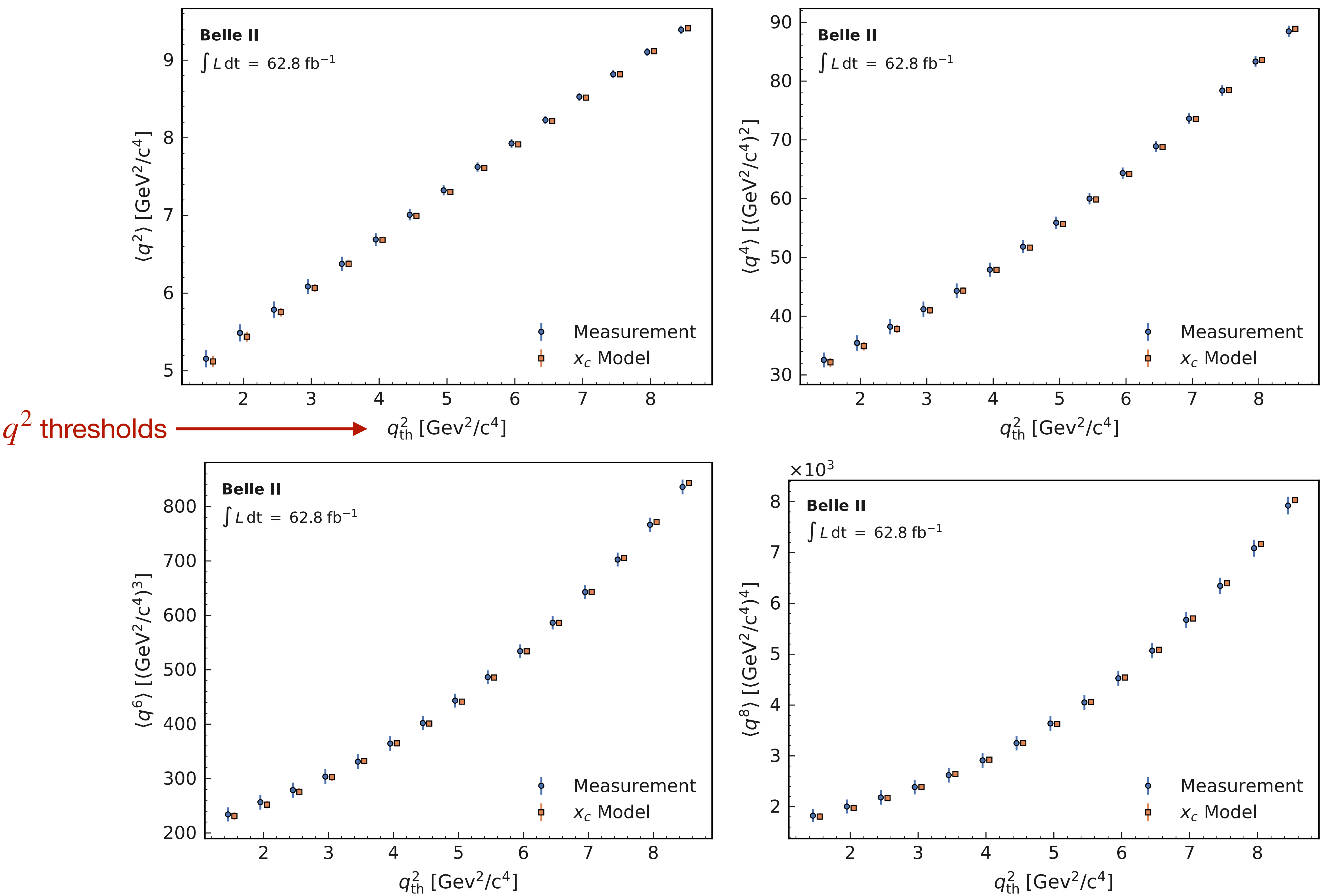
Step #4: Correct for selection effects

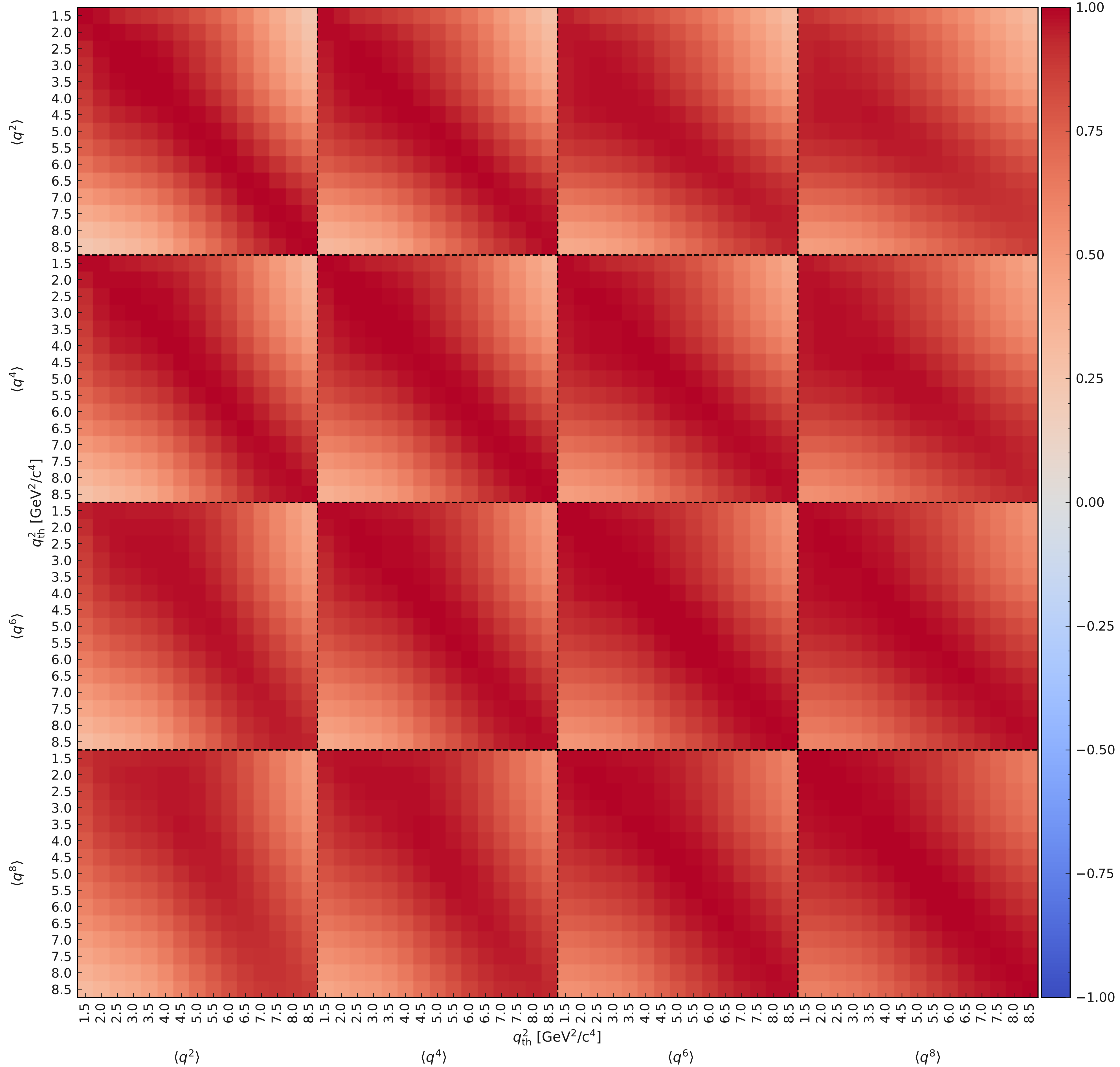
→ Repeat this for many different thresholds cuts $q^2_{th}$

# **Example:** Belle II $q^2$ spectral moments



**Statistical** plus **systematic** correlations

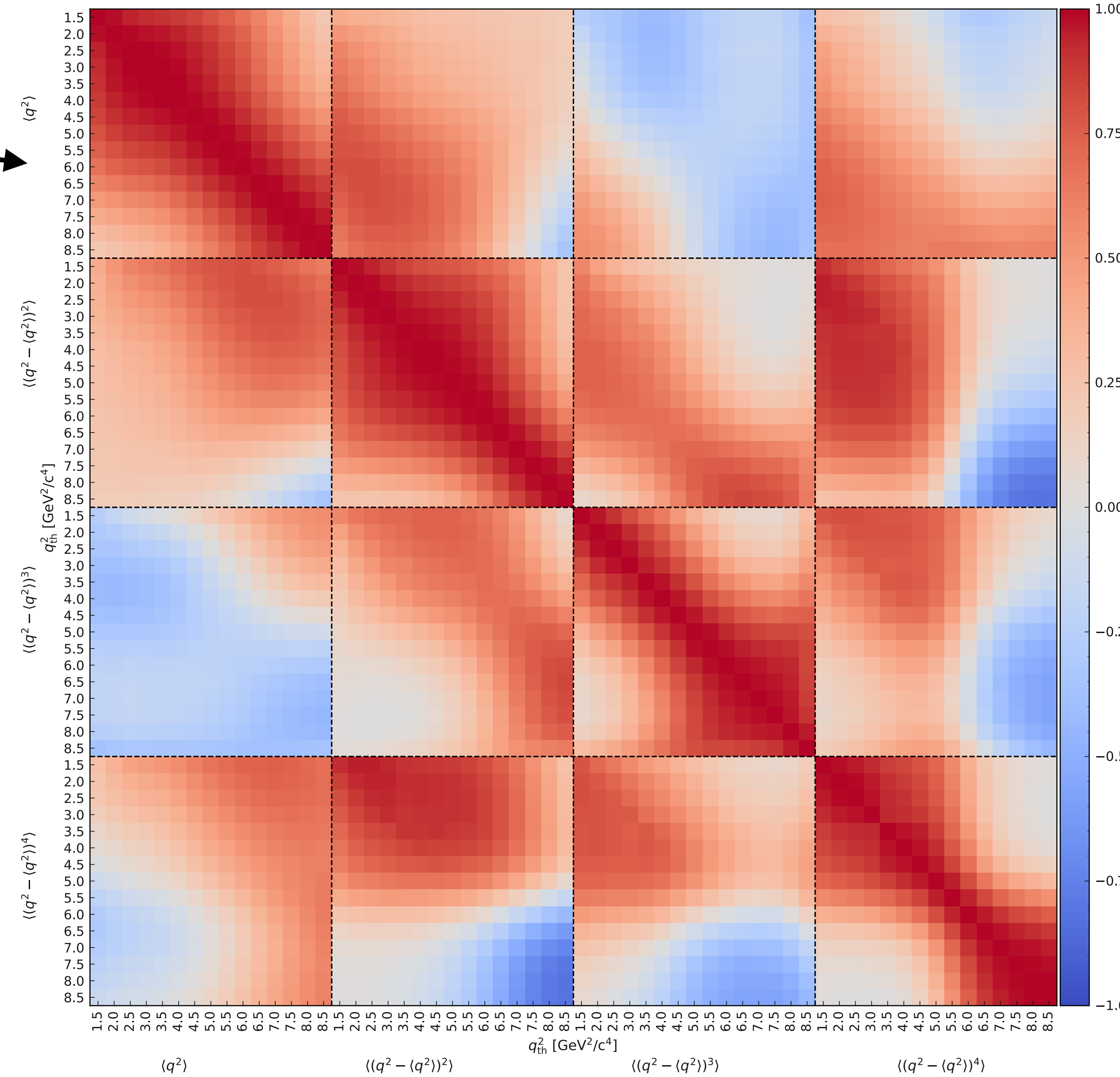
strong correlations!

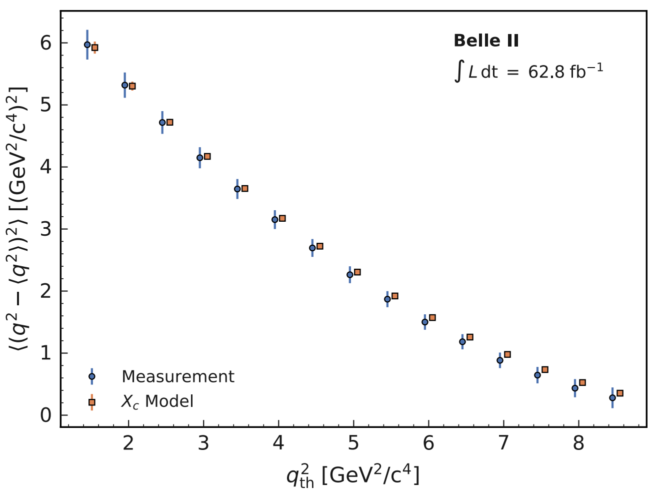


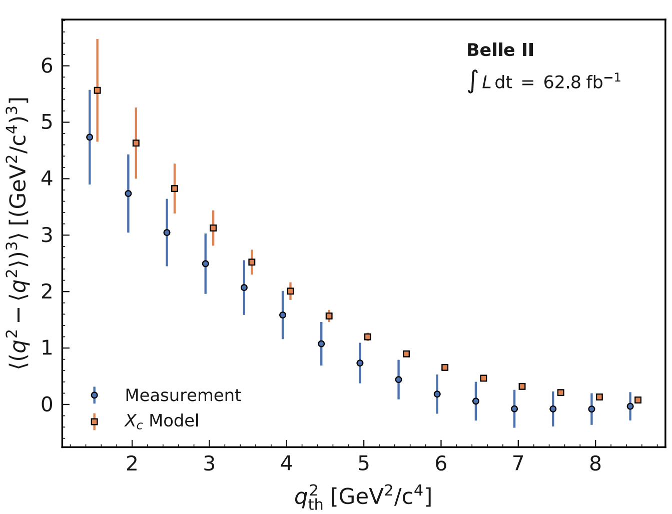
# From moments to *central moments*

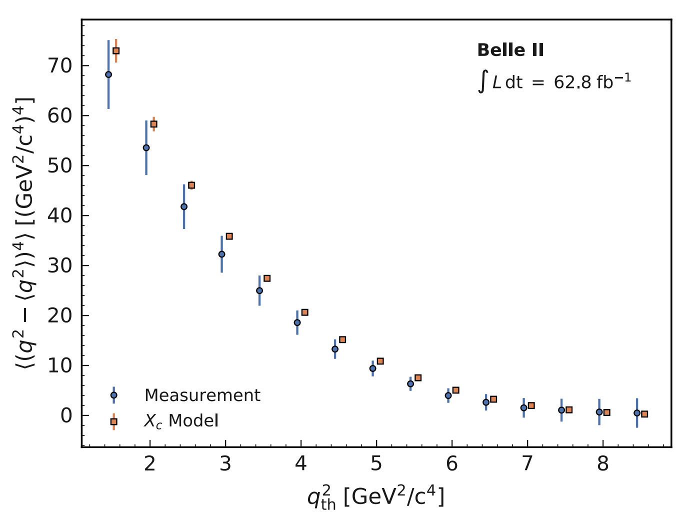
Central moments are **less** strongly correlated

$$\begin{pmatrix} \langle q^2 \rangle \\ \langle q^4 \rangle \\ \langle q^6 \rangle \\ \langle q^8 \rangle \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} \langle q^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^3 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^4 \rangle \end{pmatrix}$$



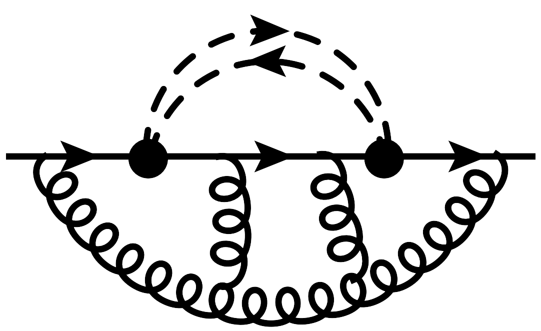



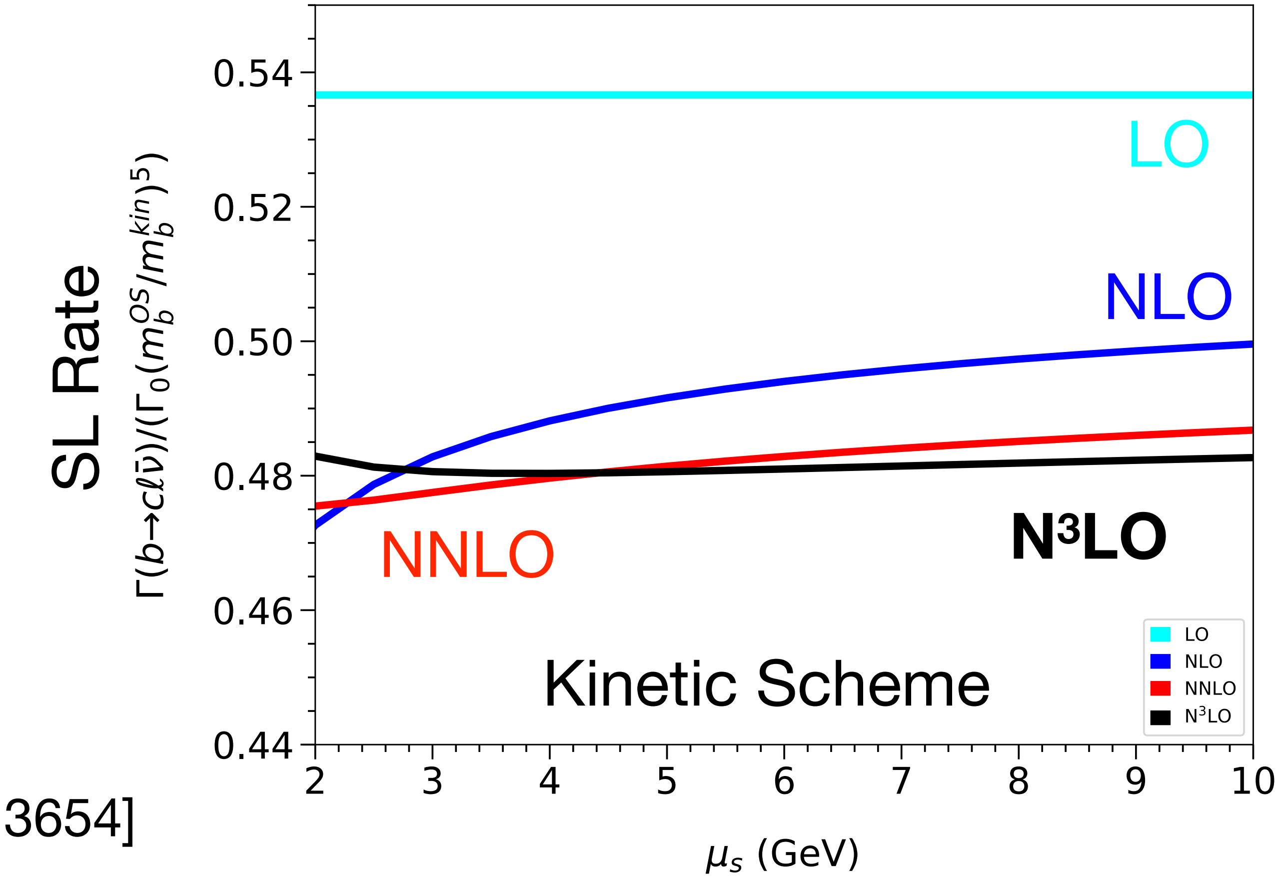

# What’s **new?**

# State-of-the-art: $|V_{cb}|$ with $E_\ell : M_X^2$

Fantastic progress on the theory side:
**semileptonic** rate @ **N³LO**!



M. Fael, K. Schönwald, M. Steinhauser
[Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]




Renormalization scale

Updated inclusive fit to $\langle E_\ell \rangle, \langle M_X \rangle$ moments:

$$|V_{cb}| = 42.16(30)_{th}(32)_{exp}(25)_{\Gamma}\ 10^{-3}$$

$$\Delta |V_{cb}|/|V_{cb}| = 1.2\,\%\,!$$

| $m_b^{kin}$ | $\overline{m}_c(2\text{GeV})$ | $\mu_\pi^2$ | $\rho_D^3$ | $\mu_G^2(m_b)$ | $\rho_{LS}^3$ | $\text{BR}_{c\ell\nu}$ | $10^3\|V_{cb}\|$ |
|---|---|---|---|---|---|---|---|
| 4.573 | 1.092 | 0.477 | 0.185 | 0.306 | -0.130 | 10.66 | 42.16 |
| 0.012 | 0.008 | 0.056 | 0.031 | 0.050 | 0.092 | 0.15 | 0.51 |
| 1 | 0.307 | -0.141 | 0.047 | 0.612 | -0.196 | -0.064 | -0.420 |
| | 1 | 0.018 | -0.010 | -0.162 | 0.048 | 0.028 | 0.061 |
| | | 1 | 0.735 | -0.054 | 0.067 | 0.172 | 0.429 |
| | | | 1 | -0.157 | -0.149 | 0.091 | 0.299 |
| | | | | 1 | 0.001 | 0.013 | -0.225 |
| | | | | | 1 | -0.033 | -0.005 |
| | | | | | | 1 | 0.684 |
| | | | | | | | 1 |

M. Bordone, B. Capdevila, P. Gambino
[Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

See also [Phys.Lett.B 829 (2022) 137068, 2202.01434] for very recent 1S fit finding $|V_{cb}| = (42.5 \pm 1.1) \times 10^{-3}$

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

**Bad news:** number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472]
(M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting **reparametrization invariance**, but **not true for every observable**

**Spectral moments :**

$v = p_B/m_B$

$$\langle M^n[w]\rangle = \int w^n(v, p_\ell, p_\nu)\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}\,\mathrm{d}\Phi$$

$w = (m_B v - q)^2 \Rightarrow \langle M_X^n\rangle$ Moments — not RPI (depends on $v$)

$w = v \cdot p_\ell \Rightarrow \langle E_\ell^n\rangle$ Moments — not RPI (depends on $v$)

$w = q^2 \Rightarrow \langle (q^2)^n\rangle$ Moments — RPI! (does not depend on $v$)

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

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Measurements of $q^2$ **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

Belle

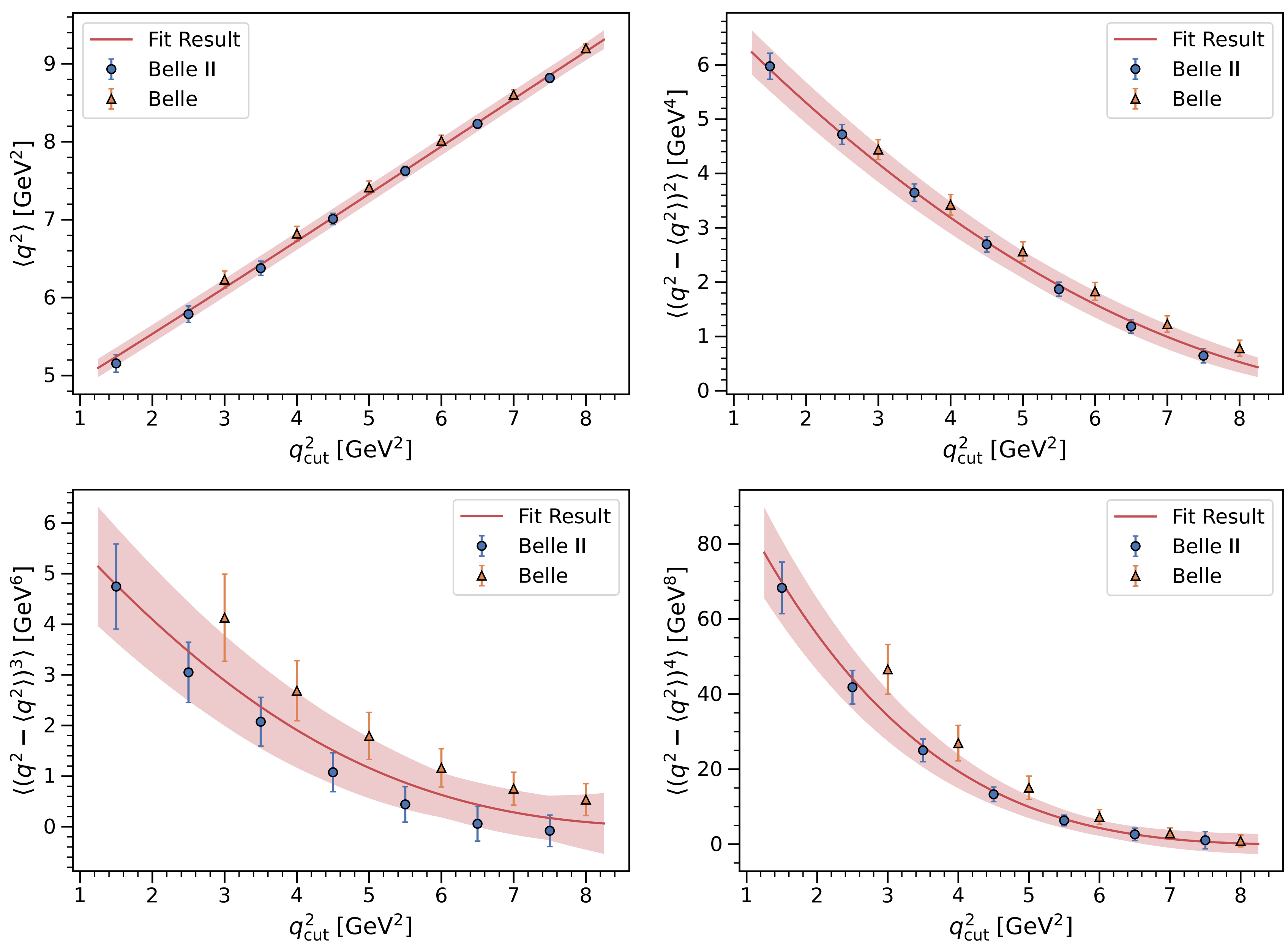
Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment
[PRD 107, 072002 (2023), arXiv:2205.06372]

Belle II

# $|V_{cb}|$ from $q^2$

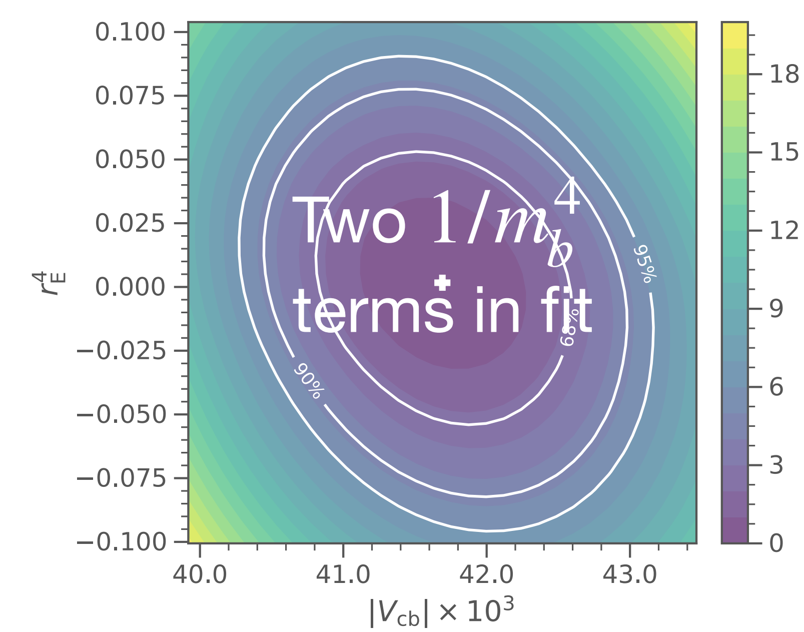
F. Bernlochner, M. Fael, K. Olschewsky, E. Persson,
R. Van Tonder, K. Vos, M. Welsch [arXiv:2205.10274]

## Extraction of $|V_{cb}|$ from $q^2$ moments:



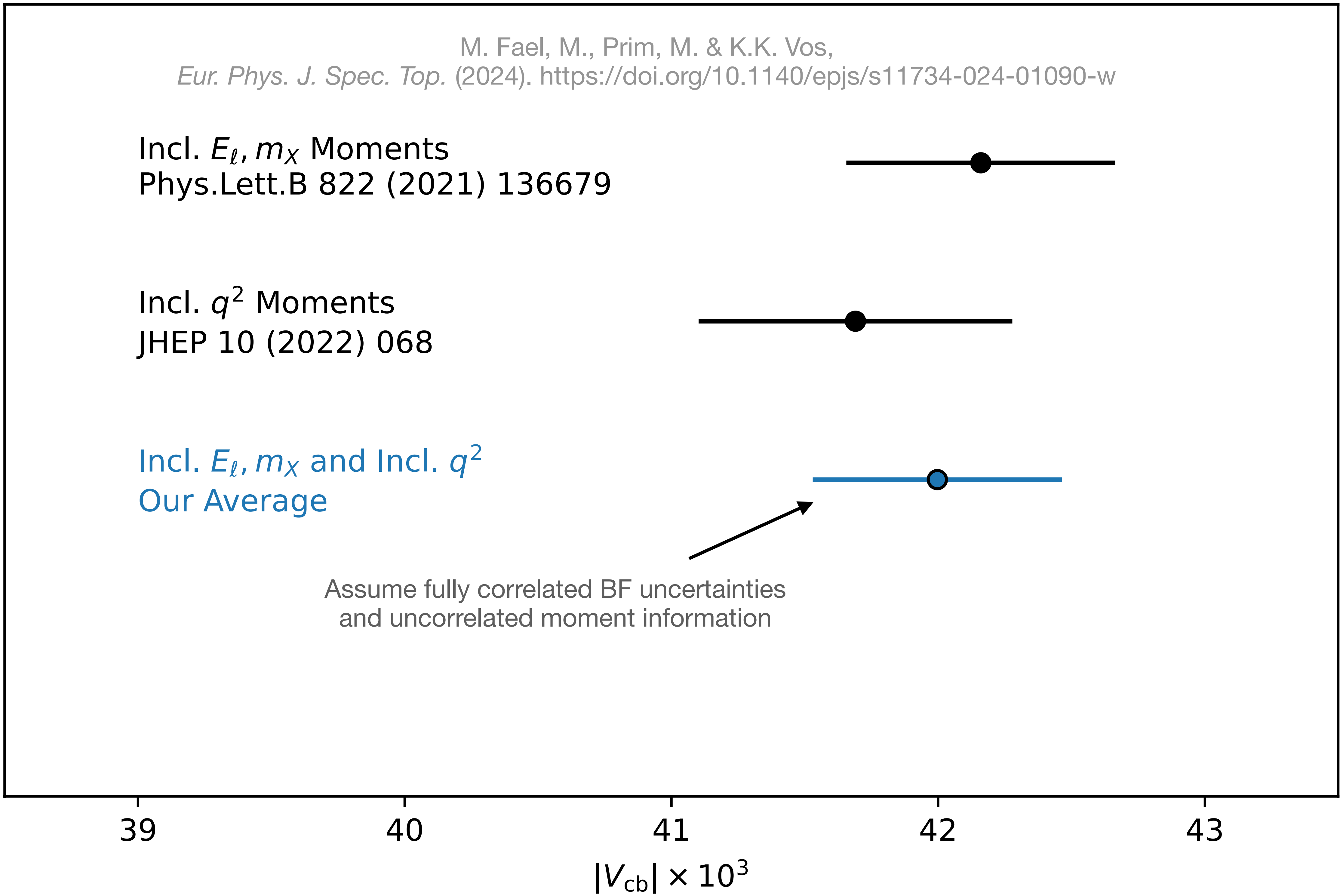

Included corrections
on the mom. predictions

| $\langle (q^2)^n \rangle$ | tree | $\alpha_s$ | $\alpha_s^2$ | $\alpha_s^3$ |
|---|---|---|---|---|
| Partonic | ✓ | ✓ | | |
| $\mu_G^2$ | ✓ | ✓ | | |
| $\rho_D^3$ | ✓ | ✓ | | |
| $1/m_b^4$ | ✓ | | | |




$$\rightarrow \quad |V_{cb}| = (41.69 \pm 0.59|_{\rm fit} \pm 0.23|_{\rm h.o.}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

# $|V_{cb}|$ from $q^2$ **versus** $E_\ell : M_X^2$



# Moments to party: $q^2 : E_\ell^B : M_X^2$

**The $q^2$ moments in inclusive semileptonic $B$ decays**

G. Finauri[a] P. Gambino[a,b,c]

https://arxiv.org/abs/2310.20324

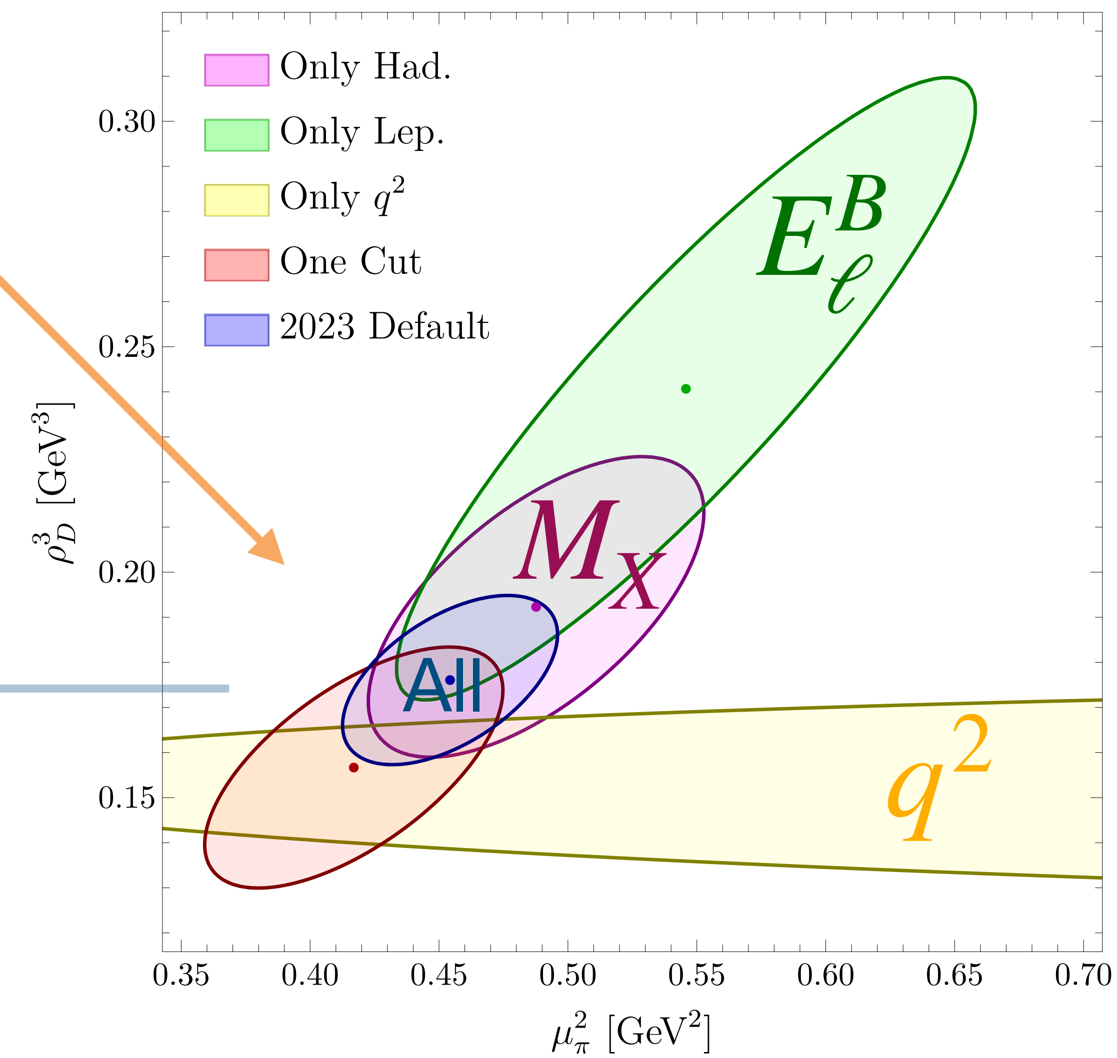
First simultaneous extraction using **all** moments

Very interesting take-away : **inclusion** of $q^2$ moments have the potential to **decrease** uncertainties on $\rho_D^3$

| | $m_b^{kin}$ | $\overline{m}_c$ | $\mu_\pi^2$ | $\mu_G^2$ | $\rho_D^3$ | $\rho_{LS}^3$ | $10^2 BR_{c\ell\nu}$ | $10^3 \lvert V_{cb} \rvert$ |
|---|---|---|---|---|---|---|---|---|
| $E_\ell^B : M_X^2$ | 4.573 | 1.092 | 0.477 | 0.306 | 0.185 | −0.130 | 10.66 | 42.16 |
| | 0.012 | 0.008 | 0.056 | 0.050 | 0.031 | 0.092 | 0.15 | 0.51 |
| $q^2 : E_\ell^B : M_X^2$ | 4.572 | 1.092 | 0.449 | 0.301 | 0.167 | −0.109 | 10.65 | 42.02 |
| | 0.012 | 0.008 | 0.042 | 0.048 | 0.018 | 0.089 | 0.15 | 0.48 |

$q^2$ moments prefer smaller value for Darwin term

also reduction, but dominated by theory errors

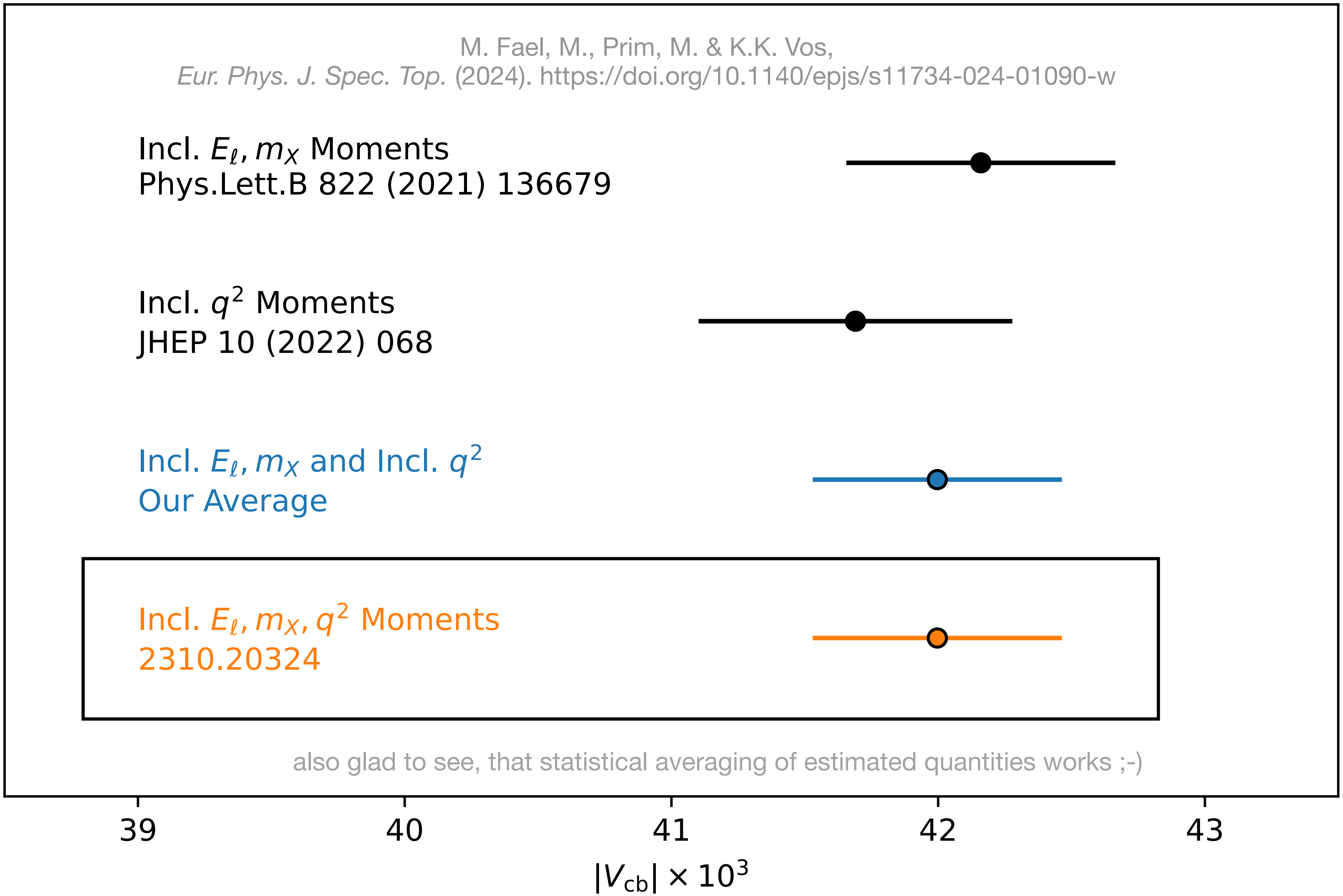



Fit includes BLM corrections ($\alpha_s^2 \beta_0$) and QED corrections for the first time ; uses updates **FLAG** input of heavy quark masses with $N_f = 2+1+1$ ⟷

$$\overline{m}_b^{(4)}(\overline{m}_b) = 4.203(11)\,\text{GeV}\,, \qquad \overline{m}_c^{(4)}(3\,\text{GeV}) = 0.989(10)\,\text{GeV}\,,$$

$$|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_\Gamma) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}\,.$$

# $|V_{cb}|$ from $q^2$ versus $E_\ell : M_X^2$ versus $q^2 : E_\ell : M_X^2$



# Interesting **future** directions

# LHCb might enter the scene

**Inclusive semileptonic $B_s^0$ meson decays at the LHC via a sum-of-exclusive modes technique: possibilities and prospects**

https://arxiv.org/abs/2312.05147

M. De Cian[a], N. Feliks[b,†], M. Rotondo[c] and K. Keri Vos[d,e]

**LHCb** records an **impressive amount** of b-hadrons of **various types** →

$$B_u : B_d : B_s : B_c : \Lambda_b \approx 40\,\% : 40\,\% : 10\,\% : 0.2\,\% : 8\,\%$$

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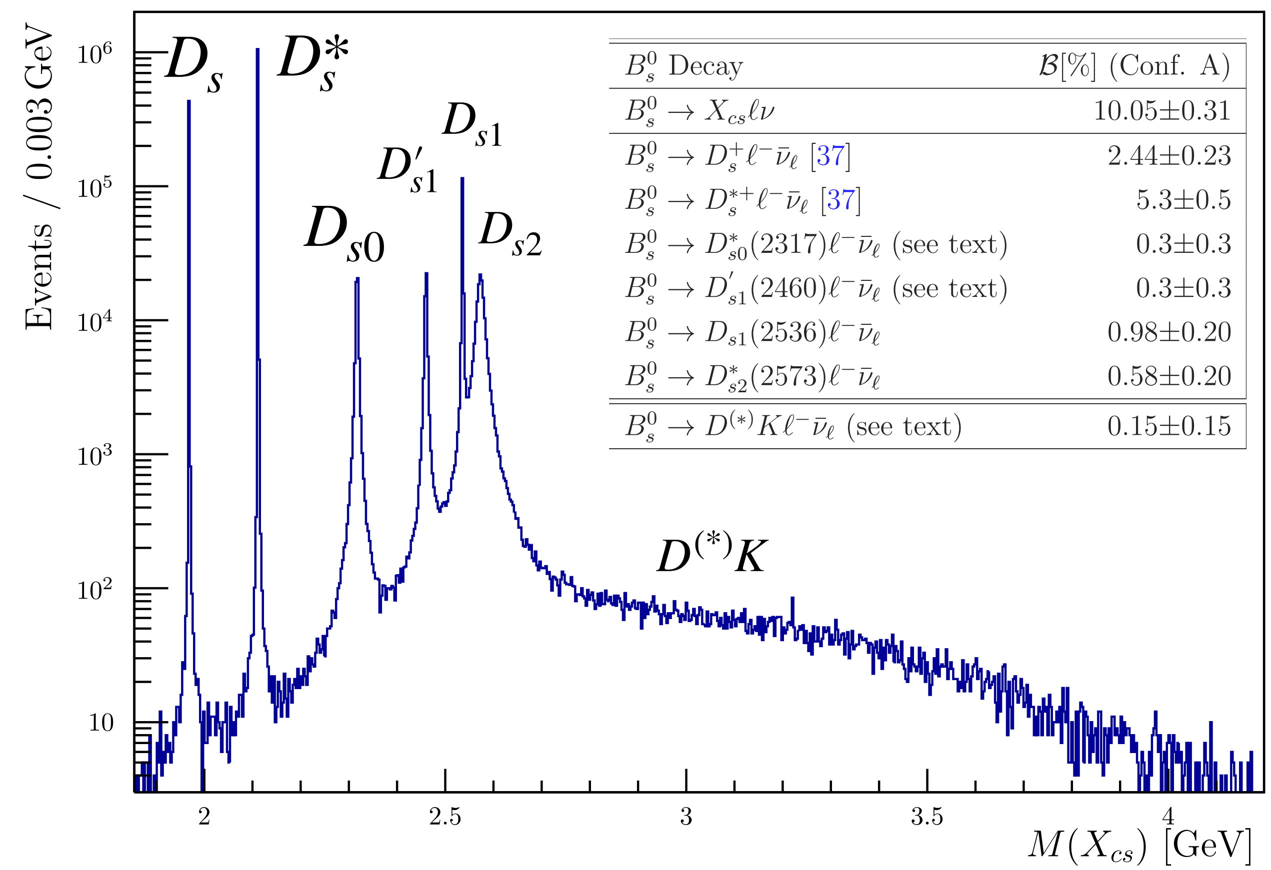
Interesting data set to study e.g. SU(3) breaking or baryon-meson differences of HQE parameters! But **how**?

**S**um over **E**xclusive **M**odes

=

Reconstruct your inclusive $B_s \to X_{cs}\ell\bar{\nu}_\ell$ system by **explicitly reconstructing** the **majority** of all **prompt final states**




| $D_{s0}^*$ | | $D'_{s1}$ | | $D_{s1}$ | | $D_{s2}^*$ | |
|---|---|---|---|---|---|---|---|
| $2317.8 \pm 0.5$ MeV | | $2459.5 \pm 0.6$ MeV | | $2535.11 \pm 0.06$ MeV | | $2569.1 \pm 0.8$ MeV | |
| $< 3.8$ MeV | | $< 3.5$ MeV | | $0.92 \pm 0.05$ MeV | | $16.9 \pm 0.7$ MeV | |
| $D_s^+\pi^0$ | $100^{+0.}_{-20}\%$ | $D_s^{*+}\pi^0$ | $48 \pm 11\%$ | $D^{*+}K_S^0$ | $85 \pm 12\%$ | $D^0K^+$ | seen |
| $D_s^+\gamma$ | $< 5\%$ | $D_s^+\gamma$ | $18 \pm 4\%$ | $D^{*0}K^+$ | $100\%$ | $D^+K_S^0$ | seen |
| $D_s^{*+}\gamma$ | $< 6$ | $D_s^+\pi^+\pi^-$ | $4.3 \pm 1.3\%$ | $D^+\pi^-K^+$ | $2.8 \pm 0.5\%$ | $D^{*+}K_S^0$ | seen |
| $D_s^{*+}\gamma$ | $< 6\%$ | $D_s^{*+}\gamma$ | $< 8\%$ | $D_s^+\pi^+\pi^-$ | seen | | |
| | | $D_{s0}^*\gamma$ | $3.7^{+5.0}_{-2.4}\%$ | $D^+K^0$ | $< 34\%$ | | |
| | | | | $D^0K^+$ | $< 12\%$ | | |

**Challenge:** need **good understanding** of

- **missing prompt exclusive contributions** to $X_{cs}$
- **correct for missing decay modes of exclusive** $D_s^{(*/**)}$

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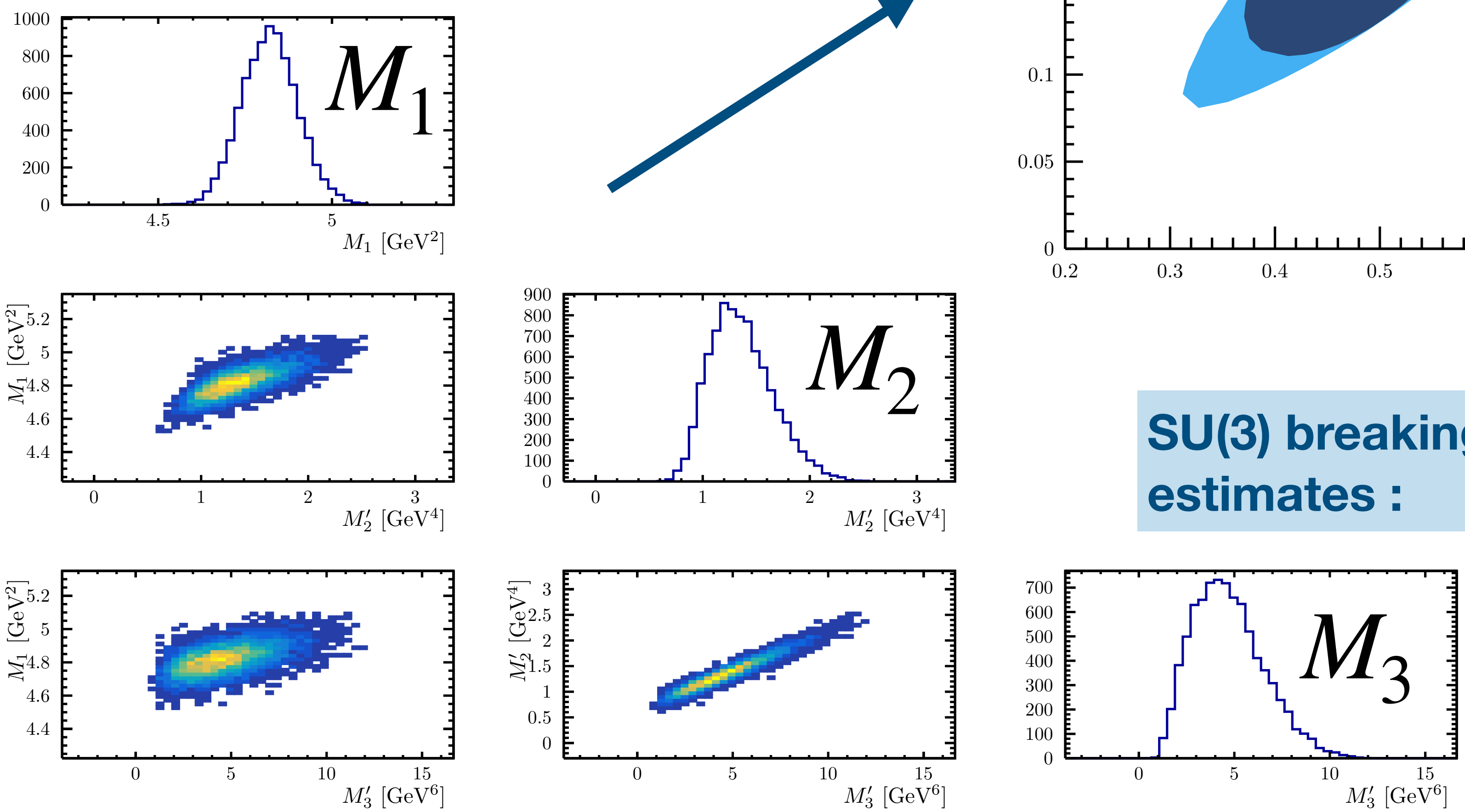
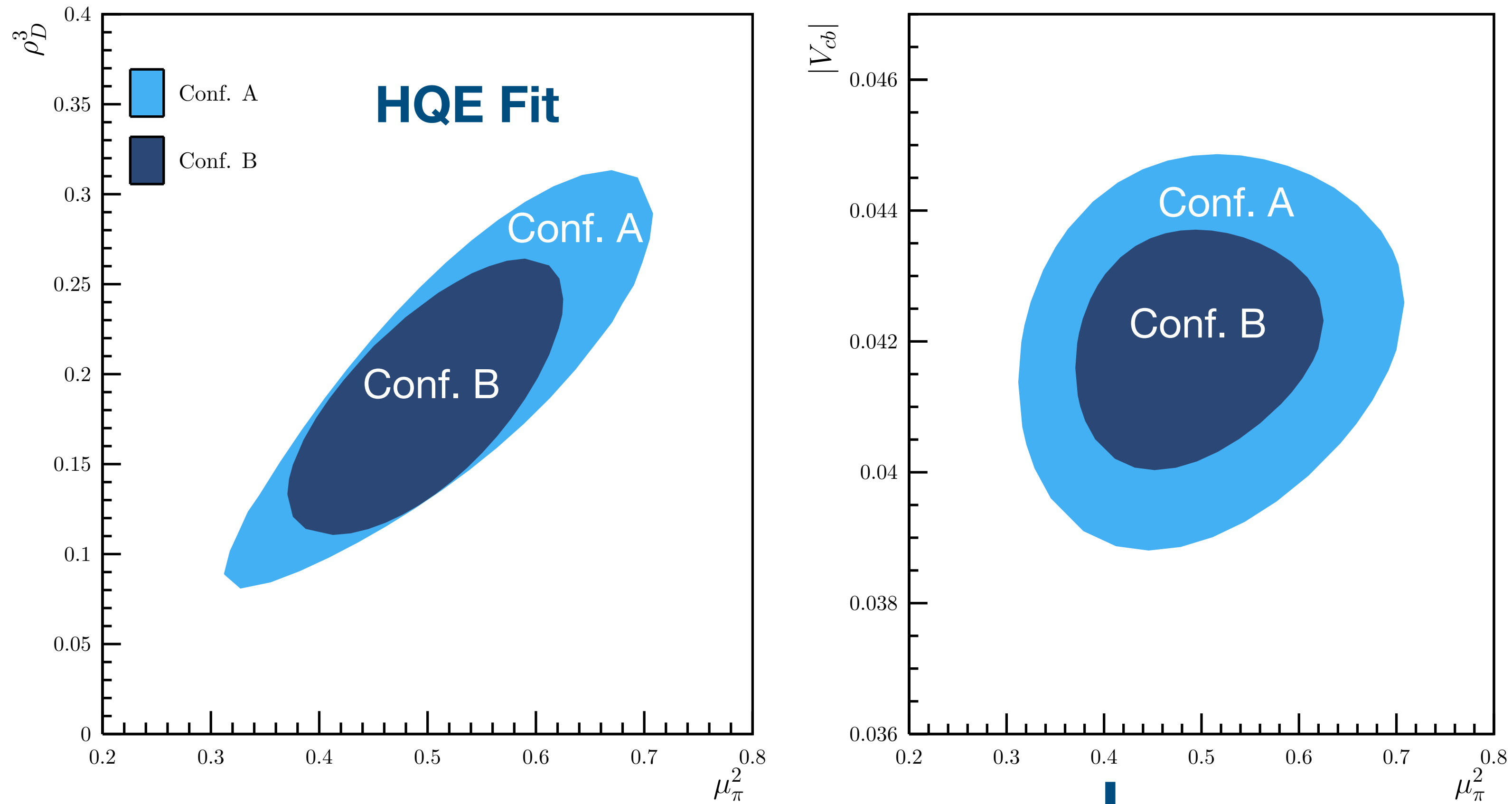
**Proof - of - concept :**




**SU(3) breaking estimates :**

$$\frac{\mu_\pi^2(B_s^0)}{\mu_\pi^2(B^0)} = 0.96\,, \qquad \frac{\rho_D^3(B_s^0)}{\rho_D^3(B^0)} = 0.86\,,$$

$$|V_{cb}| = (41.8 \pm 2.0)\cdot 10^{-3}\,.$$

# LQCD might enter the scene

**On the study of inclusive semileptonic decays of $B_s$-meson from lattice QCD** https://arxiv.org/abs/2311.09892

P. Gambino[1], S. Hashimoto[2], S. Mächler[1][3], M. Panero[1], F. Sanfilippo[4], S. Simula[4], A. Smecca[1] and N. Tantalo[5][*]

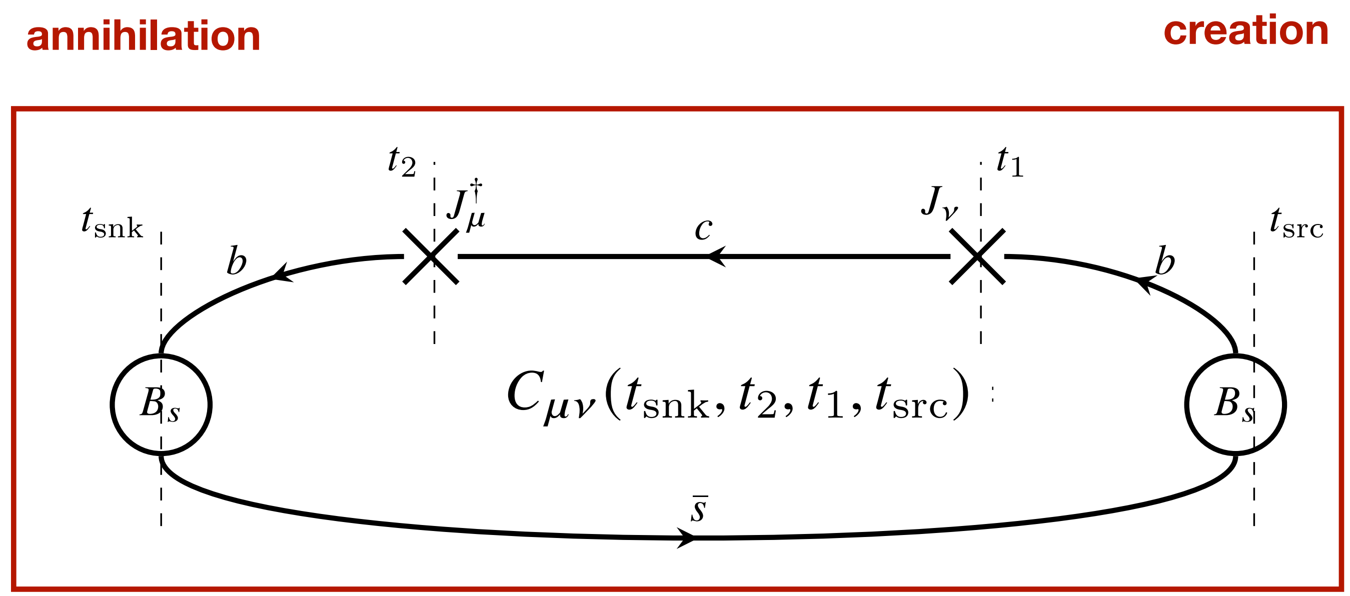
[1] *Dipartimento di Fisica, Università di Torino & INFN, Sezione di Torino - Torino, Italy*
[2] *Theory Center, Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK) - Tsukuba, Japan*
[3] *Physikinstitut, Universität Zürich - Zürich, Switzerland*
[4] *INFN, Sezione di Roma Tre - Rome, Italy*
[5] *Dipartimento di Fisica, Università di Roma "Tor Vergata" & INFN, Sezione di Roma "Tor Vergata" - Rome, Italy*

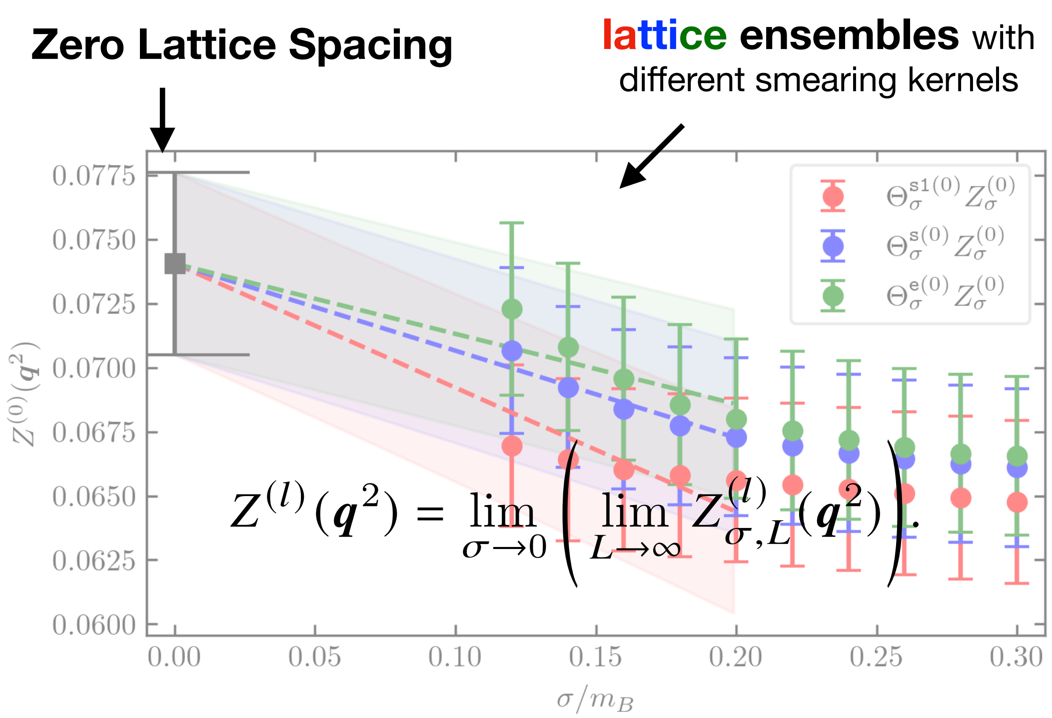
Impressive progress **understanding inclusive decays** in the framework of Lattice QCD (LQCD)

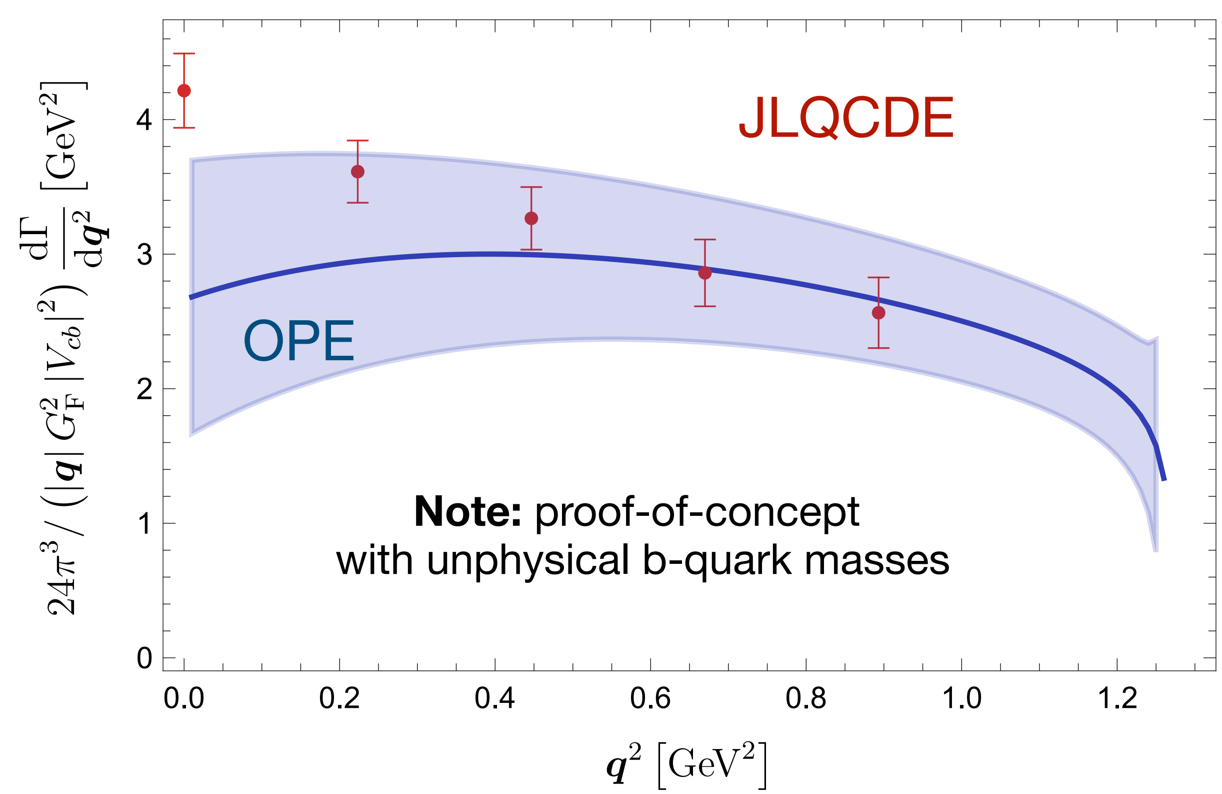
E.g. study of $B_s \to X_{cs}\ell\bar{\nu}_\ell$

$$\frac{d\Gamma}{d\boldsymbol{q}^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3|\boldsymbol{q}|}\sum_{l=0}^{2}\left(\sqrt{\boldsymbol{q}^2}\right)^{2-l} Z^{(l)}(\boldsymbol{q}^2),$$

**Structure functions**, which can be probed with lattice correlation functions



# More interesting developments! But time is running out :-(

QED effects **important** to **push precision** ;

Need more experimental results w/o FSR corrections

**QED effects in inclusive semi-leptonic $B$ decays**

Dante Bigi, Marzia Bordone,[a] Paolo Gambino,[b,c,d] Ulrich Haisch[c] and Andrea Piccione[e]

https://arxiv.org/abs/2309.02849

**Full $1/m_b^5$ !**

**Inclusive Semileptonic $b \to c\ell\bar{\nu}$ Decays to Order $1/m_b^5$**

https://arxiv.org/pdf/2311.12002.pdf

Thomas Mannel, Ilija S. Milutin

Theoretische Physik 1, Center for Particle Physics Siegen
Universität Siegen, D-57068 Siegen, Germany

**Full $\alpha_s^2$ !**

**NNLO QCD corrections to the $q^2$ spectrum of inclusive semileptonic $B$-meson decays**

https://arxiv.org/pdf/2403.03976.pdf

Matteo Fael[a] and Florian Herren[b]

[a] *Theoretical Physics Department, CERN, 1211 Geneva, Switzerland*

[b] *Physics Institute, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*


# **Discussion** items

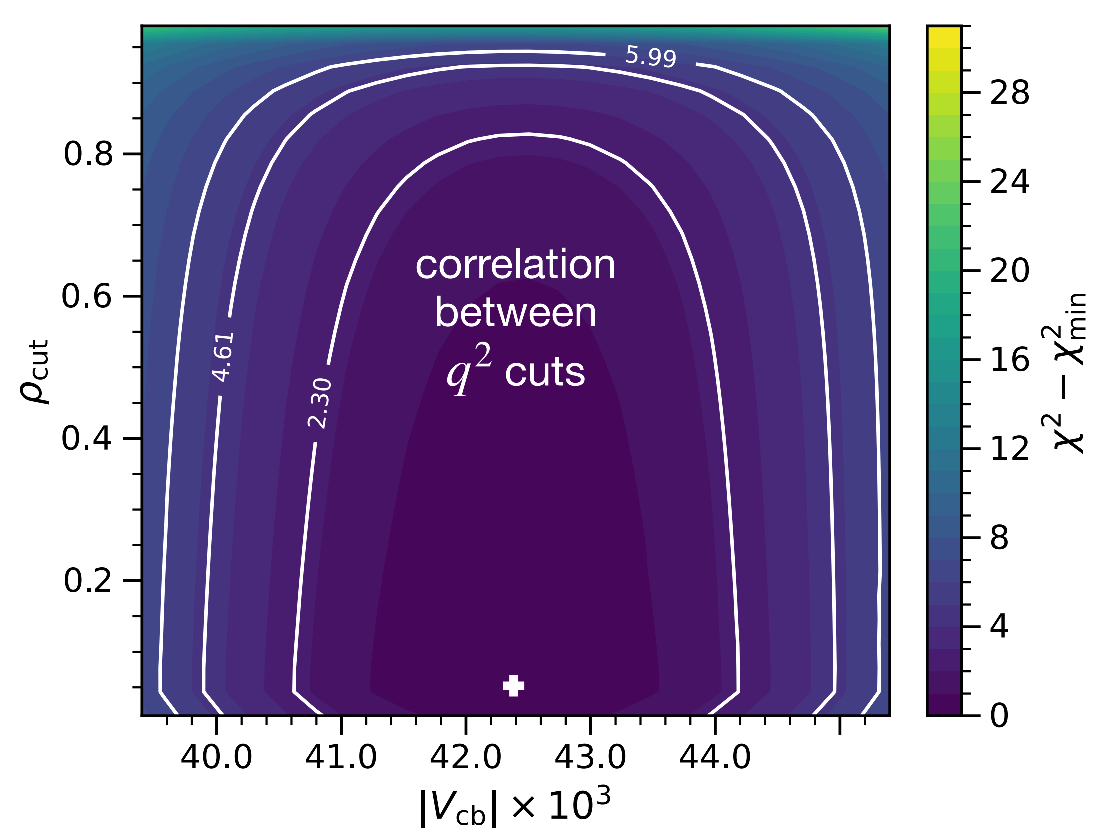
# Experimental and Theory Errors

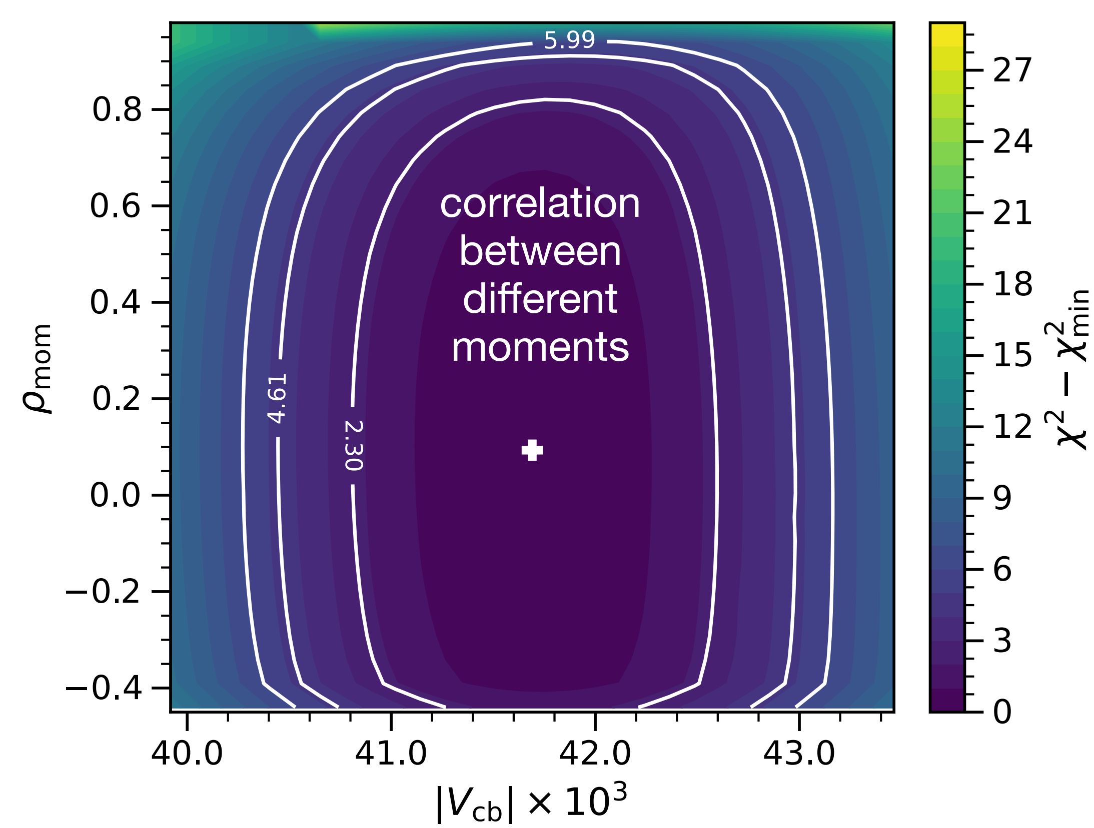
In our $V_{cb}$ fits, we currently do **not** include **experimental correlation** between moments of **different** types (e.g. $E_\ell : M_X$)

single analysis that extracts all moments simultaneously

Data is really precise and systematically limited — also **no theory correlations** between different moments

Theory correlations long-standing discussion item ; **HQE parameters depend** on them to some extend, but $V_{cb}$ only has an underlying dependence



# Oh my Darwin: SU(3) and Lifetimes

- Scenario A: large value of $\langle \mathcal{O}_6 \rangle_{B_d}$ from Ref. [239] and large $SU(3)_F$ breaking; ⟶ $E_\ell : M_X$
- Scenario B: small value of $\langle \mathcal{O}_6 \rangle_{B_d}$ from Ref. [240] and small $SU(3)_F$ breaking. ⟶ $q^2$

$$\frac{\tau(B_s)}{\tau(B_d)} = \begin{cases} 1.028 \pm 0.011 & \text{Scenario A} \\ 1.003 \pm 0.006 & \text{Scenario B} \end{cases} .$$