

# Charm Results at Belle and Belle II

On Behalf of the Belle and Belle II collaborations



# Experiments

- Asymmetric  $e^+e^-$  colliders near  $\Upsilon(4S)$  resonance

- Belle @ KEKB (1999-2010)

$$\mathcal{L}_{peak} = 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}, \mathcal{L}_{int} = 1 \text{ ab}^{-1}$$

- Belle II @ SuperKEKB (2019-current)

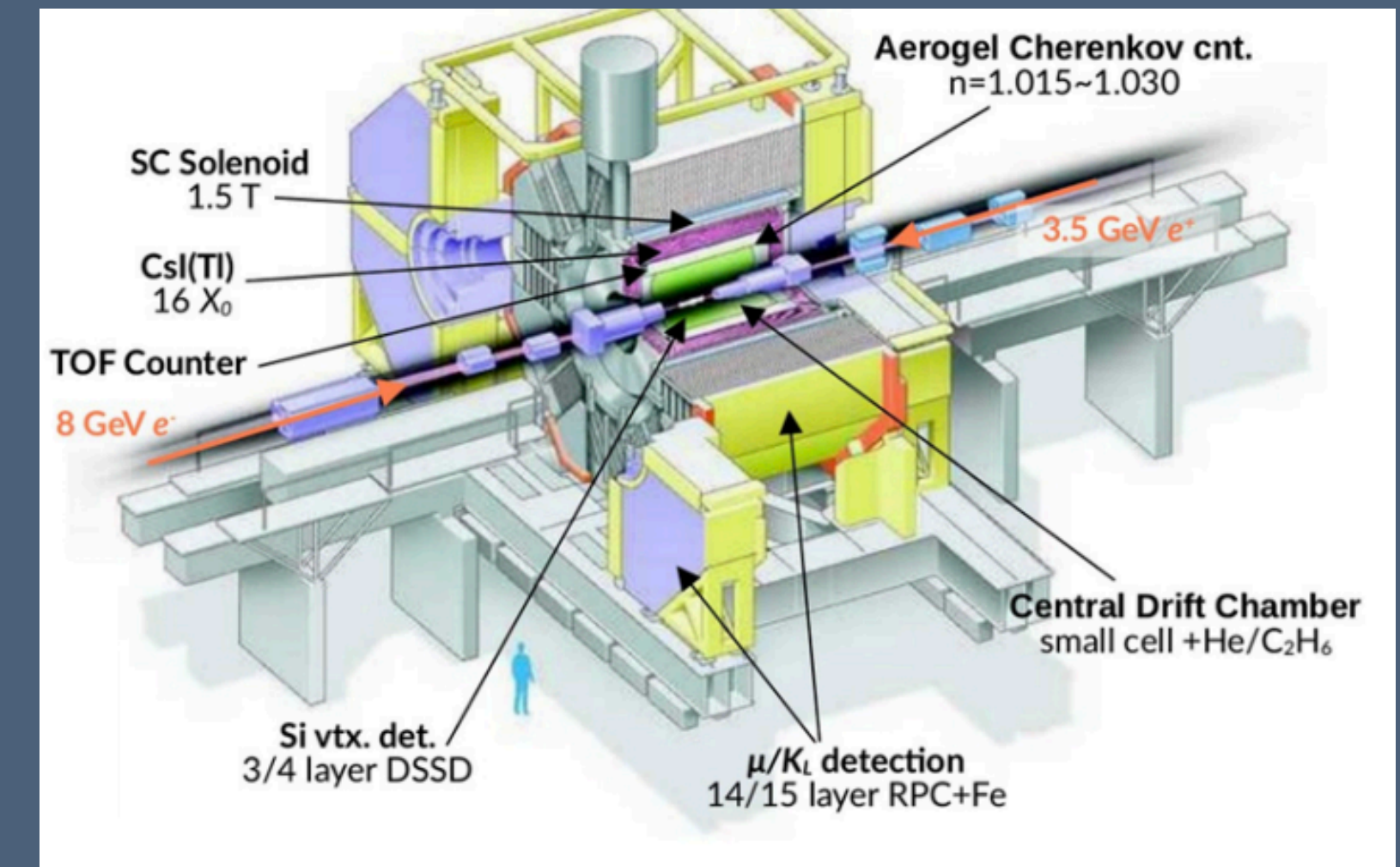
$$\mathcal{L}_{peak} = 4.7 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}, \mathcal{L}_{int} = 0.42 \text{ ab}^{-1}$$

- “Synergic” experiments

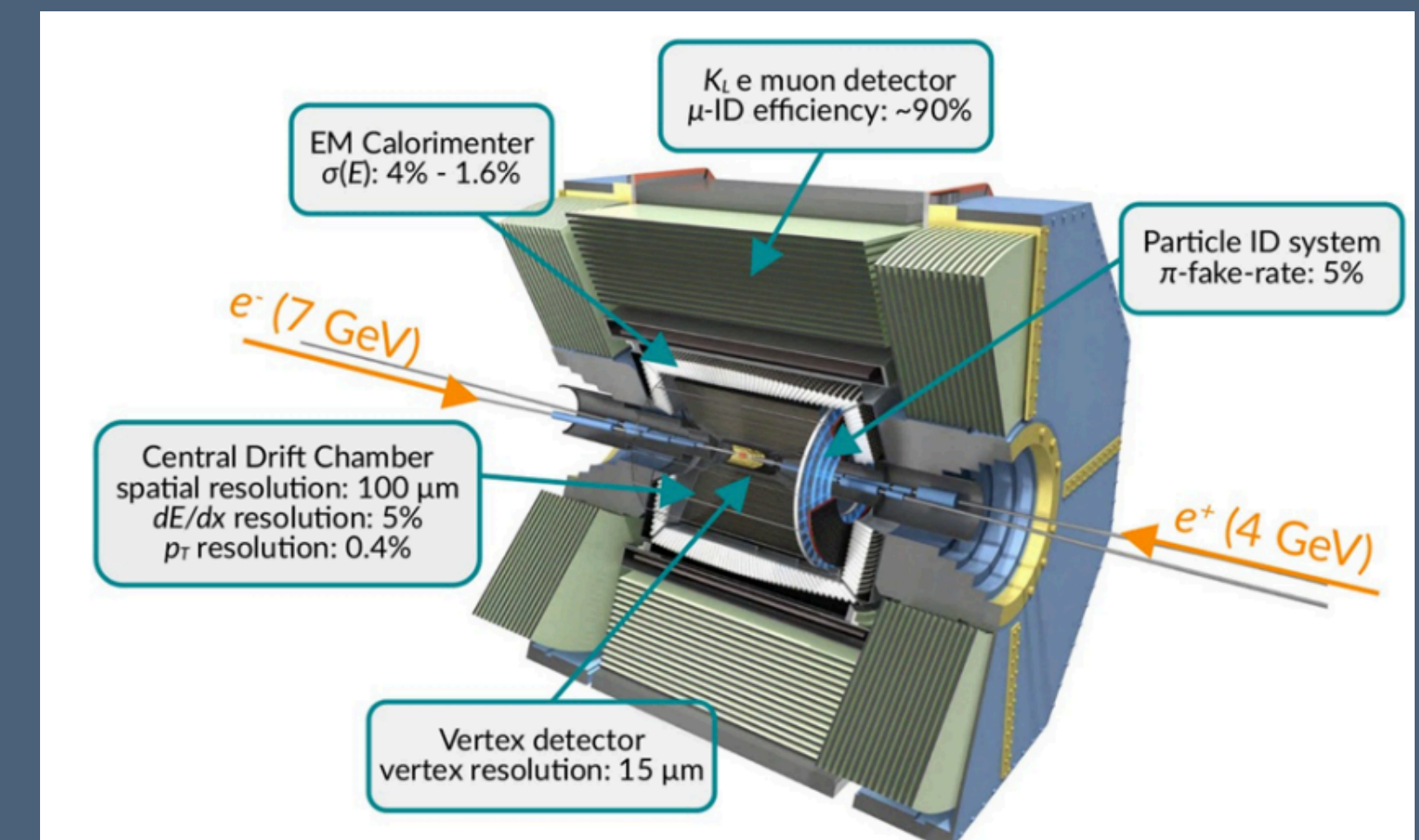
- Belle data can be analyzed with the **Belle II Analysis Software Framework** (basf2)

- Analyses can be performed with a combination of Belle and Belle II data!

Belle @ KEKB



Belle II @ SuperKEKB



# Charm Production

- Two primary mechanisms for charm production at Belle/Belle II:

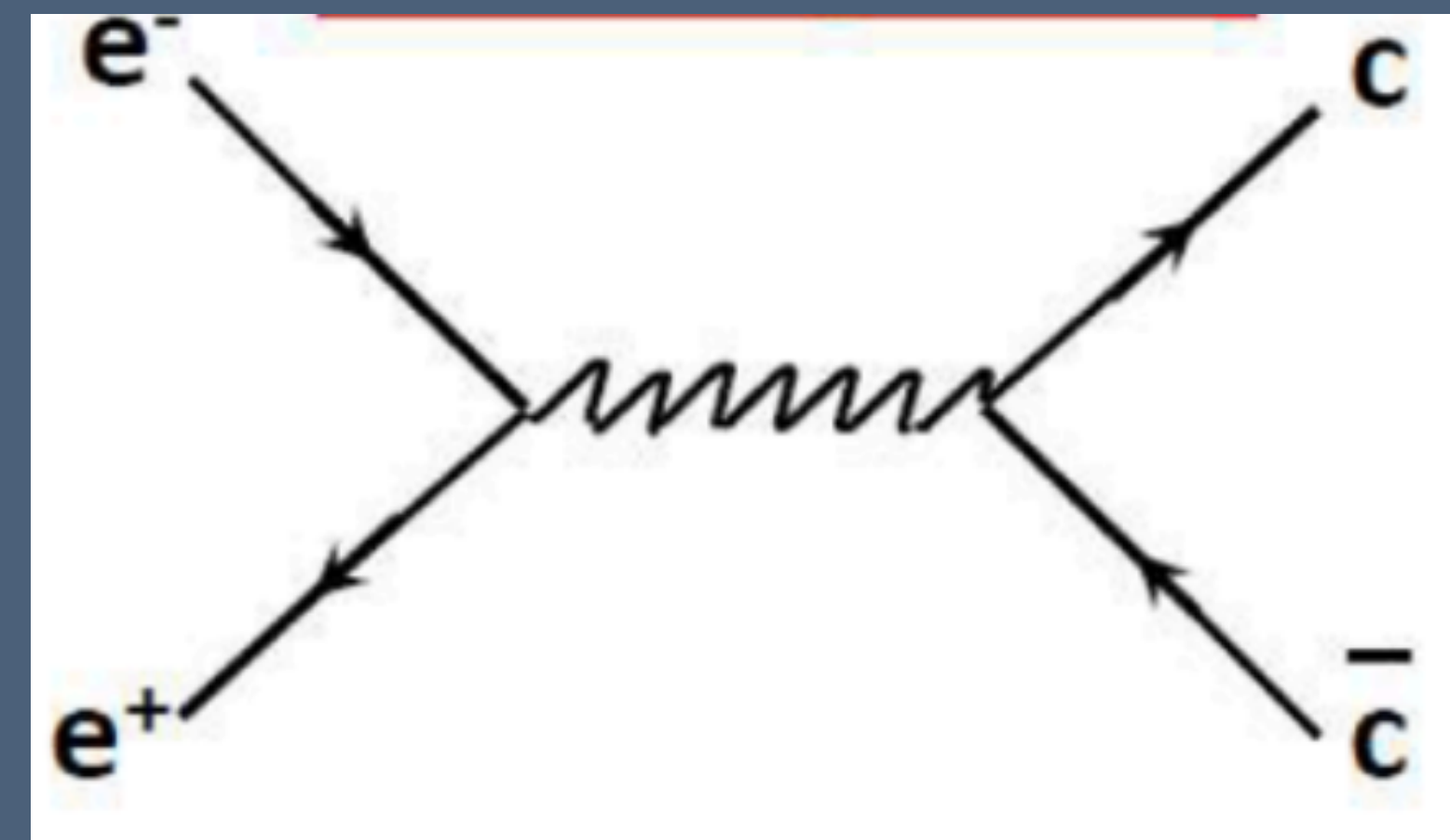
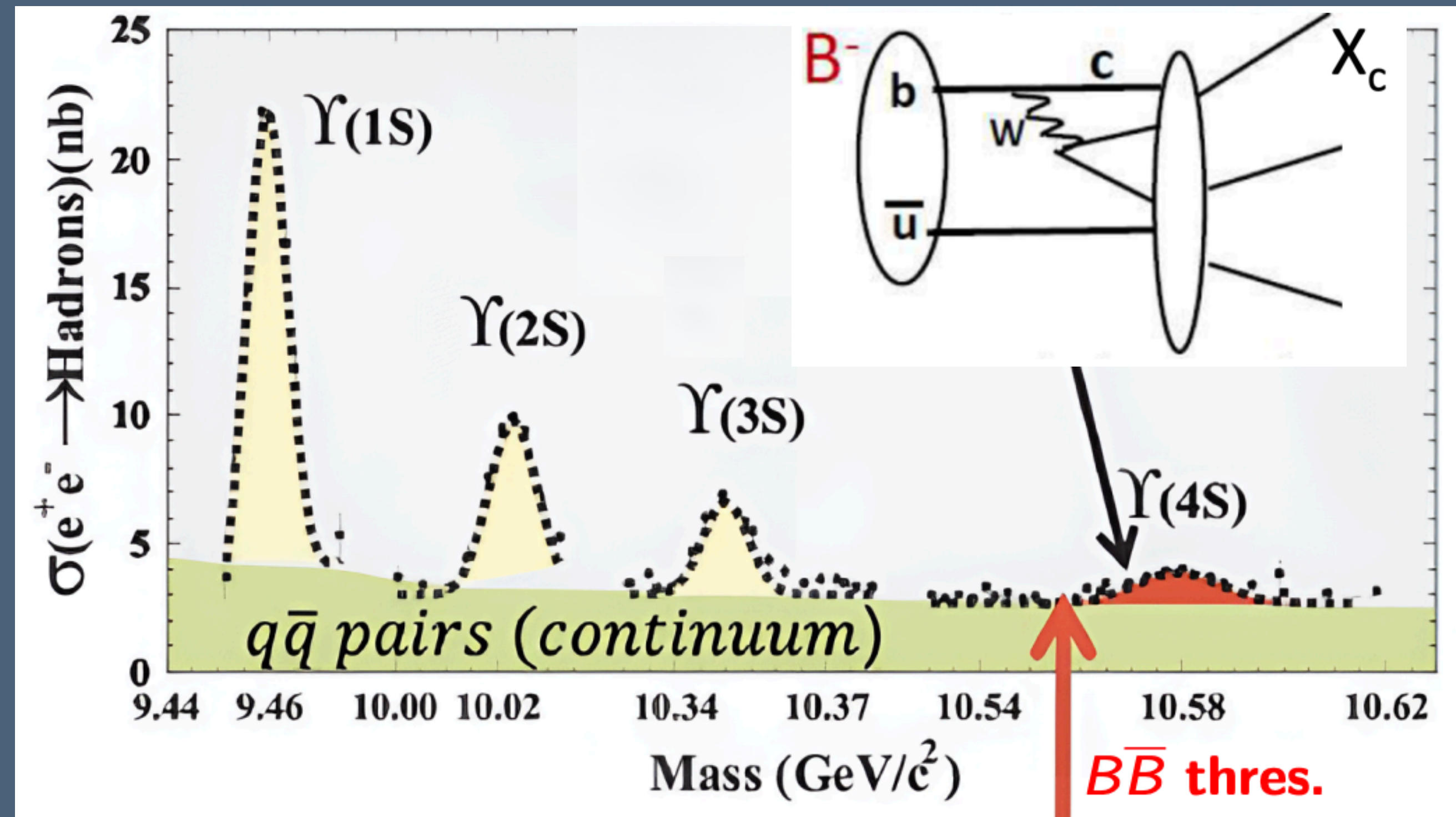
- $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B} \rightarrow X_c$

Precise  $B\bar{B}$  cross section allows for absolute measurements

- $e^+e^- \rightarrow c\bar{c} \rightarrow X_c$

Absolute measurements not possible without reference

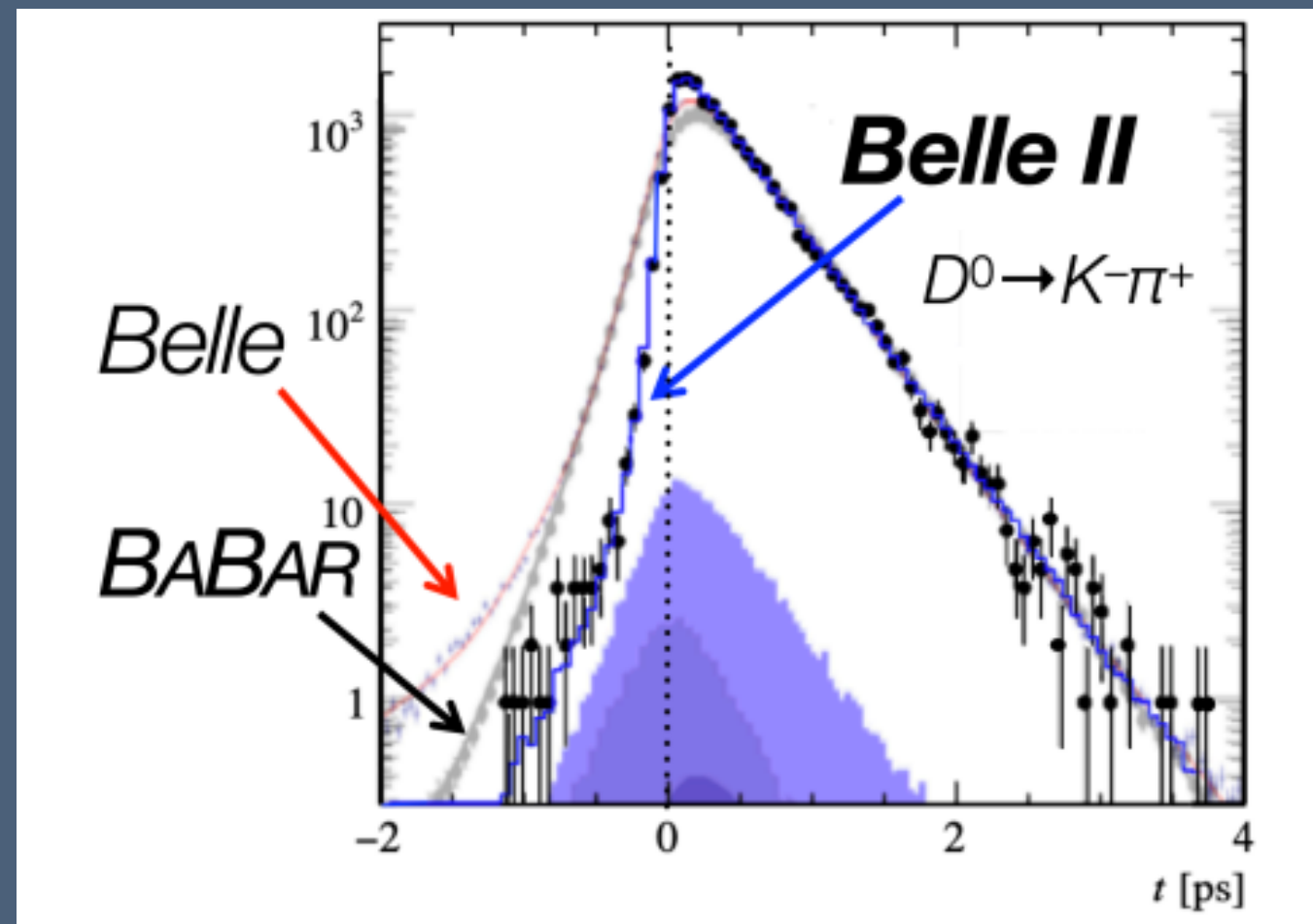
- Used for most analyses due to its **simplicity** compared to  $B\bar{B}$  processes



# Charm Lifetimes

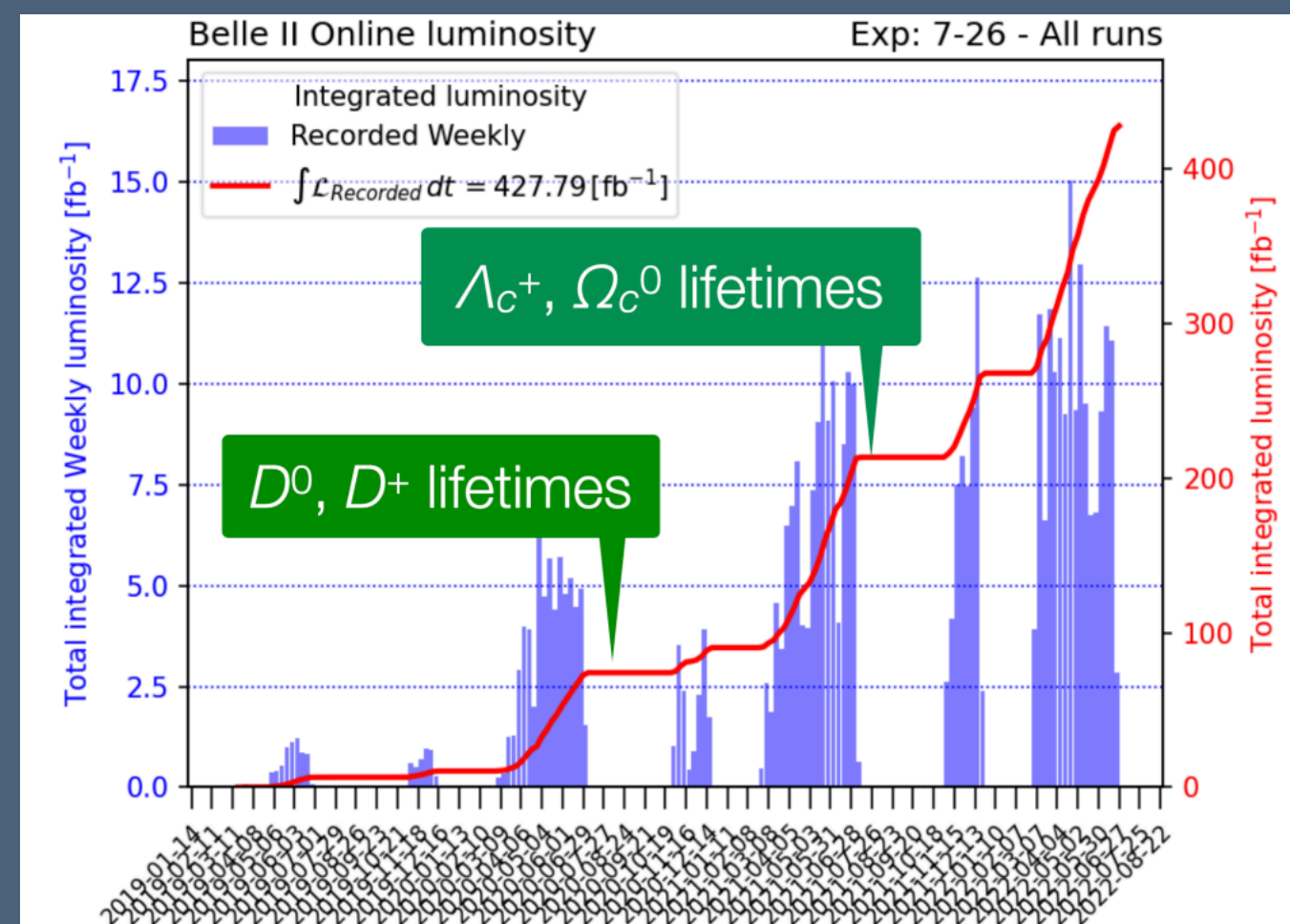
# Charm Lifetimes

## Strengthen existing theory



- Theoretically difficult to calculate due to non-perturbative effects from QCD
- Can improve theoretical understanding of QCD and provide stringent tests of the Heavy Quark Expansion (used to predict decay-widths of heavy hadrons):

$$\Gamma(H_Q \rightarrow X) = \Gamma_3 + \Gamma_5 \frac{\langle \tilde{\mathcal{O}}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \dots + 16\pi^2 \left( \tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \Gamma_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \dots \right)$$



- Belle II has precise vertexing and decay-time resolution, allowing for precise lifetime measurements
- Early dataset alone has produced **four world-leading** charm lifetime measurements and one **strong confirmation** of an LHCb result

# Charm Lifetimes

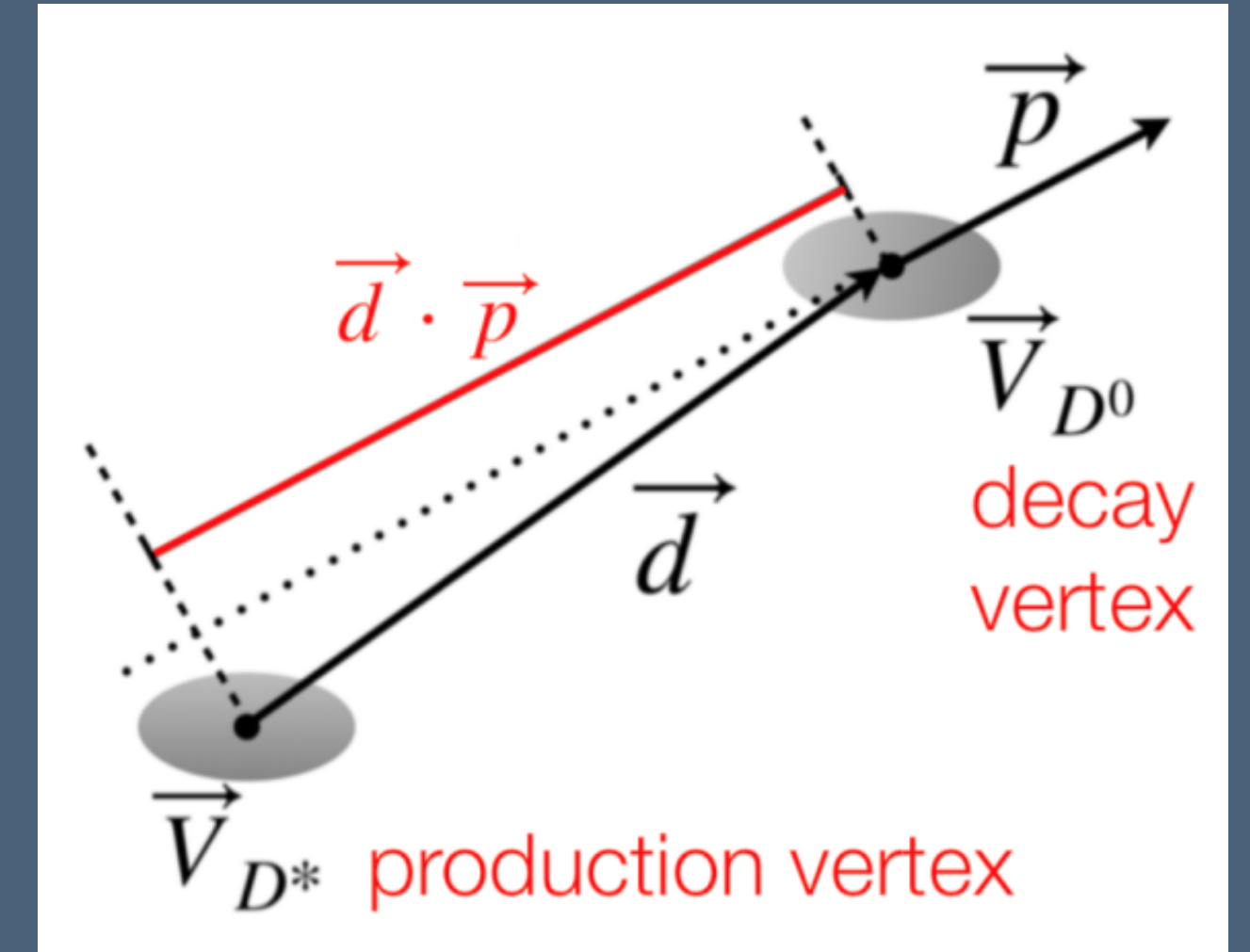
## Strengthen existing theory

- Obtained from unbinned maximum-likelihood fits to the decay-time  $t$  and the decay-time uncertainty  $\sigma_t$

$$pdf(t, \sigma_t | \tau, f, b, s_1, s_2) = pdf(t | \sigma_t, \tau, f, b, s_1, s_2) pdf(\sigma_t)$$

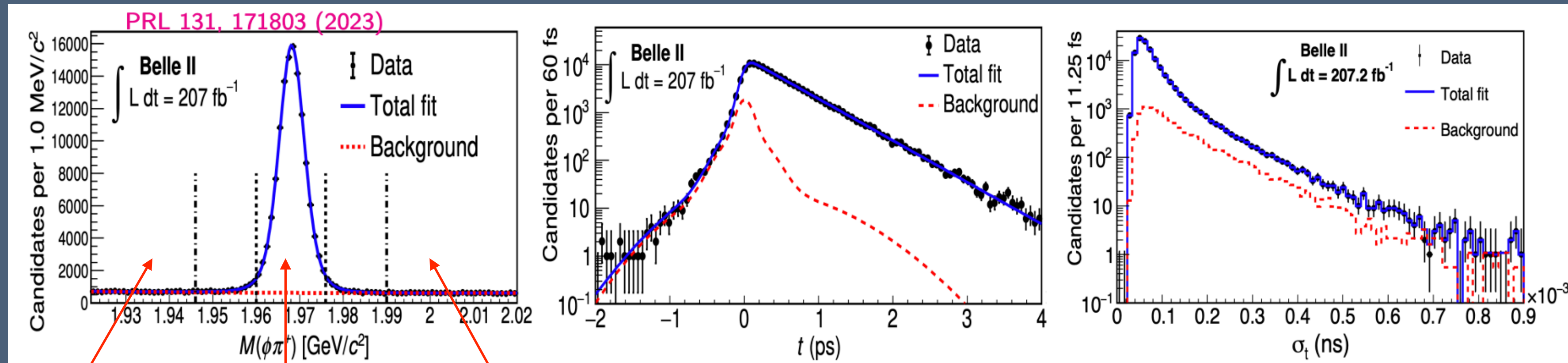
$$\propto \int_0^\infty e^{-t_{true}/\tau} R(t - t_{true} | \sigma_t, f, b, s_1, s_2) dt_{true} pdf(\sigma_t),$$

Signal pdf for  $\Lambda_c^+$  lifetime measurement



$$t = \frac{m_D}{p} \left( \vec{d} \cdot \hat{p} \right)$$

$D_s^+$

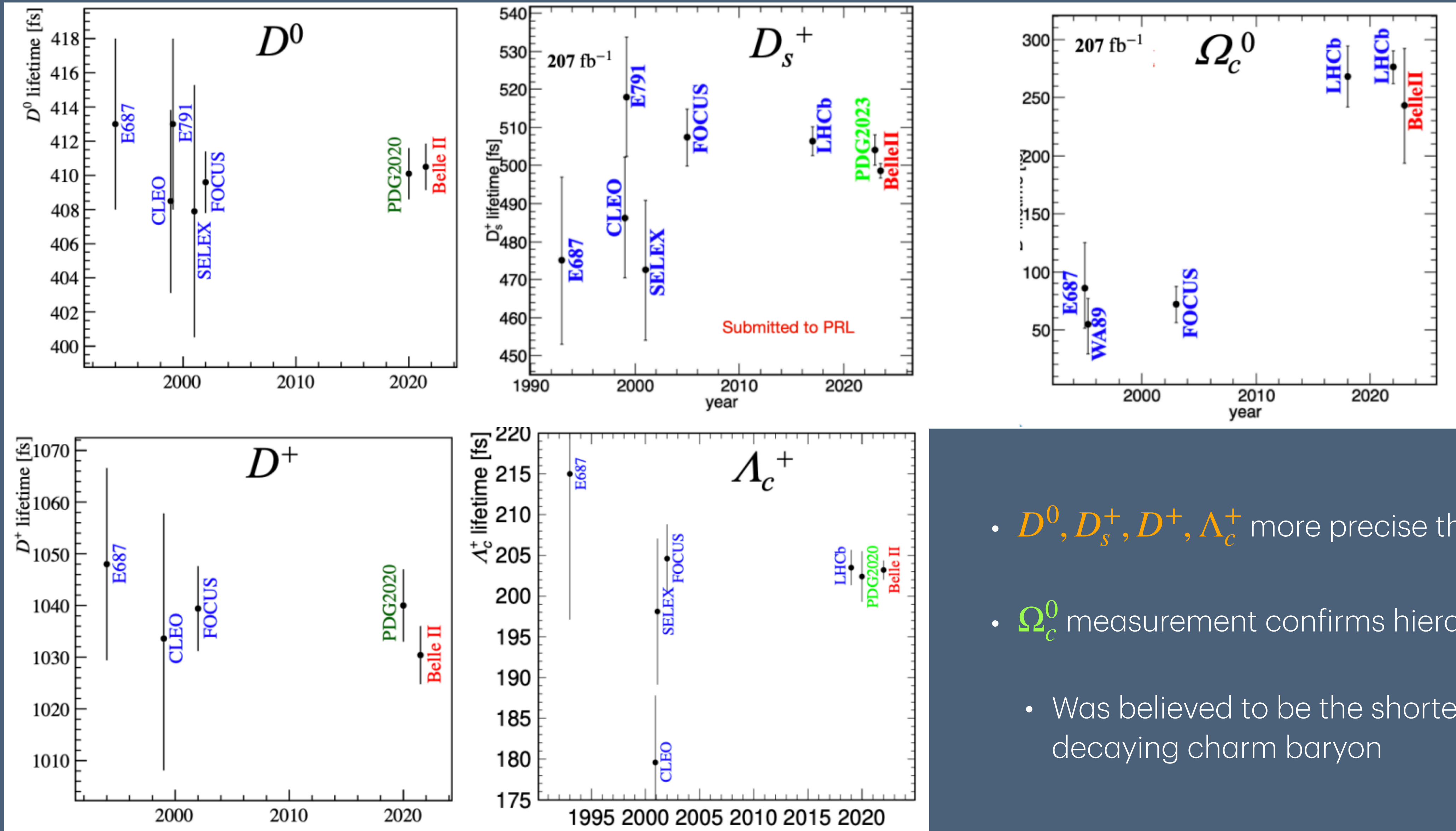


Lower sideband

Signal region

Upper sideband

# Charm Lifetime Results at Belle II



- $D^0, D_s^+, D^+, \Lambda_c^+$  more precise than world average
- $\Omega_c^0$  measurement confirms hierarchy adjustment
- Was believed to be the shortest lived weakly decaying charm baryon

# Branching Fractions



# Branching Fraction of Charm Mesons

- Cabbibo-suppressed (CS) decays provide a **strong probe for NP and CP Violation**

Belle (2023)

stat syst norm

$$\mathcal{B}(D^+ \rightarrow K^+ K^- \pi^+ \pi^0) = (7.08 \pm 0.08 \pm 0.16 \pm 0.2) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \rightarrow K^+ \pi^- \pi^+ \pi^0) = (9.44 \pm 0.34 \pm 0.28 \pm 0.32) \times 10^{-3}$$

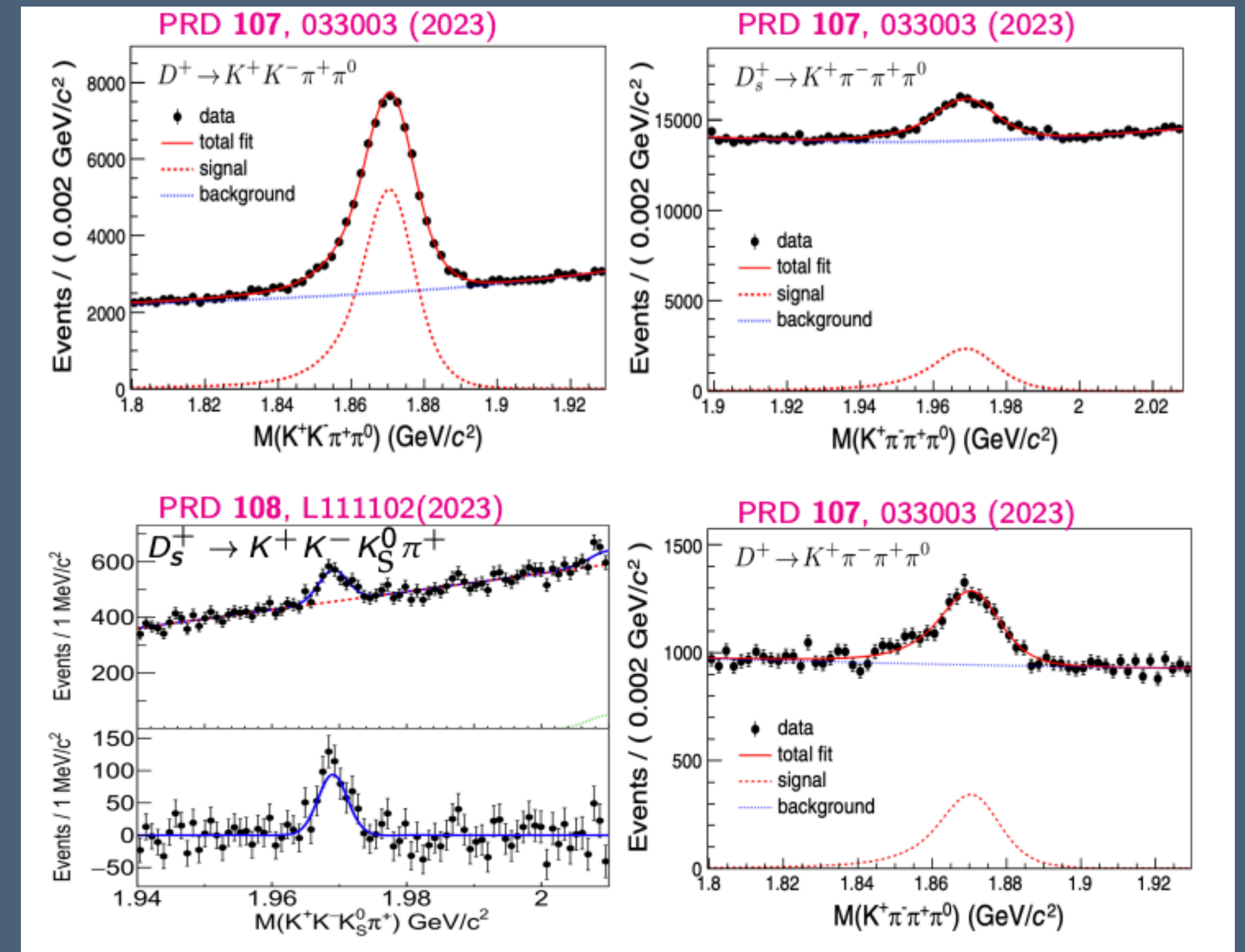
$$\mathcal{B}(D^+ \rightarrow K^+ \pi^- \pi^+ \pi^0) = (1.05 \pm 0.07 \pm 0.02 \pm 0.03) \times 10^{-3}$$

$$\mathcal{B}(D^0 \rightarrow K_s^0 K_s^0 \pi^+ \pi^-) = (4.79 \pm 0.08 \pm 0.10 \pm 0.31) \times 10^{-4}$$

Consistent with prior results but with greater precision

$$\mathcal{B}(D_s^+ \rightarrow K^+ K^- K_s^0 \pi^+) = (1.29 \pm 0.14 \pm 0.04 \pm 0.11) \times 10^{-4}$$

First measurement;  $9.2\sigma$  signal significance



Phys. Rev. D 107, 052001

# Branching Fraction of Charm Baryons

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta) = (3.14 \pm 0.35 \pm 0.17 \pm 0.25) \times 10^{-3}$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta') = (4.16 \pm 0.75 \pm 0.17 \pm 0.25) \times 10^{-3}$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda K^+) = (6.57 \pm 0.17 \pm 0.11 \pm 0.35) \times 10^{-4}$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 K^+) = (3.58 \pm 0.19 \pm 0.06 \pm 0.19) \times 10^{-4}$$

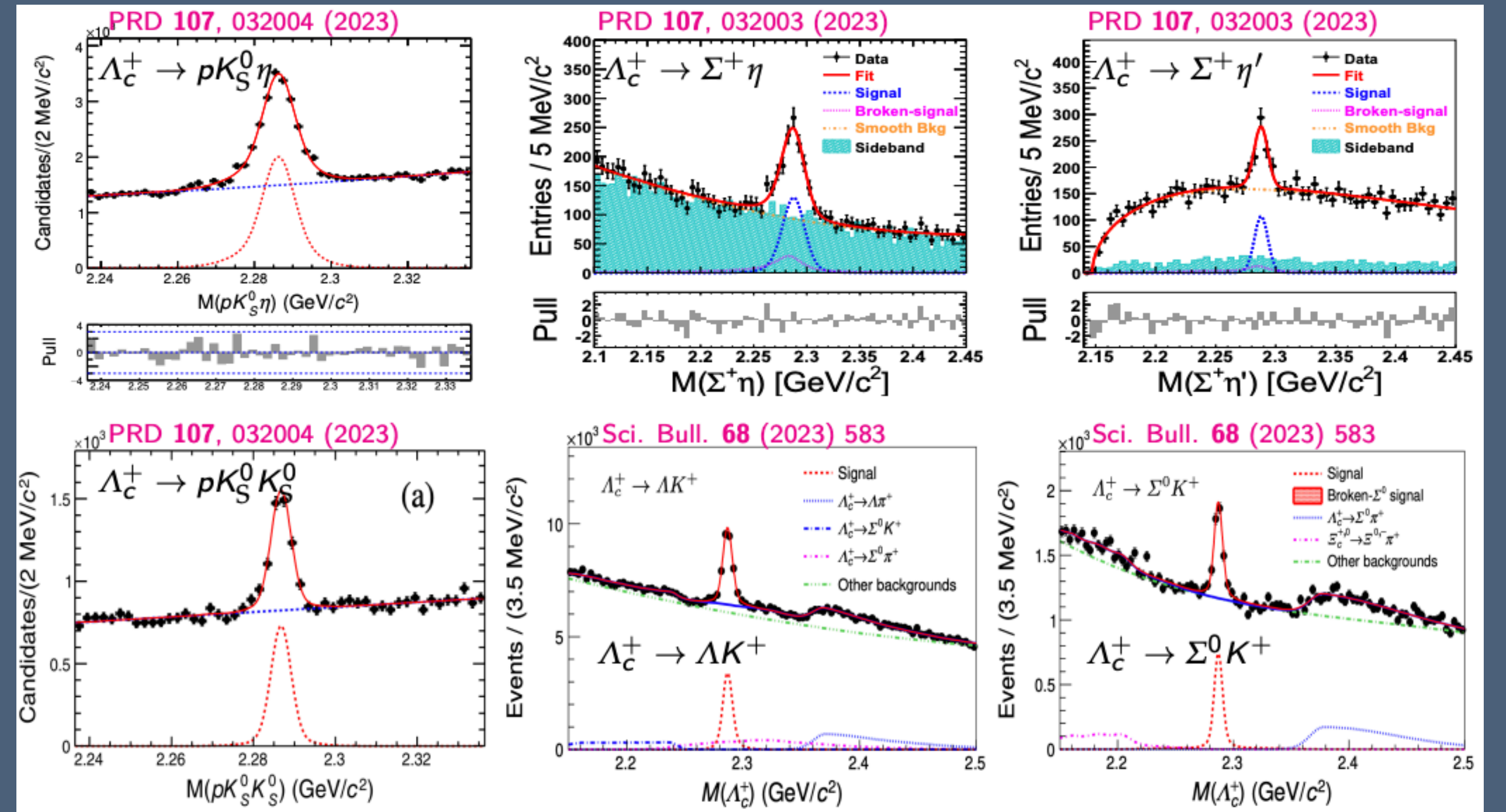
Agrees with prior results within  $2\sigma$ , but with best precision

$$\mathcal{B}(\Lambda_c^+ \rightarrow p K_s^0 K_s^0) = (2.35 \pm 0.12 \pm 0.07 \pm 0.12) \times 10^{-4}$$

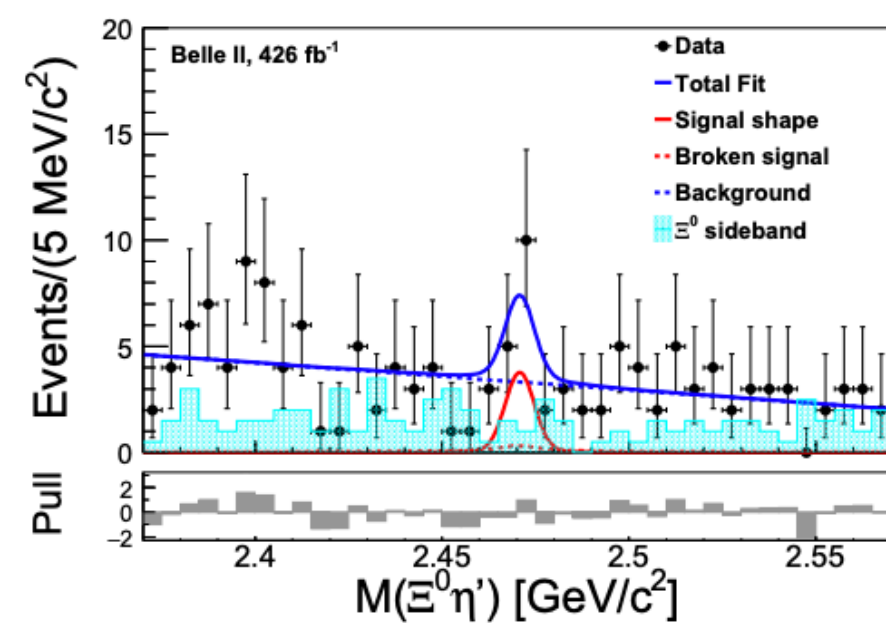
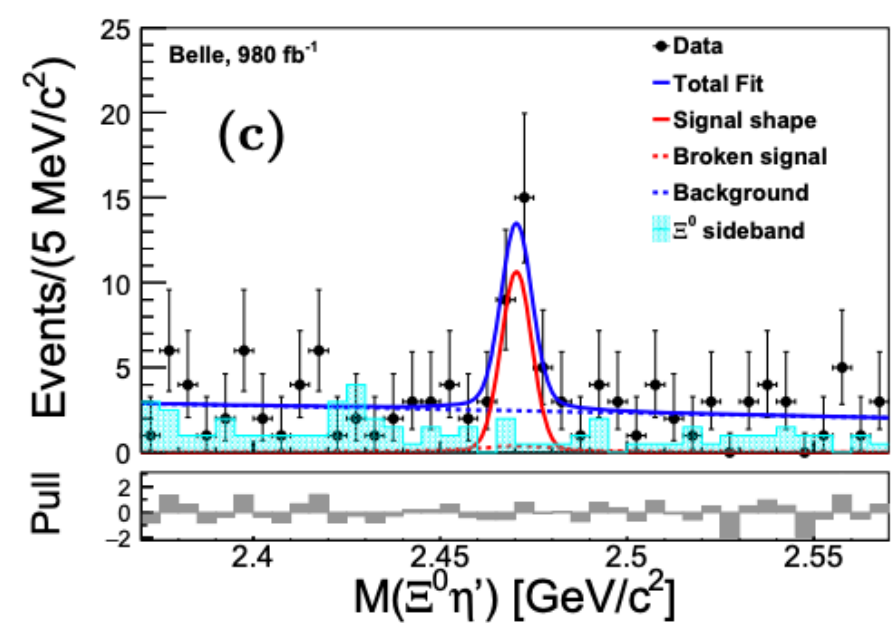
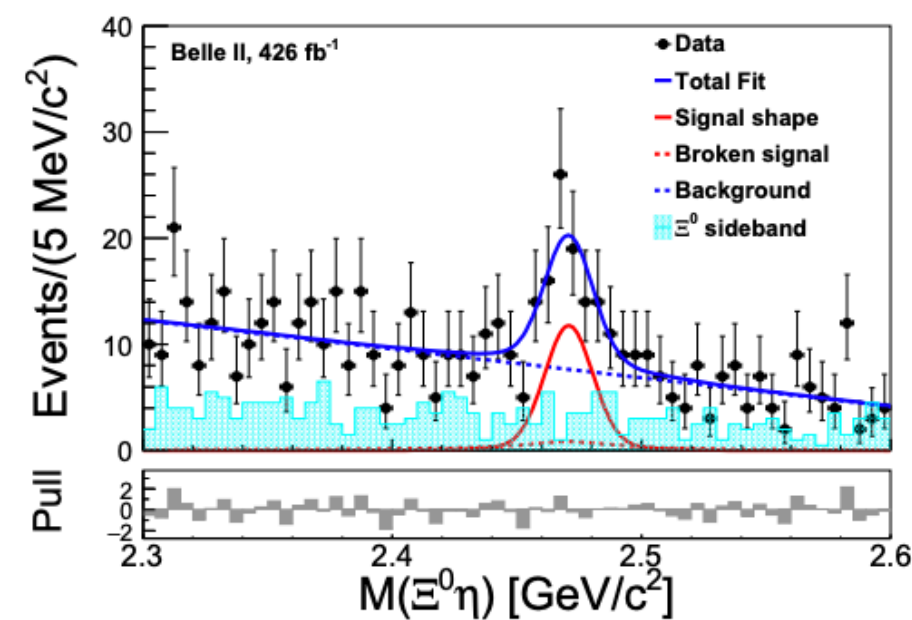
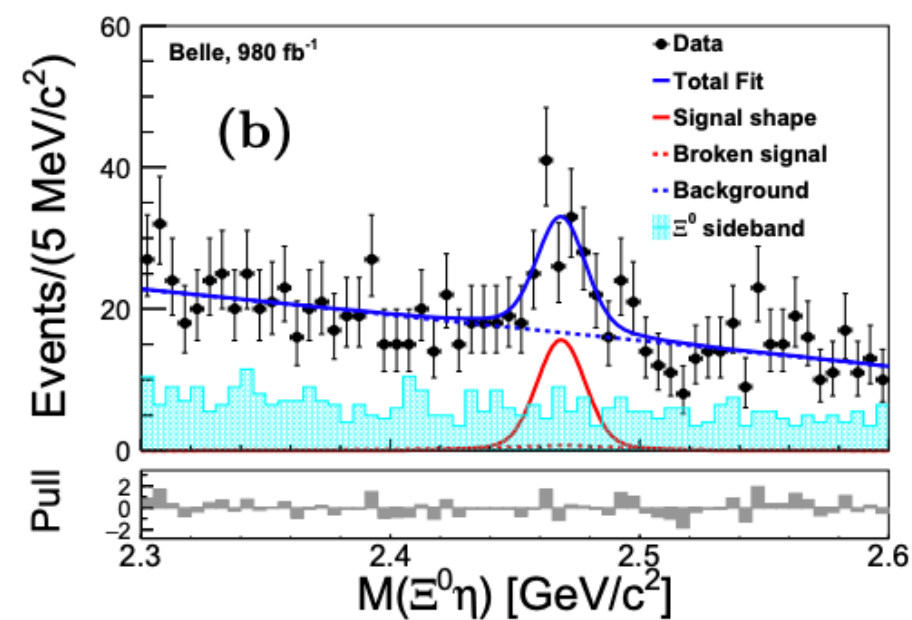
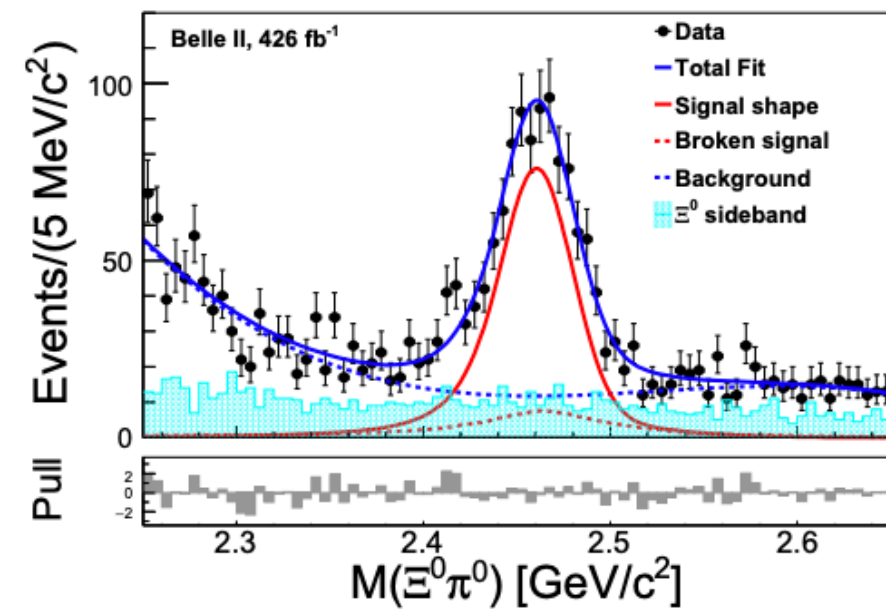
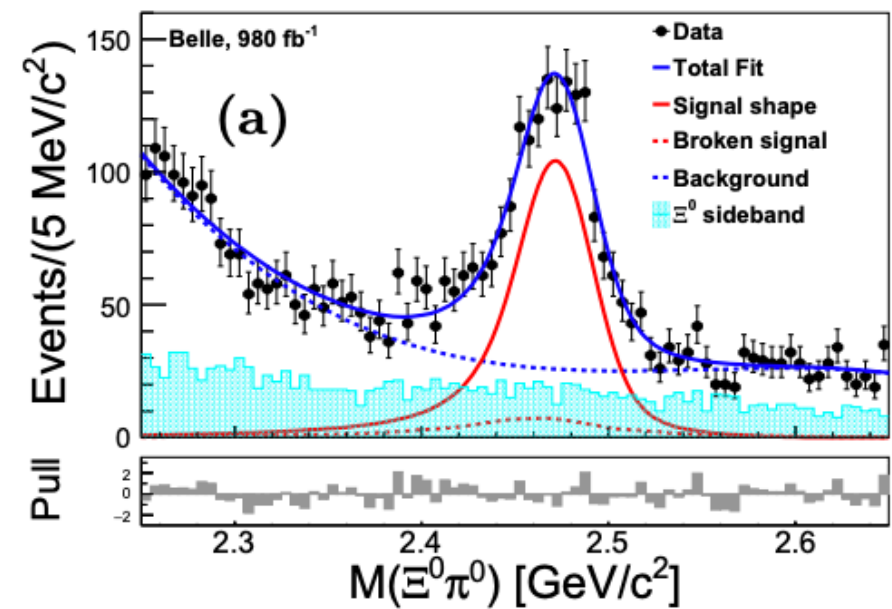
Agrees with prior results; threefold improvement in precision

$$\mathcal{B}(\Lambda_c^+ \rightarrow p K_s^0 \eta) = (4.35 \pm 0.10 \pm 0.20 \pm 0.22) \times 10^{-3}$$

First measurement;  $> 10\sigma$  statistical significance



# Branching Fraction of Charm Baryons



$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0) = (6.9 \pm 0.3 \pm 0.5 \pm 1.5) \times 10^{-3}$$

Belle 2024

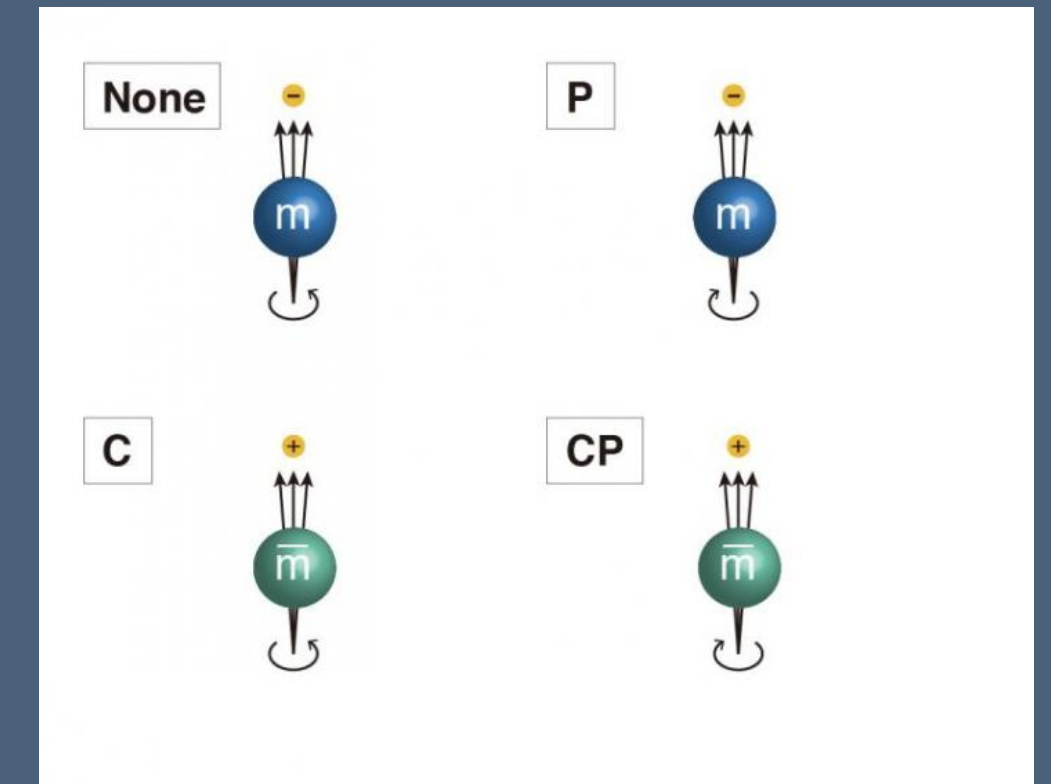
$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta) = (1.6 \pm 0.2 \pm 0.2 \pm 0.4) \times 10^{-3}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta') = (1.2 \pm 0.3 \pm 0.1 \pm 0.3) \times 10^{-3}$$

First synergic measurement; rules out various models and favors  $SU(3)_F$  – breaking models

# Search for CPV in the Charm Sector

# Introduction to CP Violation



- SM → Violated via complex phase in **Cabbibo-Kobayashi-Maskawa (CKM)** matrix

- Strength characterized by the **Jarlskog invariant:**

$$\mathcal{J} = \text{Im}[V_{us}V_{cb}V_{ub}^*V_{cs}^*] = A^2\lambda^6\eta(1 - \lambda^2/2) + \mathcal{O}(\lambda^{10}) \approx 10^{-5}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM matrix

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(4)$$

Wolfenstein Parameterization to  $\mathcal{O}(4)$

- Insufficient to produce large-scale matter-antimatter asymmetry
- CPV observed in all meson flavor sectors, but not baryon sector
- Charm baryons → sensitive probe for NP

# CPV in the Charm Sector

- $\Delta A_{CP}$  first observed between  $D^0 \rightarrow \pi^+\pi^-$  and  $D^0 \rightarrow K^+K^-$  decays (LHCb 2019, arXiv:1903.08726)
  - Combining this with time-integrated CP asymmetry  $\mathcal{A}^{CP}(K^-K^+)$  yields
    - $a_{CP}^{dir}(K^-K^+) = (7.7 \pm 5.7) \times 10^{-4}$   
(LHCb 2022, arXiv:2209.03179)
    - $a_{CP}^{dir}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$
  - Effect due to charm hadrons is  $\approx \mathcal{O}(10^{-3})$  or less (PRD 86, 036012; PRD 104, 073003)
  - Searches for other sources of CPV in the charm sector are ongoing via **complementary** observables
    1. T-odd asymmetry ( $a_{CP}^{T-odd}$ ) measurements
    2. Asymmetry ( $A_{CP}$ ) measurements



# T-odd asymmetries in four-body decays

- Define a **T-odd observable**  $C_T = \vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$  where 1,2,3 correspond to three of the four final state particles in a four-body decay
- $C_T$  should be symmetric about zero; otherwise indicates T violation (TV)
- Quantify asymmetry via

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}$$

$$\bar{A}_T = \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)}$$

*Implies CPV via CPT Theorem!*



Can be nonzero due to T violation **or** strong phases — take difference to remove phase effects

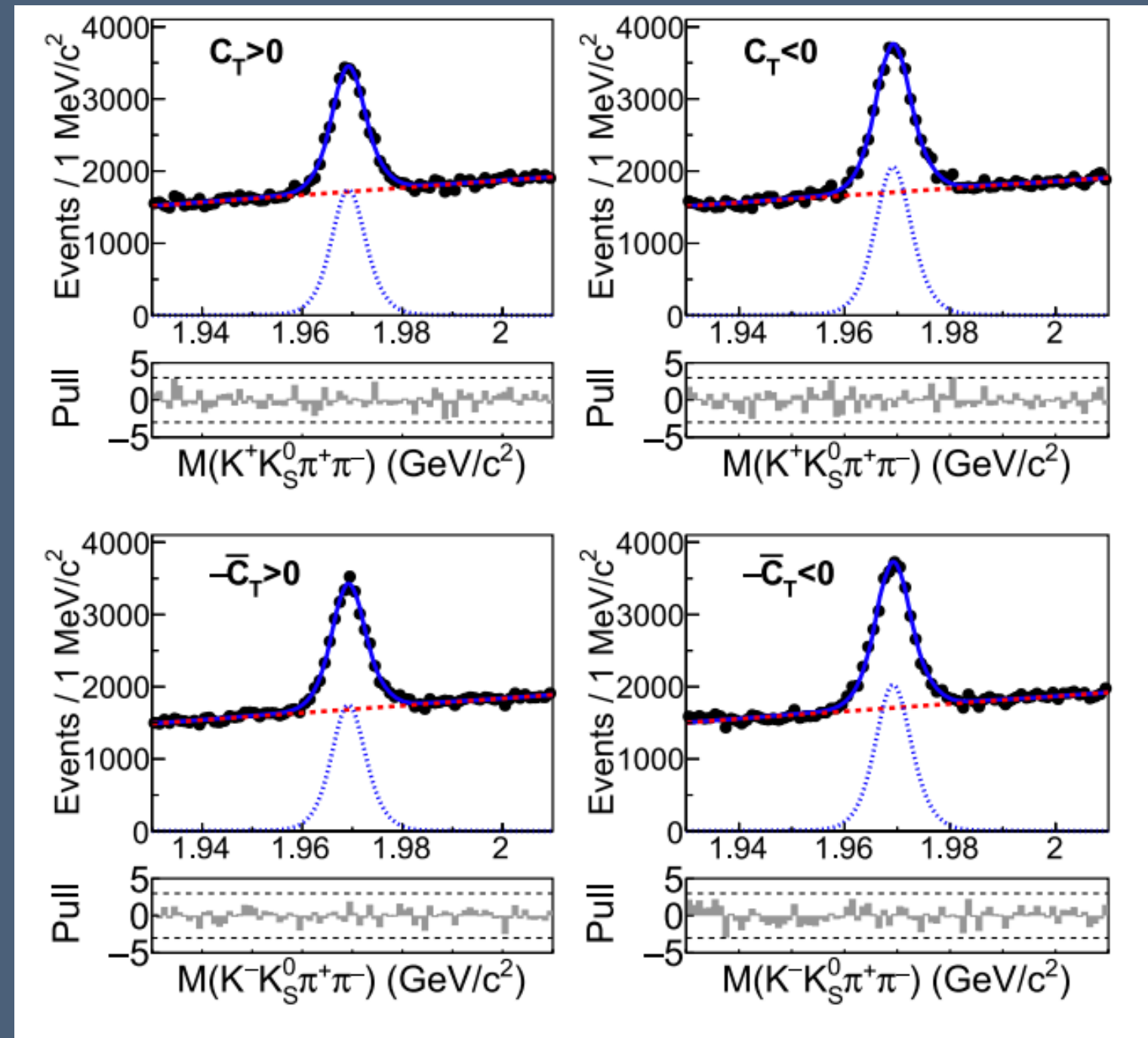
$$a_{CP}^{T-odd} = \frac{1}{2}(A_T - \bar{A}_T)$$

# T-odd asymmetries in $D_{(s)}^+ \rightarrow K^+ K_S^0 h^+ h^-$

-World leading precision

-Dominated by statistical uncertainty

-No direct CPV evidence



Mode	$A_T$ (%)	$a_{CP}^{T\text{-odd}}$ (%)
$D^+ \rightarrow K^+ K_S^0 \pi^+ \pi^-$	$(3.67 \pm 1.23)$	$(0.34 \pm 0.87)$
$D_s^+ \rightarrow K^+ K_S^0 \pi^+ \pi^-$	$(-8.31 \pm 8.89)$	$(-0.46 \pm 0.63)$
$D^+ \rightarrow K^+ K^- K_S^0 \pi^+$	$(-1.40 \pm 4.23)$	$(-3.34 \pm 2.66)$

Sys ~ 0.35%

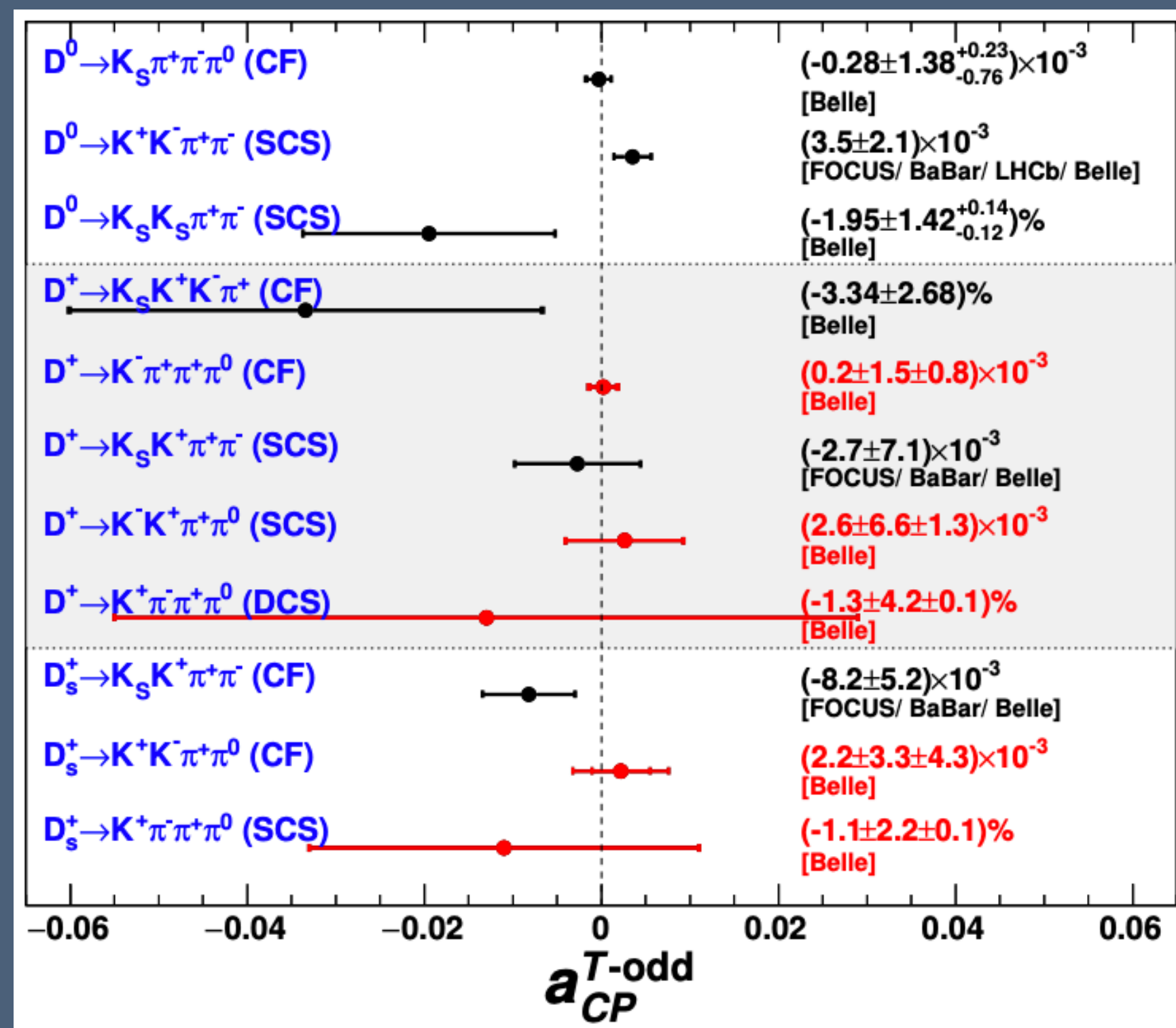


# T-odd asymmetries in $D_{(s)}^+ \rightarrow Kh\pi^+\pi^0$

-First measurements

-Strong precision

-No direct CPV evidence



Decay	$D^+ \rightarrow f$			$D_s^+ \rightarrow f$	
	$K^+ K^- \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^+ K^- \pi^+ \pi^0$
$N_D$	$27284 \pm 254$	$2062 \pm 127$	$438432 \pm 947$	$15197 \pm 484$	$167357 \pm 786$
$N_{\bar{D}}$	$27177 \pm 255$	$2044 \pm 125$	$450667 \pm 961$	$14945 \pm 479$	$167064 \pm 788$
$A_T$ (%)	$+3.63 \pm 0.93$	$-0.4 \pm 6.0$	$-0.76 \pm 0.22$	$+1.4 \pm 3.2$	$+2.96 \pm 0.47$
$a_{CP}^{T-odd}$ (%)	$+0.26 \pm 0.66$	$-1.3 \pm 4.2$	$+0.02 \pm 0.15$	$-1.1 \pm 2.2$	$+0.22 \pm 0.33$

# Direct CPV via Raw Asymmetry Measurements

- The raw asymmetry for  $\Lambda_c^+ \rightarrow \Lambda K^+$  is given by

$$A_{raw}(\Lambda_c^+ \rightarrow \Lambda K^+) \approx A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Lambda K^+) + A_{CP}^{dir}(\Lambda \rightarrow p\pi^-) + A_e^\Lambda + A_e^{K^+} + A_{FB}^{\Lambda_c^+}$$

production forward-backward asymmetry

( $\gamma - Z^0$  interference/higher order QED)

CP Asymmetry

Detection Asymmetry (efficiency differences between charge conjugates)

- $A_e^{K^+}$  contribution is reduced by weighting  $w_{\Lambda_c, \bar{\Lambda}_c} = 1 \mp A_e^{K^+}[\cos\theta, p_T]$

- Use a control mode,  $\Lambda_c^+ \rightarrow \Lambda\pi^+$ , to cancel out terms

- $\Delta A_{raw} = A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Lambda K^+) - A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Lambda\pi^+) = A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Lambda K^+)$  (measuring  $\Delta A_{raw}$  is sufficient!)

# Direct CPV via Raw Asymmetry Measurements

- Measure

$$A_{raw}(\Lambda_c^+ \rightarrow \Lambda K^+) = \frac{N(\Lambda_c^+ \rightarrow \Lambda K^+) - N(\bar{\Lambda}_c^- \rightarrow \bar{\Lambda} K^-)}{N(\Lambda_c^+ \rightarrow \Lambda K^+) + N(\bar{\Lambda}_c^- \rightarrow \bar{\Lambda} K^-)}$$

(similarly for control mode and  $\Lambda_c^+ \rightarrow \Sigma^0 K^+$  with  $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$  control)

Science Bulletin 68 (2023) 583

- **Results from Belle, 2023**

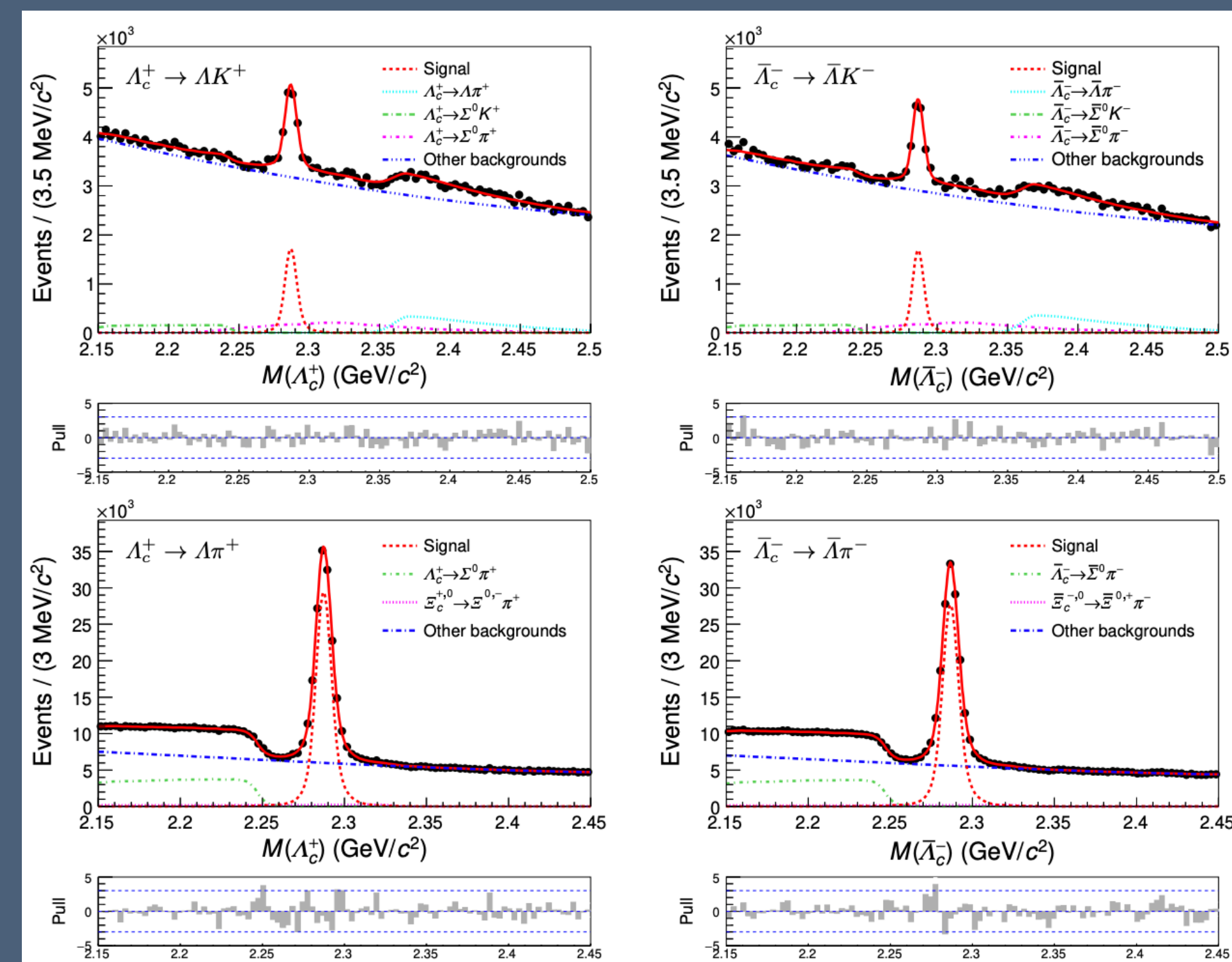
- first measurements of 2-body SCS charm decays, dominated by statistical uncertainty) — no evidence of CPV

- $A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Lambda K^+) = (2.1 \pm 2.6 \pm 0.1) \%$

- $A_{CP}^{dir}(\Lambda_c^+ \rightarrow \Sigma^0 K^+) = (2.5 \pm 5.4 \pm 0.4) \%$

- $A_{CP}^{dir}(D^0 \rightarrow K_s^0 K_s^0 \pi^+ \pi^-) = [-2.51 \pm 1.44(\text{stat})_{-0.10}^{+0.11}(\text{syst})] \%$

- Same paper measured  $a_{CP}^{T-odd} = [-1.95 \pm 1.42(\text{stat})_{-0.12}^{+0.14}(\text{syst})] \%$  Phys. Rev. D 107, 052001



# Exotic Searches

# Search for $\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^-$ at Belle

## Lepton Flavor Universality (LFU)

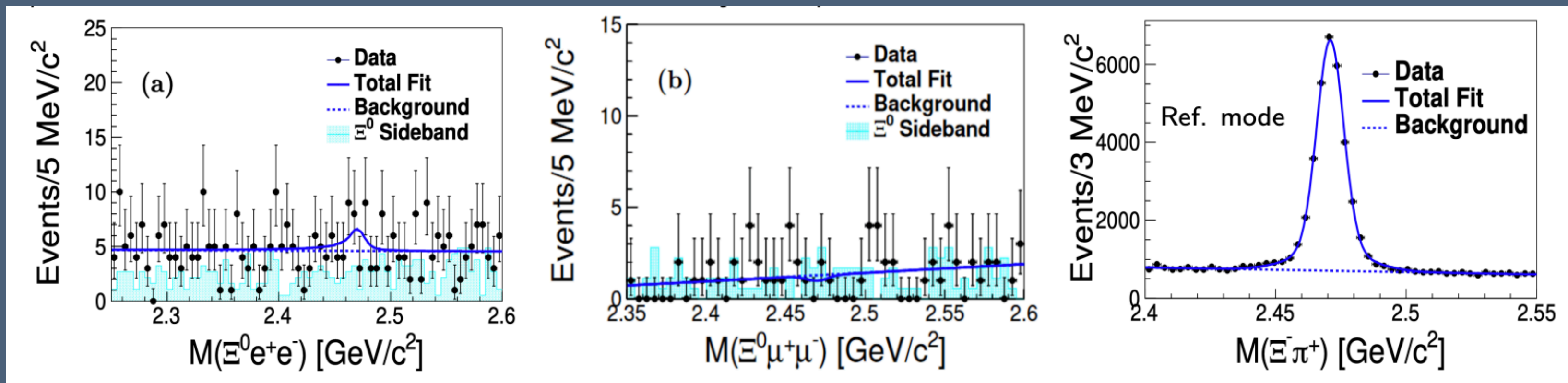
Results:

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 e^+ e^-) < 9.9 \times 10^{-5}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \mu^+ \mu^-) < 6.5 \times 10^{-5}$$

- SM  $\rightarrow$  each lepton flavor equally likely to interact with the weak force
- Search for  $\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^-$ , where  $\ell = e, \mu$ , occurred in Dec 2023 (90% CL)
  - First search for a FCNC semi-leptonic decay without neutrinos (sensitive to hamiltonian helicity structure through W-exchange diagrams)

arXiv:2312.02580



No signal observed but consistent with SM:

$$\mathcal{B}_{SM}(\Xi_c \rightarrow \Xi^0 e^+ e^-) < 2.35 \times 10^{-6}$$

$$\mathcal{B}_{SM}(\Xi_c \rightarrow \Xi^0 \mu^+ \mu^-) < 2.25 \times 10^{-6}$$

PRD103(2021):013007

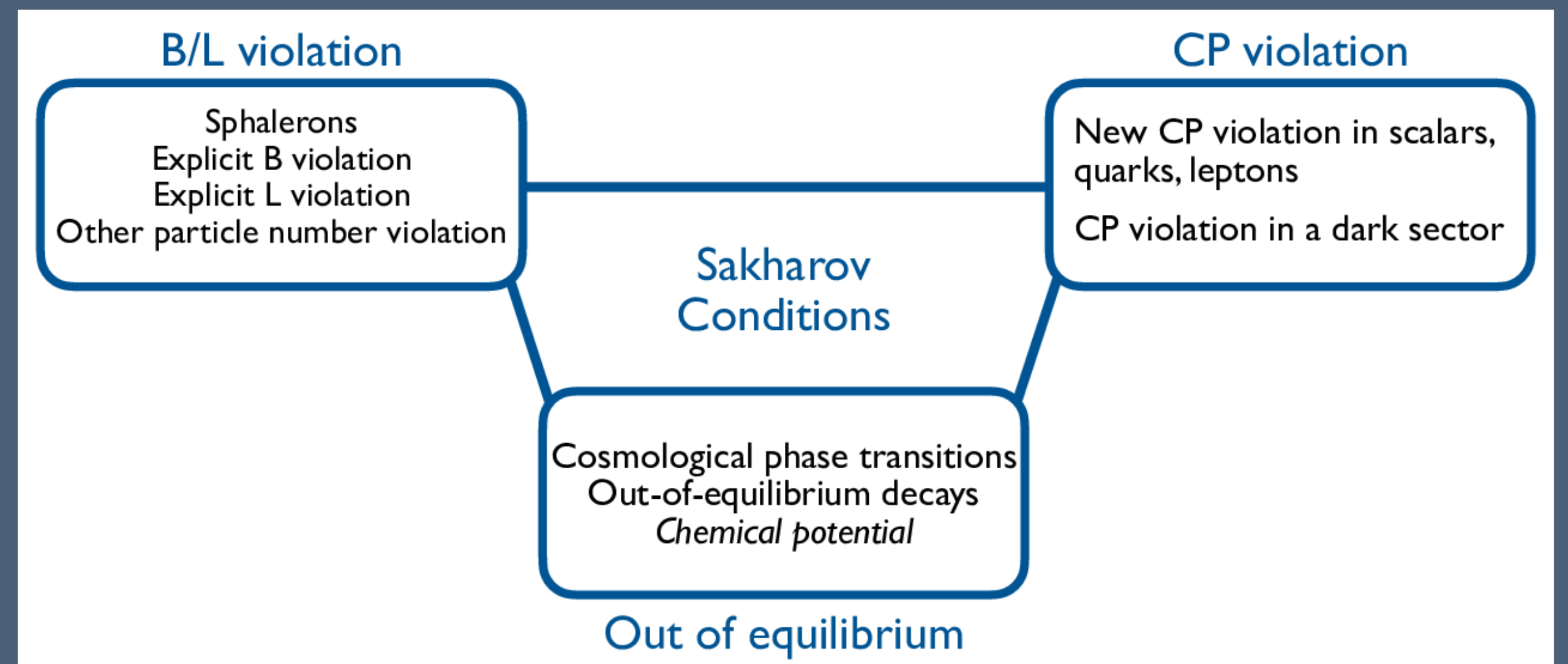
# Search for $D \rightarrow p\ell^-$ at Belle

## Test of Baryon Number Violation (BNV)

- BNV: One of Sakharov's conditions for a matter-dominated universe
- BESIII (2022, 90% Confidence Level (CL)) [10.1103/PhysRevD.105.032006](https://arxiv.org/abs/10.1103/PhysRevD.105.032006)

- $\mathcal{B}(D^0 \rightarrow \bar{p}e^+) < 1.2 \times 10^{-6}$

- $\mathcal{B}(D^0 \rightarrow pe^-) < 2.2 \times 10^{-6}$



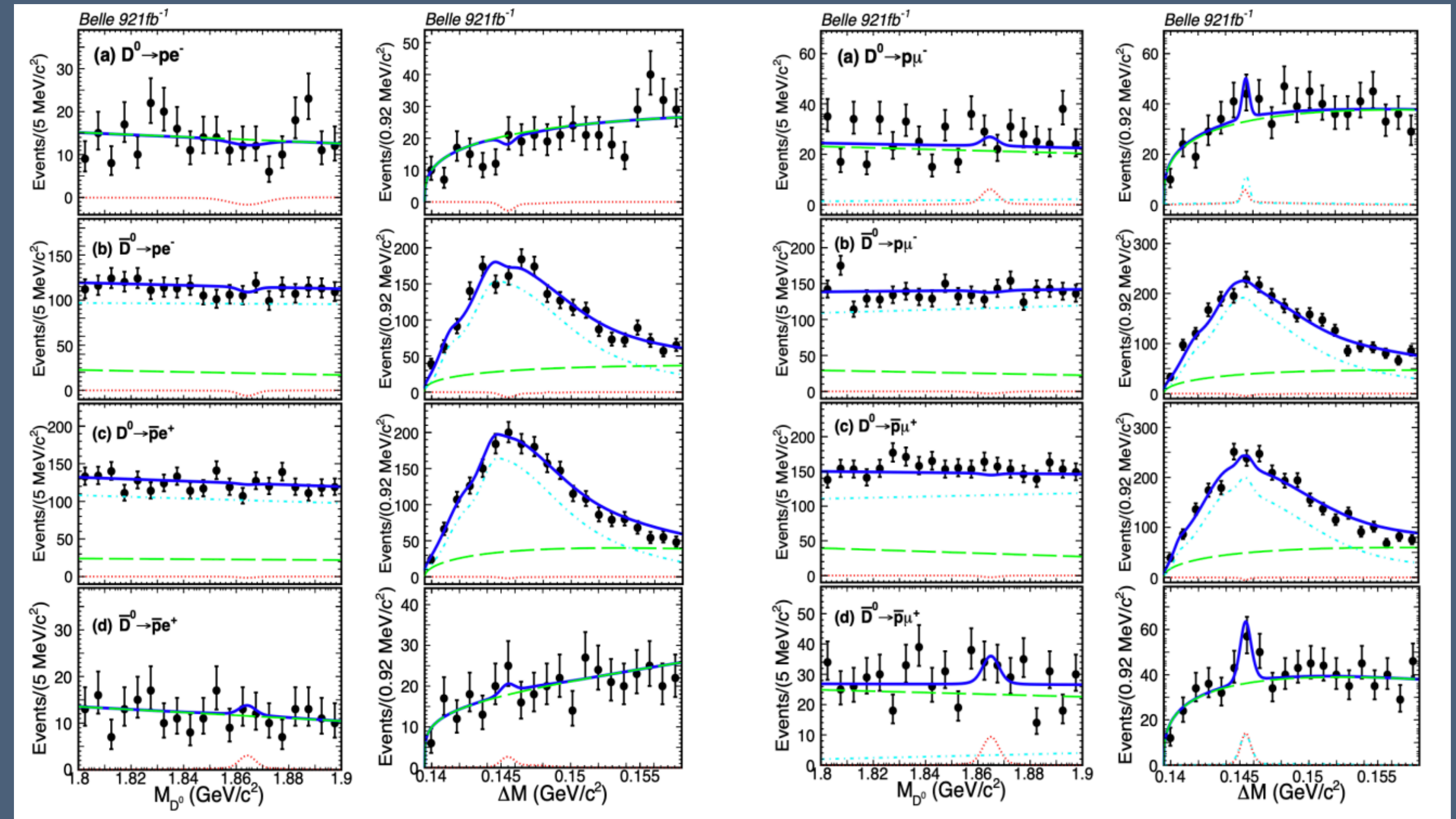
# Search for $D \rightarrow p\ell$ at Belle

## Test of Baryon Number Violation (BNV)

- Belle (2024)

TABLE I. Reconstruction efficiency ( $\epsilon$ ), signal yield ( $N_S$ ), signal significance ( $\mathcal{S}$ ), upper limit on the signal yield ( $N_{pl}^{UL}$ ), and branching fraction ( $\mathcal{B}$ ) at 90% confidence level for each decay mode.

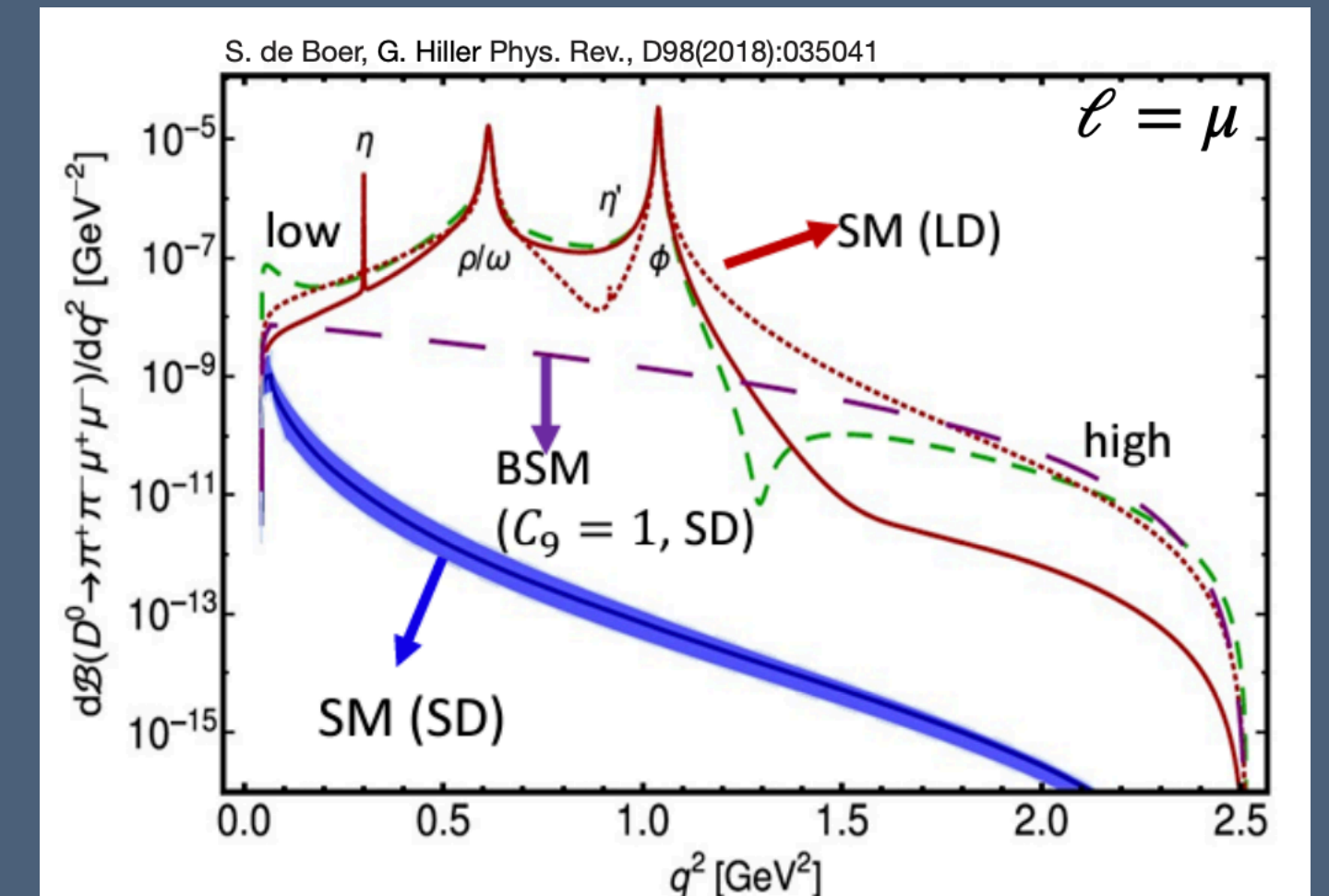
Decay mode	$\epsilon$ (%)	$N_S$	$\mathcal{S}$ ( $\sigma$ )	$N_{pl}^{UL}$	$\mathcal{B} \times 10^{-7}$
$D^0 \rightarrow pe^-$	10.2	$-6.4 \pm 8.5$	—	17.5	$< 5.5$
$\bar{D}^0 \rightarrow pe^-$	10.2	$-18.4 \pm 23.0$	—	22.0	$< 6.9$
$D^0 \rightarrow \bar{p}e^+$	9.7	$-4.7 \pm 23.0$	—	22.0	$< 7.2$
$\bar{D}^0 \rightarrow \bar{p}e^+$	9.6	$7.1 \pm 9.0$	0.6	23.0	$< 7.6$
$D^0 \rightarrow p\mu^-$	10.7	$11.0 \pm 23.0$	0.9	17.1	$< 5.1$
$\bar{D}^0 \rightarrow p\mu^-$	10.7	$-10.8 \pm 27.0$	—	21.8	$< 6.5$
$D^0 \rightarrow \bar{p}\mu^+$	10.5	$-4.5 \pm 14.0$	—	21.1	$< 6.3$
$\bar{D}^0 \rightarrow \bar{p}\mu^+$	10.4	$16.7 \pm 8.8$	1.6	21.4	$< 6.5$



90% CL upper limits; most precise for  $e$  channels and first measurement for  $\mu$  channels

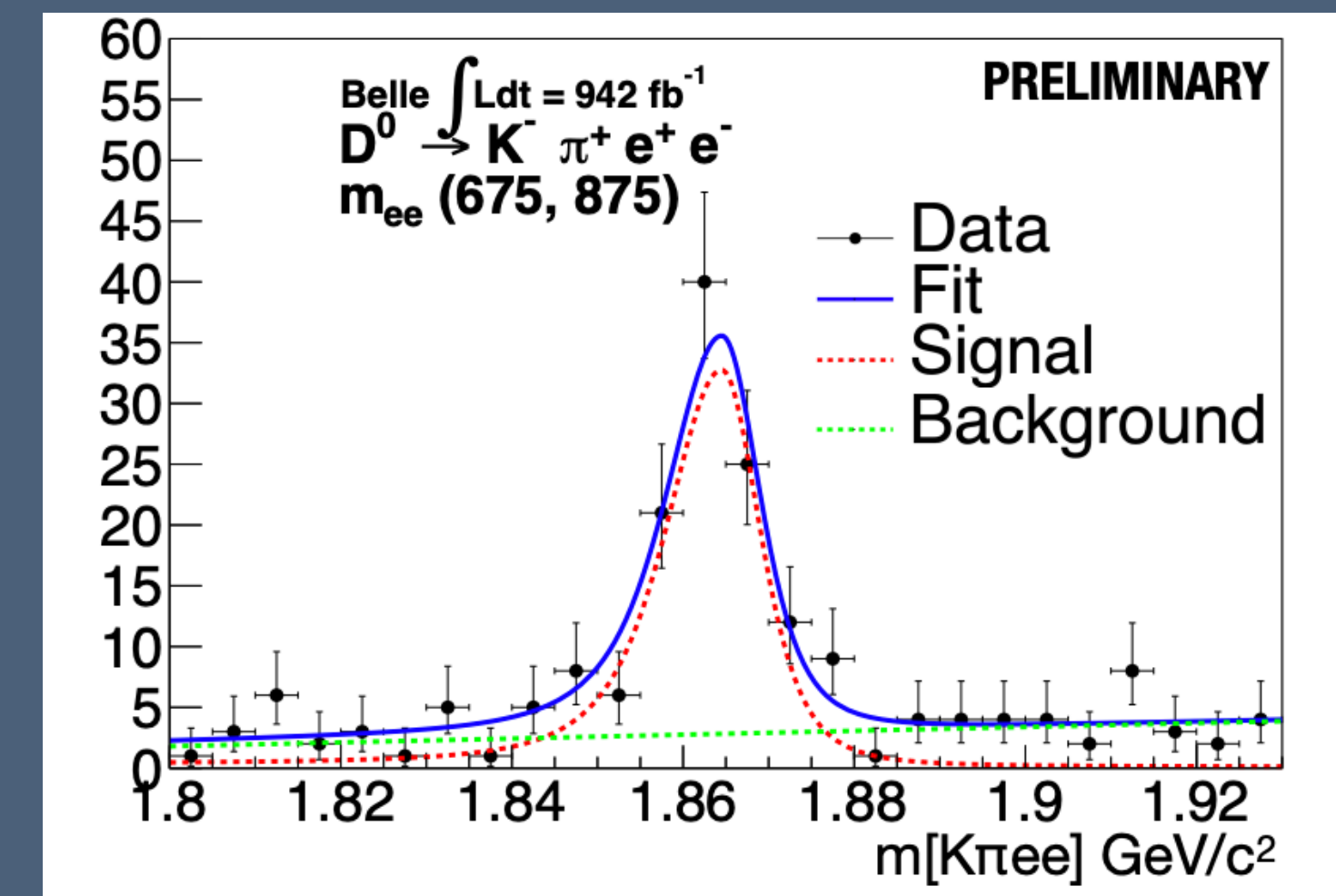
# Search for $D^0 \rightarrow hh'e^+e^-$ at Belle

- FCNC process with  $c \rightarrow u\ell\ell$  (suppressed in SM)
- Distinct  $q^2 = m^2(e^+e^-)$  resonances
- Near resonance dominated by SM (BR) and BSM may be visible far from resonances



- **New** Belle results

- Signal in  $\rho/\omega$  region:  $\mathcal{B}(D^0 \rightarrow K\pi e^+e^-) = (39.6 \pm 4.5 \pm 2.9) \times 10^{-7}$  ( $11.8\sigma$  significance), matches BABAR with higher precision and SM expectations
- 90% CL upper limits set at  $(2 - 8) \times 10^{-7}$  for other regions



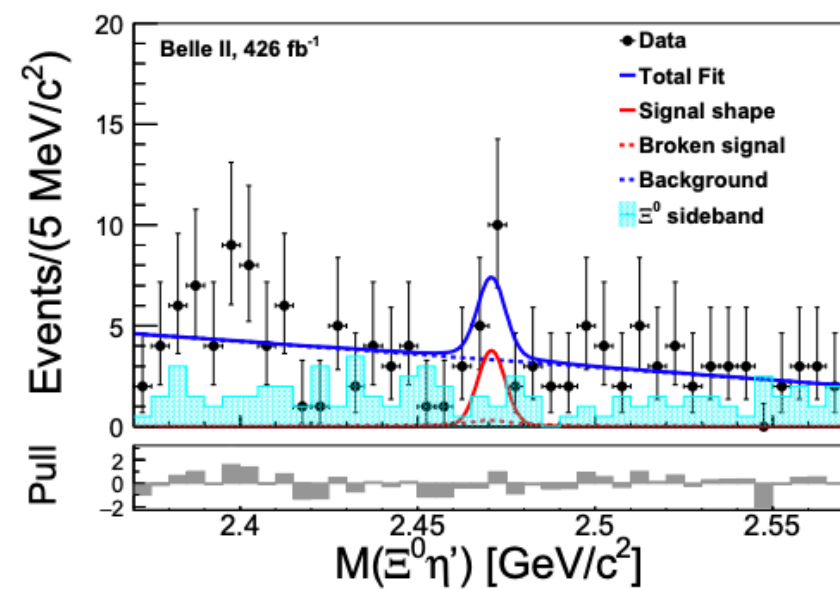
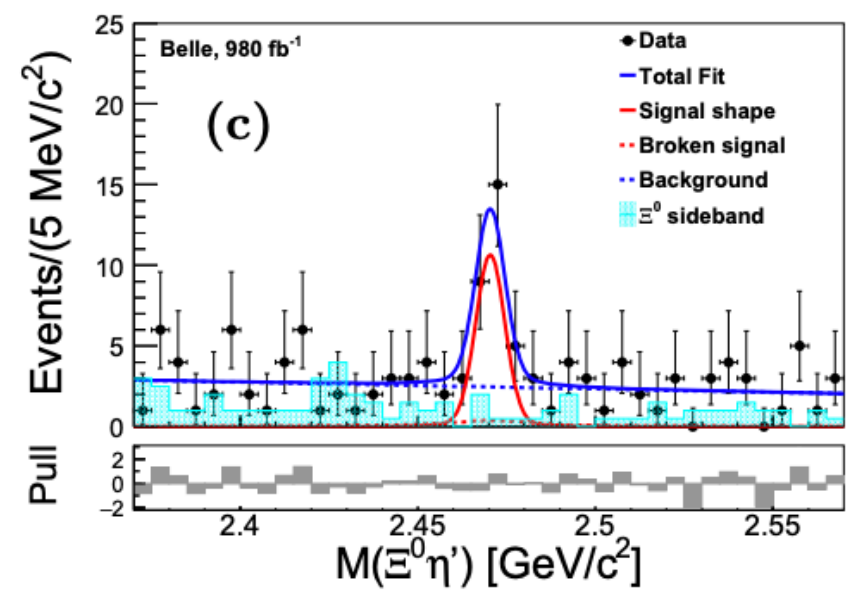
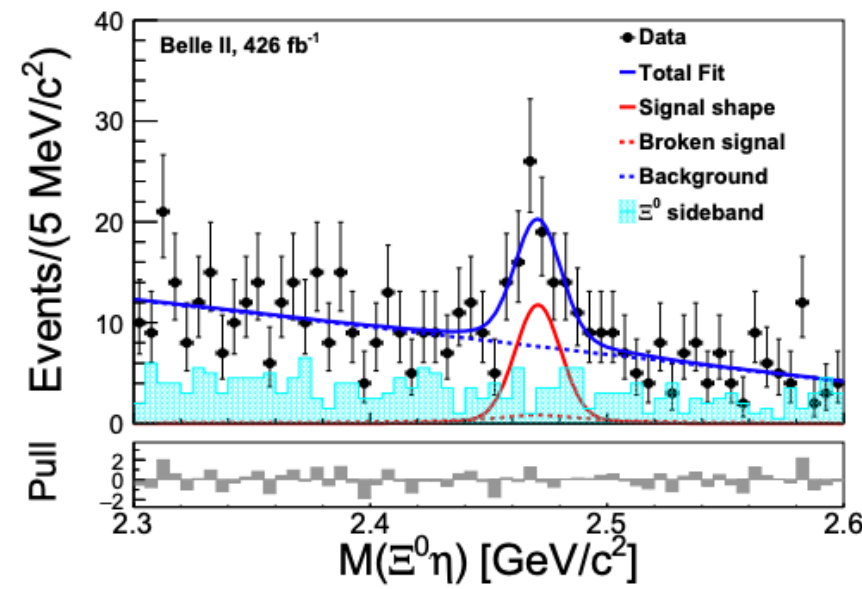
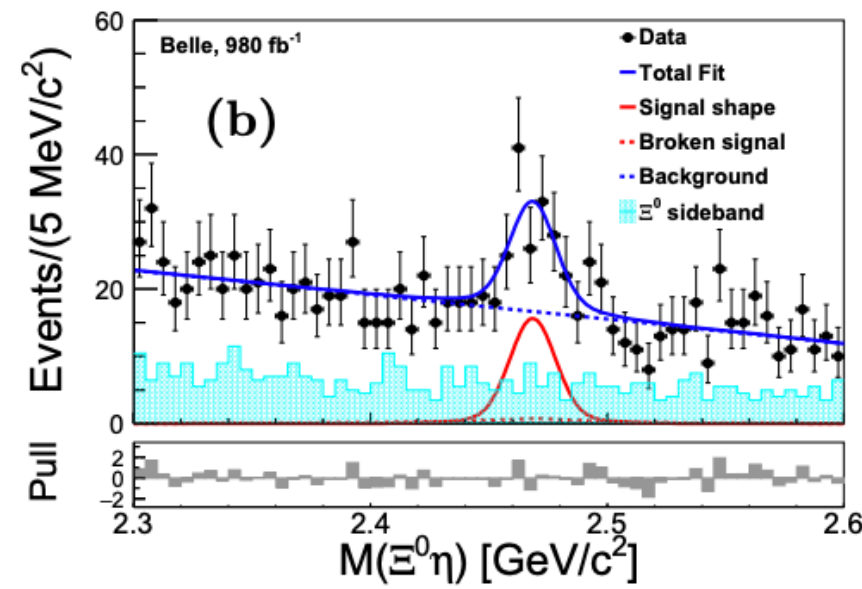
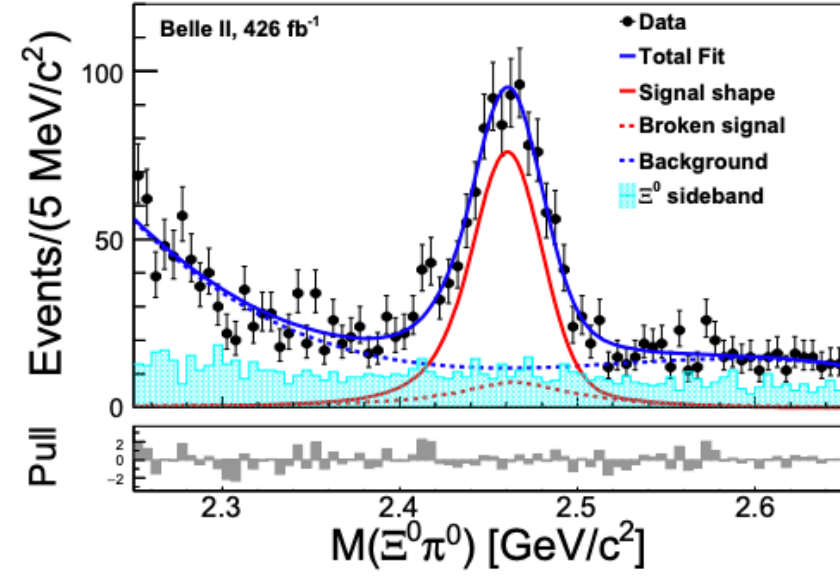
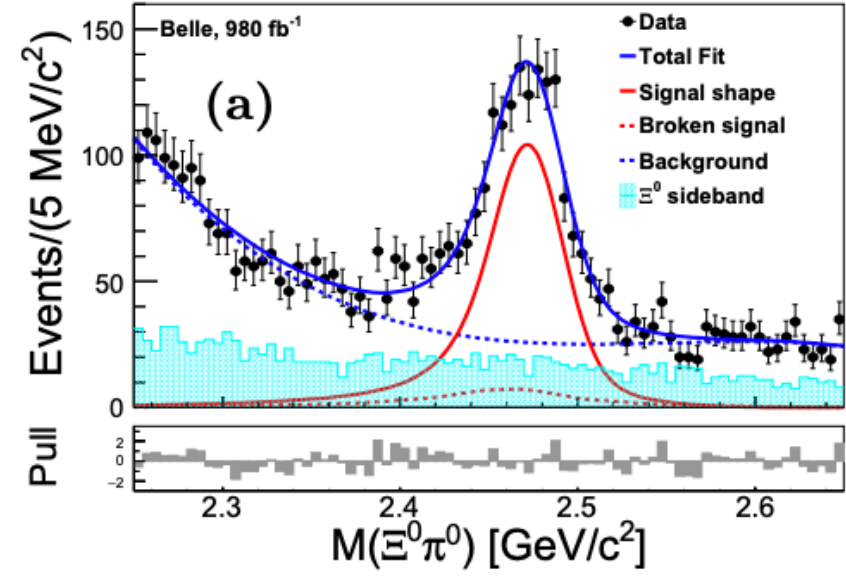


# Summary



- Belle stopped data production **nearly 15 years ago**, yet still boasts a large charm sample
- Belle II has resumed data taking after Long Shutdown 1 (LS1) and provides a smaller charm sample with **increased capability for precision measurements**. Eventually, the size will be comparable as well.
  - High precision → strong capabilities for measuring **lifetimes and branching fractions**
- Large charm samples allow **probes into NP** through CPV and BPV
- Belle and Belle II have produced **several world-leading measurements** in the charm sector

Backup



Reference	Model	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta)$	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta')$	$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$
Körner, Krämer [5]	quark	0.5	3.2	11.6	0.92
Xu, Kamal [7]	pole	7.7	-	-	0.92
Cheng, Tseng [8]	pole	3.8	-	-	-0.78
Cheng, Tseng [8]	CA	17.1	-	-	0.54
Żenczykowski [9]	pole	6.9	1.0	9.0	0.21
Ivanov <i>et al.</i> [6]	quark	0.5	3.7	4.1	0.94
Sharma, Verma [11]	CA	-	-	-	-0.8
Geng <i>et al.</i> [12]	SU(3) <sub>F</sub>	$4.3 \pm 0.9$	$1.7^{+1.0}_{-1.7}$	$8.6^{+11.0}_{-6.3}$	-
Geng <i>et al.</i> [13]	SU(3) <sub>F</sub>	$7.6 \pm 1.0$	$10.3 \pm 2.0$	$9.1 \pm 4.1$	$-1.00^{+0.07}_{-0.00}$
Zhao <i>et al.</i> [14]	SU(3) <sub>F</sub>	$4.7 \pm 0.9$	$8.3 \pm 2.3$	$7.2 \pm 1.9$	-
Zou <i>et al.</i> [10]	pole	18.2	26.7	-	-0.77
Huang <i>et al.</i> [15]	SU(3) <sub>F</sub>	$2.56 \pm 0.93$	-	-	$-0.23 \pm 0.60$
Hsiao <i>et al.</i> [16]	SU(3) <sub>F</sub>	$6.0 \pm 1.2$	$4.2^{+1.6}_{-1.3}$	-	-
Hsiao <i>et al.</i> [16]	SU(3) <sub>F</sub> -breaking	$3.6 \pm 1.2$	$7.3 \pm 3.2$	-	-
Zhong <i>et al.</i> [17]	SU(3) <sub>F</sub>	$1.13^{+0.59}_{-0.49}$	$1.56 \pm 1.92$	$0.683^{+3.272}_{-3.268}$	$0.50^{+0.37}_{-0.35}$
<b>best fit</b> → Zhong <i>et al.</i> [17]	SU(3) <sub>F</sub> -breaking	$7.74^{+2.52}_{-2.32}$	$2.43^{+2.79}_{-2.90}$	$1.63^{+5.09}_{-5.14}$	$-0.29^{+0.20}_{-0.17}$
Xing <i>et al.</i> [18]	SU(3) <sub>F</sub>	$1.30 \pm 0.51$	-	-	$-0.28 \pm 0.18$

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